Time dependence of the magnetic moment of high-temperature superconductors

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(Submitted 3 October 1988)
Zh. Eksp. Teor. Fiz. **95**, 1108–1112 (March 1989)

The magnetic moment of high-temperature superconductors decreases logarithmically with time. The temperature dependence of the coefficient of this logarithm has a maximum. Assuming that there exist two types of pinning centers, one can explain this dependence in the framework of the Anderson theory of thermal creep of Abrikosov vortices. The temperature dependence of the critical current is also discussed.

In high-temperature superconductors a slow logarithmic decrease of the trapped magnetic flux with time is observed.[1–5] This decrease follows from Anderson's theory for the thermal creep of Abrikosov vortices. Another model postulates that the boundaries between twins are weak Josephson links.[7] This model is similar to a spin-glass model. In spin glasses one also observes a logarithmic dependence of the magnetic moment on time. Experimental papers usually give the ratio

$$r=-\frac{1}{M}\frac{dM}{d\ln t}, \tag{1}$$

where M is the magnetic moment of the superconductor.

The temperature dependence of this ratio has a characteristic maximum at a temperature of 30–40° in a field of 500 G. In weak fields the position of the maximum shifts toward higher temperatures. In spin glasses such a maximum is not observed. The Anderson theory also yields a monotonic increase of r with temperature. Therefore, in Ref. 3 it is stated that the decrease of r with temperature is definitely not in accord with the picture of creep of Abrikosov vortices.

Below it is shown that a slight generalization of the Anderson model makes it possible to explain the maximum in the temperature dependence of r. The generalization consists in assuming the existence of two types of pinning centers—a large number of weak centers with a comparatively low activation energy, and a small number of strong centers with a high activation energy. At low temperatures the main contribution to the pinning and to the creep is given by the weak centers, and the ratio r increases monotonically with temperature. At high temperatures, during the experiment the lattice near the weak pinning centers has time to reach a thermal equilibrium state, the weak centers are "switched off," and the critical current is determined by the strong centers, at which the creep is significantly weaker.

If a current smaller than the critical current flows through the superconductor, the vortex lattice is in a metastable state. Thermal fluctuations lead to the result that small regions of the lattice execute thermal hopping.

If the current is close to the critical current, the height of the energy barrier is equal to

$$E=u_m(1-J/J_c)^\alpha, \tag{2}$$

where $\alpha > 0$ is a certain index that depends on the distribution of the barriers. The experiments are usually described starting from the phenomenological formula of Anderson, with $\alpha = 1$.

If we assume that the potential energy of each region executing hopping can be described approximately by a single coordinate q, then for a current close to the critical current this potential energy has the form of a cubic parabola:

$$u(q)=\frac{u_m q}{4q_m}\left(3\left(1-\frac{J}{J_c}\right)-\left(\frac{q}{q_m}\right)^2\right), \tag{3}$$

and the barrier height will be determined by formula (2) with $\alpha = 3/2$.

The probability of thermal hopping across this barrier is proportional to $\exp(-E/T)$. As a result of such hopping the current in the sample decreases:

$$dJ/dt=-\gamma e^{-E/T}. \tag{4}$$

Here the coefficient γ depends on the size of the sample. Solving this equation, with logarithmic accuracy we obtain

$$E=T\ln \omega t, \tag{5}$$

where $\omega = \gamma(dE/dJ)/T$. Rough estimates give $\omega \sim 10^5$–10^{10} sec^{-1}.

After a sufficiently strong magnetic field is switched on the critical current flows through the entire sample. This current attenuates with time. From the formulas (3) and (5) we obtain

$$J=J_c\left(1-\left(\frac{T\ln\omega t}{u_m}\right)^{1/\alpha}\right). \tag{6}$$

The magnetic moment of the sample is proportional to the current. Therefore,

$$r=-\frac{1}{M}\frac{dM}{d\ln t}=\frac{1}{\alpha}\left(\frac{T}{u_m}\right)^{1/\alpha}(\ln\omega t)^{1/\alpha-1}. \tag{7}$$

Since the effective depth u_m of the potential wells usually decreases with increase of temperature, formula (7) gives a monotonic increase of the coefficient r with temperature. This statement is valid if all the pinning centers have values of u_m of the same order of magnitude. If there are two types of pinning centers, with high barriers $u_m = u_1$ and low barriers $u_m = u_2$, the dependence (7) is valid only at low temperatures, when $E = T\ln\omega t \ll u_2$, the current is close to the critical current, and formula (7) has the form

$$r=\frac{1}{\alpha}\frac{T^{1/\alpha}}{(\ln\omega t)^{1-1/\alpha}}\sum_i \rho_i u_i^{-1/\alpha}\left\{\sum_i \rho_i\left(1-\left(\frac{T\ln\omega t}{u_i}\right)^{1/\alpha}\right)\right\}^{-1} \tag{8}$$

where ρ_i is the relative contribution of the ith pinning centers to the critical current. At a temperature $T > u_2/\ln\omega t$ the shallow pinning centers are "switched off" and we can

 0038-5646/89/030639-03$04.00

again use formula (7), in which u_1 must be substituted for u_m. If $u_1 \gg u_2$, the ratio r will be smaller at high temperatures than at low temperatures, where formula (8) is applicable. In the model under consideration a sharp jump of r occurs at temperature $T = u_2/\ln\omega t$. In experiment one observes a maximum and a smooth decrease in the region of high temperatures. There are three factors that make it possible to explain the smooth decrease. The first is that the weak centers have a spread of values of u_i, and, therefore, not all of them are switched on at once, but only those with a small value of u_i. The formula (8) can be used at high temperatures as well, but with allowance for only those centers for which $u_i > T\ln\omega t$. Thus, with increase of temperature there is an effective increase of u_m, and r decreases.

The second factor is that the magnetic field in the sample has a spatially nonuniform distribution. If u_m depends on the magnetic field, the switching on of the pinning centers also occurs gradually. The magnetic-field distribution depends on the history—on the order in which the temperature and magnetic field were varied. Therefore, the ratio r can also depend on the history. The third possible cause is that at a current much smaller than the critical current the barrier height is not equal to u_m but tends to infinity. This is due to the fact that for $J \ll J_c$ the system of vortices should undergo strong rearrangement in order to go over to a state with lower energy than the initial state. These states can be separated by a high barrier. For fields close to H_{c1}, when the interaction between vortices can be disregarded, Vinokur and Feĭgel'man postulated that this dependence has the form $E = u_m (J_c/J)^{1/4}$. This assertion is based on the result that was obtained for a dislocation in Ref. 8. Taking formula (5) into account, we obtain

$$J = J_{c1} + J_{c2}(u_m/T \ln \omega t)^4,$$

where J_{c1} is the contribution from the strong centers, and depends weakly on time. At high temperatures the second term is small and the ratio $r \propto T^{-4}$. At the present time there is no quantitative theory of creep, especially for the case of collective pinning, and therefore it is difficult to say which of these causes makes the main contribution to the comparatively smooth decrease of r with increase of temperature. In any case, the maximum of r is reached at a temperature $\sim u_m/\ln\omega t$. It is possible that with increase of the magnetic field the effective u decreases and the position of the maximum shifts toward lower temperatures, in accordance with experiment. The value of the ratio r at the point of the maximum is equal to const$/\ln\omega t$. For the characteristic times of the experiment ($t \sim 1$ min), $\omega t \sim 10^6$–10^{10} and $\ln\omega t \sim 10$–20, so that $r_{\max} \sim 10^{-1}$, in agreement with the experimental value.

The physical cause of the pinning of vortices in oxide superconductors may be the randomly arranged oxygen atoms. An individual atom interacts weakly with the vortex lattice and cannot create a metastable state. However, a large concentration of randomly arranged atoms destroys the long-range order in the vortex lattice.[9] The size of the region of short-range order depends on the pinning force and the elastic moduli of the vortex lattice[9] and is much greater than the distance between pinning centers. The number N of pinning centers in the region with short-range order is large ($N \gg 1$). Such regions are weakly correlated and, in thermal hopping, hop independently. Therefore, the height of the potential barrier for the motion of such a region through randomly arranged pinning centers is proportional to $N^{1/2}$ and can be comparatively large, although the critical current for such collective pinning is usually small.[10,11]

Another cause of pinning could be intersections of boundaries between twins.[12] The critical current in this case should depend on the number and structure of these boundaries. Such pinning centers give rise to plastic deformation of the vortex lattice. For these centers the criterion of Labusch[13] is fulfilled. In this case single-particle pinning arises, when the average pinning force is proportional to the number of centers. These strong pinning centers could be clusters of oxygen atoms in a region of size of order ξ, or other defects induced, e.g., by irradiation of the substance by fast particles.[14] At present it is not clear for which pinning centers the barrier energy is large and for which it is small.

Above, we considered a picture of thermal creep of Abrikosov vortices. It is possible that the boundaries between twins are weak links.[7] The temporal relaxation of the magnetic flux in this case is determined by the motion of Josephson vortices. The qualitative picture in this case is the same as for the motion of Abrikosov vortices. Quantitatively, however, the dynamics of a random Josephson medium has been little studied. The thermodynamic properties of granulated superconductors have been studied in many papers, e.g., Refs. 15–17. The dynamical properties were studied in Ref. 18, in which, however, a model of Josephson junctions with long-range interaction in the region of temperatures close to T_c was considered. Mathematically, the model of granulated superconductors in a magnetic field is similar to the model of a spin glass. However, direct comparison of experiments in superconductors with the experimental results on spin glasses and with certain theoretical papers[19] is not possible, since the magnetic moment and magnetic field in these systems have different physical meanings.

It is known[20,21] that in high-temperature superconductors the critical current found by magnetic measurements decreases very rapidly with increase of temperature. For example, for $T = 45$ K $= T_c/2$, the current decreases by a factor of 10 (Ref. 20). In some experiments[21] an exponential dependence of the trapped magnetic moment on the temperature is observed. As is well known, for $T \ll T_c$ all the superconducting parameters usually approach a constant value. Therefore, the strong temperature dependence of the current in this region is not easy to understand. Anderson[6] showed that creep leads to a decrease of the magnetic moment with temperature. This is connected with the fact that over the time of measurement of the magnetic moment (~ 1 min) the current in the sample decreases because of creep. As estimates show, for ordinary superconductors this decrease is negligibly small[22] and the creep has a very weak effect on the temperature dependence of the current. For high-temperature superconductors we have different values of the parameters and the estimates must be made afresh. From (6) and (7) we have

$$\frac{J - J_c}{J_c} \sim r(T) \ln \omega t.$$

Usually, $\ln\omega t \sim 10$–20. For high-temperature superconductors, $r \approx 0.05$ at $T \sim 10$–20 K. Thus, from a simple approximation it can be seen that at $T\sim 10$–20 K the difference of the measured current from the critical current becomes of the order of the critical current itself. Here and above, by the critical current we mean the maximum current without allowance for fluctuations. This current determines the magnetic moment at the initial time ($\sim 10^{-5}$ sec). It is possible to measure this critical current from the volt-ampere characteristics. Here the current is determined from the appearance of the threshold voltage V_c, which is usually rather large, corresponding to the small time scale $\sim 10^{-5}$ sec. Direct resistive measurements of the current are less sensitive to creep than magnetic measurements. At present we do not know of any resistive measurements of the critical current in single crystals of high-temperature superconductors (the results of measurements on polycrystals are determined by the weak links between the granules). The critical current measured in films (from the volt-ampere characteristic) turns out to be of the same order as that in single crystals, and depends only weakly on the temperature for $T\ll T_c$, in agreement with the assumption of strong creep; however, we do not know of results of magnetic measurements on films. The fact that during the experiment the current decreases and can be found to be much smaller than the critical current does not contradict the fact that the current change seen in experiment is rather small, since the time dependence of the current in creep is logarithmic. Analogous arguments about large creep in high-temperature superconductors were put forward in Ref. 23.

In creep the activation energy

$$E=u_m(T)E_1(J/J_c(T)),$$

and $E(J)$ for $J\to J_c$ is determined by formula (2). However, for small currents this dependence is different. The task of theory is to determine the dependence of the energy on the current for all currents. As discussed above, $E_1(J/J_c)$ can tend to infinity as $J/J_c\to 0$. If we know the dependence of the energy on the current, we obtain from Eq. (5) the temperature dependence and time dependence of the measured current. Thus,

$$r=-T/J\frac{\partial E}{\partial J}=-E(J)/J\frac{\partial E}{\partial J}\ln\omega t. \qquad (9)$$

Under the assumption that at low temperatures ($T\ll T_c$) u_m and J_c do not depend on the temperature, the temperature dependence of the measured current is determined entirely by creep. In this region we can relate the temperature dependence and time dependence of the measured current. Differentiating (5) with respect to the temperature and time, we obtain

$$T\frac{\partial J}{\partial T}=\ln\omega t\frac{\partial J}{\partial \ln t}. \qquad (10)$$

In this formula all derivatives are taken at the same time. Then the temperature dependence of ω in the argument of the logarithm can be neglected because of the large magnitude of the logarithm itself. For the same reason, it is not so important whether the magnetic moment is measured after a minute or after, say, an hour, and $\ln\omega t$ can be regarded as a constant.

The only experimental data known to us in which one sample displays temperature dependence and time dependence of the current[2] agree with formula (10). In order to determine more fully the role of creep in high-temperature superconductors further experiments are needed.

The authors are grateful to M. Feĭgel'man for useful discussions.

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Translated by P. J. Shepherd