Hypersonic solitons in metals

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A study is made of the evolution of a short longitudinal hypersound pulse in a metal in the case of a quantum interaction with resonant electrons. It is shown that, in principle, self-induced transparency can appear under certain conditions and then the pulse can travel without attenuation at a velocity different from that of sound. The system of resonance-approximation equations formulated to describe such pulse evolution is exactly integrable. The Zakharov-Shabat representation is obtained for this system of equations by the inverse scattering method and one-soliton, two-soliton, and breather solutions are obtained. This mathematical formalism can be used to describe a resonance interaction between hf pulses of any physical nature and an electron system.

The problem of nonlinear attenuation of sound in metals has been discussed in the literature on many occasions (for a review see Ref. 1). It has been established that the most favorable conditions for nonlinear behavior are obtained in a region of strong spatial dispersion when the source of nonlinearity is the nonequilibrium nature of the distribution of a small group of resonant electrons. Low values of the longitudinal velocity of these electrons moving in phase with a wave ensure that their motion is disturbed by the field much more strongly than the motion of other electrons and the distribution in the resonance region may be far from equilibrium, whereas electrons associated with the rest of the Fermi surface will be close to equilibrium. This is precisely the momentum nonlinearity mechanism described in Ref. 2 using the semiclassical approximation. In the quantum case discussed in Ref. 3, when the wave vector of an acoustic wave is comparable with the characteristic longitudinal momentum of resonant electrons, an important role is played by modification of the electron wave functions accompanied by the appearance of a specific energy band structure of the electrons.[4]

The nonlinearity reduces the attenuation and at a fixed amplitude the scale of the effect is inversely proportional to the natural width of the resonance. In the usual case of a continuous acoustic signal,[1] when the time scale of changes in the amplitude of sound is the greatest, the width of a resonance is governed by the electron collision time τ which occurs explicitly in the expression describing the nonlinear attenuation. In the opposite limit of a short acoustic pulse, when the pulse length T satisfies

$$\omega^{-1} \ll T \ll \tau, \tag{1}$$

the role of the factor governing the resonance width is now played by T and the nonlinear attenuation problem can be solved in the collisionless limit.[5]

The main conclusion reached in the present study is that in the case described by Eq. (1) under certain conditions we can expect total acoustic "bleaching" of the metal similar to the self-induced transparency (SIT) effect in nonlinear optics,[6] so that an acoustic pulse is transformed into an undamped soliton moving at a velocity other than the phase velocity of sound. It follows from the scale of the electron collision time in metals that such a soliton can be observed only in the hypersonic range of frequencies.

The nonlinear bleaching of a resonantly absorbing medium, first investigated for two-level atoms,[7] is based on the specific behavior of a two-level system in an external field of the resonance frequency inducing periodic changes in the populations of the levels at a low frequency proportional to the field amplitude.[8] Similar dynamics of interband transitions in systems with a spectrum of the semiconductor type[9] leads to the SIT effect, which can in principle occur also in such systems (difficulties in realizating SIT in real semiconductors are discussed in Ref. 10). The SIT effect for acoustic waves in a system with a gap in the quasiparticle spectrum is fundamentally identical with the optical SIT and, bearing in mind much narrower ranges of frequencies available in acoustics ($\omega < 10^{11}\ \mathrm{s}^{-1}$), it can be expected in such narrow-gap systems as superconductors[11] or ^{3}He in the superfluid state.[12]

The situation is completely different in metals because the absorption of sound is due to intraband transitions. The dynamics of these transitions is qualitatively different from that of interband transitions and the acoustic analog of the SIT seems at first sight to be impossible. In fact, as is known from Ref. 4, the spectrum of electrons near a resonance exhibits a forbidden band under the influence of a periodic field; the appearance of this band is due to an exponential rise of the wave functions. However, in the case of the quadratic dispersion law considered in Ref. 4, this rise is related to a specific property of a resonance transition, namely it is accompanied by backward reflection of an electron in a reference system moving with the acoustic wave. In the case of a more complex dispersion law the scattering may be in the forward direction and then, as demonstrated by the calculations reported below, the exponential rise of the wave functions changes to oscillations necessary for the establishment of the SIT effect.

The SIT in optics is mathematically due to the integrability of the relevant system of equations.[13] The equations for acoustic solitons are also integrable and in a certain sense represent a generalization of the SIT equations allowing for the finite momentum transferred to an electron as a result of its scattering by an acoustic wave. The structure of the equations of the associated linear problem, which is the basis for the application of the inverse scattering method, is analogous to the well-known (in the theory of solitons) three-wave system expanded to allow for the interaction of an

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acoustic wave with an infinite number of the electron degrees of freedom. This generalized three-wave structure is, in our opinion, of universal nature and it describes, subject to the conditions of Eq. (1), nonlinear evolution of fields of different physical nature when such fields interact resonantly with the electron system.

The present paper is organized as follows. The first section provides a derivation of the resonance-approximation equations which describe the nonlinear evolution of a short pulse of longitudinal sound in a metal subject to the condition (1). The second section identifies the conditions for the existence of soliton solutions and analyzes a model three-wave system. The third section applies a method which factors the McCall-Hahn ansatz used to obtain a one-soliton solution. It gives the explicit form of operators of the associated problem which is used in the Appendix to obtain breather and two-soliton solutions. The third section provides a discussion of the properties of acoustic solitons and the feasibility of experimentally observing these solitons.

1. DERIVATION OF THE RESONANCE-APPROXIMATION EQUATIONS

Propagation of longitudinal sound in a metal is described by an equation from the theory of elasticity

$$(\partial_{tt}^2-s^2\partial_{xx}^2)u=\frac{1}{\rho_0}F, \tag{2}$$

where $u(x,t)$ is the displacement of the lattice; ρ_0 is the density of the metal; $F(x,t)$ is the force exerted by electrons on the lattice and represents a linear functional of the electron distribution. In the hf limit this force is known to consist of resonant and nonresonant parts. In accordance with the above discussion, the resonance term is a source of nonlinearity in Eq. (2) and it can be calculated exactly allowing for the quantum nature of the resonant electrons, whereas the nonresonant part of the force can be calculated in the linear approximation with semiclassical precision. It is known from the linear theory that the nonresonant part of the force contributes to the renormalization of the velocity of sound and is assumed to be included in s. The resonant force is dissipative in the linear approximation and becomes dispersive in the SIT case.[6] The dispersion of the velocity of sound due to nonequilibrium of the nonresonant electrons is a quantity of higher order (in the parameter s/v_F) than the contribution of the resonant electrons and it will be ignored.[15] The smallness of the resonance force means that the changes in the amplitude and phase of the wave induced by this force occur slowly on the scale of the wave period:

$$\partial_x u(x,t)={}^1/_2[U_0(\chi,t)e^{iqx}+U_0^*(\chi,t)e^{-iqx}],$$
$$\chi=x-st,\quad \partial_t U_0\ll\omega U_0,\quad \partial_\chi U_0\ll qU_0. \tag{3}$$

The explicit form of the resonant force will be described by introducing the density matrix for resonant electrons $\hat\rho(\mathbf{p}_\perp;x,x',t)$, in which the conserved transverse quasimomentum is treated separately and an allowance is made for the contribution of only those values of the longitudinal quasimomentum p_x which lie in the resonance region (for simplicity, we shall assume that this region is in the vicinity of $p_x=0$). Following the treatment of Ref. 15, we obtain an expression for the force ($\hbar=1$)

$$F_{res}(x,t)=\int\frac{d\mathbf{p}_\perp}{4\pi^3}\Lambda(p_\perp,0)\,\partial_x\hat\rho(\mathbf{p}_\perp;x,x,t), \tag{4}$$

where Λ is the anisotropic part of the deformation potential: $\Lambda=\lambda_{xx}-\langle\lambda_{xx}\rangle$. In deriving this expression the electric field is excluded because of the electrical neutrality condition; the inertial term and the corrections to the deformation potential made in passing to the laboratory reference system are small in the resonance region and are ignored.

Since we are going to discuss the evolution of a short wave packet of Eq. (1), we shall find $\hat\rho$ using the following collisionless transport equation

$$i\partial_t\hat\rho=[\hat h,\hat\rho],\quad \hat h=\varepsilon_\parallel(\mathbf{p}_\perp,\hat p_x)+\Lambda(\mathbf{p}_\perp,0)\,\partial_x u. \tag{5}$$

The interaction with sound in the Hamiltonian of this equation is included using the same approximations as those employed in dealing with the force of Eq. (4), and the kinetic energy operator $\varepsilon_\parallel(\mathbf{p}_\perp,\hat p_x)$ is obtained by expanding the full dispersion law in terms of small quantities $p_x\sim q\ll p_F$:

$$\varepsilon(\mathbf{p})\approx\varepsilon_\perp(\mathbf{p}_\perp,0)+\varepsilon_\parallel(\mathbf{p}_\perp,p_x).$$

Relaxation of the electron nonequilibrium resulting from the interaction with sound occurs far from the packet and its details are important only to the extent that they affect the distribution of electrons which return to the resonance region. If we assume that these electrons have managed to relax completely before returning to the resonance region, we can formulate the initial equilibrium condition for Eq. (5):

$$\hat\rho(\mathbf{p}_\perp;x,x',-\infty)=\int dp_x n_F(\varepsilon_\perp+\varepsilon_\parallel)\exp[ip_x(x-x')]. \tag{6}$$

The formal solution of Eq. (5) satisfying the initial condition (6) is

$$\hat\rho(\mathbf{p}_\perp;x,x',t)=\int dp_x n_F(\varepsilon_\perp+\varepsilon_\parallel)\psi(p_x;x,t)\psi^*(p_x;x',t),$$
$$\psi(p_x;x,-\infty)=\exp(-i\varepsilon_\parallel t+ip_x x) \tag{7}$$

is the solution of the wave equation with the Hamiltonian (5).

For a constant sound amplitude the exact resonance condition is

$$\varepsilon_\parallel(p_x+q)-\varepsilon_\parallel(p_x)=\omega.$$

It is convenient to rewrite it, by inverting the function $\varepsilon_\parallel(p_x)$, which yields an equation for the energy $\tilde\varepsilon$ of an electron in a reference system linked to an acoustic wave:

$$p_+(\tilde\varepsilon)-p_-(\tilde\varepsilon)=q,\quad \tilde\varepsilon=\varepsilon_\parallel-p_x s; \tag{8}$$

For simplicity we shall assume that the scattering is of the single-channel type. Reducing the Hamiltonian of Eq. (5) to the stationary form by the Galilean approximation, we shall seek its wave functions in the direct vicinity to the exact resonance $\delta\tilde\varepsilon\ll\tilde\varepsilon_0$, where $\tilde\varepsilon_0$ is the solution of Eq. (8), in the form

$$\psi_\varkappa(x,t)=(A_\varkappa e^{ip_+x}+B_\varkappa e^{ip_-x})e^{-i\varepsilon_0 t},\quad \varkappa=\pm1, \tag{9}$$

where $A(\chi,t)$ and $B(\chi,t)$ are slowly varying functions satisfying the initial conditions in the limit $T\to-\infty$:

$$U_0(\chi,-\infty)=0,$$
$$A_+(\chi,-\infty)=\exp(i\delta p_+\chi-i\delta\tilde\varepsilon t),\quad B_+(\chi,-\infty)=0, \tag{10}$$
$$A_-(\chi,-\infty)=0,\quad B_-(\chi,-\infty)=\exp(i\delta p_-\chi-i\delta\tilde\varepsilon t);$$
$$\delta p_\varkappa=\delta\tilde\varepsilon/\tilde v_\varkappa,\quad \tilde v_\varkappa=v_\varkappa-s=d\tilde\varepsilon/dp_\varkappa.$$

Retaining the representation of the wave functions in the form of Eq. (9) in the case of slow variation of the amplitude of sound described by Eq. (3) and substituting it into the wave equation, we find that (after averaging over the fast variable χ) the result is the following system of equations for the coefficients A and B:

$$i\partial_t\begin{pmatrix}A\\B\end{pmatrix}=\begin{pmatrix}\tilde{v}_+\hat{p}, & {}^1/_2\Lambda U_0\\ {}^1/_2\Lambda U_0^*, & \tilde{v}_-\hat{p}\end{pmatrix}\begin{pmatrix}A\\B\end{pmatrix},\qquad \hat{p}=-i\frac{\partial}{\partial\chi}. \quad (11)$$

It is clear from these equations that the requirement that A and B vary slowly is equivalent to the inequality $\Lambda U_0 \ll q\tilde{v}_\varkappa$, which implies that the width of the first resonance is small compared with the separation between this resonance and the next one, and is the condition associated with the quantum nature of the problem. At high amplitudes the resonances overlap and the problem becomes semiclassical.

Collecting Eqs. (7), (9), (4), and (3) and substituting them into Eq. (2), we find that averaging with respect to the fast variable yields the following equation for the complex amplitude U_0:

$$i\partial_t U_0=\frac{q^2}{\rho_0\omega}\int\frac{d\mathbf{p}_\perp}{4\pi^3}\Lambda\sum_{\varkappa j}\int\frac{d\delta\tilde{\varepsilon}_j}{|\tilde{v}_{\varkappa j}|}n_F \times\left[\varepsilon_\perp+\tilde{\varepsilon}_j+sp_{\varkappa j}+\delta\tilde{\varepsilon}_j\left(1+\frac{s}{\tilde{v}_{\varkappa j}}\right)\right]A_{\varkappa j}B_{\varkappa j}^*, \quad (12)$$

where the index j labels the solutions of the system (8).

Equations (11) and (12) represent the complete system of resonance-approximation equations describing nonlinear evolution of a short pulse of longitudinal sound propagating in a metal. The initial conditions of Eq. (10) correspond to the problem of a pulse in infinite space. In a real experimental situation we have a half-space at the boundary of which a signal of given shape

$$U_0(x=0)=f(t),\qquad f(\pm\infty)=0$$

is excited and the conditions for A and B of Eq. (10) are still specified in the limit $t\to-\infty$ only for electrons with $v_\varkappa<0$ moving from the interior of the metal to the surface; these electrons are scattered on the surface and no longer participate in the resonance. On the other hand, electrons with $v_\varkappa>0$ reach the resonance region as a result of elastic scattering of equilibrium nonresonant electrons on the surface of a metal; the condition (10) for these electrons retains its original form but it now applies at the $x=0$ boundary.

In the linear approximation the process of pulse evolution simply represents the Landau damping effect. Linearization of the equations about the ground state of Eq. (10) and the usual application of the Laplace transformation yields the following result in the limit $x\to\infty$:

$$U_0\propto\exp\left(-\frac{\Gamma x}{2s}\right),\qquad \Gamma=\frac{q^2}{\rho_0}\int\frac{d\mathbf{p}_\perp}{4\pi^2}\frac{\delta(\varepsilon_\perp-\varepsilon_F)\Lambda^2(\mathbf{p}_\perp)}{|v_+-v_-|}. \quad (13)$$

Allowing for the nonlinearity we find that the asymptotic behavior of such an acoustic pulse is more complicated: under certain conditions it can transform into a soliton or a group of solitons which represent the exact solutions of the problem described by Eqs. (10)–(12). The necessary conditions for the existence of soliton solutions are easiest to derive from an analysis of a model system of equations describing the interaction of an acoustic pulse with the only pair of electron degrees of freedom.

2. THREE-WAVE MODEL

This model is obtained by formal replacement of the Fermi function in Eq. (12) with the δ function, which selects one value from the set of the phase parameters $\delta\tilde{\varepsilon}$, $\mathbf{p}_\perp$, $\varkappa$ and j:

$$i(\partial_t+\tilde{v}_+\partial_\chi)A=\Phi B,\qquad i(\partial_t+\tilde{v}_-\partial_\chi)B=\Phi^*A,$$
$$i\partial_t\Phi=g^2AB^*,\qquad \Phi={}^1/_2\Lambda U_0,\qquad g^2=\text{const}. \quad (14)$$

The system (14) consists of equations known from the theory of solitons: it is exactly integrable and describes three-wave interactions (Ref. 14).[1)] The conditions for the existence of the soliton solutions can be found by considering the simplest one-soliton solution which is readily found by direct integration.

We shall seek the self-similar solution which depends on a variable $\zeta=\chi-\tilde{v}t$ and which contains a real function $\Phi(\zeta)$ that decreases at infinity. Substitution of a variable

$$\theta(\zeta)=\int_{-\infty}^{\zeta}d\zeta'\,\Phi(\zeta') \quad (15)$$

reduces the first pair of the equations in the system (14) to two identical equations for A and B

$$d^2B/d^2\theta+\Omega^2B=0,\qquad \Omega^{-2}=(\tilde{v}_+-\tilde{v})(\tilde{v}_--\tilde{v}), \quad (16)$$

from which it follows that the nature of the electron amplitudes depend strongly on the sign of Ω^2: when this sign is negative, the amplitudes rise exponentially, but for the positive sign the amplitudes oscillate. The solution for Φ which decreases at infinity can exist only for an oscillatory solution of Eq. (16). It follows that solitons moving at low velocities $|\tilde{v}|<|\tilde{v}_\varkappa|$ can exist only if $\tilde{v}_+\tilde{v}_->0$. We can easily see that the dispersion law $\tilde{\varepsilon}(p_x)$, which can ensure that this condition is satisfied, should have an even number of extrema between the points p_+ and p_- of Eq. (8), i.e., it should be nonconvex. At high soliton velocities there are no limitations on the electron spectrum. In the usual formulation of the problem of nonlinear attenuation of sound,[3,4] postulating that $\Phi=\text{const}$ and $\tilde{v}=0$, and that the dispersion law is quadratic with $\tilde{v}_+\tilde{v}_-<0$, we see that the condition $\Omega^2<0$ for exponential growth of the solutions of Eq. (14) is satisfied, and this gives rise to an acoustic gap in the resonance region.

The solution of Eq. (16) is governed by the initial condition (10) and depends on $\varkappa$. Let us assume that $\delta\tilde{\varepsilon}=0$ and $\varkappa=-1$. In this case the initial conditions $A_-(0)=0$ and $B_-(0)=1$ correspond to a stable homogeneous solution with $\Phi=0$, and they yield an inhomogeneous solution of the type

$$B_-=\cos\Omega\theta,\quad A_-=-i\Omega(\tilde{v}_--\tilde{v})\sin\Omega\theta. \quad (17)$$

Substitution of this solution in the third equation of the system (14) gives

$$(\partial_\zeta\theta)^2=\frac{\tilde{v}_+-\tilde{v}}{\tilde{v}}g^2\sin^2\Omega\theta, \quad (18)$$

the solution of which exists if $\tilde{v}(\tilde{v}_+-\tilde{v})>0$. A comparison of this inequality with $\Omega^2>0$ obtained earlier yields the following system of conditions:

$$\tilde{v}_+,\ \tilde{v}_-<\tilde{v}<0;\qquad 0<\tilde{v}<\tilde{v}_+,\ \tilde{v}_-. \quad (19)$$

It therefore follows that only a "slow" soliton can exist above a stable ground state.

This solution models a general situation in the initial equations in the case of an uninverted electron distribution. An inverted distribution corresponds to the case $\varkappa=+1$, when similar calculations yield a different system of conditions:

$$\tilde{v}>0,\ \tilde{v}_+,\ \tilde{v}_-;\qquad \tilde{v}<0,\ \tilde{v}_+,\ \tilde{v}_-. \tag{20}$$

The above inequalities admit a "fast" soliton, which is compatible with an arbitrary electron dispersion law.

3. ONE-SOLITON SOLUTION OF THE COMPLETE SYSTEM OF EQUATIONS

The existence of a soliton solution of the complete system of equations (10)–(12) presupposes a matching of the electron degrees of freedom such that after the passage of an acoustic pulse all of them recover the initial state; in other words, the soliton shape should correspond to the nonreflection potential of Eq. (11). As in the case of the Schrödinger equation, the nonreflection potential in Eq. (11) exists for all the values of the mismatch $\delta\tilde{\varepsilon}$ and both signs of $\varkappa$; mode locking with different values of $v_\varkappa$ and Λ is impossible. Hence, we obtained an additional restriction on the possibility of appearance of an acoustic soliton: the resonance velocities and the component of the deformation potential responsible for the interaction should be isotropic in respect of the transverse quasimomentum, and Eq. (8) should have just one solution.

Within the limits of these restrictions the system of equations (10)–(12) has a representation which is standard for exactly integrable systems and represents a commutation relationship $[\hat{\mathscr{L}}_\chi, \hat{\mathscr{L}}_t]=0$ for two linear operators of the type[14]

$$\hat{\mathscr{L}}_\chi=-i\partial_\chi+\hat{U}+\lambda\hat{J},\qquad \hat{\mathscr{L}}_t=-i\partial_t+\hat{V}+\lambda\hat{I}. \tag{21}$$

The matrices occurring in these operators act in the space of infinite-dimensional vector functions of the type

$$\Psi^+=(\psi_1^*,\ \langle\psi_2^*|,\ \psi_3^*),$$

where the first and third elements are scalar functions and the second element, separated by the Dirac bracket, represents two consecutive infinite series of elements labeled by the variables ($\varkappa=+1, \delta p_+$) and ($\varkappa=-1, \delta p_-$).[2)] When this notation is adopted, the explicit form of the matrices in Eq. (21) is similar to the matrices in the usual three-wave problem[13]:

$$\hat{U}=\begin{pmatrix} 0, & k_+\langle gA|, & \Phi/(\tilde{v}_+\tilde{v}_-)^{1/2} \\ \alpha\beta k_+|gA^*\rangle, & 0, & \alpha k_-|gB^*\rangle \\ \Phi^*/(\tilde{v}_+\tilde{v}_-)^{1/2}, & \beta k_-\langle gB|, & 0 \end{pmatrix},$$

$$\hat{V}=\begin{pmatrix} 0, & -\tilde{v}_+k_+\langle gA|, & 0 \\ -\alpha\beta\tilde{v}_+k_+|gA^*\rangle, & 0, & -\alpha\tilde{v}_-k_-|gB^*\rangle \\ 0, & -\beta\tilde{v}_-k_-\langle gB|, & 0. \end{pmatrix},$$

$$\hat{J}=\mathrm{diag}(\tilde{v}_-, 0\ldots 0, \tilde{v}_+),\qquad \hat{I}=\tilde{v}_+\tilde{v}_-\,\mathrm{diag}(0, 1\ldots 1, 0),$$

$$k_\pm=|\tilde{v}_\pm(\tilde{v}_+-\tilde{v}_-)|^{-1/2},\ \alpha=\mathrm{sign}(\tilde{v}_- - \tilde{v}_+),\ \beta=\mathrm{sign}\,\tilde{v}_+,$$

$$g^2(\varkappa,\delta p_\varkappa)=\frac{q^2\Lambda^2}{2\rho_0\omega}\int\frac{d\mathbf{p}_\perp}{4\pi^3}n_F(\varepsilon_\perp,\varkappa,\delta p_\varkappa). \tag{22}$$

In the algebraic operations the products of the blocks within the Dirac brackets can be expanded in accordance with the rules

$$\langle\ldots\rangle=\sum_\varkappa\int_{-\infty}^{\infty}d\delta p_\varkappa(\ldots).$$

The system (22) is written down for the case of interest when $\tilde{v}_+\tilde{v}_->0$; in the nonsoliton case $\tilde{v}_+\tilde{v}_-<0$ the representation described by the system (22) is still valid when the signs of the elements of the first columns of the matrices $\hat{U}$ and $\hat{V}$ are reversed.

It is worth noting that the representation given by the system (22) becomes invalid when the velocities are identical: $\tilde{v}_+=\tilde{v}_-$. The existence of this singularity follows also from the expression for the linear attenuation coefficient of Eq. (13) which becomes infinite for identical velocities. Formally, this divergence can be compensated by assuming that the resonant phase volume in g^2 vanishes; in this case after going to the limit the system (10)–(12) becomes equivalent to the equations for the optical SIT. A realistic example of such degeneracy of the generalized three-wave structure of Eqs. (10)–(12) is a semiconductor near its threshold.[11]

The existence of the representation described by Eqs. (21) and (22) allows us, in principle, to obtain all the soliton solutions (10)–(12) by the inverse scattering method. The technique used to obtain these solutions with the help of one of the variants of the "dressing" method,[14,16] is described in the Appendix by considering the examples of breather and two-soliton solutions. The one-soliton solutions can be obtained by a less cumbersome method using the McCall-Hahn factorization ansatz.[7]

Bearing in mind that in the one-soliton solution the quantity Φ is a function of just one variable ζ and assuming the absence of phase modulation, we go over in Eq. (11) to the stationary equation

$$\begin{pmatrix} A \\ B \end{pmatrix}(\chi,t)=\exp\left[it\left(1-\frac{\tilde{v}}{\tilde{v}_\varkappa}\right)\right]\begin{pmatrix} a \\ b \end{pmatrix}(\zeta), \tag{23}$$

$$\begin{pmatrix} i(\tilde{v}_+-\tilde{v})\partial_\zeta+(1-\tilde{v}/\tilde{v}_\varkappa)\delta\tilde{\varepsilon}, & \Phi \\ \Phi, & i(\tilde{v}_--\tilde{v})\partial_\zeta+(1-\tilde{v}/\tilde{v}_\varkappa)\delta\tilde{\varepsilon} \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}=0.$$

In solving the system (23) we follow Ref. 11 and replace the complex two-component vector (a,b) with a real four-vector $\gamma=(\gamma_0, \gamma_x, \gamma_y, \gamma_z)$, which is composed of bilinear combinations of the components of the vector (a,b):

$$\gamma_0={}^1/_2(|a|^2+|b|^2),\qquad \gamma_z={}^1/_2(|a|^2-|b|^2),$$
$$\gamma_x+i\gamma_y=ab^*,\qquad \gamma_x^2+\gamma_y^2+\gamma_z^2-\gamma_0^2=0. \tag{24}$$

In terms of the vector γ the system (23) can be rewritten as follows:

$$\partial_\eta\gamma_x=\delta_\varkappa\gamma_y,\qquad \partial_\eta\gamma_0=-2\Delta\Phi\gamma_y,\qquad \partial_\eta\gamma_z=2w\Phi\gamma_y,$$
$$\partial_\eta\gamma_y=-\delta_\varkappa\gamma_x-2w\Phi\gamma_z-2\Delta\Phi\gamma_0, \tag{25}$$

where the following notation is used for the sake of brevity:

$$\eta=\Omega^2\zeta,\qquad \delta_\varkappa=(\tilde{v}_+-\tilde{v}_-)(1-\tilde{v}/\tilde{v}_\varkappa)\delta\tilde{\varepsilon},$$
$$\Delta={}^1/_2(\tilde{v}_+-\tilde{v}_-),\qquad w={}^1/_2(\tilde{v}_++\tilde{v}_-)-\tilde{v}. \tag{26}$$

The initial conditions of Eq. (10) for the vector γ become

$$\gamma_x=\gamma_y=0,\qquad \gamma_0={}^1/_2,\qquad \gamma_z=\varkappa/2. \tag{27}$$

Following Ref. 7, we represent γ_y in the factorized form

$$\gamma_y = f(\delta_\varkappa)\partial_\eta\Phi; \tag{28}$$

then, Eqs. (25) subject to the initial conditions of Eq. (27) yield expressions for the remaining components of the vector γ:

$$\gamma_x = \delta_\varkappa f(\delta_\varkappa)\Phi, \quad \gamma_z = \varkappa/2 + wf(\delta_\varkappa)\Phi^2,$$
$$\gamma_0 = {}^1/_2 + \Delta f(\delta_\varkappa)\Phi^2. \tag{29}$$

Substitution of the solution described by Eqs. (28) and (29) into the identity (24), which is satisfied by γ_i, and separation of the variables in this identity gives two equations for finding the functions $f(\delta_\varkappa)$ and $\Phi(\eta)$:

$$\delta_\varkappa^2 + \frac{\varkappa}{f}(\tilde{v}_\varkappa - \tilde{v}) = D = \text{const}, \tag{30}$$

$$(\partial_\eta\Phi)^2 + D\Phi^2 + \Omega^{-2}\Phi^4 = 0. \tag{31}$$

Equation (31) has solutions which vanish at infinity only if

$$\Omega^2 > 0, \quad D = -d^2 < 0. \tag{32}$$

Going back to the variable ζ, we obtain

$$\Phi = \frac{1}{\Omega L}\,\mathrm{ch}^{-1}\left(\frac{\zeta}{L}\right), \quad L = \frac{1}{\Omega^2 d}. \tag{33}$$

The dispersion equation for the soliton velocity $\tilde{v}$ is obtained by substituting the solution described by Eqs. (28)–(30), subject to the definition of Eq. (24), into Eq. (12):

$$\tilde{v} = \frac{q^2}{2\rho_0\omega}\int\frac{d\mathbf{p}_\perp}{4\pi^3}\sum_\varkappa \frac{\varkappa}{2} n_F(\varepsilon_\perp + \bar{\varepsilon} + p_\varkappa s)\int\frac{d\delta\bar{\varepsilon}}{|\tilde{v}_\varkappa|}\frac{\tilde{v}_\varkappa - \tilde{v}}{\delta_\varkappa^2 + d^2}. \tag{34}$$

In this expression the Fermi function, which depends smoothly on the mismatch, is taken outside the integral with respect to $\delta\bar{\varepsilon}$ in accordance with the main initial assumption that the resonance is narrow [Eq. (1)]. The contribution to the right-hand side of Eq. (34) originating from γ_x of Eq. (29) then vanishes (in this approximation) because γ_x is odd in of $\delta_\varkappa$,, which justifies the assumption of no phase modulation. Integrating Eq. (34) and bearing in mind Eq. (13) for the linear attenuation coefficient, we obtain

$$\tilde{v} = {}^1/_2\Gamma L\,\mathrm{sign}(\tilde{v}_\varkappa - \tilde{v}). \tag{35}$$

The solution of Eq. (35) exists only if

$$\tilde{v}(\tilde{v}_\varkappa - \tilde{v}) > 0,$$

which together with Eq. (32) reproduces the conditions for the existence of a soliton above the stable ground state of Eq. (19), which is obtained in the three-wave model framework.

We shall now rewrite the final expressions for the soliton characteristics [Eqs. (33) and (35)] by introducing the soliton velocity in the laboratory reference system $v = \tilde{v} + s$ as well as the duration on the lifetime $T = L/v$:

$$\Phi = \Phi_m\,\mathrm{ch}^{-1}\left[\frac{1}{T}\left(t - \frac{x}{v}\right)\right], \quad \Phi_m = \frac{1}{T\Omega v},$$
$$v = \frac{s}{1 - {}^1/_2\Gamma T\,\mathrm{sign}(v_\varkappa - s)}. \tag{36}$$

Figure 1 plots the soliton amplitude and velocity as functions of the lifetime.

4. DISCUSSION OF RESULTS

The acoustic soliton of Eq. (36) resembles a 2π pulse considered in the optical theory of the SIT effect[6]: it has the same spatial configuration and for $v_\varkappa < s$ its velocity depends in the same way on the duration. However, there are also some qualitative differences. There is a maximum lifetime

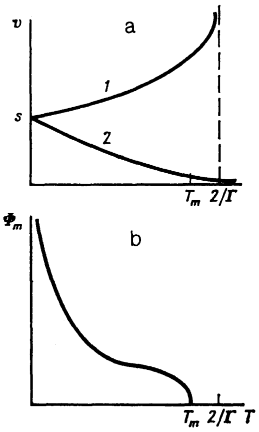

FIG. 1. Dependence of the velocity (a) and amplitude (b) of an acoustic soliton on its duration. Curve 1 in Fig. 1a corresponds to $v\pm > s$ and curve 2 to $v\pm < s$.

$$T_m = (2/\Gamma)\,|1 - s/v_0|,$$

where v_0 is the value of $v_\varkappa$ closest to s, and supersonic velocities are permissible. These features follow directly from the inequalities of Eq. (19) and are due to the electron transport mechanism for the soliton energy: in the time during which the energy of the leading edge of a pulse is transferred to electrons and is returned by electrons to the trailing edge, electrons manage to travel a certain distance. Therefore, in a reference system linked to an acoustic wave a soliton moves in the same direction as the electrons and its velocity cannot exceed the electron velocities. It therefore follows that if a resonant pair of electrons overtakes an acoustic wave, the soliton will move at a supersonic velocity, but if such a pair lags behind an acoustic wave, then the soliton velocity will be subsonic; the limitations on the soliton velocity are communicated through the dependence $v(T)$ of Eq. (36) to the soliton pulse length. Since the characteristic velocities of resonant electrons in a reference system moving with an acoustic wave are of order q/m, the deviation of the soliton velocity from the velocity of sound at frequencies close to the limit of $\omega \sim 10^{11}\,\mathrm{s}^{-1}$ can reach a value on the order of the velocity itself:

$$v - s \sim \omega/ms \sim s.$$

The conditions under which acoustic solitons can exist impose certain restrictions on the material and on the parameters of an acoustic signal. The main requirement on the material, which follows from the preceding sections, is that its dispersion law should have the following properties: the velocities of electrons in a resonant pair should have the same sign in a reference system moving with the sound; all the existing resonance pairs should have identical characteristics (in terms of the transverse quasimomentum, identical and isotropic longitudinal velocities and isotropic component of the deformation potential which is responsible for the interaction).

We shall now consider an electron spectrum which has all these properties. We shall assume that the constant-energy surfaces near the Fermi level are spherical and that a ring-shaped trough with a characteristic transverse size $\sim q$ appears along the meridian of a constant-energy sphere. This trough should be oriented at right-angles to the direction of

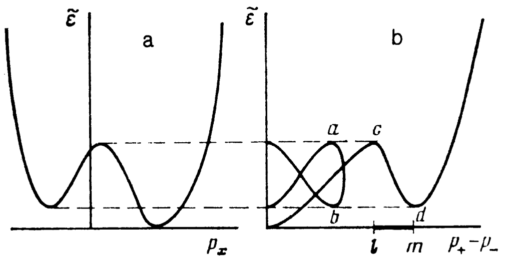

FIG. 2. a) Longitudinal dispersion law of resonant electrons in a reference system linked to an acoustic wave. b) Pair distances along the horizontal between all the points of the graph $\tilde{\varepsilon}(p_x)$; pairs of points with the same sign of the derivative correspond to the sections *ab* and *cd*, and the thick segment on the abscissa identifies the interval of the allowed wave vectors.

propagation of sound. Then, the function $\varepsilon_{\parallel}(p_x)$ in Eq. (5) can be regarded qualitatively as having the configuration of a biquadratic parabola $\alpha p_x^4 - \beta p_x^2$; Fig. 2a shows a plot of the corresponding function $\tilde{\varepsilon}(p_x)$ of Eq. (8) in the case of a filled Fermi surface. Figure 2b gives pair distances along the horizontal between all the points in the plot representing $\tilde{\varepsilon}(p_x)$; this plot makes it possible to determine, for a given value of q, all the resonant pairs by applying Eq. (8). In the sections *ab* and *cd* the resonance velocities have the required identical sign, but the condition of uniqueness corresponds only to the section *cd* characterized by $v_x > 2$. Therefore, if we select the wave vector of sound inside the interval corresponding to the projection of the section cd on the abscissa, we obtain a supersonic soliton. In reality, this situation is closest to a semimetal with a small dumbbell-shaped Fermi surface.

The condition that the pulse length should be short compared with the mean free time of electrons [Eq. (1)] imposes stringent requirements on the purity of a metal or semimetal: even in the case of hypersound with $\omega \sim 10^{10}$–$10^{11}\ \mathrm{s}^{-1}$ the electron collision time should not be less than $\tau \sim 10^{-8}$–10^{-9} s. The amplitude of a pulse injected into a metal sample should be long enough so that a soliton can form: the amplitude should be of the order of $\lambda qu \sim 1/T$; on the other hand, the amplitude should not be so high that the quantum nature of the interaction of sound with electrons is lost: $\lambda qu \ll q^2/m$. The other condition is compatible with Eq. (1) if $q^2/m \gg 1/\tau$.

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APPENDIX

The existence of the commutation representation $[\hat{\mathscr{L}}_\chi, \hat{\mathscr{L}}_t] = 0$ of the investigated nonlinear equation makes it possible, in principle, to solve the Cauchy problem using some modification of the inverse scattering method because of the one-to-one correspondence in the associated problem

$$\hat{\mathscr{L}}_\chi\Psi = -i\partial_\chi\Psi + \hat{U}\Psi + \lambda\hat{J}\Psi = 0,$$
$$\hat{\mathscr{L}}_t\Psi = -i\partial_t\Psi + \hat{V}\Psi + \lambda\hat{I}\Psi = 0 \qquad \text{(A1)}$$

between the dependence of the scattering data on the spectral parameter and the coordinate dependence of the potential matrix.

We limit ourselves to constructing particular solutions of the soliton type using the system (10)–(12) and the variant of the method which makes it possible to construct solutions of this type from the initial data by algebraic procedures.[14,16] The potential matrix $\hat{U}$, corresponding to N soliton solutions of the initial system, gives rise to eigenfunctions in the associated problem and these eigenfunctions as functions of the spectral parameter λ have only pole-type singularities. Bearing this point in mind, we represent the fundamental matrix $\hat{\Psi}$ of the solutions of the system (A1) in the form

$$\hat{\Psi}(\chi, t, \lambda) = \hat{S}(\chi, t, \lambda)\hat{\Psi}_0(\chi, t, \lambda),$$
$$\hat{S}(\chi, t, \lambda) = 1 + \sum_{i=1}^{N}(\lambda-\lambda_i)^{-1}\hat{A}_i(\chi, t), \qquad \text{(A2)}$$

where $\hat{\Psi}_0$ is the fundamental matrix of the solutions of "free" equations in which the matrices $\hat{U}_0$ and $\hat{V}_0$ are obtained by substituting in Eq. (22) the initial values of the functions A, B and Φ in the limit $t = -\infty$ [Eq. (10)]. It should be stressed that these matrices contain nonzero elements.

The equation for $\hat{S}$, which follows from Eqs. (A1) and (A2) in the limit $\lambda \to \infty$, determines the algebraic relationship between $\hat{U}$ and $\hat{A}_i$:

$$\hat{U} = \hat{U}_0 + \left[\hat{J}, \sum_{i=1}^{N}\hat{A}_i\right], \qquad \text{(A3)}$$

and its residues at the points λ_i yield a system of equations for the determination of $\hat{A}_i$:

$$i\partial_\chi\hat{A}_i = [\hat{U}_0, \hat{A}_i] + \lambda_i[\hat{A}_i, \hat{J}] + \left[\hat{J}, \sum_{k=1}^{N}\hat{A}_k\right]\hat{A}_i. \qquad \text{(A4)}$$

It therefore follows that the selection of the pole dependence (A2) of the eigenfunctions (A1) on the spectral parameter determines the coordinate dependence of $\hat{A}_i$ and, on the basis of Eq. (A3), also the $2N$-parameter solution of the initial problem. The existence of one imaginary pole $\hat{S}(\lambda)$ corresponds to the solution (36) given in the text above. We shall now find the explicit solution for the case of two poles.

Using the symmetry of the matrices $\hat{U}$ and $\hat{V}$,

$$\hat{U}^+ = \hat{B}\hat{U}\hat{B}, \quad \hat{V}^+ = \hat{B}\hat{V}\hat{B}, \quad \hat{B} = \mathrm{diag}(1, \alpha\beta\ldots\alpha\beta, 1), \qquad \text{(A5)}$$

we can easily demonstrate that the fundamental matrix of the solutions $\hat{\Psi}$ and, therefore, the matrix $\hat{S}$ have the property of B unitarity:.

$$\hat{S}^{-1}(\lambda) = \hat{B}\hat{S}^+(\lambda^*)\hat{B}. \qquad \text{(A6)}$$

which means that if λ_i is a pole of $\hat{S}(\lambda)$, then λ_i^* is zero. Let us assume that

$$\hat{S}(\lambda_i^*)x_i = 0, \quad i = 1, 2, \qquad \text{(A7)}$$

and that $\hat{A}_i$ is a bivector[3)]:

$$\hat{A}_i = z_i \otimes x_i^+\hat{B}. \qquad \text{(A8)}$$

It then follows from Eqs. (A2), (A7), and (A8) that

$$\hat{A}_i=(C_{11}C_{22}-C_{12}C_{21})^{-1}(C_{ji}x_j\otimes x_i^+\hat{B}-C_{jj}x_i\otimes x_i^+\hat{B}),$$
$$C_{ij}=-(\lambda_i-\lambda_j^*)^{-1}(x_i^+,\ \hat{B}x_j). \tag{A9}$$

A direct check shows that if x_i satisfies the "free" equations

$$\hat{\mathscr{L}}_{0x}(\lambda_i^*)x_i=0,\quad \hat{\mathscr{L}}_{0t}(\lambda_i^*)x_i=0, \tag{A10}$$

then $\hat{A}_i$ of Eq. (A9) satisfies the system (A4) and the matrix $\hat{S}(\lambda_i^*)$ of Eq. (A2) satisfies the condition (A6).

We can find the solutions of the "free" equations (A10) by rewriting the first of them in terms of components and utilizing the explicit form of the functions $\hat{U}$ and $\hat{V}$ of Eq. (22) and the initial conditions of Eq. (10):

$$\begin{aligned}&(-i\partial_\chi-\lambda\tilde{v}_-)\psi_{01}+\langle g_+k_+e^{i\delta p_+(\chi-\tilde{v}_+t)}\delta_{\varkappa,1}|\psi_{02}\rangle=0,\\&-i\partial_\chi|\psi_{02}\rangle+|\alpha\beta g_+k_+e^{-i\delta p_+(\chi-\tilde{v}_+t)}\delta_{\varkappa,1}\rangle\psi_{01}\\&+|\alpha g_-k_-e^{-i\delta p_-(\chi-\tilde{v}_-t)}\delta_{\varkappa,-1}\rangle\psi_{03}=0,\\&(-i\partial_\chi-\lambda\tilde{v}_+)\psi_{03}+\langle\beta g_-k_-e^{i\delta p_-(\chi-\tilde{v}_-t)}\delta_{\varkappa,-1}|\psi_{02}\rangle.\end{aligned} \tag{A11}$$

We can easily see that the system of equations separates into two independent subsystems, of which one couples ψ_{01} to the first half of the block $|\psi_{02}\rangle$ corresponding to $\varkappa=+1$, while the second couples ψ_{03} to the second half of the same block corresponding to $\varkappa=-1$. The dispersion equations for the subsystems are

$$q_\varkappa=\lambda\tilde{v}_{-\varkappa}-\alpha\beta k_\varkappa^2\int d\delta p_\varkappa\frac{g_\varkappa^2}{q_\varkappa-\delta p_\varkappa}. \tag{A12}$$

Similar dispersion relationships are also obtained for the second equation in Eq. (A10). The soliton solutions of the initial equations are obtained if we select the vector x_i as a linear combination of independent solutions of Eq. (A10):

$$x_i=a_i\Psi_{0+}(\lambda_i^*)+b_i\Psi_{0-}(\lambda_i^*),$$
$$\Psi_{0\varkappa}(\lambda)=\exp[iq_\varkappa(\chi-\tilde{v}_\varkappa t)-i\lambda\tilde{v}_+\tilde{v}_-t]\begin{pmatrix}\delta_{\varkappa,1}\\|\varphi_\varkappa\rangle\\\delta_{\varkappa,-1}\end{pmatrix}, \tag{A13}$$
$$|\varphi_\varkappa(\varkappa')\rangle=\alpha\beta^{(1+\varkappa)/2}\left|\frac{k_\varkappa g_\varkappa}{q_\varkappa-\delta p_\varkappa}\exp[-i\delta p_\varkappa(\chi-\tilde{v}_\varkappa t)]\delta_{\varkappa\varkappa'}\right\rangle,$$

where a_i' and b_i' are arbitrary constants.

Substituting Eqs. (A12) and (A13) into Eq. (A9), we find with the aid of Eq. (A3) that the potential Φ representing

$$\Phi=(\tilde{v}_+\tilde{v}_-)^{1/2}(\tilde{v}_--\tilde{v}_+)(\hat{A}_1+\hat{A}_2)_{13}$$

can be described by the following final expressions.

1. In the case of a breather characterized by $\lambda_2=-\lambda_i^*$, $a_1=a_2$, $b_1=b_2$, we have

$$\Phi=4\Phi_1c^{1/2}\frac{\mathrm{ch}\,X_1\cos X_2\cos\varphi-\mathrm{sh}\,X_1\sin X_2\sin\varphi}{\mathrm{ch}\,2X_1+c\cos 2X_2},$$
$$X_1=\frac{1}{T_1}\left(t-\frac{x}{v_1}\right)+X_{01},\quad X_2=\frac{s}{v_1T_2}\left(t-\frac{x}{s}\right)+X_{02}, \tag{A14}$$
$$c=\prod_\varkappa\frac{(v_\varkappa^2+\nu^2(v_{-\varkappa}-v)^2)^{1/2}}{v_\varkappa-s},\quad \nu=\frac{T_2}{T_1}.$$

2. A two-soliton solution characterized by $\lambda_i=-\lambda_i^*$ has the potential

$$\Phi=\frac{c_1(\Phi_1\,\mathrm{ch}^{-1}X_1+e^{i\varphi}\Phi_2\,\mathrm{ch}^{-1}X_2)}{\Phi_1^2+\Phi_2^2-c_2\,\mathrm{th}\,X_1\,\mathrm{th}\,X_2-c_3\,\mathrm{ch}^{-1}X_1\,\mathrm{ch}^{-1}X_2},$$
$$X_i=\frac{1}{T_i}\left(t-\frac{x}{v_i}\right)+X_{0i}, \tag{A15}$$
$$c_1^2=\Phi_1^4+\Phi_2^4-(\mu_+^2+\mu_-^2)\Phi_1^2\Phi_2^2,$$
$$c_2=(\mu_++\mu_-)\Phi_1\Phi_2,\ c_3=2\Phi_1\Phi_2\cos\varphi,$$
$$\mu_\varkappa^2=\frac{(v_\varkappa-v_1)(v_{-\varkappa}-v_2)}{(v_{-\varkappa}-v_1)(v_\varkappa-v_2)}.$$

In Eqs. (A14) and (A15) the quantities X_{0i} and φ are constants, $\Phi_i=\Phi_m(T_i)$, and $v_i=v(T_i)$ is described by the expressions in Eq. (36). We can easily show that Eqs. (A15) and (36) are consistent: when solitons travel to infinity a two-soliton configuration should split into two independent solitons. In fact, in the limit $X_1\to\infty$ or $X_2\to\infty$, (A15) reduces to Eq. (36).

[1)]The difference between this problem and the usual formulation of the three-wave problem, which gives different soliton solutions, is due to the assumption of a nonzero ground state of Eq. (10).
[2)]The idea of this representation was put forward by A. V. Mikhaĭlov.
[3)]Here, the symbol ⊗ denotes a tensor product.

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