Cyclotron absorption and emission in a strongly magnetized plasma

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Cyclotron absorption and emission along an arbitrarily strong magnetic field in a tenuous electron-positron plasma is considered on the basis of an exact quantum-relativistic treatment. The maser effect is rigorously proved to be impossible in this case. Simple expressions are obtained for the absorption coefficient and emissivity of a thermal plasma. It is shown that the nonequidistance of the Landau levels for an anisotropic plasma-particle distribution function may lead to a new type of oscillations in the cyclotron absorption and emission spectra.

1. It is known[1] that two circularly polarized transverse modes can propagate along the magnetic field in a plasma. The dispersion equation for these modes has the form

$$(ck/\Omega)^2=1+4\pi(\chi_{xx}\pm i\chi_{xy}), \tag{1}$$

where Ω is the frequency, $\mathbf{k}$ is the wave vector of the mode, and $\chi_{ik}=\chi_{ik}(\Omega,\mathbf{k})$ is the polarizability tensor of the plasma in a system of coordinates with z-axis along $\mathbf{B}$. In a tenuous ($|\chi_{ik}|\ll 1$) plasma $k\approx\Omega/c$, and one can find from Eq. (1) the absorption coefficients of the modes

$$\mu_{1,2}=2\,\mathrm{Im}\,k_{1,2}=\frac{4\pi\Omega}{c}\,\mathrm{Im}(\chi_{xx}\pm i\chi_{xy}). \tag{2}$$

The quantum relativistic polarizability tensor of a magnetized electron-positron plasma was obtained by Svetozarova and Tsytovich.[2] For longitudinal propagation, the anti-Hermitian parts of the tensor components, averaged over the spin variables, have the form[3]

$$\mathrm{Im}\begin{Bmatrix}\chi_{xx}\\ i\chi_{xy}\end{Bmatrix}=\frac{\pi^2\alpha\lambda_c^3 b}{2\omega^2}\iint_{-\infty}^{+\infty}dp_z\,dp_z'\sum_{n,n'=0}^{\infty}\left(1-\frac{p_zp_z'}{\varepsilon\varepsilon'}\right)\delta(\varepsilon'-\varepsilon-\omega)\times\begin{Bmatrix}(\delta_{n',n+1}+\delta_{n',n-1})[(f_--f_-')\delta(p_z'-p_z-q_z)+(f_+-f_+')\delta(p_z'-p_z+q_z)]\\ (\delta_{n',n+1}-\delta_{n',n-1})[(f_--f_-')\delta(p_z'-p_z-q_z)-(f_+-f_+')\delta(p_z'-p_z+q_z)]\end{Bmatrix} \tag{3}$$

where $\alpha=e^2/\hbar c$, $\lambda_c=\hbar/mc$, $b=\hbar\Omega_B/mc^2=B/4.41\times10^{13}$ G ($\Omega_B=eB/mc$ is the cyclotron frequency), and $\omega=\hbar\Omega/mc^2$ and $\mathbf{q}=\hbar\mathbf{k}/mc$ are the dimensionless frequency and wave vector of the photon, respectively. The longitudinal momenta of the particles in units of mc are p_z and p_z', n and n' are the numbers of the Landau levels,

$$\varepsilon=\varepsilon_n(p_z)=(1+2nb+p_z^2)^{1/2} \tag{4}$$

is the particle energy in units of mc^2, $\varepsilon'=\varepsilon_n'(p_z')$, $f_\mp=f_n^{(\mp)}(p_z)$ and $f_\mp'=f_n'^{(\mp)}(p_z')$ are the electron ($-$) and positron ($+$) distribution functions, normalized to the particle density:

$$\pi b\int_{-\infty}^{+\infty}dp_z\left[f_0^{(\mp)}(p_z)+2\sum_{n=1}^{\infty}f_n^{(\mp)}(p_z)\right]=N_\mp. \tag{5}$$

To calculate Eq. (2) with the help of Eq. (3) we use the conservation laws

$$\varepsilon'=\varepsilon+\omega,\quad p_z'=p_z\pm q_z \tag{6}$$

and the selection rules $n'=n\pm1$. Supposing that $q_z=\omega$, we find that Eqs. (4) and (6) are satisfied only for $n'=n+1$ and determine the energies and longitudinal momenta of the electrons and positrons taking part in the absorption of photons of frequency ω:

$$\begin{Bmatrix}\varepsilon\\ \varepsilon'\end{Bmatrix}=\frac{\omega^2+b^2}{2\omega b}+\begin{Bmatrix}n\\ n'\end{Bmatrix}\omega, \tag{7}$$

$$\begin{Bmatrix}p_z\\ p_z'\end{Bmatrix}=\frac{\omega^2-b^2}{2\omega b}+\begin{Bmatrix}n\\ n'\end{Bmatrix}\omega. \tag{8}$$

Integrating over p_z and p_z' and summing over n', we obtain

$$\mu=\frac{4\pi^3\alpha\lambda_c^2 b}{\omega}\sum_{n=0}^{\infty}(1+2n)(f-f'). \tag{9}$$

Here, we have deliberately dropped the mode indices (1,2) and the ($\mp$) of the distribution functions because the absorption of mode 1 is due exclusively to the electrons and that of mode 2 to the positrons of the plasma (since the directions of the circular polarizations of the modes and of the Larmor rotation of the particles are equal). The distribution functions f and f' in Eq. (9) are determined by the resonance values (8) of the longitudinal momenta, and $n'=n+1$. Using the detailed-balance principle, we find the emissivity

$$j=\frac{1}{2}\alpha mc^2 b\omega^2\sum_{n'=1}^{\infty}(2n'-1)f'. \tag{10}$$

Since in Eq. (8) the longitudinal momentum p_z' differs from p_z by replacement of n by n', it follows that f' depends on $n'=n+1$ in the same way as f depends on n. Thus, in Eq. (9) $f=f_n$, $f'=f_{n+1}$, and all terms connected with stimulated emission are cancelled by some of the term responsible for the absorption:

$$\sum_{n=0}^{\infty}(1+2n)(f_n-f_{n+1})=f_0-f_1+3f_1-3f_2+\ldots=f_0+2\sum_{n=1}^{\infty}f_n. \tag{11}$$

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Hence, the absorption coefficient for longitudinal propagation of photons in a tenuous plasma is given by the formula

$$\mu=\frac{4\pi^3\alpha\lambda_c^2 b}{\omega}\left(f_0+2\sum_{n=1}^{\infty} f_n\right) \tag{12}$$

and *is positive for any distribution function*. This result occurs because the energies of the resonant particles (7) form an equidistant spectrum (Gaponov[4] called attention to this fact for weakly relativistic electrons). Thus, radiation along the magnetic field interacts with particles of the tenuous plasma as with harmonic oscillators which, as is known from Ref. (5), are incapable of maser amplification.

It is easy to obtain from Eqs. (8), (10), and (12) the classical relativistic expression

$$\mu_{cl}=\frac{8\pi^3 e^2}{mc\Omega}\int_0^{\infty} dp_\perp p_\perp f(p_\perp,p_z), \tag{13}$$

$$j_{cl}=\frac{e^2\Omega^2}{2\Omega_B c}\int_0^{\infty} dp_\perp p_\perp^3 f(p_\perp,p_z), \tag{14}$$

where

$$p_z=\frac{\Omega^2-\Omega_B^2}{2\Omega\Omega_B}+\frac{\Omega p_\perp^2}{2\Omega_B}, \tag{15}$$

in which the classical distribution function is normalized by the condition

$$2\pi\int_{-\infty}^{+\infty} dp_z\int_0^{\infty} dp_\perp p_\perp f(p_\perp,p_z)=N. \tag{16}$$

We use these formulas to analyze two important particular cases.

2. *The thermal relativistic distribution of the particles* is given by

$$f_n(p_z)=NA^{-1}\exp(-\varepsilon/t), \tag{17}$$

where

$$A=2\pi b\left[K_1(t^{-1})+2\sum_{n=1}^{\infty}(1+2nb)^{1/2}K_1(t^{-1}(1+2nb)^{1/2})\right] \tag{18}$$

is a normalization constant, $\varepsilon=\varepsilon_n(p_z)$, $t=T/mc^2$ is the dimensionless temperature, and K_1 is a modified Bessel function of the second kind. In this case summation over the Landau levels in Eqs. (12) and (10) leads to the following results:

$$\mu=\frac{4\pi^3\alpha\lambda_c^2 Nb}{A\omega}\exp\left(-\frac{\omega^2+b^2}{2\omega bt}\right)\operatorname{cth}\left(\frac{\omega}{2t}\right), \tag{19}$$

$$j=\frac{mc^2\omega^3}{8\pi^3\lambda_c^2}\left[\exp\left(\frac{\omega}{t}\right)-1\right]^{-1}\mu. \tag{20}$$

Expressions (19) and (20) reduce to the classical ones in two cases: $b\ll 2t(1+2t)$, when the normalization constant (18) becomes classical, $A_{cl}=4\pi tK_2(t^{-1})$, and $\omega\ll t$, when it is possible to neglect quantum effects in the frequency dependences of μ and j. For $\omega\ll t$ quantization of the transverse particle motion decreases μ and j by a factor A/A_{cl} and becomes important when the classical approach undervalues the phase volume occupied by the particles in the quantizing magnetic field. For $\omega\gtrsim t$ both the quantum recoil and the quantization of the transverse motion become important because resonance particles occupy lower Landau levels (even for $A=A_{cl}$, when many levels are populated). These effects increase μ by a factor $\sim\omega/2t$ and decrease j by a factor $\sim(2t^2/\omega^2)\exp(\omega/t)$.

3. *The anisotropic particle distribution* is often[1,6,7] specified as

$$f_n(p_z)=\frac{N\operatorname{th}(b/2t_\perp)}{\pi b(2\pi t_\parallel)^{1/2}}\exp\left(-\frac{nb}{t_\perp}-\frac{p_z^2}{2t_\parallel}\right), \tag{21}$$

where $t_\parallel=T_\parallel/mc^2$ and $t_\perp=T_\perp/mc^2$ which have in the nonrelativistic case ($b,t_\parallel,t_\perp\ll 1$) the sense of dimensionless longitudinal and transverse temperatures.

Summation over the Landau levels in Eqs. (12) and (10) for such a distribution function is not possible analytically, but the expressions for μ and j can be transformed into

$$\mu=\frac{4\pi^{5/2}\alpha\lambda_c^2 N}{(2t_\parallel)^{1/2}\omega}\operatorname{th}\left(\frac{b}{2t_\perp}\right)\left\{\exp\left[-\left(\frac{\omega^2-\omega_0^2}{2\omega\gamma_0}\right)^2\right]+2\sum_{n=1}^{\infty}\exp\left[-\left(\frac{\omega^2-\omega_n^2}{2\omega\gamma_n}\right)^2-\frac{nb}{t_\perp}\right]\right\}, \tag{22}$$

$$j=\frac{\alpha mc^2N\omega^2}{2\pi^{3/2}(2t_\parallel)^{1/2}}\operatorname{th}\left(\frac{b}{2t_\perp}\right)\sum_{n=1}^{\infty}(2n-1)\exp\left[-\left(\frac{\omega^2-\omega_n^2}{2\omega\gamma_n}\right)^2-\frac{nb}{t_\perp}\right], \tag{23}$$

where

$$\omega_n=b(1+2nb)^{-1/2},\quad \gamma_n=b(2t_\parallel)^{1/2}(1+2nb)^{-1}. \tag{24}$$

These equations describe the spectra of cyclotron absorption and emission as superpositions of separate lines connected with the absorption (emission) of photons in transitions of particles with the nth Landau level to the level $n+1$ ($n-1$). The line centers are situated at successively decreasing frequencies ($\omega_{n+1}<\omega_n$ due to the nonequidistance of levels), while the broadening is due to the parallel particle motion. The meaning of the parameters ω_n and γ_n is clarified in Fig. 1, in which the radiative transitions of particles between Landau levels is shown. The frequencies of absorbed and radiated photons in transitions of electrons from the state (n,p_z) are determined by the conservation laws (6) which take the form

$$\omega=\varepsilon_{n+\nu}(p_z+q_z)-\varepsilon_n(p_z),\quad \nu=1,2,\ldots \tag{25}$$

for absorption, and

$$\omega=\varepsilon_n(p_z)-\varepsilon_{n-\nu}(p_z-q_z),\quad \nu=1,2,\ldots,n \tag{26}$$

for emission. For longitudinal propagation in a tenuous plasma ($\nu=1$, $q_z=\omega$), we obtain from Eqs. (25) and (26) the same frequency for absorbed and emitted photons,

$$\omega_a=\omega_e=\omega_n(p_z)=b[\varepsilon_n(p_z)-p_z]^{-1}. \tag{27}$$

The central line frequencies in Eqs. (22) and (23) correspond to the maximally populated states with $p_z=0$, namely, $\omega_n=\omega_n(0)$. Transitions from these states are indi-

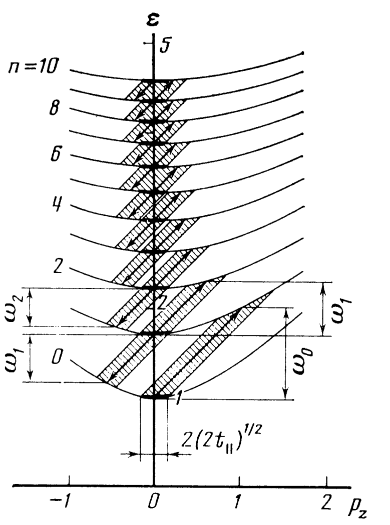

FIG. 1. Transitions between Landau levels, due to emission and absorption of photons along the magnetic field in a tenuous plasma with a strongly anisotropic particle distribution, for typically small parallel momenta ($|p_z| \lesssim (2t_\parallel)^{1/2} \ll b$) and many populated levels ($b \ll t_\perp$). Transitions falling in nonintersecting hatched bands form quantum relativistic oscillations of cyclotron absorption and emission spectra.

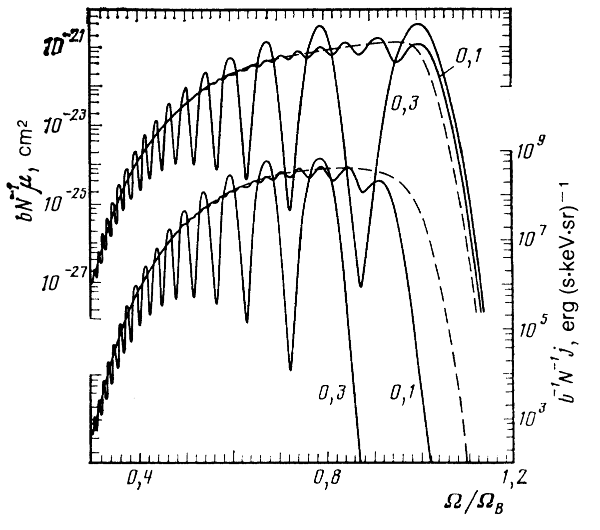

FIG. 2. Spectra of cyclotron absorption and emission along the magnetic field in a plasma with an anisotropic temperature ($t_\perp = 0.3$, $t_\parallel = 5\times 10^{-4}$) for various values of b (the numbers close to the curves). With decreasing b the oscillations smooth out and the spectra approach classical ones (indicated by dashed lines).

cated by arrows. The linewidths, $\gamma_n = [\omega_n((2t_\parallel)^{1/2}) - \omega_n(-(2t_\parallel)^{1/2})]/2$, are determined by the thermal spread of electrons in the nth level $|p_z| \lesssim (2t_\parallel)^{1/2}$. Transitions forming cyclotron absorption and emission lines are shown by the hatched bands. One can see that for sufficiently small $t_\parallel$ these bands for some of the lowest Landau levels do not cross each other, i.e., the corresponding spectral lines do not overlap. A necessary condition for this is $\omega_n - \omega_{n+1} \gtrsim \gamma_n$, which for $n=0$ gives $b \gtrsim 2(2t_\parallel)^{1/2}((1+2t_\parallel)^{1/2} + (2t_\parallel)^{1/2})$. On the other hand, neighboring lines have comparable amplitudes if $b \lesssim t_\perp$. Thus, for lines with

$$2(2t_\parallel)^{1/2}((1+2t_\parallel)^{1/2}+(2t_\parallel)^{1/2}) \ll b \ll t_\perp \tag{28}$$

the cyclotron absorption and emission spectra have pronounced oscillatory form (Fig. 2). The condition for the onset of oscillations (28) requires a strongly anisotropic distribution function such that the transverse degrees of freedom of the particles are excited much more strongly than the longitudinal. Under astrophysical conditions such a situation can arise, for example, in the accretion of plasma onto the magnetic poles of neutron stars.[8]

In the classical limit [for $b \ll \min(t_\parallel{}^{1/2}, t_\parallel, t_\perp)$], the lines closely overlap each other and Eqs. (22) and (23) reduce to the expressions

$$\mu_{cl} = \frac{\pi\Omega_p{}^2\Omega_B}{2c\Omega^2 t_\perp}\exp(y^2-z^2)\,\mathrm{erfc}(y), \tag{29}$$

$$j_{cl} = \frac{\Omega_p{}^2\Omega_B m(2t_\parallel)^{1/2}}{16\pi^{5/2}ct_\perp}\exp(-z^2)\,[1-\pi^{1/2}y\exp(y^2)\,\mathrm{erfc}(y)], \tag{30}$$

where $\Omega_p = (4\pi e^2N/m)^{1/2}$ is the plasma frequency,

$$z = \frac{\Omega^2-\Omega_B{}^2}{2\Omega\Omega_B(2t_\parallel)^{1/2}}, \quad y = z + \frac{\Omega_B}{\Omega}\left(\frac{t_\parallel}{2t_\perp{}^2}\right)^{1/2},$$

$$\mathrm{erfc}(y) = \frac{2}{\pi^{1/2}}\int_y^\infty \exp(-u^2)\,du. \tag{31}$$

The profiles μ_{cl} and j_{cl} are sharply asymmetric in the case of strong anisotropy when $t_\parallel$ and $t_\perp$ satisfy inequality (28). For $\Omega > \Omega_B[1+(2t_\parallel)^{1/2}]$, we have

$$y \gg 1, \quad \mathrm{erfc}(y) \approx \frac{\exp(-y^2)}{\pi^{1/2}y}\left(1-\frac{1}{2y}\right)$$

so that $\mu_{cl}, j_{cl} \propto \exp(-z^2)$, i.e., they are broadened by the linear Doppler effect due to thermal motion of particles along the magnetic field. For $\Omega < \Omega_B[1-(2t_\parallel)^{1/2}]$ we have

$$y \ll -1, \quad \mathrm{erfc}(y) \approx \pi^{1/2},$$

therefore

$$\mu_{cl}, j_{cl} \propto \exp(y^2-z^2) = \exp\left(\frac{\Omega^2-\Omega_B{}^2}{2\Omega^2 t_\perp} + \frac{\Omega_B{}^2 t_\parallel}{2\Omega^2 t_\perp{}^2}\right),$$

i.e., the broadening is determined by the quadratic Doppler effect due primarily to the perpendicular motion of the particles.

In conclusion we note that the effect of quantum relativistic oscillations should also be manifested in the propagation of radiation at an arbitrary angle ϑ to the magnetic field. Assuming $q_z = \omega\cos\vartheta$, we find from Eqs. (25) and (26) that two groups of different characteristic frequencies exist for $\vartheta \neq 0$

$$\omega_{a,e} = \{\mp(\varepsilon - p_z\cos\vartheta) \pm [(\varepsilon - p_z\cos\vartheta)^2 \pm 2\nu b\sin^2\vartheta]^{1/2}\}/\sin^2\vartheta. \tag{32}$$

In the cyclotron emission-spectrum oscillations should appear at frequencies ω_e, and in the absorption spectrum at both frequency groups, ω_a and ω_e. Because the frequencies ω_e correspond to stimulated emission of photons in this case, the spectral dependence of μ should produce on them dips that can have negative values under certain conditions. Thus, for $\vartheta \neq 0$ the maser effect, which is strictly forbidden for $\vartheta = 0$, is possible in a tenuous plasma with an anisotropic distribution function.

The results are directly applicable to the study of processes in the magnetospheres of neutron stars where the magnetic field reaches 10^{11}–10^{13} G. They may also be useful for magneto-optical semiconductors with carriers of small effective mass if their energy dispersion can be modeled by Eq. (4).

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