Coherent amplification of phase-modulated ultrashort laser pulses

V. D. Taranukhin and S. Yu. Ten

M. V. Lomonosov Moscow State University

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We conduct a theoretical investigation of the amplification of phase-modulated ultrashort laser pulses, making rigorous allowance for the dispersive properties of the amplifying medium. We analyze distortions (during the amplification process) of the amplitude and phase characteristics of pulses modulated by a linear chirp. It is shown that these distortions are qualitatively different for different parts of a pulse. For the trailing part of a pulse, the presence of a chirp leads to a synchronous oscillatory modulation of the intensity and instantaneous frequency of the radiation, which can be used, in particular, to generate picosecond pulse packets with terahertz repetition rates. Upon further amplification, this modulation is smoothed out, and the original pulse shape is reestablished, but not the chirp. Analytic calculations are illustrated by numerically modeling the amplification of picosecond pulses in a high-pressure CO_2 amplifier, taking into account the rotational structure of the CO_2 molecule gain spectrum. The existence of this structure means that in addition to effects associated with an isolated line, phase modulation of the input pulses induces qualitatively new behavior.

Achievements in the production of high-power laser fields have heightened interest in the theory of the amplification of ultrashort coherent pulses, especially those that are linearly chirped[1)] at a high rate (the parameter $k_{ch} = \Delta\omega_t \tau_p \gg 1$, where $\Delta\omega_\parallel$ is the total pulse bandwidth and τ_p is its duration). The amplification of such pulses to a level below the self-focusing and breakdown intensity of an active medium,[2)] and their subsequent compression,[1] enables one to obtain ultrashort pulses and at the same time efficiently tap the stored energy in the amplifier, thus producing ultrahigh-power laser fields. Recent experiments have been the first to implement the amplification of chirped pulses.[2] There is, however, no corresponding theory.

Typical gain behavior for ultrashort pulses is due to coherent effects,[3,4] on the one hand, and to dispersion and the specific structure of the gain band in the active medium,[5] on the other. A number of novel features associated with such structure have been detected during investigations of the amplification of picosecond pulses in high-pressure CO_2 amplifiers[6–8] and in XeCl excimer amplifiers.[9] Phase modulation of the amplified pulses induces additional peculiarities in this process.

The amplification of ultrashort phase-modulated pulses has been investigated previously in either the balance-equations approximation (transverse relaxation time satisfied $T_2 \ll \tau_p$),[10] or in the approximation $T_2 \gg \tau_p$, yielding a self-induced transparency problem.[11] In the first case, apart from neglecting coherent effects, one also allows approximately for the dispersive properties of the medium to lowest order in dispersion theory, as a rule. With these same constraints, one usually also solves for the propagation of phase-modulated pulses in passive media.[12–14]

In self-induced transparency problems, only stationary (soliton) solutions are studied; it has been shown that for phase-modulated pulses, the "area" theorem then fails to hold, and the parameters of a propagating pulse change in a complicated manner.[15] In the limit infinite pulse trains ought to be produced[16,17] but of course in real amplifiers this does not happen.

For phase-modulated pulses with an arbitrary ratio between the times T_2 and τ_p, the problem has only been addressed numerically,[18,19] and even then only to first order in two-level media. Numerical modeling has helped to establish that chirping the input pulses markedly affects their modulation envelope. The lack of an analytic solution, however, has hampered a full interpretation of the various regularities that have been revealed. Furthermore, distortion of the pulse phase modulation during the amplification (attenuation) process has remained almost entirely unexamined.

At the same time, this very question is fundamental to the implementation of a method for amplifying chirped pulses.[2] It is also quite important in the interpretation of a great many experiments (dealing with the amplification of picosecond infrared pulses in CO_2 amplifiers,[7] for example) for which there is a definite lack of consistency between theoretical and experimental results.[7,20]

In the present paper, we develop a theory of the amplification of coherent, phase-modulated pulses with an arbitrary ratio between T_2 and τ_p. We explore both the amplitude and frequency characteristics of the amplified pulses. We have also carried out numerical modeling of amplification of picosecond pulses in high-pressure CO_2 amplifiers, taking both saturation and the actual structure of the CO_2 gain spectrum into account. In addition to effects expected for an isolated gain line, the presence of this structure is responsible for the appearance of qualitatively new behavior. The theory developed for a two-level medium, however, also enables one to interpret this novel behavior.

THE WEAK-GAIN CASE

The equations describing the coherent amplification of ultrashort pulses, taking account of the composite structure of the gain band, are of the form

$$\frac{\partial \varepsilon}{\partial z} = -i\frac{2\pi\omega_0}{c\tilde{n}}\sum_j \mathscr{P}_j - \frac{\alpha_0}{2}\varepsilon, \tag{1}$$

$$\frac{\partial \mathscr{P}_j}{\partial t} + \left[i(\omega_0-\Omega_j)+\frac{1}{T_2}\right]\mathscr{P}_j = \frac{id_j^2}{\hbar}n_j\varepsilon, \tag{2}$$

$$\frac{\partial n_j}{\partial t} = \frac{1}{\hbar}\,\mathrm{Im}\,(\varepsilon\mathscr{P}_j^*) - \frac{n_j-n_{j0}}{T_{1j}}, \tag{3}$$

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where ε and ω_0 are the slowly-varying amplitude and carrier frequency of the radiation. $\mathscr{P}_j$, d_j, n_j, and Ω_j are the polarization, dipole moment, population inversion, and transition rate of an individual component of the gain spectrum, $\tilde{n}$ is the refractive index, with allowance for nonresonant transitions, α_0 is the linear loss coefficient per unit length, T_{1j} and n_{j0} are the longitudinal relaxation time and equilibrium population inversion for a j-transition, z is the length of the gain medium, and t is the time in a comoving coordinate system.

As will become clear shortly, the basic behavior induced by the phase modulation of the amplified pulses is manifest even in the linear gain regime (n_j approximately constant). In this linear approximation, let us first consider a two-level medium. The Riemann method enables us to put the general solution of the system (1), (2) in the form

$$\varepsilon(t,z)=\left\{\varepsilon_0(t)+\int_0^\infty \varepsilon_0\left(t-\frac{T_2y^2}{4A}\right)\times\exp\left[-\frac{y^2}{4A}(1+i\Delta)\right]I_1(y)\,dy\right\}\exp\left(-\frac{\alpha_0 z}{2}\right), \quad (4)$$

where I_1 is modified Bessel function, $\Delta=(\omega_0-\Omega)T_2$, $A=g_0z/2$, g_0 is the gain per unit length for a weak and fairly lengthy signal, and $\varepsilon_0(t)$ is the pulse amplitude at $z=0$.

We begin our investigation of Eq. (4) with the weak-gain case, $A\ll 1$ (or in general, $A\ll T_2/\tau_p$), for which it is possible to derive a number of exact, finite expressions for an input pulse of the special form

$$\varepsilon_0(x)=\begin{cases}\varepsilon_{00}x^{2n+1}\exp[-x+ik_{\rm ph}(x-x_0)^2/2], & x\geqslant 0,\\ 0, & x<0,\end{cases} \quad (5)$$

where $x=t/T_2$, $k_{\rm ph}$ is a coefficient related to the chirp for the input pulse, n is an integer, and x_0 marks the moment in time when the instantaneous frequency satisfies $\omega_{\rm inst}=\omega_0$, for which two pertinent case can be identified: 1) a "symmetric" chirp, where $x_0=2n+1$ corresponds to the middle of the pulse (this case occurs for phase-modulated pulses in Kerr media—in fiber optics, for example[1]); 2) an "asymmetric" chirp, where $x_0=0$ comes at the beginning of the pulse (this type of phase modulation is possible in plasma media[21,22]).

For $A\ll T_2/\tau_p$, we may put $I_1(y)\simeq y/2$ in (4), yielding

$$\varepsilon(x)=\left\{\varepsilon_0(x)+A\int_0^x \varepsilon_0(x-y)\exp[-y(1+i\Delta)]\,dy\right\}\times\exp(-\alpha_0 z/2) \quad (4a)$$

and Eq. (5) then gives

$$\varepsilon=\varepsilon_{00}\exp(-x)\left\{x^{2n+1}\exp[ik_{\rm ph}(x-x_0)^2/2]+A\exp[i(k_{\rm ph}x_0^2/2-x\Delta)]\times\int_0^x y^{2n+1}\exp[i(\Delta-k_{\rm ph}x_0)y+ik_{\rm ph}y^2/2]\,dy\right\}\exp(-\alpha_0 z/2). \quad (6)$$

The integral in (6) can be expressed in terms of elementary functions for $\Delta=k_{\rm ph}x_0$ (and in particular at exact resonance, $\Delta=0$, or for an asymmetric chirp):

$$\varepsilon=\varepsilon_0(x)\left\{1-iA\sum_{m=0}^{n-1}\frac{(2i)^m n!}{(n-m)!\,x^{2m+1}k_{\rm ph}^{m+1}}-\frac{i(2i)^n n!\,A}{x^{2n+1}k_{\rm ph}^{n+1}}\left[1-\exp\left(-\frac{ik_{\rm ph}x^2}{2}\right)\right]\right\}\exp\left(-\frac{\alpha_0 z}{2}\right). \quad (7)$$

In the general case, the expressions for the pulse intensity I and its instantaneous frequency $\omega_{\rm inst}$ are quite unwieldy. In the special case $n=0$, they become (henceforth we assume $k_{\rm ph}\gg 1$)

$$I\approx I_0(x)\,[1+(2A/k_{\rm ph}x)\sin(k_{\rm ph}x^2/2)]\,e^{-\alpha_0 z}, \quad (8)$$

$$\omega_{\rm inst}T_2\approx k_{\rm ph}(x-x_0)-A\sin(k_{\rm ph}x^2/2), \quad (9)$$

These expressions imply that a chirp on the input pulses leads to synchronous oscillatory modulation of their envelope and instantaneous frequency. The depth of modulation increases with the length of the gain medium, and its frequency increases through the end of a pulse. At the same time, toward the end of a pulse, the depth of modulation of the pulse intensity falls off, and is inversely proportional to $k_{\rm ph}$. Typically, the number of subpulses in the resulting pulse train depends on neither the length z of the gain medium nor the duration τ_p of the input pulse, and is uniquely determined solely by the chirp factor $k_{\rm ch}$, which is simply related to $k_{\rm ph}$ by $k_{\rm ch}=k_{\rm ph}(\tau_p/T_2)^2$. With increasing τ_p (i.e., increasing n), the depth of modulation of both the intensity and instantaneous frequency decrease (transition to incoherent gain regime).

Now consider the case $\Delta\neq k_{\rm ph}x_0$. For $k_{\rm ph}\gg 1$, the method of stationary phase[23] is applicable to the evaluation of the integral in Eq. (6). The point at which the phase of the integrand in (6) becomes stationary, $y_0=x_0-\Delta/k_{\rm ph}$ may lie either inside or outside the limits of integration, depending on whether x is less than or greater than $x_0-\Delta/k_{\rm ph}$, and this then means that there will be two different ways of calculating the value of the integral, one toward the beginning and one toward the end of the pulse. In the latter case (which is the same as the leading edge of the pulse for a symmetric chirp and $\Delta=0$), calculating the integral with the stationary phase point outside the limits of integration,[23] we obtain

$$\varepsilon\approx\varepsilon_0(x)\,[1+iA/k_{\rm ph}(y_0-x)]\exp(-\alpha_0 z/2). \quad (10)$$

For the final segment of the pulse (coinciding with the tailing edge of the pulse for a symmetric chirp and $\Delta=0$), the solution takes the form

$$\varepsilon\approx\varepsilon_0(x)\{1+A(2\pi/|k_{\rm ph}|)^{1/2}(y_0/x)^{2n+1}\times\exp[-ik_{\rm ph}(x-y_0)^2/2\pm i\pi/4]\}\exp(-\alpha_0 z/2), \quad (11)$$

where the choice of sign follows the sign of $k_{\rm ph}$.

The point $x_0-\Delta/k_{\rm ph}$ thus divides the pulse in two. At pulse startup ($x<x_0-\Delta/k_{\rm ph}$), even though there may be some distortion of the pulse phase and amplitude, it is not oscillatory. Passing on to $x>x_0-\Delta/k_{\rm ph}$, we find that oscillatory modulation has developed in both the pulse envelope,

$$I\approx I_0(x)\left\{1+2A\left(\frac{2\pi}{|k_{\rm ph}|}\right)^{1/2}\left(\frac{y_0}{x}\right)^{2n+1}\times\cos\left[\frac{k_{\rm ph}(x-y_0)^2}{2}\mp\frac{\pi}{4}\right]\right\}\exp(-\alpha_0 z), \quad (12)$$

and in its instantaneous frequency,

$$\omega_{\rm inst} T_2 \approx k_{\rm ph}(x-x_0) - A(2\pi|k_{\rm ph}|)^{1/2}(x-y_0)(y_0/x)^{2n+1}\cos[k_{\rm ph}(x-y_0)^2/2 \mp \pi/4]. \qquad (13)$$

The basic features of this modulation are the same as for $\Delta = k_{\rm ph} x_0$.

In the weak-gain approximation, we can also analyze gain lines with complex structure. By successive approximations [starting with the field value ε at $z=0$ in (2)], we obtain to first order

$$\varepsilon^{(1)} = \Big[\varepsilon_0(x) + \int_0^\infty \varepsilon_0(x-y)\exp(-y) \times \sum_j A_j \exp(-iy\Delta_j)\,dy\Big]\exp(-\alpha_0 z/2), \qquad (14)$$

where the subscript j identifies quantities corresponding to the jth component of the gain spectrum. Higher-order approximations follow in a straightforward manner. Note that the first approximation, Eq. (14), is equal to (4a) for a two-level medium.

Without going into detail, we point out that a complex gain line with temporal pulse structure gives rise to additional modulation through the factor $\Sigma_j \exp(-iy\Delta_j)$ (quantum beats). For a uniformly spaced gain spectrum, this modulation is regular in nature,[6] and can "interfere" quite effectively with the modulation due to the chirp. The outcome of this interference depends critically on the gain-line structure, so it only makes sense to conduct a detailed investigation of this case for a specific system (see below).

ARBITRARY GAIN

The method of stationary phase also facilitates an analysis of a system with arbitrary gain (4), and with arbitrarily shaped input pulses.

$$\varepsilon_0(x) = a_0(x)\exp[ik_{\rm ph}(x-x_0)^2/2] \qquad (15)$$

(here it is most convenient to assume that the pulse is centered at $x=0$; then $x_0=0$ corresponds to a symmetric chirp, and $x_0 = -\tau_p/2T_2$ to an asymmetric chirp). Note also that the assumption of a linear chirp is not essential to the argument: for $k_{\rm ph} \gg 1$, the method can be used with arbitrary phase modulation.

Substituting (15) into (4), we obtain

$$\varepsilon = \varepsilon_0(x)\Big\{1 + \frac{1}{2}\int_0^\infty \frac{a_0(x-y/4A)}{a_0(x)}\frac{I_1(y^{1/2})}{y^{1/2}}\exp\Big(-\frac{y}{4A}\Big) \times \exp\Big[i\frac{k_{\rm ph}}{4A}\Big(\frac{y^2}{8A} - y\delta\Big)\Big]dy\Big\}\exp\Big(-\frac{\alpha_0 z}{2}\Big), \qquad (16)$$

where $\delta = x - x_0 + \Delta/k_{\rm ph}$. The integral in (16) has one stationary phase point $y_1 = 4A\delta$, which may lie either inside or outside the limit of integration, depending on the sign of δ.

In the initial stage of the pulse ($\delta < 0$), for a fast enough falloff in amplitude $a_0(x)$ as $x \to \infty$, the method of stationary phase applied to (16) yields

$$\varepsilon \approx \varepsilon_0(x)(1 - iA/k_{\rm ph}\delta)\exp(-\alpha_0 z/2), \qquad (17)$$

which is the same as Eq. (10), the latter having been derived with $A \ll T_2/\tau_p$. In (17), however, we may also examine a system with fairly high gain. Deformation of the envelope and instantaneous frequency of the pulse is not oscillatory for either small or large gain values A, and variations in $\omega_{\rm inst}$ are small in either case:

$$\Delta(\omega_{\rm inst} T_2) \approx A/k_{\rm ph}\delta^2 \ll 1 \quad (A \ll 1),$$

$$\Delta(\omega_{\rm inst} T_2) \approx k_{\rm ph}/A \ll 1 \quad (A \gg 1),$$

which suggests that in the linear stage of amplification, the chirp is approximately preserved in this part of the pulse.

In the final part of the pulse ($\delta > 0$), the point y_1 lies within the limits of integration, and the method of stationary phase yields

$$\varepsilon \approx \varepsilon_0(x)\{1 + B\exp[-ik_{\rm ph}\delta^2/2 \pm i\pi/4]\}\exp(-\alpha_0 z/2), \qquad (18)$$

where

$$B = \Big(\frac{2\pi A}{\delta|k_{\rm ph}|}\Big)^{1/2}\frac{a_0(x_0 - \Delta/k_\Phi)}{a_0(x)} I_1(2(A\delta)^{1/2})e^{-\delta},$$

and the sign in the exponent is the same as the sign of $k_{\rm ph}$. Thus, for $A \ll 1$, the pulse intensity and instantaneous frequency are

$$I \approx I_0(x)\{1 + 2B\cos[k_{\rm ph}\delta^2/2 \mp \pi/4]\}\exp(-\alpha_0 z), \qquad (19)$$

$$\omega_{\rm inst} T_2 \approx k_{\rm ph}(x - x_0) - Bk_{\rm ph}\delta\cos[k_{\rm ph}\delta^2/2 \mp \pi/4] \qquad (20)$$

and for $A \gg 1$,

$$I \approx I_0(x)B^2[1 + 2B^{-1}\cos(k_{\rm ph}\delta^2/2 \mp \pi/4)]\exp(-\alpha_0 z), \qquad (21)$$

$$\omega_{\rm inst} T_2 \approx k_{\rm ph}(x - x_0) - k_{\rm ph}\delta[1 - B^{-1}\cos(k_{\rm ph}\delta^2/2 \mp \pi/4)]. \qquad (22)$$

Clearly, in the initial stage of amplification ($A \ll 1$), the final part of the pulse exhibits synchronous oscillatory modulation of both its amplitude and instantaneous frequency. This modulation is of the same nature as in the special cases considered above: the rate of oscillation increases toward the end of the pulse, its relative amplitude is proportional to $g_0 z$ and $|k_{\rm ph}|^{-1/2}$, and the number of subpulses depends on neither the length of the gain medium nor the input pulse duration, but is determined soley by the value of $k_{\rm ch}$.

For long gain paths z (assuming that the system remains in the linear gain regime), the relative amplitude of these oscillations begins to decline rapidly—there is a smoothing of the amplitude modulation of the pulse, and "destruction" of the chirp:

$$\omega_{\rm inst} T_2 \approx -\Delta - k_{\rm ph}B^{-1}\delta\cos(k_{\rm ph}\delta^2/2 \mp \pi/4) \approx -\Delta. \qquad (23)$$

In the linear stage of chirp amplification, then, the input pulse engenders an oscillatory modulation of the intensity and instantaneous frequency. Upon further amplification, this modulation is smoothed out. The original pulse shape is thereby restored, but not the chirp. For compression of amplified pulses to occur,[2] it is therefore necessary that the transition to the nonlinear gain regime (or considerable amplification of the input pulses) occur before the chirp is destroyed.

AMPLIFICATION OF PICOSECOND PULSES IN A HIGH-PRESSURE CO_2 AMPLIFIER ($n_j \neq$ const)

The full system of equations (1)–(3), with both the composite structure of the gain band and saturation taken into account, can only be investigated numerically, and therefore only for specific systems. Here we consider the amplification of picosecond infrared pulses in a TE-mode CO_2 amplifier. The j components of the gain band in (1)–(3) then correspond to a number of vibration-rotation transitions of the CO_2 molecule which exhibit inversion and which are "covered" by the spectrum of the pulse being amplified. In our calculations, we took account of 10–16 such transitions in the vicinity of the $P(20)$ line of the $(00^01)-(10^00)$ band of CO_2.

Numerical modeling was performed for pulses with a duration $\tau_p = 10$–100 psec and various coefficients k_{ch}. We also varied the value of ω_0 over the gain band and the pressure p of the active medium in the amplifier.

As will become apparent, much of the behavior observed in isolated-line gain media is preserved in a composite gain band. Moreover, the theory developed for an isolated line also turns out to be useful in interpreting the qualitatively new phenomena that arise in a real TE-mode CO_2 amplifier.

In Fig. 1 we have charted the evolution of a pulse of duration $\tau_p = 12$ psec, with an asymmetric chirp and a frequency ω_0 that matches $\omega_{P(20)}$, the frequency of the center line of the $P(20)$ gain band of CO_2. It is quite apparent how oscillatory modulation takes shape in the pulse envelope (for k_{ch} under these conditions, the pulse shape remains practically unchanged). It is also clear that this modulation is manifested most strongly toward the end of the pulse (at the beginning of the pulse, both amplitude and instantaneous frequency distortions are small). The reasoning goes as follows. According to (14), we may write for the pulse intensity in the initial stage of amplification

$$I(x) \propto \left\{ |\varepsilon_0(x)|^2 + 2\,\mathrm{Re}\left[\varepsilon_0(x) \sum_j \Delta\varepsilon_j^*\right]\right\}, \qquad (24)$$

$$\Delta\varepsilon_j = A_j \int_0^\infty \varepsilon_0(x-y)\exp[-y(1+i\Delta_j)]\,dy,$$

where $\Delta\varepsilon_j$ is an integral that takes the contribution of the j-transition of the gain band into account. The summation in (24) contains two types of terms:

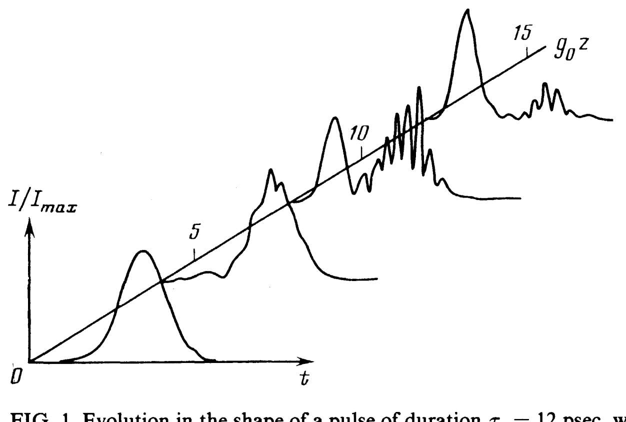

FIG. 1. Evolution in the shape of a pulse of duration $\tau_p = 12$ psec, with peak intensity $I_{in} = 10^4$ W/cm^2, $\omega_0 = \omega_{P(20)}$, and an asymmetric chirp with $k_{ch} \approx 23$, during amplification in a TE-mode CO_2 amplifier ($p = 10$ atm).

$$\sum_j = \sum_j (\omega_{inst} > \Omega_j) + \sum_j (\omega_{inst} < \Omega_j). \qquad (25)$$

When a chirp is present, terms in the first of the sums in (25) lead to oscillatory modulation of the pulse parameters (when $k_{ch} \gg 1$, the stationary phase point y in the corresponding integrals lies between the limits of integration). Terms in the second sum, on the other hand, do not give rise to oscillatory modulation of the pulse parameters. In the latter case, the point y_0 lies outside the limits of integration, and the contribution of an individual term $\Delta\varepsilon_j \sim k_{ch}^{-1}$ is much smaller than in the first sum, where $\Delta\varepsilon_j \sim k_{ch}^{-1/2}$. On the whole, however, the ratio of the contributions from the two sums depend on the number of terms N_1 and N_2 in each ($N_1 + N_2 = N_b$ where N_b is the constant overall number of actual j transitions in the gain band).

At the beginning of a pulse, the contribution from the second (nonoscillatory) sum dominates. Indeed, for the sake of definiteness, let us consider an asymmetric chirp, $k_{ch} > 0$, and a frequency ω_0 equal to the lowest-frequency j transition in the gain spectrum. Then right at the beginning of the pulse, $N_1 = 0$ and $N_2 = N_b$. Near the end of the pulse, N_2 will have decreased, while on the other hand N_1 will have grown and the contribution of the oscillatory part to the pulse intensity will have become the dominant factor. This contribution can be written out approximately as

$$I_{osc} \propto \sum_{n=0}^{N_1} e^{-y_0} \cos\left(\frac{k_{ph}}{2} y_0^2 - \frac{\pi}{4}\right), \qquad (26)$$

where

$$y_0 = (\delta\omega + n\Delta\Omega_0)\,\tau_p^2/T_2 k_{ch},$$

$\delta\omega$ is the offset of the instantaneous frequency $\omega_{inst}(t)$ from the (currently) closest frequency $\Omega_j < \omega_{inst}$, and $\Delta\Omega_0$ is the frequency separation between neighboring j-transitions. It is fairly easy to see that if the parameter $q = \tau_p^2 \Delta\Omega_0 / T_2 k_{ch}$ is small enough, the form taken by the oscillatory modulation of the intensity will be determined by the superposition of a certain number of sinusoids of approximately the same amplitude, but displaced in time with respect to each other (both the period and depth of modulation will then depend critically on the relationship between the parameters k_{ch}, T_2, and τ_p). The example presented in Fig. 1 ($\Delta\Omega_0 \approx 1.8$ cm^{-1}, $T_2 \approx 8$ psec) corresponds to $q \approx 0.3$. Note that for such relatively short pulses, weak discrimination between contributions from various j-transitions in (26) at $p = 10$ atm occurs by the time $k_{ch} \gtrsim 10$.

Now consider the situation when there is good discrimination. For $q \gtrsim 1$, the main contribution to (26) at a given instant in time comes from just a single j-transition. After a time $\Delta t_j = qT_2$, the main contribution will come from the next j-transition, and so on. This occurs when either the pressure in the gain medium is increased ($q \sim p$) or the amplified pulsed is made longer ($q \sim \tau_p^2$). Thus, for pulses with $\tau_p = 30$ psec and $p = 10$ atm, there should be good discrimination even at the rather high value $k_{ch} \lesssim 40$.

Notice that only a few periods of oscillation are excited by an individual j-transition in the time Δt_j. In fact, since these oscillations have a period $\tau_{osc} \approx \tau_p (4\pi/k_{ch})^{1/2}$, we find that even for the minimum possible k_{ch} (as determined

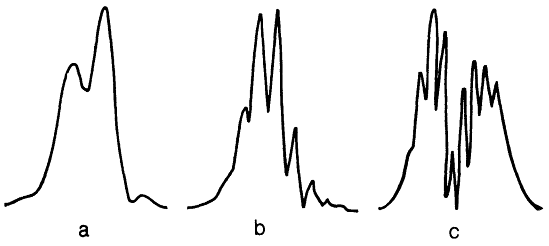

FIG. 2. Pulse shape for a duration $\tau_p = 30$ psec, $I_{in} = 10^4$ W/cm^2, $\omega_0 = \omega_{P(20)}$, and an asymmetric chirp with a) $k_{ch} \approx 10$, b) $k_{ch} \approx 23$, and c) $k_{ch} \approx 31$, for $g_0 z \approx 8$ and $p = 10$ atm (for $k_{ch} \approx 0$, the pulse envelope is an undistorted Gaussian).

by requiring that $\tau_{osc} = \tau_p$) we have $\Delta t_j / \tau_{osc} \approx \tau_p$ (psec)/33. Furthermore, in the case of good discrimination, these oscillations will have almost completely died out by the end of the time interval Δt_j. For $q \gtrsim 1$, therefore, the period of both the amplitude and frequency modulation of the pulse should equal Δt_j. The number of subpulses $N_{AM} = \tau_p / \Delta t_j$ formed as a result of the amplitude modulation will then be

$$N_{AM} = k_{ch} / \Delta\Omega_0 \tau_p. \qquad (27)$$

As in the case of a single gain line, $N_{AM} \sim k_{ch}$ (Fig. 2). In (27), however, there is a substantial dependence on the pulse duration which is absent from the single-line case. This dependence has been confirmed by numerical modeling. For example, for fixed values of $k_{ch} = 20$–40 and pulses with $\tau_p = 60$ psec, the value of N_{AM} is half that for pulses with $\tau_p = 30$ psec.

Note also that just as for a single gain line, there is synchronization of the amplitude and frequency modulation of the pulse (Fig. 3).

In Fig. 4a, we show the evolution of a pulse with good discrimination ($q \approx 1.8$). The interference between pulse modulation due to the chirp and modulation associated with the beating between different j lines of the CO_2 molecule is apparent,[6] with a period of $2\pi/\Delta\Omega_0 \approx 18.5$ psec. A comparison with the case $k_{ch} = 0$ (Fig. 4b) shows that this modulation results in deeper modulation of the pulse envelope, and its developing at a later stage of amplification.

Finally, we note that for sufficiently small $k_{ch} < 4\pi$, the pulse envelope and its instantaneous frequency are hardly distorted by the chirp, and efficient compression of the amplified pulses is therefore feasible. At the same time, obtaining ultrashort pulses with deep amplitude modulation at large k_{ch} may be looked upon as the generation of picosecond pulse trains with a high repetion rate ($\sim 10^{12}$ Hz), a feat with important applications.[18,24] The repetition rate within the pulse train is controlled in the latter case by varying the chirp rate on the input pulses.

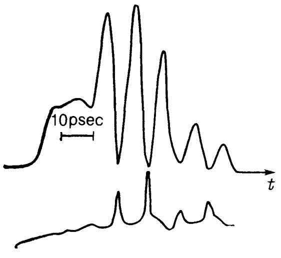

FIG. 3. Time dependence (upper) and instantaneous frequency (lower) for a flat-topped pulse, $\tau_p = 36$ psec, $I_{in} = 10^6$ W/cm^2, $\omega_0 = \omega_{P(20)}$, and a symmetric chirp with $k_{ch} \approx 46$, for $g_0 z \approx 8$ and $p = 10$ atm.

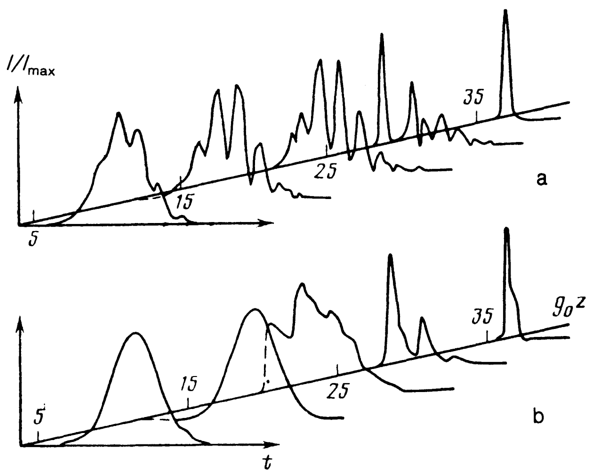

FIG. 4. Evolution of a pulse with $\tau_p = 30$ psec, $I_{in} = 10^4$ W/cm^2, $\omega_0 = \omega_{P(28)}$ and an asymmetric chirp with a) $k_{ch} = 23$ and b) $k_{ch} = 0$ for $p = 10$ atm.

To summarize, we point out that our approach enables one to analyze the amplification of ultrashort phase-modulated pulses in other gain media as well, and, for example, to study the efficiency (and feasibility) of proposed amplification techniques for chirped pulses.[2] This approach can also prove useful in research (and particularly in the interpretation of results) on coherent spectroscopy,[25,26] since from the present point of view, the difference in the linear stage between passive and active media is not a fundamental one. On the other hand, the results that we have obtained invite inquiry into the feasibility of deliberate control of the shape of ultrashort laser pulses by chirping them, so as to obtain, for example, picosecond pulse trains with terahertz repetition rates.

[1)]In the foreign literature and lately in the Soviet literature as well, the term "chirp" has been used to refer to linearly frequency-modulated radiation.

[2)]Self-focusing and breakdown are fundamental limiting factors in the generation of high-power laser radiation.

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