Field splitting of a cw superluminescence line

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It is shown that atomic levels undergo splitting as a result of a nonlinear interaction of a superluminescence field with an active transition and this gives rise to a discrete substructure of the line profile.

1. INTRODUCTION

Nonlinear transformation of a superluminescence spectrum and stimulated Raman scattering (STRS) under quasi-cw conditions has been discussed frequently in the literature.[1–5] Under such conditions the duration of the radiation pulses is considerably longer than the dephasing time of the atomic polarization and a discrete quasiperiodic structure is observed in the spectrum. The structure found in the emission spectrum of a nitrogen laser is attributed in Ref. 2 to the splitting of atomic levels of an active transition because of the interaction with a superluminescence field. It is pointed out in Ref. 5 that the overall width of the discrete structure of the STRS spectrum exceeds considerably the value predicted by the linear theory and this is due to perturbation of atomic levels by the STRS field. However, a self-consistent theoretical description of superluminescence line splitting due to the field perturbation of atomic levels is not yet available.

The approximation employed below is based on the fact that under linear conditions a superluminescence line exhibits its narrowing because of the frequency dispersion of the gain. For example, in the case of homogeneous broadening the width $\Gamma_{\rm eff}$ of a luminescence line is related to the linewidth of a single atom by the expression

$$\Gamma_{eff}=2\Gamma(\ln 2/\alpha_0 z)^{1/2}, \tag{1}$$

where α_0 is the gain at the line center and z is the length of the path in which the amplification takes place. In the region of the medium where the gain is high, $\alpha_0 z \gg 1$, and the change in the line profile because of the nonlinear interaction is still small, superluminescence can be regarded as a process of simultaneous propagation in an amplifying medium of a quasimonochromatic strong field at the frequency of the investigated transition characterized by a width $\Gamma_{\rm eff} < 2\Gamma$ and a weak wide-band field at the side frequencies. According to Ref. 6, the gain for the weak field at a frequency ω_μ in the presence of a strong monochromatic field is

$$\alpha_\mu=\alpha\left\{L(\omega)-2|G|^2\,\mathrm{Re}\left[\frac{1+\Gamma/(\Gamma-i\omega)}{(\Gamma-i\omega)(\gamma-i\omega)+4|G|^2}\right]\right\}, \tag{2}$$

where

$$\alpha=\frac{4\pi k|d_{21}|^2 n_0}{\Gamma\hbar(1+4|G|^2/\gamma\Gamma)}; \tag{3}$$

d_{21} is the matrix element of the dipole moment; n_0 is the unsaturated difference between the level populations; $G=d_{21}E/2\hbar$; E is the amplitude of the strong field; Γ and γ are the relaxation constants of the off-diagonal and diagonal elements of the ρ matrix; $\omega=\omega_\mu-\omega_{21}$; $L(\omega)=\Gamma^2/(\Gamma^2+\omega^2)$. If $2|G|>\Gamma$, the gain described by Eq. (2) has maxima at frequencies $\omega=0,\ \pm 2|G|$. The gain maxima correspond to transitions between atomic levels split by the strong field. The profile of the superluminescence line cannot be described if we only know the profile, because several other factors still have to be allowed for. Firstly, under high-gain conditions a strong field varies considerably with the coordinate z and the separations between the gain maxima are different in different parts of the medium. The line profile can be obtained if we integrate contributions from all parts of the medium. Secondly, a weak field has a continuous spectrum and in the case of an optically dense medium we have to allow for the parametric interaction of the spectral components at mirror-image frequencies when the fields at the frequencies ω_{21} and ω_μ induce polarization at $2\omega_{21}-\omega_\mu$. Finally, we must allow explicitly for the fact that superluminescence is created from a δ-correlated noise due to spontaneous emission. As shown below, these factors do not mask the manifestations of the splitting of the levels in the superluminescence spectrum. As before, there are well-resolved maxima at frequencies $\omega=0,\ \pm 2|G|$. This type of spectrum is retained up to distances at which the side maxima reach saturation values.

2. PROFILE OF A SUPERLUMINESCENCE LINE IN THE LINEAR CASE

We shall now consider a medium which amplifies radiation traveling along the z axis. Ignoring the effects which appear because of the finite transverse dimensions, we shall average the field over the transverse cross section:

$$E(z)=s^{-1}\int ds E(\mathbf{r}). \tag{4}$$

The polarization of the medium must also be averaged. We shall consider the unidirectional problem. We shall begin with a system of equations for the Fourier amplitudes of the envelopes of the field and for the elements of the ρ matrix:

$$(d/dz-i\omega/c)G(\omega)=ip\rho_{21}(\omega), \tag{5}$$

$$(\Gamma-i\omega)\rho_{21}(\omega)=-i\int_{-\infty}^{\infty}d\omega_1 G(\omega-\omega_1)n(\omega_1)+\sigma(\omega), \tag{6}$$

$$(\gamma-i\omega)n(\omega)=\gamma n_0\delta(\omega)-4\,\mathrm{Re}\left[i\int_{-\infty}^{\infty}d\omega_1 G^*(\omega-\omega_1)\rho_{21}(\omega_1)\right], \tag{7}$$

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where $p=2\pi k\,|d_{21}|^2\hbar^{-1}$; $n=n_2-n_1$; $n_j=\rho_{jj}$; $j=1,2$. Equation (6) for the off-diagonal elements of the ρ matrix includes a random source $\sigma(\omega)$ describing spontaneous emission. The first term on the right-hand side of Eq. (7) for the difference between the populations describes pumping which is constant in time. Random sources are assumed to be δ-correlated:

$$\langle\sigma(\omega,\ z)\sigma^*(\omega',\ z')\rangle=\Gamma n_2(s\pi)^{-1}\delta(\omega+\omega')\delta(z-z'). \quad (8)$$

The spectral density of the steady-state process is given by

$$\langle G(\omega)G^*(\omega')\rangle=g(\omega)\delta(\omega+\omega'). \quad (9)$$

We shall find the spectral density $g(\omega)$ in the approximation linear in the field when the field term in Eq. (7) is ignored. We then have

$$g(\omega)=g_0[\exp(\alpha(\omega)z)-1], \quad (10)$$

where $g_0=n_2k\,|d_{21}|^2/n_0\hbar s$, $\alpha(\omega)=\alpha L(\omega)$. The distribution described by Eq. (10) is of width Γ_{eff} given by Eq. (1).

3. PROFILE OF A SUPERLUMINESCENCE LINE IN THE NONLINEAR CASE

We shall allow for the nonlinearity in a part of the medium defined by $z\geqslant z_0$, $\alpha_0z_0\gg1$, where the width of the central peak is considerably less than 2Γ, but the nonlinearity has not yet modified the spectral distribution substantially, and the field at the side frequencies $|\omega|>\Gamma$ is weak. Then, the field of the central peak can be regarded approximately as monochromatic $G_0(\omega)=G\delta(\omega)$, which corresponds to a monochromatic component of the polarization $\rho_{21}^0(\omega)=\rho_{21}^0\delta(\omega)$. In solving Eqs. (6) and (7) we can ignore the influence of the source since the condition $\alpha_0z_0\gg1$ means that for $z\geqslant z_0$ the stimulated emission at a frequency $|\omega|\lesssim\Gamma_{\text{eff}}/2$ predominates over the spontaneous emission, and the contribution of the spontaneous processes is exponentially small. Subject to this comment, we have

$$\rho_{21}{}^0=(-i/\Gamma)Gn^0,\quad n^0=n_0(1+4|G|^2/\gamma\Gamma)^{-1}. \quad (11)$$

In the case of a weak wide-band field at the side frequencies we can adopt the standard procedure (see Ref. 6). Linearization of Eqs. (6) and (7) in respect of the amplitudes of the weak fields at mirror-image frequencies and of the corresponding off-diagonal elements of the ρ matrix yields

$$(\Gamma-i\omega)\rho_{21}(\omega)=-iGn(\omega)-iG(\omega)n^0+\sigma(\omega), \quad (12)$$

$$(\Gamma-i\omega)\rho_{12}(\omega)=iG^*n(\omega)+iG^*(\omega)n^0+\sigma^*(\omega), \quad (13)$$

$$(\gamma-i\omega)n(\omega)=2[-iG^*\rho_{21}(\omega)+iG\rho_{12}(\omega)-iG^*(\omega)\rho_{21}{}^0+iG(\omega)\rho_{12}{}^0]. \quad (14)$$

Using the system of equations (12)–(14) to derive expressions for $\rho_{12}(\omega)$ and $\rho_{21}(\omega)$ and applying Eq. (11), we obtain the following system of equations for the field amplitudes:

$$\frac{d}{dz}G=\frac{\alpha}{2}G, \quad (15)$$

$$\left(-\frac{i\omega}{c}+\frac{d}{dz}\right)G(\omega)=\frac{\alpha}{2D}\left[\left(\Gamma-i\omega+\frac{i2|G|^2\omega}{\Gamma(\gamma-i\omega)}\right)G(\omega)+\frac{2G^2(2\Gamma-i\omega)}{\Gamma(\gamma-i\omega)}G^*(\omega)\right]+iq\left[\left(\Gamma-i\omega+\frac{2|G|^2}{\gamma-i\omega}\right)\sigma(\omega)+\frac{2G^2}{\gamma-i\omega}\sigma^*(\omega)\right], \quad (16)$$

$$\left(-\frac{i\omega}{c}+\frac{d}{dz}\right)G^*(\omega)=\frac{\alpha}{2D}\left[\left(\Gamma-i\omega+\frac{i2|G|^2\omega}{\Gamma(\gamma-i\omega)}\right)G^*(\omega)-\frac{2G^{*2}(2\Gamma-i\omega)}{\Gamma(\gamma-i\omega)}G(\omega)\right]-iq\left[\left(\Gamma-i\omega+\frac{2|G|^2}{\gamma-i\omega}\right)\sigma^*(\omega)+\frac{2G^{*2}}{\gamma-i\omega}\sigma(\omega)\right], \quad (17)$$

where

$$D=\Gamma^{-1}(\Gamma-i\omega)[\Gamma-i\omega+4|G|^2/(\gamma-i\omega)],$$

$$q=2\pi k|d_{21}|^2h^{-1}(\Gamma-i\omega)D^{-1}.$$

We shall now substitute in Eq. (15) the expressions for the field in the form $G=|G|\exp(i\Phi)$, where Φ is the undetermined phase. According to Eq. (15), Φ is independent of the coordinate z. We shall introduce normal waves

$$G_\pm(\omega)=G(\omega)e^{-i\Phi}\pm G^*(\omega)e^{i\Phi}, \quad (18)$$

which satisfy the following equations:

$$\frac{d}{dz}G_\pm(\omega)=\lambda_\pm'G_\pm(\omega)\pm f_\pm, \quad (19)$$

where

$$\lambda_\pm'=-\frac{i\omega}{c}+\frac{\alpha}{2D}\left[\Gamma-i\omega+\frac{i2|G|^2\omega}{\Gamma(\gamma-i\omega)}\mp\frac{2|G|^2(2\Gamma-i\omega)}{\Gamma(\gamma-i\omega)}\right],$$

$$f_+=i2\pi k|d_{21}|(\Gamma-i\omega)(Dh\Gamma)^{-1}(d_{21}{}^*\sigma(\omega)e^{-i\Phi}-d_{21}\sigma^*(\omega)e^{i\Phi}),$$

$$f_-=i2\pi k|d_{21}|[h(\Gamma-i\omega)]^{-1}(d_{21}{}^*\sigma(\omega)e^{-i\Phi}+d_{21}\sigma^*(\omega)e^{i\Phi}).$$

The solution of Eq. (19) is

$$G_\pm(\omega)=G_\pm(\omega,z_0)\exp\lambda_\pm+\exp\lambda_\pm\int_{z_0}^{z}dz'\exp(-\lambda_\pm(z'))f_\pm(z'), \quad (20)$$

where

$$\lambda_\pm=\int_{z_0}^{z}dz'\lambda_\pm'(z'). \quad (21)$$

The integrals in Eq. (21) can be calculated by expressing the differential dz' from Eq. (15) in the form

$$dz'=\alpha^{-1}d|G|^2/|G|^2. \quad (22)$$

Using the relationship (22), we shall now carry out integration in Eq. (21), which yields

$$\lambda_+=\frac{i\omega}{c}(z-z_0)+\frac{\Gamma}{2(\Gamma-i\omega)}\ln\left|\frac{G}{G(z_0)}\right|^2+\frac{1}{2}\left(1+\frac{\Gamma}{\Gamma-i\omega}\right)\ln\left(\frac{\Gamma-i\omega+4|G(z_0)|^2/(\gamma-i\omega)}{\Gamma-i\omega+4|G|^2/(\gamma-i\omega)}\right), \quad (23)$$

$$\lambda_-=\frac{i\omega}{c}(z-z_0)+\frac{\Gamma}{2(\Gamma-i\omega)}\ln\left|\frac{G}{G(z_0)}\right|^2. \qquad (24)$$

The expression for the spectral density at the side frequencies is obtained if we ignore the amplitude fluctuations and assume that the linear approximation is still valid at the point z_0, i.e., that the spectral density at this point is described by Eq. (10). We can then easily show that the expression for $\langle|G(z_0)|^2\rangle$ is of the form

$$\langle|G(z_0)|^2\rangle=|G(z_0)|^2=\pi\Gamma g_0[\exp(\alpha_0 z_0)-1]. \qquad (25)$$

Under these assumptions the spectral density at the side frequencies given by the expression

$$g(\omega)=g_0\left[\frac{|G|^2}{\pi\Gamma g_0}\right]^{L(\omega)}\left\{1+\left[\frac{Q(0)}{Q(|G|^2)}\right]^{(1+L(\omega))/2}\right.$$
$$\times\exp[\varphi(|G|^2)]$$
$$\times\left[1+(4\pi\Gamma g_0)^{L(\omega)}\Gamma^2\frac{(\gamma^2+\omega^2)^{(1-L(\omega))/2}}{(\Gamma^2+\omega^2)^{(1+L(\omega))/2}}\right.$$
$$\left.\left.\times\int\limits_{4|G(z_0)|^2}^{4|G|^2}dy\frac{(1+y/\Gamma\gamma)\text{txp}[-\varphi(y)]}{[Q(y)]^{(1-L(\omega))/2}y^{(1+L(\omega))}}\right]\right\}, \qquad (26)$$

where

$$\varphi(|G|^2)=\frac{\omega}{\Gamma}L(\omega)\left\{\text{arctg}\left[\frac{\gamma\Gamma+4|G|^2-\omega^2}{(\gamma+\Gamma)\omega}\right]\right.$$
$$\left.-\text{arctg}\left[\frac{\gamma\Gamma-\omega^2}{(\gamma+\Gamma)\omega}\right]\right\},$$
$$Q(|G|^2)=(\gamma\Gamma+4|G|^2-\omega^2)^2+(\gamma+\Gamma)^2\omega^2.$$

When the splitting of the atomic levels is large so that $2|G|>\Gamma$, the integral in Eq. (26) can be calculated approximately. We have to distinguish then two regions $|\omega|\leqslant 2|G|$ and $|\omega|>2|G|$. In the first, the integrand has at the point $y=\omega^2$ a sharp maximum corresponding to the maximum of the function $Q^{-1/2}(y)$. The functions $\exp[-\varphi(y)]$ and $[Q^{1/2}(y)y]^{-L(\omega)}$ vary slowly within the integration interval and they can be taken outside the integral sign at the argument value $y=\omega^2$. The remaining factor can be integrated. Equation (26) then transforms to

$$g(\omega)=g_0\left[\frac{|G|^2}{\pi\Gamma g_0}\right]^{L(\omega)}$$
$$\times\left\{1+\left[\frac{Q(0)}{Q(|G|^2)}\right]^{(1+L(\omega))/2}\exp[\varphi(|G|^2)]\right.$$
$$\times\left[1+\left(\frac{4g_0\pi\Gamma}{\omega^2}\right)^{L(\omega)}\frac{\Gamma}{\gamma}\exp[-\varphi(\omega^2)]\right.$$
$$\left.\left.\times\ln\left(\frac{Q^{1/2}(|G|^2)+4|G|^2-\omega^2}{Q^{1/2}(0)-\omega^2}\right)\right]\right\}. \qquad (27)$$

In the case described by $|\omega|>2|G|$ the slowly varying factors in the integrand depend so weakly on y that they can be regarded as constants and then integration of Eq. (26) gives

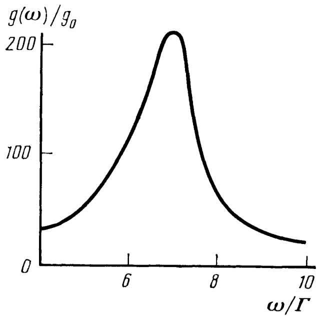

FIG. 1. Profiles of the side components in the superluminescence spectrum calculated for $4|G|^2=50\Gamma^2$, $\pi\Gamma g_0=10^{-7}\Gamma^2$, and $\gamma=0.1\Gamma$.

$$g(\omega)=g_0\left[\frac{|G|^2}{\pi\Gamma g_0}\right]^{L(\omega)}\left\{1+\left[\frac{Q(0)}{Q(|G|^2)}\right]^{(1+L(\omega))/2}\left[\exp\varphi(|G|^2)\right.\right.$$
$$\left.\left.+\frac{\Gamma}{\gamma}\ln\left(\frac{Q^{1/2}(|G|^2)+4|G|^2-\omega^2}{Q^{1/2}(0)-\omega^2}\right)\right]\right\}. \qquad (28)$$

The spectral distribution described by Eqs. (27) and (28) is given in Fig. 1, which demonstrates that at frequencies $|\omega|\approx 2|G|$ there are strong maxima. Equations (27) and (28) can be used readily to estimate the width and amplitude of the maxima

$$\Delta\omega\approx 3^{1/2}(\Gamma+\gamma),\quad g(2|G|)\approx g_0\frac{2|G|}{\gamma}\ln\left(\frac{4|G|}{\gamma+\Gamma}\right) \qquad (29)$$

4. CONCLUSIONS

Narrowing of the spectrum in the linear region at the onset of the nonlinear interaction makes a superluminescence field quasimonochromatic and causes splitting of the atomic levels of the active transition. Side maxima appear in the superluminescence spectrum. Amplification of the side components may increase them to saturation values. When the splitting is large, the side components interact with atoms in the same way as quasimonochromatic fields. The superluminescence spectrum then becomes multicomponent and equidistant.

The expressions obtained above can be used also to describe an STRS line in the approximation of a given constant monochromatic exciting field, which can be done by making the substitution $d_{21}\rightarrow d_R E_R$, where d_R is a reduced matrix element and E_R is the amplitude of the exciting field.

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[1]G. V. Abrosimov, Opt. Spektrosk. **31**, 106 (1971) [Opt. Spectrosc. (USSR) **31**, 54 (1971)].

[2]V. I. Ishchenko, V. N. Lisitsyn, A. M. Razhev, et al., Pis'ma Zh. Eksp. Teor. Fiz. **19**, 669 (1974) [JETP Lett. **19**, 346 (1974)].

[3]F. A. Korolev, A. I. Odintsov, N. T. Turkin, and V. V. Yakunin, Kvantovaya Elektron. (Moscow) **2**, 413 (1975) [Sov. J. Quantum Electron. **5**, 237 (1975)].

[4]I. L. Klyukach and R. I. Sokolovskiĭ, Zh. Eksp. Teor. Fiz. **71**, 424 (1976) [Sov. Phys. JETP **44**, 223 (1976)].

[5]E. A. Morozova and A. I. Sokolovskaya, Kvantovaya Elektron. (Moscow) **4**, 2052 (1977) [Sov. J. Quantum Electron. **7**, 1179 (1977)].

[6]S. G. Rautian, G. I. Smirnov, and A. M. Shalagin, *Nonlinear Resonances in the Spectra of Atoms and Molecules* [in Russian], Nauka, Novosibirsk (1979).

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