Fluctuations and diffusion in a weakly heated electron gas

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(Submitted 10 December 1988)
Zh. Eksp. Teor. Fiz. **95**, 1701–1710 (May 1989)

Results are presented of a study of the diffusion, of the spectral density of the current fluctuations, and of the correlation tensor that characterizes the degree of correlation (the non-ideality) of a weakly heated nonequilibrium electron gas. The investigations were made at intermediate electron densities, when the electron-electron collisions compete with the energy relaxation of the electron gas. It is found that under the conditions considered the nonequilibrium correction to the diffusion tensor is of the same order as the correlation tensor, so that the chances of observing the latter in experiment are improved.

1. INTRODUCTION

It is known that a system in thermodynamic equilibrium satisfies the fluctuation—dissipation theorem or the Callen–Welton relation,[1] which connect the fluctuation characteristics of the system with its dissipation characteristics. These relations are invalid to some degree or another in a nonequilibrium system. It was assumed for a long time that in a nonequilibrium electron gas only one of these relations is satisfied, the so-called Price relation[2]

$$(\delta j_i\,\delta j_k)_{\omega\to 0}=\frac{e^2 n}{V_0}(D_{ik}+D_{ki}), \tag{1.1}$$

which connects the spectral density $(\delta j_i \delta j_k)_\omega$ of the homogeneous current fluctuations with the diffusion tensor D_{ik}. In (1.1) V_0 is the normalization volume and n the electron density. However, after a consistent theory of fluctuations in a nonequilibrium electron gas was developed,[3,4] it became clear that the Price relation is also violated. It was found that the latter violation is due to electron-electron collisions that generate, in the nonequilibrium state, a certain electron-electron correlation. For a nonequilibrium gas it is therefore necessary to replace (1.1) by (see, e.g., Ref. 5)

$$(\delta j_i\,\delta j_k)_{\omega\to 0}=\frac{e^2 n}{V_0}(D_{ik}+D_{ki}-\Delta_{ik}). \tag{1.2}$$

The additional tensor[1)] Δ_{ik} introduced by this relation is, by virtue of the foregoing, a characteristic of the degree of correlation (or non-ideality) of the nonequilibrium electron gas.

To our knowledge, there are no experimental studies of the tensor Δ_{ik}. Theoretical estimates of this tensor were obtained either in the electron-temperature approximation (see, e.g., Ref. 5) or by using a Maxwell distribution with drift.[6] These estimates pertain to the region of high electron densities, when the electron-electron collisions prevail over the relaxation processes due to interaction with the heat bath.

We present here the results of a study of the diffusion, of the current-fluctuations spectral density, and of the tensor Δ_{ik} at intermediate electron densities, where the electron-electron collisions compete with the energy relaxation, i.e., in the region

$$\tau_p \ll \tau_{ee} \sim \tau_\varepsilon. \tag{1.3}$$

Here τ_p and τ_ε denote the times characterizing the momentum and energy relaxation, and τ_{ee} is the characteristic electron-electron collision time.

We have carried out a numerical study of these kinetic characteristics of an electron gas under weak heating conditions. This study was prompted by the following factors. First, interelectron Coulomb scattering has a singularity at low energies and is therefore more substantial for weak than for strong heating. Second, a well-developed experimental technique for measuring the kinetic properties of warm electrons in semiconductors is available at present,[7] so that identification of specific parameter ranges is of interest from the standpoint of an experimental observation of the violation of relation (1.1).

We shall show that the Price relation can be significantly violated in the considered region of intermediate densities (the tensor Δ_{ik} is of the same order as the nonequilibrium correction to the diffusion tensor D_{ik}). This violation, however is due not strictly speaking to the additional correlation generated by the electron-electron collisions (as is the case for high densities), but to the dependence of the form of the stationary distribution function on the electron density.

2. KINETIC EQUATION UNDER WEAK HEATING CONDITIONS

We present in this section the notation and equations which will be needed below (a detailed derivation can be found, e.g., in Ref. 8) and which are connected with the field expansion of the solution of the stationary kinetic equation:

$$e\mathbf{E}\frac{\partial}{\partial \mathbf{p}}F(\mathbf{p})=S(F). \tag{2.1}$$

Here $S(F)$ is the collisional term of the equation and includes both the interaction of the electrons in the heat bath and the electron–electron scattering. To be specific, we consider the case when the electron is heated by a constant electric field $\mathbf{E}$ directed along the p_z axis. We shall calculate the nonequilibrium field corrections to the characteristic accurate to E^2 inclusive, and represent therefore the stationary distribution function by the expansion

$$F(\mathbf{p})=\Phi(x)+\frac{p_z}{p}\varphi(x)$$

$$=\Phi_0(x)+\frac{p_z}{p}\left(\frac{E}{E^*}\right)\varphi_1(x)+\left(\frac{E}{E^*}\right)^2\Phi_2(x). \tag{2.2}$$

For convenience (to make all distribution-function components of the same dimension) we have introduced here the constant E^* with the dimension of the electric field, and use also the dimensionless electron energy

 0038-5646/89/050982-06$04.00

$$x=\varepsilon/kT \tag{2.3}$$

(T is the heat-bath temperature). The function $\Phi_0(x)$ is the equilibrium distribution function:

$$\Phi_0(x)=e^{-x}. \tag{2.4}$$

The equations for the two other components, the symmetric $\Phi_2(x)$ and the antisymmetric $\varphi_1(x)$, are obtained by substituting the expansion (2.2) in the kinetic equation (2.1) and suitably averaging the latter over the angle variables. We shall find it convenient hereafter to denote the linearized collisional operators by $S_0(\Phi_2)/\tau_\varepsilon$ and $S_1(\varphi_1)/\tau_p$ for the symmetric and anti-symmetric parts, respectively. The operators $S_0(\Phi_2)$ and $S_1(\varphi_1)$ were chosen to be dimensionless, and the quantities τ_p and τ_ε can be easily expressed in terms of constants that characterize the intensities of the actual scattering mechanisms. We choose the constant E^* in the form

$$eE^*=[3mkT/(2\tau_p\tau_\varepsilon)]^{1/2}. \tag{2.5}$$

The above expressions for the distribution-function components take then the following dimensionless form

$$S_1(\varphi_1)=(3\delta)^{1/2}x^{1/2}\frac{\partial}{\partial x}\Phi_0, \tag{2.6}$$

$$S_0(\Phi_2)=-\frac{1}{(3\delta)^{1/2}}\frac{1}{x^{1/2}}\frac{\partial}{\partial x}(x\varphi_1), \tag{2.7}$$

where the parameter

$$\delta=\tau_p/\tau_\varepsilon \tag{2.8}$$

characterizes the degree of inelasticity of the scattering.

Owing to the constraint (1.3) used by us we take into account, in place of the total two-particle collision operator

$$W_p(F,F)=\int d^3p'\,W_{pp'}(F,F), \tag{2.9}$$

where

$$W_{pp'}(F,F)=\frac{4ne^4}{\epsilon^2}\int d^3q\frac{1}{q^4}\delta[\varepsilon(\mathbf{p}+\mathbf{q})+\varepsilon(\mathbf{p}'-\mathbf{q}) -\varepsilon(\mathbf{p})-\varepsilon(\mathbf{p}')][F(\mathbf{p}+\mathbf{q})F(\mathbf{p}'-\mathbf{q})-F(\mathbf{p})F(\mathbf{p}')] \tag{2.10}$$

(ε is the dielectric constant), only its symmetric part, which we include in Eq. (2.7). We use here the collision operator in the so-called Landau form, which takes suitable account of the scattering with small momentum change, which is most significant in the case of weak heating. Following a procedure described in detail in Ref. 9, we transform (2.10) into

$$W_{pp'}(F,F)=W_0\sum_{\alpha\beta}\left(\frac{\partial}{\partial p_{\alpha'}}-\frac{\partial}{\partial p_\alpha}\right)K_{\alpha\beta}(|\mathbf{p}-\mathbf{p}'|) \times\left(\frac{\partial}{\partial p_\beta}-\frac{\partial}{\partial p_{\beta'}}\right)F(\mathbf{p})F(\mathbf{p}') \tag{2.11}$$

where

$$K_{\alpha\beta}(q)=\frac{1}{q}\left\{\delta_{\alpha\beta}-\frac{q_\alpha q_\beta}{q^2}\right\}, \tag{2.12}$$

$$W_0=\frac{2\pi nme^4}{\epsilon^2}\Lambda. \tag{2.13}$$

We assume that the Coulomb logarithm is equal to

$$\Lambda=\ln\frac{3(ekT)^{3/2}}{2\pi^{1/2}n^{1/2}e^3}. \tag{2.14}$$

Now, substituting the expansion (2.2) in (2.11) and next in (2.9), linearizing the obtained expression with respect to Φ_2, averaging over the angle variables, and changing to dimensionless variables, we obtain the final expression for the electron-electron collision operator:

$$S_0^{ee}(\Phi_2)=w\frac{1}{x^{1/2}}\frac{d}{dx}\left\{\left(\frac{d\Phi_2}{dx}+\Phi_2\right)\int_0^x dx'\,x'^{1/2}e^{-x'} +e^{-x}\left[\int_0^x dx'x'^{1/2}\Phi_2(x')-\frac{2}{3}\left(x^{3/2}\int_x^\infty dx'\Phi_2(x') +\int_0^x dx'x'^{3/2}\Phi_2(x')\right)\right]\right\}, \tag{2.15}$$

which we have taken into account in Eq. (2.7).

The constant

$$w=8W_0\tau_\varepsilon/\pi^{1/2}(2mkT)^{3/2} \tag{2.16}$$

characterizes the relative intensity of the electron-electron scattering compared with the heat-bath mechanisms of energy relaxation. The electron-electron scattering is not taken into account in (2.6) [by virtue of (1.3)]. Moreover, the antisymmetric collision operator is written in the relaxation-time approximation

$$S_1(\varphi_1)=-\frac{1}{\tau(x)}\varphi_1(x). \tag{2.17}$$

This enables us to solve Eq. (2.6) right away and obtain, after substituting the solution in (2.7), the equation

$$S_0(\Phi_2)=\frac{1}{x^{1/2}}\frac{d}{dx}(x^{3/2}\tau(x)e^{-x}) \tag{2.18}$$

for the symmetric nonequilibrium correction to the distribution function. It is necessary to add to this equation the normalization condition

$$\int_0^\infty dx x^{1/2}\Phi_2(x)=0. \tag{2.19}$$

It was Eq. (2.18) which we have investigated by numerical methods.

3. THE DIFFUSION COEFFICIENT

We apply now the procedure of the preceding section to the diffusion coefficient.

The diffusion coefficient, defined by the expression for the diffusion current

$$j_i=-D_{ik}\frac{\partial n}{\partial r_k}, \tag{3.1}$$

can be written, following Ref. 5, in the form

$$D_{ik}=\frac{1}{N_0}\int d^3p\,v_i\Psi_k(\mathbf{p}), \tag{3.2}$$

where N_0 is the normalization integral of the equilibrium distribution function (2.4), and the function $\Psi_k(\mathbf{p})$ satisfies the following kinetic equation:

$$e\mathbf{E}\frac{\partial}{\partial \mathbf{p}}\Psi_k(\mathbf{p})-S^L(\Psi_k)=(v_k-V_k)\frac{\partial}{\partial n}nF(\mathbf{p}), \tag{3.3}$$

where

$$V_k=\frac{1}{N_0}\frac{\partial}{\partial n}n\int d^3p\, v_kF(\mathbf{p}), \tag{3.4}$$

and $S^L(\Psi_k)$ denotes the linearized collision operator. The function $\Psi_k(\mathbf{p})$ satisfies the zero normalization condition

$$\int d^3p\,\Psi_k(\mathbf{p})=0. \tag{3.5}$$

Confining ourselves, as in the preceding section, to an approximation up to E^2 inclusive, we write for $\Psi_k(\mathbf{p})$ the expansion

$$\Psi_k(\mathbf{p})=\frac{p_k}{p}\chi(x)+\delta_{kz}\left\{\Psi(x)+\frac{p_z}{p}\psi(x)\right\}$$
$$=\frac{p_k}{p}\left\{\chi_0+\left(\frac{E}{E^*}\right)^2\chi_2\right\}+\delta_{kz}\left\{\frac{E}{E^*}\Psi_1+\frac{p_z}{p}\left(\frac{E}{E^*}\right)^2\psi_2\right\}. \tag{3.6}$$

Substituting now this expansion, as well as the expansion (2.2), in Eq. (3.3) we obtain after the appropriate averaging and after changing to dimensionless variables the following systems of equations:

$$\chi(x)=\tau_p\left(\frac{2kT}{m}\right)^{1/2}x^{1/2}\tau(x)\frac{\partial}{\partial n}n\Phi(x), \tag{3.7}$$

$$\psi_2(x)=-(3\delta)^{1/2}\tau(x)x^{1/2}\frac{\partial}{\partial x}\Psi_1(x), \tag{3.8}$$

$$S_0(\Psi_1)=\left(\frac{1}{3\delta}\right)^{1/2}\left(\frac{2kT}{m}\right)^{1/2}\tau_p\left\{\left[\frac{2}{\pi^{1/2}}I_0-x\tau(x)\right]e^{-x}+\frac{1}{x^{1/2}}\frac{d}{dx}(x\tau(x)e^{-x})\right\}, \tag{3.9}$$

where

$$I_0=\int_0^\infty dx x^{1/2}\tau(x)e^{-x}. \tag{3.10}$$

Substituting the expansion (3.6) in the definition (3.2), we express the diffusion tensor in the form:

$$D_{ik}=\delta_{ik}\frac{2}{3}\left(\frac{2kT}{\pi m}\right)^{1/2}\times\int_0^\infty dx x\left\{\chi_0(x)+\left(\frac{E}{E^*}\right)^2[\chi_2(x)+\delta_{kz}\psi_2(x)]\right\}. \tag{3.11}$$

We introduce now for convenience a certain dimensionless function $\Phi_2(x)$ defined as

$$S_0(\Phi_2)=\left[x\tau(x)-\frac{2}{\pi^{1/2}}I_0\right]e^{-x} \tag{3.12}$$

and satisfying, just as the function $\Phi_2(x)$ in (2.19), a zero normalization condition. It is now easy to verify that the solution of (3.9) can be written in the form

$$\Psi_1=\left(\frac{1}{3\delta}\right)^{1/2}\left(\frac{2kT}{m}\right)^{1/2}\tau_p\{\Phi_2-\Phi_2\}. \tag{3.13}$$

Substituting this expression in (3.8) and substituting next (3.7) and (3.8) in (3.11), we obtain the final expression for the nonzero diagonal components of the diffusion tensor D_{ik}. It is convenient to write it in the form

$$D_{xx}=D_{yy}=D_0(1+\gamma_\perp(E/E^*)^2), \tag{3.14}$$

$$D_{zz}=D_0(1+\gamma_\parallel(E/E^*)^2), \tag{3.15}$$

where

$$D_0=\frac{4kT}{3\pi^{1/2}m}\tau_pI_0 \tag{3.16}$$

is the equilibrium isotropic diffusion coefficient, and the dimensionless coefficients

$$\gamma_\perp=\frac{1}{I_0}\int_0^\infty dx x^{3/2}\tau(x)\frac{\partial}{\partial n}(n\Phi_2), \tag{3.17}$$

$$\gamma_\parallel=\gamma_\perp+\frac{1}{I_0}\int_0^\infty dx x^{3/2}\tau(x)\frac{d}{dx}(\Phi_2-\Phi_2) \tag{3.18}$$

characterize the nonequilibrium corrections to the diffusion tensor.

The differentiation in (3.17) with respect to the density is indicative of the dependence of the form of the nonequilibrium increment to the stationary distribution function on the inter-electron collisions.

To find the nonequilibrium corrections (3.17) and (3.18) to the diffusion coefficient we must solve Eqs. (3.12) and (2.18) numerically.

4. SPECTRAL DENSITY OF CURRENT FLUCTUATIONS

A definition of the spectral density of homogeneous current fluctuations is the expression (1.2) above. For the additional correlator we write, following Ref. 5, the expression

$$\Delta_{ik}=A_{ik}+B_{ik}, \tag{4.1}$$

where

$$A_{ik}=\frac{1}{N_0^2}\int d^3p\int d^3p'\, v_iv_k'S_p^{-1}S_{p'}^{-1}W_{pp'}(F,F), \tag{4.2}$$

$$B_{ik}=-\frac{1}{N_0}\int d^3p\left\{S_p^{-1}n\frac{\partial}{\partial n}F(\mathbf{p})[v_iv_k'+v_i'v_k]\right\}_{v'=v}. \tag{4.3}$$

The symbol S_p^{-1} denotes here formally an operator inverse to the linearized kinetic operator, i.e., to the operator in the left-hand side of Eq. (3.3). By virtue of the approximation (1.3) we need use for the correlation operator $W_{pp'}(F,F)$ only the symmetric part of the expression (2.11) used above, in the Landau form, for this operator which acts on the symmetric component of the function $\Phi(x)$.

We see from (4.2) and (4.3) that the additional tensor Δ_{ik} consists of two different parts. One of them, A_{ik}, is due to the electronic-states correlation generated in the electron-electron-scattering process, as evidenced by the presence of the operator $W_{pp'}(F,F)$ in the integrand of (4.2). It is just to this part of the tensor Δ_{ik} that principal attention was paid in

earlier studies (see, e.g., Ref. 5), inasmuch as in the electron temperature approximation (which is valid in the region $\tau_{ee} \ll \tau_\varepsilon$) B_{ik} is simply zero. Only the quantity A_{ik} was likewise investigated in Ref. 10, where the violation of the Price relation under strong heating condition was studied by the Monte Carlo method and where this violation was found for high densities ($n \gtrsim 10^{18}$ cm^{-3}). The second part B_{ik} is likewise due to electron-electron collisions. Their influence here, as we see from (4.3), is differently manifest, namely, as a dependence of the form of the stationary distribution function $F(\mathbf{p})$ on the intensity of the electron-electron collisions (via the dependence on the density n).

Substituting the symmetric part of the expansion (2.2) in (4.2) and (4.3), we readily verify that the first nonvanishing terms are proportional exactly to the correction, proportional to E^2, of interest to us, since

$$W_{pp'}(\Phi_0, \Phi_0)=0, \tag{4.4}$$

$$n\frac{\partial}{\partial n}\Phi_0=0. \tag{4.5}$$

It suffices therefore to take the inverse operators S_p^{-1} into account in the zeroth order in E. In addition, it is evident from the structure of the integrals in the indicated expressions that only the antisymmetric part of the opertors S_p^{-1} need be considered. We can write then, in accordance with the approximation used by us,

$$S_p^{-1}=\tau_p\tau(x). \tag{4.6}$$

Thus, substituting (4.6) in (4.2) and (4.3), integrating over the angle variables, and changing to dimensionless quantities, we obtain as the final result:

$$B_{ik}=\delta_{ik}D_0\gamma_B(E/E^*)^2, \tag{4.7}$$

where

$$\gamma_B=\frac{2}{I_0}n\frac{\partial}{\partial n}\int_0^\infty dx x^{1/2}\tau(x)\Phi_2(x), \tag{4.8}$$

and

$$A_{ik}=\delta_{ik}D_0\gamma_A(E/E^*)^2, \tag{4.9}$$

where

$$\gamma_A=\frac{4}{5}w\delta\int_0^\infty dx x^{1/2}\int_0^\infty dx' x'^{1/2}[\Phi_2'(x)+\Phi_2(x)]e^{-x'}$$

$$\times[\tau(x)\tau'(x')-\tau'(x)\tau(x')]\frac{x_<^2}{x_>^{1/2}}. \tag{4.10}$$

The smaller of the quantities x and x' should be substituted here for $x_<$, and the larger for $x_>$.

We see thus from (4.7)–(4.10) that the tensor Δ_{ik} receives in the weak-heating region an isotropic contribution, proportional to E^2, indicative of the degree of violation of the Price relation. In the case of the quasi-elastic scattering mechanism $\delta \ll 1$ (this is precisely what is assumed in the case (1.3) considered by us), it is necessary to include in the calculated tensor Δ_{ik} only the term B_{ik}, since the expression for γ_A contains the parametrically small quantity δ. Under weak-heating conditions, consequently, violation of the Price relation at intermediate densities is possible because of the density dependence of the stationary distribution function.

5. RESULTS OF NUMERICAL CALCULATIONS AND THEIR DISCUSSION

For an actual estimate of the extent to which the Price relation can be significantly violated in the weak-heating region, we cite in this section some numerical-calculation results.

We have calculated the coefficients $\gamma_\parallel$, $\gamma_\perp$, and γ_B for quasi-elastic electron-gas interaction, when the equations for the symmetric parts of the distribution functions (2.18) and (3.12) contain the electron-electron-collision operator (2.15) and the operator of electron collisions with a heat bath of the Davydov type:

$$S_0(\Phi_2)=\frac{1}{x^{1/2}}\frac{\partial}{\partial x}\left\{x^2\left[\frac{\partial}{\partial x}\Phi_2+\Phi_2\right]\right\}. \tag{5.1}$$

The momentum-relaxation time was estimated by the expression

$$\tau(x)=x^{-1/2}. \tag{5.2}$$

Approximation by quasi-elastic collisions is frequently used to describe the electron gas both in a gas-discharge plasma and in semiconductors. Since (5.1) is a differential operator, and the integral part of (2.15) is a Volterra operator, Eqs. (2.18) and (3.12) can be reduced to a system of differential equations, and these were calculated by the Runge–Kutta method. The dependences of the coefficients $\gamma_\parallel$, $\gamma_\perp$, and γ_B on the only parameter w, calculated for this model, are shown in Fig. 1. The parameter w [Eq. (2.16)] is indicative, as noted earlier, of the relative intensity of the electron-electron scattering compared with the heat-bath energy relaxation. In the limit as $w \to 0$, the coefficients $\gamma_\parallel$ and $\gamma_\perp$ go over into the values obtained without allowance for electron-electron collisions, while as $w \to \infty$ we have values that agree with the result of the electron-temperature approximation. The coefficients γ_B, as should be expected, vanishes in both limiting cases. It has a maximum when the energy relaxation rate produced inside the electron system by electron-electron collisions is of the same order as the energy relaxation rate in the heat bath. This is attested to, in particular, by estimates of the times τ_ε and τ_{ee}, which are equal at $w \approx 6$. The maximum of the γ_B dependence corresponds to approximately the same value.

It can be seen from Fig. 1 that in the case of quasi-elastic

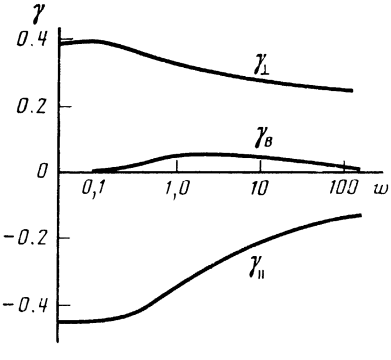

FIG. 1. Plots of the coefficients $\gamma_\perp$, $\gamma_\parallel$, and γ_B against the parameter w that determines the relative intensity of the electron-electron scattering compared with the interaction with the thermostat.

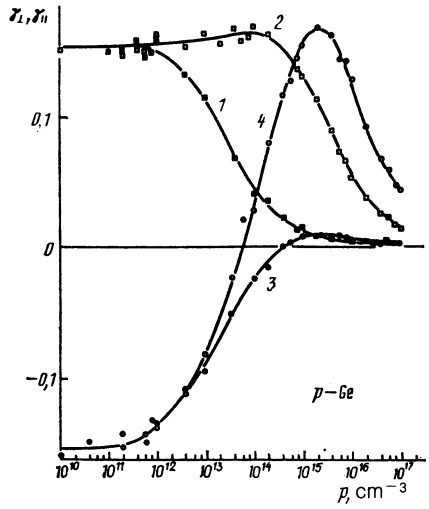

FIG. 2. Plots of $\gamma_\perp$ (curves 1 and 2) and $\gamma_\parallel$ (curves 3 and 4) against the hole density in p-Ge at $T = 80$ K, with (1, 3) and without (2, 4) allowance for electron-electron scattering.

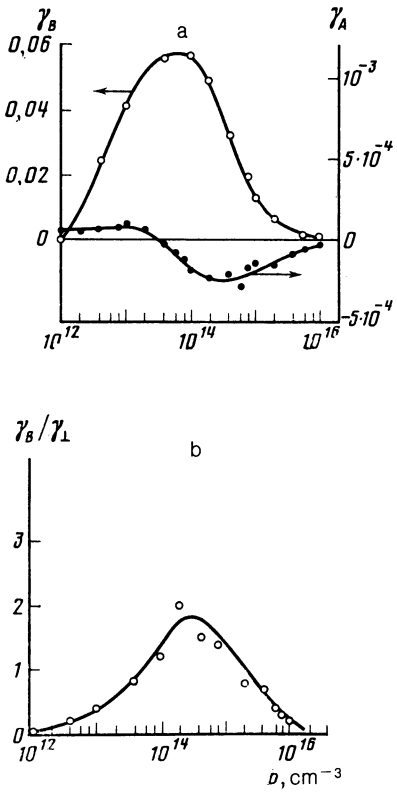

FIG. 3. Plots of γ_A and γ_B (a) and of the ratio $\gamma_B/\gamma_\perp$ against the hole density in p-Ge at $T = 80$ K.

scattering the diffusion coefficients (the plots of $\gamma_\perp$ and $\gamma_\parallel$) is sensitive to the presence of electron-electron collisions and violation of the Price relation (curve γ_B) also occurs but is small, on the order of 10% (compared with the corrections to the diffusion tensor).

It is known that in the case of low-temperature scattering of electrons by optical phonons in semiconductors the interelectron collisions provide a new energy-relaxation channel (the scattering mechanism called composite[1)] sets in) and influence thereby substantially both the energy relaxation itself and the form of the stationary distribution function. It is natural to assume that in this situation the coefficients $\gamma_\parallel$, $\gamma_\perp$, and γ_B turn out to be more sensitive to the presence of electron-electron collisions. To illustrate this possibility we computed these coefficients for semiconductors with parameters corresponding to the heavy-hole band in p-Ge. We took into account in (2.7), besides the interelectron scattering, the scattering by acoustic and optical phonons. In addition to the last two mechanisms, we took into account in the obtained momentum relaxation time $\tau(x)$ also scattering by ionized impurities. We used in the calculation an effective mass $0.3m_0$, a sound velocity $5.4\cdot10^3$ m/s, a density $5.33\cdot10^3$ kg/m^3, a deformation-potential constant 6 eV, a constant of coupling with the optical phonons $9\cdot10^{10}$ eV/m, a characteristic optical-phonon temperature 430 K, and a relative dielectric constant 16.

The calculation was performed for a compensated semiconductor at $T = 80$ K. Equations (2.18) and (3.12) were solved by a previously proposed[8] two-particle Monte Carlo method and the electron-electron collisions were taken into account by an iteration procedure described in detail in Ref. 12.

The calculation results are shown in Figs. 2 and 3. Figure 2 shows the dependences of $\gamma_\perp$ and $\gamma_\parallel$ on the hole density p (the density is a measure of the electron–electron-collision intensity). The reversed sign of the correction to the longitudinal diffusion coefficient $\gamma_\parallel$ is due to the hole scattering by ionized impurities. Comparing the results calculated without and with allowance for electron-electron collisions, we see that at intermediate densities ($p \sim 10^{14}$–10^{16} cm^{-3}) these collisions influence the diffusion coefficient substantially. Note that the influence of this type of scattering on other parameters of warm electrons was noted also earlier.[12,13]

Since the collision operator is not quasi-elastic in the presence of scattering by optical phonons, we have plotted in Fig. 3a not only the coefficient γ_B but also γ_A. In the region of intermediate densities the coefficient γ_A is still small compared with γ_B, apparently an indirect indication that the inequality $\tau_\varepsilon \gg \tau_p$ holds in this case. Figure 3b shows the ratio of the coefficient γ_B, which is indicative of the violation of the Price relation, and the nonequilibrium correction to the diffusion coefficient $\gamma_\perp$. It is seen to be substantial at $p \sim 10^{14}$–10^{15} cm^{-3}. This leads to the hope of being able to observe in experiment violations of the Price relation in semiconductors at intermediate densities under conditions of weak heating, and possibly also strong.

The authors thank R. Katilyus for a discussion of the results.

[1)]It is sometimes called the correlation tensor.

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Translated by J. G. Adashko