Lamb shift of Rydberg atoms in a cavity

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The alteration of the Lamb shift in the levels of a Rydberg atom in a cavity due to resonance between the atomic transition and an eigenmode of the cavity is investigated. The Lamb shift in the cavity is spatially modulated. The magnitude of the Lamb shift can be controlled by changing the parameters of the cavity or by electron beam scanning near the electric field antinodes. The dependence of the shift on the quantum numbers and nuclear charge can vary significantly, and in some cases a restructuring of the levels of the whole Coulomb multiplet can occur. The effective Hamiltonian and density of states for a Rydberg atom interacting via image forces with a plane metallic surface are found.

1. INTRODUCTION

In a number of recent papers[1–4] the variation of the probability of spontaneous emission of a Rydberg atom in a cavity has been investigated. This effect is due to the variation of the density of photon states in the cavity in the region of frequencies at which transitions occur, in comparison with the case of free space. In this case, if the transition frequency is close to an eigenmode of the cavity, there arises a pronounced amplification of the spontaneous emission. If the transition frequency lies below the fundamental mode of the cavity, a strong suppression of spontaneous emission is observed. (In the case of an ideal cavity the spontaneous emission is completely suppressed.)

More recently theoretical[5,6] and experimental[7] confirmation have appeared that in the cavity (confocal or Fabry–Perot) an observable (of the order of 1 MHz) change in the Lamb shift (for $2s$–$2p$-states of the hdyrogen atom) takes place. It is our aim to examine the effect of the cavity on the Lamb shift (LS) of the Rydberg states of atoms.

Alteration of the Lamb shift is most pronounced in cases of confocal or closed cavities. We will investigate the latter case on account of its simplicity and easy visualizability. Generalization to the case of a confocal cavity does not introduce any fundamental difficulties. In a closed cavity with a sufficiently high Q factor the density of photon states consists of a set of quite sharp peaks. (In waveguides the density of states has root singularities, which correspond to the thresholds of propagation of the various modes. In an open cavity the density of states passes through a discontinuity at each threshold frequency for the appearance of a new mode.)

The most noteworthy effect occurs in the case which obtains when the eigenfrequency of the cavity is close to the frequency of an atomic transition. In what follows we will restrict the discussion to the case of a rectangular cavity.

It should be noted that in the radio frequency region, which is the region we will be considering, the effect is substantially less than in the optical range. But, thanks to the high Q factor of radio frequency cavities, the effect nevertheless can become observable. In contrast with, say, a confocal cavity, where the density of photon modes is changed only in a small solid angle $\Delta\Omega_k$ in comparison with free space,[8] in a closed cavity the density of photon states changes radically. As a result, for atomic states whose virtual transitions are in resonance with a mode (which can be degenerate) of the cavity, the LS is changed in an extraordinarily strong way. In particular, it can exceed the fine structure splitting. As a consequence, the character of the dependence of the LS on the quantum numbers and the nuclear charge Z is changed in an essential way. Here the magnitude of the Lamb shift depends on the detuning of the frequency of the virtual transition ω_{nn}' from the cavity frequency k, which is close to ω_{nn}'.

2. SHIFT IN FREE SPACE

As is well known,[9] the LS in free space can be represented as a sum of two contributions due to the high- and the low-frequency virtual transitions. The high-frequency contribution for the Rydberg states is of the order of

$$E^{>}\sim m_e Z^4\alpha^5/n^3l^2.$$

(In this article we will use the system of units $\hbar=c=1$.) The low-frequency part of the LS $E^{<}$ can be separated into two components to which the virtual transitions contribute, specifically, to levels lying either above or below the given level. Both components are of the order of

$$\delta E_{\pm}^{<}\sim m_e Z^4\alpha^5/n^5,$$

but they enter into $E^{<}$ with opposite signs, and in the case of Rydberg states they almost completely cancel, so that the total contribution is approximately n times smaller: $E^{<}\sim m_e Z^4\alpha^5/n^6$. This is especially pronounced for those states which are close to circular ($n-l-1\ll n$). In ths case for $E^{<}$ we have

$$E^{<}\approx(1-{}^4/_3\ln 2)\,m_e Z^4\alpha^5/n^6. \tag{1}$$

The correction $E^{>}$ in the case of almost circular states is equal to

$$|E^{>}|\approx m_e Z^4\alpha^5/4\pi n^5. \tag{2}$$

Thus we see that in free space the LS is determined mainly by the high-frequency correction (2):

$$|E^{<}/E^{>}|\sim 1/n. \tag{3}$$

In a number of recent articles[10–13] relatively simple theoretical schemes have been proposed to describe the LS, which use the apparatus of nonrelativistic quantum mechanics. We also will adhere to this approach, which is entirely adequate

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for the problem under consideration. In fact, as will be shown below, the main effect consists in a modification of the low-energy part of the LS.

3. MODIFICATION OF THE SHIFT OF THE LEVELS OF A RYDBERG ATOM

Let us consider the effect of the cavity on the LS of Rydberg atoms located in it. In the first place, the effect of the cavity is manifested in an alteration of the boundary conditions for the electromagnetic field (in comparison with free space). In the cavity the density of states $\rho(k)$ of the electromagnetic field has δ-function-like singularities at the eigenfrequencies.

The effect under discussion involves the resonance of one of the virtual transitions with an eigenmode of the cavity. For a high Q factor of the cavity the resonant contribution $E_r^<$ to the low-frequency part of the shift can substantially exceed not only the value of $E^<$ in free space, but also the low-frequency part of the shift $E^>$.

In the cavity also changed are the fine structure levels, the polarization of the vacuum, and the boundary conditions on the Coulomb wave function of the electron in the atom. All these effects are exponentially small (of the order of $\exp(-a_n/L)$, where L is characteristic dimension of the cavity and $a_n = n^2/m_e Z\alpha$ is the radius of an orbit of the Rydberg atom). Thus, the main part of the modification of the levels is associated with the contribution of the resonant transition.

Let us consider the Hamiltonian of the system consisting of atom + electromagnetic field in the cavity:

$$H = \frac{1}{2m_e}(\mathbf{p}+e\mathbf{A})^2 - e\Phi(\mathbf{r}) + \sum_{k,\lambda} k\hat{a}_{k\lambda}{}^+\hat{a}_{k\lambda}, \tag{4}$$

where $\mathbf{A}$ is the vector potential of the electromagnetic field of the cavity, $\Phi(\mathbf{r}) = Ze/r$ is the nuclear potential, $\mathbf{k} = \pi(\nu_x/L_x, \nu_y/L_y, \nu_z/L_z)$ are the eigenmodes of the cavity (ν_j are positive integers), and λ indicates the polarization states, but in the case of a cubic cavity—also the various spatial configurations of the degenerate mode with wavenumber k.

The operator of the vector potential in the cavity can be expanded in a complete set of standng waves:

$$\mathbf{A}(\mathbf{R}) = \sum_k \left(\frac{2\pi}{k}\right)^{1/2} \sum_\lambda (\mathbf{u}_{k\lambda}(\mathbf{R})\hat{a}_{k\lambda} + \mathbf{u}_{k\lambda}^*(\mathbf{R})\hat{a}_{k\lambda}{}^+), \tag{5}$$

where the functions $\mathbf{u}_{k\lambda}$ are orthonormal:

$$\int\limits_{0<R_i<L_i} \mathbf{u}_{k\lambda}\mathbf{u}_{k\lambda}^*\, d\mathbf{R} = \delta_{kk'}\delta_{\lambda\lambda'} \tag{6}$$

and satisfy boundary conditions which correspond to the cavity walls being perfectly conducting. We impose the transverse gauge $\nabla\cdot\mathbf{A} = 0$ on the vector potential.

In our analysis of the spectrum of the Hamiltonian (4) we will make a number of simplifying assumptions. First, we assume that the frequency of one of the electronic transitions $\omega_{nn}{}'$ is close to the frequency of one of the eigenmodes of the cavity k. We will call the quantity $\Delta = \omega_{nn'} - k$ the mismatch. In this case it is sufficient in Hamiltonian (4) to keep only the contribution of the resonant mode with the given wave vector k. For definiteness let us assume that the transition $n \to n' = n - \nu$ ($\nu \sim 1$) is the resonant one.

Second, we require that the condition of the applicability of the dipole approximation be fulfilled: $ka_n \ll 1$, enabling us to consider the electromagnetic field to be constant inside the orbit of the atom. The interaction Hamiltonian in the dipole approxmation can be represented in the equivalent form: $V_{\rm int} = \mathbf{dE}$.

Third, we will assume that the cavity is ideal. (For a superconducting resonator in the microwave range the Q factor can reach values in the range $\sim 10^8$–10^9). For cavities with a finite Q factor $Q = k/2\gamma$, where γ is the damping, it is possible to take the damping into account by introducing into all the formulas an "effective detuning" $\tilde{\Delta} = (\gamma^2 + \Delta^2)^{1/2}$. In addition, the entire treatment will be carried out for zero temperature, or, more accurately, in the absence of real photons in the cavity. Generalization to the case in which real photons are present in the cavity does not introduce any fundamental difficulties.

With the aim of enhancing the interesting effect, we assume that the atom is located in an antinode of the electric field $\mathbf{E}$ (for definiteness we assume that the atom is located in the center of the cavity).

The virtual transition $n \to n - \nu$ turns out to be in the radio frequency microwave region only for Rydberg atoms. To describe them we can use the quasiclassical approximation. Three limiting cases are possible.

a) Sharp tuning: $\Delta \ll \omega_f$, where $\omega_f = m_e Z^4\alpha^4/n^3 l^2$ is the characteristic scale of the fine structure. In this case the problem reduces to a two-level problem.

b) Relatively coarse tuning: $\omega_f \leqslant \Delta \ll \omega_n/n$, where $\omega_n = m_e Z^2\alpha^2/n^3$ is the Rydberg frequency, and ω_n/n is the characteristic scale of the anharmonicity of the nonrelativistic Coulomb spectrum. In this case it is necessary to allow for virtual transitions between the two Coulomb multiplets (n-layers). This case is somewhat more complicated since it requires the solution of the secular equation for a multiply (approximately) degenerate system. We will consider this in more detail below.

c) Coarse tuning: $\omega_n \gg \Delta \gtrsim \omega_n/n$. In this case in some neighborhood of the given n-layer the nonrelativistic Coulomb spectrum can be considered as almost equidistant (with accuracy Δ). Some of the Coulomb n-layers fall into resonance at once. The magnitude of the effect in this case is substantially less in comparison with the two previous cases.

4. THE CASE OF RELATIVELY COARSE TUNING: REDUCTION OF THE SECULAR EQUATION

In the transition from the state $|nlm\rangle$ to the state $|n'l'm'\rangle$ the electron radiates a virtual photon in the state $|1_{k\lambda}\rangle$, which is then reabsorbed by the same electron. Thus, the initial (= final) state and the intermediate states of the atom + field system have the form $|0\rangle|nlm\rangle$ and $|1_{k\lambda}\rangle|n'l'm'\rangle$, respectively.

We assume that the condition of relatively course tuning $\omega_f \lesssim \Delta \ll \omega_n/n$ is fulfilled, i.e., the states of the Coulomb multiplet can be taken to be degenerate with the same or greater accuracy than the states which are coupled by the virtual photon transition. There is thus a subspace of almost degenerate states with dimension $D = n^2 + q(n')^2$, where q is the multiplicity of the degeneracy of the resonant photon mode. When the perturbation is turned on a strong mixing of states from the indicated subspace takes place, which leads to the necessity of constructing a valid wave function in the zeroth approximation:

$$|\psi^{(0)}\rangle=\sum_{lm} C_{lm}|0\rangle nlm\rangle+\sum_{l'm'\lambda} C_{l'm'\lambda}|1_{k\lambda}\rangle|n'l'm'\rangle. \qquad (7)$$

After substituting Eq. (7) into Eq. (4), we obtain the following system of equations for the coefficients C_{lm} and $C_{l'm'\lambda}$:

$$E_r C_{lm}=\sum_{l'm'\lambda} V_{lm}^{l'm'\lambda} C_{l'm'\lambda}, \qquad E'_r C_{l'm'\lambda}=\sum_{\overline{lm}} V_{l'm'\lambda}^{\overline{lm}} C_{\overline{lm}}, \qquad (8)$$

where

$$V_{lm}^{l'm'\lambda}=\langle n'l'm'|\langle 1_{k\lambda}|V_{int}|0\rangle|nlm\rangle, \qquad E'_r=E_r+\Delta.$$

The system of equation (8), which is of dimension $D=n^2+q(n')^2$, can be reduced to a system of order n^2 by eliminating the coefficients $C_{l'm'\lambda}$:

$$E_r E'_r C_{lm}=\sum_{\overline{lm}} W_{lm}^{\overline{lm}} C_{\overline{lm}}. \qquad (9)$$

Here $W_{lm}^{\overline{lm}}$ are the matrix elements of the operator

$$\hat{W}=P_{n,0}\hat{V}_{int}P_{n',1}\hat{V}_{int}P_{n,0}.$$

The operators $P_{n,0}$ and $P_{n',1}$ project onto the subspace of the states of the form $|0\rangle|nlm\rangle$ and $|1_{k\lambda}\rangle|n'l'm'\rangle$, respectively.

Thus, the secular equation for the effective Hamiltonian $\hat{W}$, acting within the n-layer, is given by

$$\det\|\hat{W}-E_rE'_r\|=0. \qquad (9')$$

In what follows, for definiteness we will consider only a rectangular cavity. In this case the operator $\hat{W}$ is diagonal in m.

The Lamb shift E_r can be expressed in terms of the corresponding eigenvalue of the operator $\hat{W}$:

$$E_r(n,m,s)=\operatorname{sign}\Delta\left(\left(\Delta^2/4+W(n,m,s)\right)^{1/2}-|\Delta|/2\right), \qquad (10)$$

where s is a new "good" quantum number, which orders the levels with given n and m in energy. In the limit $|\Delta|\ll W(n,m,s)$ we find

$$|E_r(n,m,s)|\approx(W(n,m,s))^{1/2}, \qquad (11)$$

which gives an upper bound for the effect.

We note that in the case of sharp tuning ($\Delta\ll\omega_f$) there is no mixing and the correction is determined by the diagonal matrix elements of the effective Hamiltonian $\hat{W}$:

$$|E_r(n,m,l)|\approx(W_{lm}{}^{lm})^{1/2}. \qquad (12)$$

In the opposite limiting case, $W(n,m,s)\ll|\Delta|$, Eq. (10) takes the form of a second-order perturbation theory correction:

$$E_r(n,m,l)\approx W(n,m,s)/\Delta, \quad |\Delta|\gg W(n,m,s). \qquad (13)$$

5. QUASICLASSICAL ASYMPTOTIC BEHAVIOR OF THE SPECTRUM OF THE EFFECTIVE HAMILTONIAN

Let us consider the case of a rectangular cavity with sides L_x, L_y, and L_z. Let us assume that the electric field of the resonant mode is directed along the z axis. The effective Hamiltonian takes the form

$$\hat{W}_{re}=C_{re}P_n\hat{z}P_{n'}\hat{z}P_n, \qquad (14)$$

where

$$C_{re}=16\pi\alpha\frac{\omega_{nn'}^2}{kL_xL_yL_z}\left(1-\left(\frac{k_z}{k}\right)^2\right).$$

For a cubic cavity there are, generally speaking, some degenerate modes with various directions $\mathbf{k}$. After summing over all the degenerate modes, we obtain an isotropic effective Hamiltonian

$$\hat{W}_{cub}=C_{cub}\sum_{j=1}^{3}P_n\hat{r}_jP_{n'}\hat{r}_jP_n, \qquad (15)$$

where

$$C_{cub}=16\pi\alpha\varkappa\frac{\omega_{nn'}^2}{kL^3}\left(1-\left(\frac{k_0}{k}\right)^2\right).$$

Here k_0 is one of the components of the vector $\mathbf{k}$ and $\varkappa$ is a numerical constant. For example, when an antinode of the electric field is located at the center of the cavity, two components of the vector $\mathbf{N}=(\nu_x,\nu_y,\nu_z)$ should be even, and one, say ν_0, should be odd. Then $k_0=\pi\nu_0/2$ and the constant $\varkappa$ is equal to unity if the remaining components of $\mathbf{N}$ coincide, and equal to two in the opposite case.

From Eq. (14) in the case $L_x\sim L_y\sim L_z$ and Eq. (15) it follows that for almost circular states the eigenvalues of the operator $\hat{W}$ are on the order of

$$W\sim m_e{}^2Z^6\alpha^7/n^8,$$

from which we obtain the following estimate for the Lamb shift:

$$E_r\sim m_eZ^3\alpha^{7/2}/n^4, \qquad (16)$$

which is $n/Z\alpha^{3/2}$ times greater than the Lamb shift in free space. Of course, the upper bound (16) is not attainable in practice. However, this estimate gives a proper account of the character of the dependence of the modified LS on n and Z.

In the case of coarse tuning ($\Delta\sim\omega_n/n$) no increase like expression (16), takes place in the LS in comparison with its value in free space. The dependence of the LS on n and Z in this case is the same as in free space.

To investigate the quasiclassical asymptotic behavior of the spectrum of operators (14) and (15), we make use of the quasiclassical formula for the reduced matrix elements $\langle nl\|\hat{\mathbf{r}}\|n'l'\rangle$ (see Ref. 14). Transforming to action–angle variables $l\leftrightarrow\theta$, we obtain

$$W_{re}(m,l,\theta)=C_{re}(\varepsilon_0(l)+\varepsilon_2(l)\cos 2\theta)(1-(m/l)^2), \qquad (17)$$

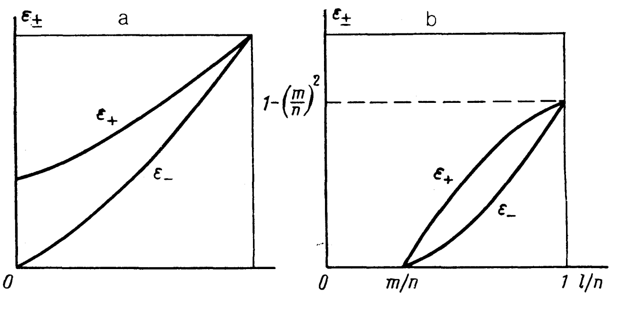

FIG. 1. Curves of the quasipotential bounding the region accessible to classical trajectories (qualitative picture): a) the case of a cubic cavity $\varepsilon_\pm=\varepsilon_0\pm\varepsilon_2$, b) the case of a rectangular cavity $\varepsilon_\pm=(\varepsilon_0\pm\varepsilon_2)(1-(m/l)^2)$.

$$W_{\mathrm{cub}}(l,\theta) = C_{\mathrm{cub}}(\varepsilon_0(l)+\varepsilon_2(l)\cos 2\theta), \tag{18}$$

where the functions $\varepsilon_{0,2}(l)$ are expressed in terms of the Bessel function whose argument is the eccentricity $a = (1-(l/n)^2)^{1/2}$ multiplied by $\nu \equiv n - n'$

$$\begin{aligned}\varepsilon_0(l) &= \frac{1}{\nu^2}(J_{\nu-1}^2(\nu a) - 2J_\nu^2(\nu a) + J_{\nu+1}^2(\nu a)),\\ \varepsilon_2(l) &= \frac{1}{\nu^2}(2J_\nu^2(\nu a) - 2J_{\nu-1}(\nu a)J_{\nu+1}(\nu a)).\end{aligned} \tag{19}$$

It is customary to refer to the functions $W_{\mathrm{re}}(\theta)$ and $W_{\mathrm{cub}}(\theta)$ as quasipotentials[15] since they bound the region of classically permissible trajectories. We will call the points where the energy level intersect the quasipotential curves turning points. The form of the quasipotential curves is shown (in dimensionless units) in Fig. 1. For the quasiclassical quantization of Hamiltonians (17) and (18) we apply the Bohr–Sommerfeld rule

$$\oint l\, d\theta = 2\pi(\tilde{s}+\gamma) \tag{20}$$

where $\tilde{s}$ is the new quantum number and γ is a numerical constant of order unity. We reorder the resulting levels in increasing energy and transform to the new quantum number $\tilde{s}\to s$, where we assume that s takes on values in the interval $|m| \leqslant s \leqslant n-1$. The quantization (20) can be carried out by numerical methods, and the results of such calculations are presented in Figs. 2 and 3. In the case of sufficiently sharp tuning, the LS in the cavity grows rapidly with increasing l, whereas in free space, on the other hand, the LS decays as $1/l^2$. The same trends are also found in the case of relatively coarse tuning. Here it is necessary to take into account the fact that l is not a good quantum number.

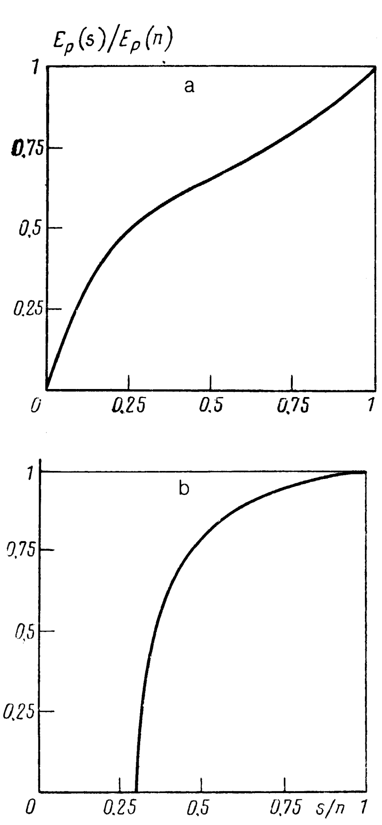

FIG. 3. Dependence of E_r on s in the limit $\Delta \ll W$ for $\nu = 1$ in the case of a cubic cavity (a), and in the limit $\Delta \gg W$ for $\nu = 1$ in the case of a rectangular cavity for $m/n = 0.3$ (b).

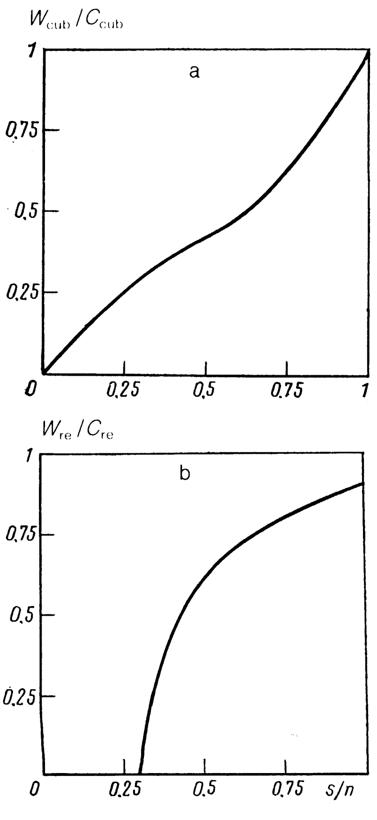

FIG. 2. Spectrum of the operator W_{cub} for $\nu = 1$ as a function of the new quantum number s in the classical limit (a), and spectrum of the operator W_{re} for $\nu = 1$ as a function of the new quantum number s is the classical limit for $m/n = 0.3$ (b).

In the case of sharp tuning the dependence of the LS on s is changed in another respect. For the most part both components are shifted in one direction (which depends on the sign of Δ), and the Lamb splitting of the doublet (in the case of almost circular states) is approximately n times smaller than the shift of the doublet as a whole. We note that the Lamb shift in the cavity can be controlled either by varying Δ by changing the parameters of the cavity or by scanning the atomic beam close to an antinode of the electric field. As was already noted, E_r depends on the location of the atom in the cavity with respect to an antinode of the resonant mode. In the adiabatic approximation of the LS $E_r(\mathbf{R})$ plays the role of a potential energy. Thus there appears a weak force $\mathbf{F} = -\nabla E_r$ tending to localize the atoms at the nodes or antinodes of the resonant mode.

6. A RYDBERG ATOM CLOSE TO A METAL SURFACE

Here we consider the van der Waals interaction of a Rydberg atom with a flat metallic surface. An experimental study of the interaction of a Rydberg atom with the walls of the cavity via van der Waals forces was recently carried out.[16]

We consider the problem of the interaction of a Rydberg atom with a flat conducting surface located a distance z_0 from the nucleus of the atom. The Hamiltonian of the system can be easily written down if we introduce image charges which create an electric field in free space which coincides with the field of the charge induced by the atom on the conducting surface.

We represent the Hamiltonian in the form $H = H_0 + V$, where

$$H_0=p^2/2m_e-\alpha/r,$$
$$V=\alpha/(4z_0(z_0+z)+r^2)^{1/2}-\alpha/4z_0-\alpha/4(z_0+z). \quad (21)$$

Here $\mathbf{r} = (x,y,z)$ is the radius vector of the electron with respect to the nucleus, and $r\equiv|\mathbf{r}|$.

We will investigate the case in which the Rydberg atom is located far enough away from the surface: $z_0 \gg n^2a_0$, where $a_0 = 1/m_e\alpha$ is the Bohr radius. We expand the perturbation operator V in the small parameter a_0n^2/z_0. The first nonvanishing correction to H_0 corresponds to the dipole approximately and is equal to

$$V_d=-\frac{\alpha}{16z_0^3}(r^2+z^2). \quad (22)$$

To calculate the effective Hamiltonian of the dipole approximation (i.e., to order n^4/z_0^3), it suffices to average Hamiltonian (22) over the rapid phase φ, which is conjugate to n. Using the Coulomb parameterization

$$r/a_0=x_0(\varphi)\mathbf{A}/|\mathbf{A}|+y_0(\varphi)\mathbf{B}/|\mathbf{B}|,$$

where

$$x_0(\varphi)=n^2(\cos\xi-A/n),\qquad y_0(\varphi)=nl\sin\xi,\qquad \varphi=\xi-\frac{A}{n}\sin\xi,$$

ζ is the eccentric anomaly, and $\mathbf{B} = [\mathbf{1A}]$, we find

$$\langle r_\mu r_\nu\rangle=\frac{n^2}{2}(5A_\mu A_\nu-\delta_{\mu\nu}A^2-l_\mu l_\nu+\delta_{\mu\nu}n^2). \quad (23)$$

Substituting the quasiclassical representation of the Runge–Lentz vector in parabolic coordinates

$$A_x={}^1/_2(n^2-(m+k)^2)^{1/2}\cos(\chi+\psi)$$
$$-{}^1/_2(n^2-(m-k)^2)^{1/2}\cos(\chi-\psi),$$
$$A_y={}^1/_2(n^2-(m+k)^2)^{1/2}\sin(\psi+\chi)$$
$$+{}^1/_2(n^2-(m-k)^2)^{1/2}\sin(\psi-\chi),$$
$$A_z=k,$$

in Eq. (23), where χ and ψ are the angular variables conjugate to the action variables k and m, we obtain the following expression for $\langle V_d\rangle_\varphi$:

$$\langle V_d\rangle_\varphi=n^2\alpha a_0^2(\varepsilon-4n^2+2m^2)/32z_0^3, \quad (24)$$

where

$$\varepsilon=-6k^2+f(n,m,k)\cos 2\chi,$$
$$f^2(n,m,k)=n^4+m^4+k^4-2(n^2m^2+n^2k^2+k^2m^2).$$

The density of states $\rho(\varepsilon)$ was found in Ref. 17 for a Hamiltonian of this type with the help of the Bohr–Sommerfeld quantization. An analogous quantum Hamiltonian, arising in the problem of an atom in a magnetic field, was introduced by Solov'ev and Braun.[18]

Following Ref. 17, we obtain for $\rho(\varepsilon)$

$$\rho(\varepsilon)=\begin{cases}\dfrac{1}{\pi}\dfrac{1}{(q_>+|q_<|)^{1/2}}K\left(\left[\dfrac{q_>}{q_>+|q_<|}\right]^{1/2}\right), & \text{region I}\\[2ex] \dfrac{1}{\pi}\dfrac{1}{q_>^{1/2}}K\left(\left[\dfrac{q_>-q_<}{q_>}\right]^{1/2}\right), & \text{region II}\end{cases} \quad (25)$$

where $q_>$ ($q_<$) is the greater (lesser) root of the equation

$$((n+m)^2-q)((n-m)^2-q)-(\varepsilon+6q)^2=0.$$

Region I corresponds to the case $q_< <0<q_>$, whereas in region II we have $0<q_< <q_>$. Region I obtains for $|\varepsilon|<\varepsilon_c$, and region II for $|\varepsilon|>\varepsilon_c$, where $\varepsilon_c = n^2 - m^2$. Region II exists only for $m\leqslant m_c = \sqrt{5/7}\,n$. In this region the classical trajectories are asymmetric and the states possess a spontaneous dipole moment whose magnitude is inversely proportional to the density of states $\rho(\varepsilon)$.

At the point $|\varepsilon| = \varepsilon_c$ the density of states $\rho(\varepsilon)$ has a logarithmic singularity, which is connected with the topological restructuring of the classical trajectories upon passing through a saddle point.

The correction to the energy levels which is caused by the image forces under resonance conditions $\omega_n \sim k$ has the same order-of-magnitude estimate $m_eZ^4\alpha^5/n^5$ as the high-frequency part of the shift in free space. However, in contrast with E_r, the given correction does not have a resonant character and can be easily isolated by varying the parameters of the cavity.

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