Self-oscillations at the long-wavelength fundamental absorption edge of a crystal

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It is shown for the first time that self-oscillations can in principle arise at the long-wavelength fundamental absorption edge of a crystal during resonant excitation of high-density excitons. The conditions for the appearance of various temporal structures in a system of coherent excitons and photons are found from the Keldysh equations generalized to the case with a coherent external pump and decays, in the spatially homogeneous case. Both regular and stochastic self-oscillations are possible in the system, depending on the parameters of the equations. A numerical simulation has been carried out. The primary bifurcations in the system have been found.

1. INTRODUCTION

Cooperative nonlinear coherent processes in optical systems have recently attracted much attention. Among these processes are self-induced transparency, optical nutation, photon echos, and optical bistability and multistability.

Risken and Numendal[1] analyzed self-oscillations in lasers on the basis of the Bloch-Maxwell equations describing the interaction of an electromagnetic field with a system of two-level atoms. It was shown that for a certain set of parameters of the system the steady-state continuous states become unstable, with the result that self-oscillations occur in the lasing intensity.

Self-oscillations in the geometry of a ring resonator and a Fabry-Perot resonator filled with two-level absorbers were studied in Refs. 2–4. It was shown that incorporating propagation effects has the consequence that a certain part of the upper branch of optical bistability, with a positive slope, goes unstable, and the steady state of the field converts into pulsating light.

Haken[5] and Oraevskiĭ[6] showed that the system of Bloch-Maxwell equations is homologous to the system of Lorenz equations, which has in addition to ordinary attractors, some singular attractive sets in phase space (strange attractors). The presence of these strange attractors is evidence of a dynamic stochastic nature in dissipative systems. The onset of dynamic chaos in optics has now been the subject of many studies. It was shown in Refs. 7–9 that the amplitude of the field which is transmitted through a ring resonator with a nonlinear medium undergoes a sequence of period-doubling bifurcations which result in the appearance of an optical turbulence. This effect has been observed experimentally.[10] The transition from periodic to chaotic behavior in bistable optical devices (nonlinear Fabry-Perot resonators and ring resonators) was studied in Refs. 11 and 12. Dynamic stochastic behavior in quantum generators was studied in Refs. 13 and 14.

Research on these phenomena in a system of excitons and biexcitons in a condensed medium was begun comparatively recently. Biexciton-exciton conversion and interexciton transitions exhibit many similarities with the model of two-level atoms. There are, on the other hand, some substantial differences. Specifically, a system of excitons and biexcitons differs from a disordered set of atoms or impurity centers in a crystal by virtue of its translational and quantum-statistics properties. It also differs in the method by which the initial state is prepared. We have studied[15–18] optical turbulence accompanying exciton-exciton and exciton-exciton transitions in semiconductors. Since excitons and biexcitons are transient excitations of a crystal, it was shown that the dynamic evolution of the corresponding quantum transitions is described by a generalized system of Lorenz equations in a four-dimensional phase space. The conditions under which that system of equations can be reduced and converted into the ordinary system of Lorenz equations were found.

The time evolution of coherent excitons and photons was studied in Refs. 19–21 for both low and high excitation levels; the pulse length was shorter than the characteristic relaxation times, and the system was a Hamiltonian system. We know that Hamiltonian systems do not have asymptotic stable states or stable limiting cycles.[22,23] Incorporating the scattering of coherent quasiparticles causes the oscillations which arise to decay, and there are no nonzero steady states of excitons or photons.[24]

In this paper we examine a new cooperative optical phenomenon: self-oscillations in the exciton part of the spectrum in which quantum transitions occur not between two levels, e.g., exciton and biexciton levels, but between the ground state of the crystal and one specific exciton level. Using a generalized system of Keldysh equations during the application of a coherent external pump, with decays of quasiparticles, in the spatially homogeneous case, we show that nonlinear periodic and stochastic self-oscillations can arise in a system of coherent excitons and photons. We find the simplest bifurcation properties of the equations and the conditions under which both regular and stochastic self-oscillations arise in the system. We have carried out a numerical simulation. We describe the transition of a system from a steady state of periodic self-oscillations (a Hopf bifurcation). It has been found that the transition of the system to a state of dynamic chaos occurs through period-doubling bifurcations.

The self-oscillations with which we are concerned here are quite different from the free oscillations of coherent excitons and photons which were studied in Refs. 19–21 and 24. Simultaneously incorporating an external pump and decays leads to the appearance of long-lived nonlinear and stochastic self-oscillations and to the appearance of complex attractors in phase space.

Coherent nonlinear effects, including self-oscillations

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in the exciton part of the spectrum, are quite different from the corresponding phenomena in two-level systems. At relatively low exciton densities, at which the excitons may be regarded as bosons, the Hamiltonian of the interaction of the excitons and photons is quadratic, and the amplitude of the electromagnetic field, E, is related linearly to the amplitude of the exciton wave: $E\sim a$. This circumstance is an important distinction between the exciton problem and the model of two-level atoms, in which the Hamiltonian for the interaction of the light with the two-level medium is cubic, and the system has a natural nonlinearity. The change in the field in Maxwell's equations is determined by the density of atoms.

In the case of excitons, the nonlinearity stems from a dynamic and kinematic exciton-exciton interaction. The development of coherent nonlinear phenomena in the exciton part of the spectrum is described by a system of Keldysh equations.[25] These are equations of the Ginzburg-Landau type and describe coherent states of excitons and photons which vary slowly in space and time. The Keldysh equations have served as the starting point for a study of many aspects of the coherent nonlinear propagation of light in dense condensed media in the exciton part of the spectrum. In particular, in Refs. 26–30 we used the Keldysh equations to construct a theory for a self-induced transparency in the exciton part of the spectrum and a theory for optical bistability in the geometry of a Fabry-Perot resonator. This bistability was observed in Ref. 31.

Since the dynamic evolution of a system of coherent excitons and photons is described by equations which are quite different from the Bloch-Maxwell equations, the periodic and stochastic self-oscillations which arise in our problem differ from the corresponding self-oscillations in two-level systems. In particular, the nonlinearity due to the exciton-exciton interaction unavoidably leads to phase modulation and to the result that the system of coherent excitons and photons evolves in a four-dimensional phase space, while the dynamic evolution of two-level systems takes place in a three-dimensional phase space, and the phase modulation is inconsequential.[5,6]

Furthermore, an optical bistability and self-oscillations are possible in the model of two-level atoms under exact-resonance conditions.[5,6] As we will see in the analysis below, these phenomena can arise in the exciton part of the spectrum only if the difference between the frequency of the external electromagnetic field and the frequency of the exciton transitions (the "detuning of the resonance") exceeds the characteristic frequencies of the problem, i.e., only if there is a frequency threshold in addition to the intensity threshold.

Because of all these facts, the periodic and stochastic self-oscillations and the corresponding attractors in phase space which arise in our problem have a more complicated structure than that in the model of two-level atoms.

2. GENERALIZED DYNAMIC EQUATIONS OF COHERENT EXCITONS AND PHOTONS

A monochromatic plane wave

$$E=E_0\exp(ikX-i\omega t) \tag{1}$$

is incident on a resonator, whose mirrors may be the polished faces of the crystal itself. This wave excites a field mode of the resonator, which is in turn coupled with excitons. The interaction of the active medium with the coherent external pump and the heat reservoir, which provides the relaxation processes, will be taken into account phenomenologically at a certain point in our analysis.

A system of equations describing coherent excitons and photons which are slightly nonuniform in space and time, without an external pump and without dissipation effects, was derived by Keldysh.[25] For waves which are propagating along the X axis this system of equations is

$$i\frac{\partial a}{\partial t}=\left[\Omega_\perp-\frac{\hbar}{2m}\frac{\partial^2}{\partial X^2}+\frac{g|a|^2}{\hbar V}\right]a-\frac{d}{\hbar}E^+, \tag{2}$$

$$c^2\frac{\partial^2E^+}{\partial X^2}-\frac{\partial^2E^+}{\partial t^2}=\frac{4\pi d}{v_0}\frac{\partial^2 a}{\partial t^2}, \tag{3}$$

where $a(X,t)$ is the amplitude of the coherent excitons, $E^+(X,t)$ is the positive-frequency part of the oscillatory electromagnetic field, g is the constant of the exciton-exciton interaction, d is the dipole moment of the transition from the ground state of the crystal to the exciton state, m is the translational mass of an exciton, v_0 is the volume of a unit cell, V is the volume of the crystal, and $\Omega_\perp$ is the limiting frequency of a transverse exciton.

We write the macroscopic amplitudes of the excitons and the field as modulated plane waves with a carrier frequency ω and a wave vector $\mathbf{k}$:

$$a(X,t)=V^{1/2}\tilde{A}\exp(ikX-i\omega t), \tag{4}$$

$$E^+(X,t)=\left(\frac{V}{v_0}\right)^{1/2}\tilde{\varepsilon}\exp(ikX-i\omega t), \tag{5}$$

where the slowly varying functions $\tilde{A}$ and $\tilde{\varepsilon}$ are the envelopes of corresponding wave packets.

At this point we adopt the approximation of slowly varying envelopes, which is valid under the conditions

$$\left|\frac{\partial\tilde{A}}{\partial t}\right|\ll\omega|\tilde{A}|,\qquad\left|\frac{\partial\tilde{A}}{\partial X}\right|\ll k|\tilde{A}|, \tag{6}$$

etc. The meaning here is that the envelopes of a wave packet are functions which are fairly smooth in comparison with the rapidly oscillating part. The envelopes change only slightly over a wavelength and over a period of the light incident on the crystal.

Substituting (4), (5) into (2), (3), assuming slowly varying amplitudes, and ignoring spatial-dispertion effects (which are inconsequential in the pertinent part of the spectrum), we find the following simplified equations for coherent excitons and photons with uniform spatial distributions:

$$\frac{d\tilde{A}}{dt}=i\left[\omega-\Omega_\perp-\frac{g}{\hbar}|\tilde{A}|^2\right]\tilde{A}-\gamma_{ex}\tilde{A}+\frac{id}{\hbar v_0^{1/2}}\tilde{\varepsilon}, \tag{7}$$

$$\frac{d\tilde{\varepsilon}}{dt}=i\frac{2\pi d\omega}{v_0^{1/2}}\tilde{A}+i\frac{\omega^2-c^2k^2}{2\omega}\tilde{\varepsilon}-\gamma\tilde{\varepsilon}+\varepsilon_0, \tag{8}$$

where γ_{ex}, γ, and ε_0—the decay constants of the excitons and photons and the amplitude of the coherent external pump—were introduced phenomenologically in Eqs. (7) and (8). These equations give a comprehensive description of the dynamic evolution of coherent excitons and photons which are distributed uniformly in a crystal when an external pump is acting and when there are decays. In the most general case, the amplitudes $\tilde{A}$ and $\tilde{\varepsilon}$ are complex quantities, so system (7), (8) consists of four independent nonlinear ordinary differential equations. These equations are, we might note, the same as the equations for the exciton and photon amplitudes in Ref. 32. The latter equations were derived rigorously on the basis of the quantum theory of fluctuations and decays from the flux part of the corresponding Fokker-Planck equation; the fluctuation terms were ignored.

At this point we switch to dimensionless quantities. We introduce

$$x=\frac{\tilde{A}}{\tilde{A}_0},\quad y=\tilde{\varepsilon}/\tilde{\varepsilon}_0,\quad \delta=(\omega-\Omega_\perp)/\gamma_{ex},$$
$$\Delta=(\omega^2-c^2k^2)/2\omega\gamma_{ex},\quad \tilde{A}_0=[\hbar\gamma_{ex}/|g|]^{1/2},$$
$$\tilde{\varepsilon}_0=(2\pi\hbar\omega)^{1/2}\tilde{A}_0,\quad \sigma=\gamma/\gamma_{ex},\quad \alpha=(\omega\Omega_0/2\gamma_{ex}^2)^{1/2},\qquad (9)$$
$$\Omega_0=4\pi d^2/\hbar v_0,\quad P=\varepsilon_0/\gamma_{ex}\tilde{\varepsilon}_0,\quad \nu=g/|g|=\pm1,\quad T=\gamma_{ex}t,$$

where $\nu = 1$ corresponds to a repulsion between excitons, and $\nu = -1$ to an attraction. Using (9), we find that system (7), (8) takes the form

$$\frac{dx}{dT}=i[\delta-\nu|x|^2]x-x+i\alpha y,\qquad (10)$$

$$\frac{dy}{dT}=i\alpha x+i\Delta y-\sigma y+P.\qquad (11)$$

Equations (10) and (11) fall in the class of nonlinear ordinary differential equations which describe open dynamic systems. For such equations, several steady-state solutions x,y are possible. Not all of the steady states, however, can be stable, depending on the relations among the parameters. Accordingly, an analysis of the solutions of Eqs. (10) and (11) involves resolving the question of the stability of the steady states. The latter are found from the condition $\dot{x}=\dot{y}=0$. In this case we find from (10), (11)

$$x_{st}=-\alpha P\frac{[\sigma(\delta-n)+i(\sigma+\alpha^2)]}{[\sigma^2(\delta-n)^2+(\sigma+\alpha^2)^2]},\qquad (12)$$

$$y_{st}=-\frac{x_{st}}{\alpha}[\delta-n+i]=P\frac{[\sigma(\delta-n)+i(\sigma+\alpha^2)][\delta-n+i]}{[\sigma^2(\delta-n)^2+(\sigma+\alpha^2)^2]},\qquad (13)$$

from which we in turn find

$$I_0=\frac{n}{\alpha^2}[\sigma^2(\delta-n)^2+(\sigma+\alpha^2)^2],\qquad (14)$$

where $I_0=|P|^2$ is the dimensionless intensity of the field which is incident on the crystal, and $n=|x|^2$ is the steady-state density of coherent excitons. Expression (14) relates the exciton density to the external pump and is essentially the equation of the theory of optical bistability in the exciton part of the spectrum and incorporates an exciton-exciton interaction.

Writing the complex amplitudes x and y in the form $x=x_3+ix_4$, $y=x_1+ix_2$, we find the following system of equations from (10), (11):

$$dx_1/dT=-\sigma x_1-\Delta x_2-\alpha x_4+P,\qquad (15)$$

$$dx_2/dT=\Delta x_1-\sigma x_2+\alpha x_3,\qquad (16)$$

$$dx_3/dT=-\alpha x_2-x_3-[\delta-(x_3^2+x_4^2)]x_4,\qquad (17)$$

$$dx_4/dT=\alpha x_1+[\delta-(x_3^2+x_4^2)]x_3-x_4.\qquad (18)$$

The system of nonlinear differential equations (15)–(18) is our basic system of equations for analyzing the possible occurrence of self-oscillations in the exciton part of the spectrum. For simplicity we will be discussing the case in which the frequency ω of the external field is equal to the natural frequency of a field mode of the resonator, $\omega = ck$ $(\Delta = 0)$. In this case the characteristic equation for the steady-states of Eqs. (15)–(18) can be written

$$\lambda^4+a_1\lambda^3+a_2\lambda^2+a_3\lambda+a_4=0,\qquad (19)$$

where

$$a_1=2(\sigma+1),\quad a_2=(\sigma+1)^2+2(\sigma+\alpha^2)+D,$$
$$a_3=2[(\sigma+1)(\sigma+\alpha^2)+\sigma D],\quad a_4=(\sigma+\alpha^2)^2+\sigma^2D,$$
$$D=(\delta-n)(\delta-3n).$$

If the steady-state solutions x_i^{st} are to be stable, all the roots of Eq. (19) must have negative real parts. A necessary and sufficient condition for this negativity is that all of the principal diagonal minors of the Hurwitz matrix be positive. The following inequalities must then hold:

$$D>D_1=-(\sigma+1)^2,\quad D>D_2=-(\alpha^2/\sigma+1)^2.\qquad (20)$$

Treating I_0 as a function of n, we easily find from (14) that under the condition

$$\delta<\delta_2=3^{1/2}(\alpha^2/\sigma+1)$$

this function is a single-valued, monotonically increasing function and has an inflection point $n_p = (2/3)\delta$; we also find

$$I_{0p}=\frac{2\delta^3\sigma^2}{27\alpha^2}\left[1+\frac{9}{\delta^2}\left(\frac{\alpha^2}{\sigma}+1\right)^2\right].$$

If $\delta > \delta_2$, the function $I_0(n)$ has two extrema (at $n = n_3$ and $n = n_4$), given by, respectively,

$$n_{3,4}=\frac{2\delta}{3}\mp\left(\frac{\delta^2}{9}-\frac{1}{3}\left(\frac{\alpha^2}{\sigma}+1\right)^2\right)^{1/2}.\qquad (21)$$

The inverse function $n(I_0)$ is triple-valued at $\delta > \delta_2$; in other words, there are three values of the exciton density in the crystal which correspond to a given value of the amplitude of the external field (Fig. 1). It is not difficult to see that region $3\rightarrow4$ of the functional dependence $n(I_0)$ is unstable for $\delta > \delta_2$, since the second of the inequalities (20) does not hold on this part of the curve.

Analysis shows that the stability of the $n(I_0)$ curve depends strongly on whether the first of inequalities (20) holds

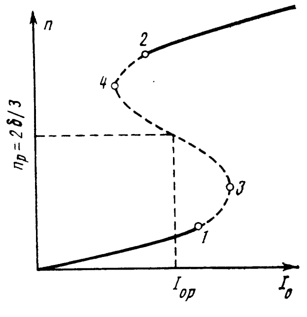

FIG. 1. The exciton density $n=|x|^2$ versus the intensity of the electromagnetic field incident on the crystal, $I_0=|P|^2$.

and on the relation between the frequency of the free linear exciton-photon nutation, α, and the parameter σ. If $\alpha > \sigma$, then $\delta_2 > \delta_1 = 3^{1/2}(\sigma + 1)$. For resonance detunings such that the relation $\delta < \delta_1$ holds, the functional dependence $n(I_0)$ is then single-valued, and a steady state is stable for all values of the external pump. Although there is no bistability in the system in the case $\delta_1 < \delta < \delta_2$, the part of the $n(I_0)$ curve in the interval $I_1 < I_0 < I_2$ goes unstable. Here I_1 and I_2 are given by

$$I_{1,2} = \frac{2\delta^3\sigma^2}{27\alpha^2}\left[1+3v\mp\left(1+\frac{w-3v}{2}\right)(1-w)^{1/2}\right],$$

$$w = 3\left(\frac{\sigma+1}{\delta}\right)^2, \quad v = 3\left(\frac{\sigma+\alpha^2}{\sigma\delta}\right)^2. \qquad (22)$$

In the case $\delta > \delta_2$, as we have already mentioned, an optical bistability arises in the system in the pump interval $I_4 < I_0 < I_3$, where

$$I_{3,4} = \frac{2\delta^3\sigma^2}{27\alpha^2}\left[1+3v\pm(1-v)^{3/2}\right]. \qquad (23)$$

On the lower and upper branches, however, instabilities arise in a certain interval of the pump. The lower branch of the curve is stable for $0 < I_0 < I_1$ and unstable for $I_1 < I_0 < I_3$. The upper branch is stable for $I_0 > I_2$ and unstable for $I_4 < I_0 < I_2$.

If the frequency of exciton-photon conversions satisfies $\alpha < \sigma$, there will again be no bistability in the system at $\delta < \delta_2$, and the entire $n(I_0)$ curve will be stable. At $\delta > \delta_2$, a bistability arises in the system; both the lower and upper branches of the curve are stable.

3. NUMERICAL SIMULATION; CONCLUSION

At present there is no standard algorithm for solving general nonlinear differential equations, and it is difficult to find analytic solutions of Eqs. (15)–(18). We have accordingly carried out a numerical simulation.

The evolution of the solutions of system of differential equations (15)–(18) depends strongly on the evolution of a small region in the phase space of this system. Treating the motion of points in the phase space as the motion of a liquid with a divergence

$$\frac{\partial \dot{x}_1}{\partial x_1}+\frac{\partial \dot{x}_2}{\partial x_2}+\frac{\partial \dot{x}_3}{\partial x_3}+\frac{\partial \dot{x}_4}{\partial x_4} = -(2\sigma+2),$$

we conclude that any small volume of the phase space tends toward zero as $T \to \infty$ at a rate which does not depend on x_i, with a time scale $(2\sigma + 2)^{-1}$. The situation is analogous to the famous Lorenz system,[33] which plays a special role in modern physics primarily in connection with the problem of turbulence. To be fair, we should point out that the "strange" behavior of the solutions of deterministic nonlinear equations has been seen even before the appearance of the work by Lorenz, in particular, in studies by the Soviet physicists Grasyuk and Oraevskiĭ.[6] The tendency of a small volume of interest to go to zero does not mean that it shrinks to a point. All of the orbits contract to a certain subset in phase space with a vanishing phase volume.

If the steady-state solutions are unstable, the attractors in phase space may be one of several things: a limit cycle, a torus, or a strange attractor. These entities correspond to nonlinear periodic, quasiperiodic, and stochastic self-oscillations in the system. A characteristic property of the latter is that the random self-excitations arise in the system not because of the introduction of random forces in the initial conditions or the action of random external forces; instead, their appearance is an internal property of the system and is related to the comlex motion of the orbits in phase space.

Figure 2 shows plots of the exciton density $|x|^2$ and the intensity of the internal electromagnetic field, $|y|^2$, along with projections of the phase orbits onto the (x_1,x_2) and (x_3,x_4) plane for $\sigma = 10$, $\alpha = 22.3$, $\delta = 20$, and $P = 85$ $(\delta_1 = 19, \delta_2 = 98)$. We see that for these parameter values nonlinear periodic self-oscillations arise in the system, and as time passes a phase orbit goes over to a stable limit cycle.

Figure 3 shows the time evolution of the internal electromagnetic field and a projection of the phase orbits onto the (x_3,x_4) plane for the values $\sigma = 10$, $\alpha = 22.9$, $\delta = 46$, and $P = 107$. We see from this figure that bifurcation occurs from one limiting cycle to another, in a process accompanied by a quadrupling of the period.

Figure 4 shows a record of the stochastic self-modulation process and corresponding projections of the phase orbits for $\sigma = 10$, $\alpha = 22.9$, $\delta = 46$, and $P = 135$. We see that random self-oscillations arise in the system. The numerical simulation shows that the onset of dynamic chaos in a system of coherent excitons and photons depends strongly on the level of the external pump in the region of instability of the system. That surface in phase space to which phase orbits contract varies with the level of the external pump. The onset of a chaotic regime of oscillations occurs through a sequence of period doublings. In contrast with Lorenz chaos, in which the stochastic oscillations and the creation of a strange attractor are associated with periodic jumps between corresponding equilibrium states, in our case the stochastic nature is a consequence of the phase modulation and thus of the appearance of a chaotic attractor in the four-dimensional phase space, which becomes filled in a complicated way with nonintersecting phase orbits.

Figure 5 shows the time evolution of $|x|^2$ and $|y|^2$ and corresponding projections of the phase orbits for conditions corresponding to the onset of optical bistability at $\delta > \delta_2 = 98$ and $P = 245$. Oscillations with two characteristic periods arise in the system. The oscillations with the smaller period are associated with the detuning of the resonance between the frequency of the external field and the frequency of the exciton transition. The larger oscillation period is associated with the natural frequency of exciton-photon conversions, modulated by the exciton-exciton interaction in a situation in which an external pump is acting and quasiparticles are decaying.

The numerical simulation shows that when the pump falls in the stability region of the lower branch of the optical-bistability curve the system goes through a few regular damped oscillations and arrives at its steady state; correspondingly, there is a stable focus in the phase space. If the external pump takes on values near the instability region of the lower branch of the optical-bistability curve, a Hopf bifurcation occurs; i.e., there is a transition from a focus to a limiting cycle. The numerical simulation shows that the lower branch of the optical bistability is stable in a very small region near the point at which the instability arises. Small deviations from the steady state in the direction of a stronger

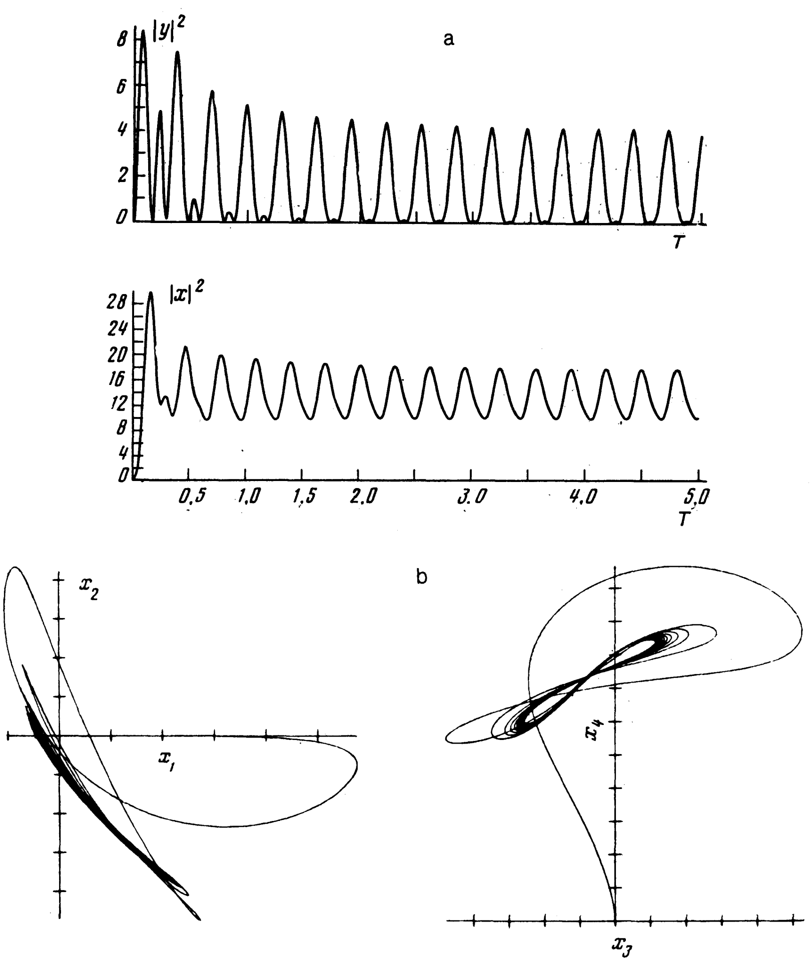

FIG. 2. Time evolution of the exciton density $|x|^2$, and the intensity of the electromagnetic field, $|y|^2$; projections of the phase orbits onto the (x_1,x_2) and (x_3,x_4) planes for $\sigma = 10$, $\alpha = 22.3$, $\delta = 20$, and $P = 85$. A stable limiting cycle appears.

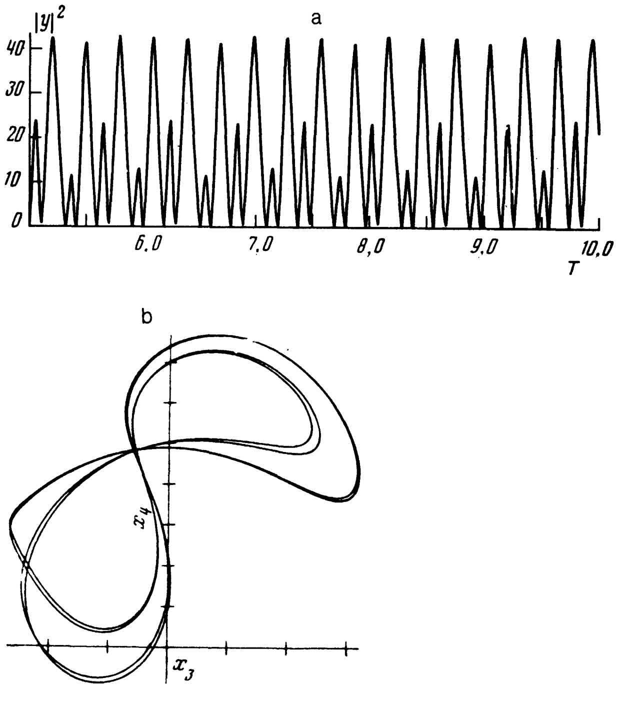

FIG. 3. Time evolution of the internal electromagnetic field; phase protrait of a bifurcation of a limiting cycle after a quadrupling of the period in the (x_3,x_4) plane for $\sigma = 10$, $\alpha = 22.9$, $\delta = 46$, and $P = 107$.

pump result in self-oscillations in the system. In the case in which the upper branch of the optical-bistability curve is stable, while the lower branch is unstable, the system will go through a few regular oscillations and then into a state of stable focus; i.e., an inverse Hopf bifurcation will occur. The region of stability of the upper optical-bistability curve is far larger than the lower region.

We have a few comments regarding the possibility of experimentally observing self-oscillations in the exciton part of the spectrum. As we mentioned above, instabilities arise if the exciton-photon conversion constant α is larger than the decay parameter $\sigma(\alpha > \sigma)$, i.e., if there is a clearly expressed polariton effect in the system. The optical bistability which was first observed among excitons in systems consisting of layers of GaAs and AlGaAs (Refs. 34–37) stems from a change in the refractive index of a crystal with the light intensity. It is caused by bleaching of an exciton resonance due to screening of the Coulomb interaction,[38] not by an exciton-exciton interaction. Optical bistability resulting from the exciton-exciton interaction has been seen experimentally in a GaSe cyrstal.[31] We know that the polariton effect is extremely weak in this crystal, so that observing self-oscillations here is highly unlikely. The most likely place to observe this effect is in the CdS crystal, in which the polariton effect is most pronounced.

We conclude with some numerical estimates for crys-

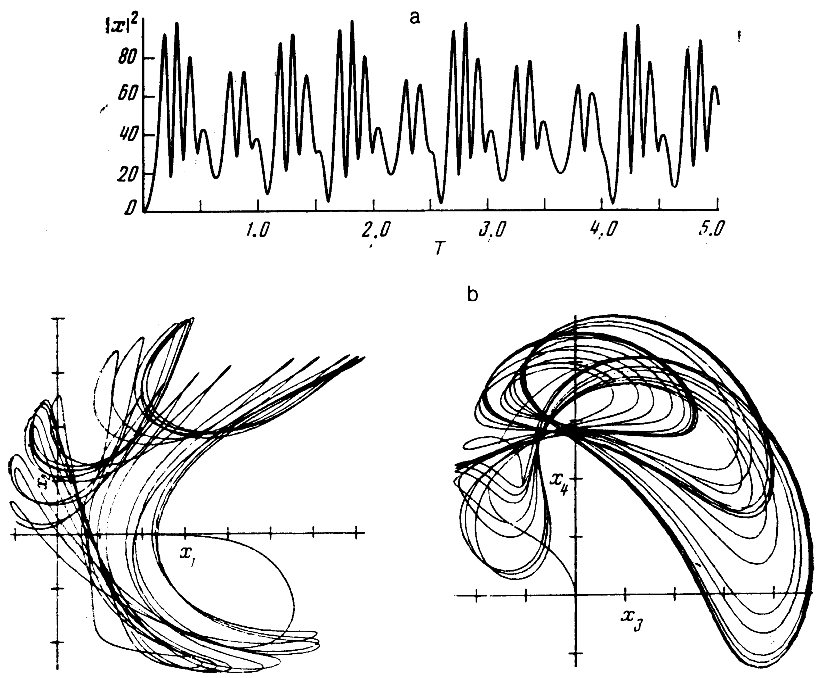

FIG. 4. Time evolution of a stochastic self-modulation process; projections of phase orbits in the (x_1,x_2) and (x_3,x_4) plane for $\sigma = 10$, $\alpha = 22.9$, $\delta = 46$, and $P = 135$.

tals of the CdS type: $g = 2.4 \cdot 10^{-32}$ erg·cm³, $\gamma_{ex} \sim 3 \cdot 10^{11}$ s^{-1}, $\gamma \sim 3 \cdot 10^{12}$ s^{-1} ($\sigma = 10$), and $\Omega_0 = 10^{-4} \cdot \Omega_\perp = 4 \cdot 10^{11}$ s^{-1}. We find that the critical exciton densities and the critical power levels of the light incident on the crystal—at which regular and stochastic self-oscillations can be observed—are $n_{ex1} \sim 10^{17}$ cm^{-3}, $n_{ex2} \sim 8 \cdot 10^{17}$ cm^{-3}, $I_1 \sim 10$ MW/cm² and $I_2 \sim 100$ MW/cm², respectively. The density of excitons at which a Mott transition occurs in the CdS

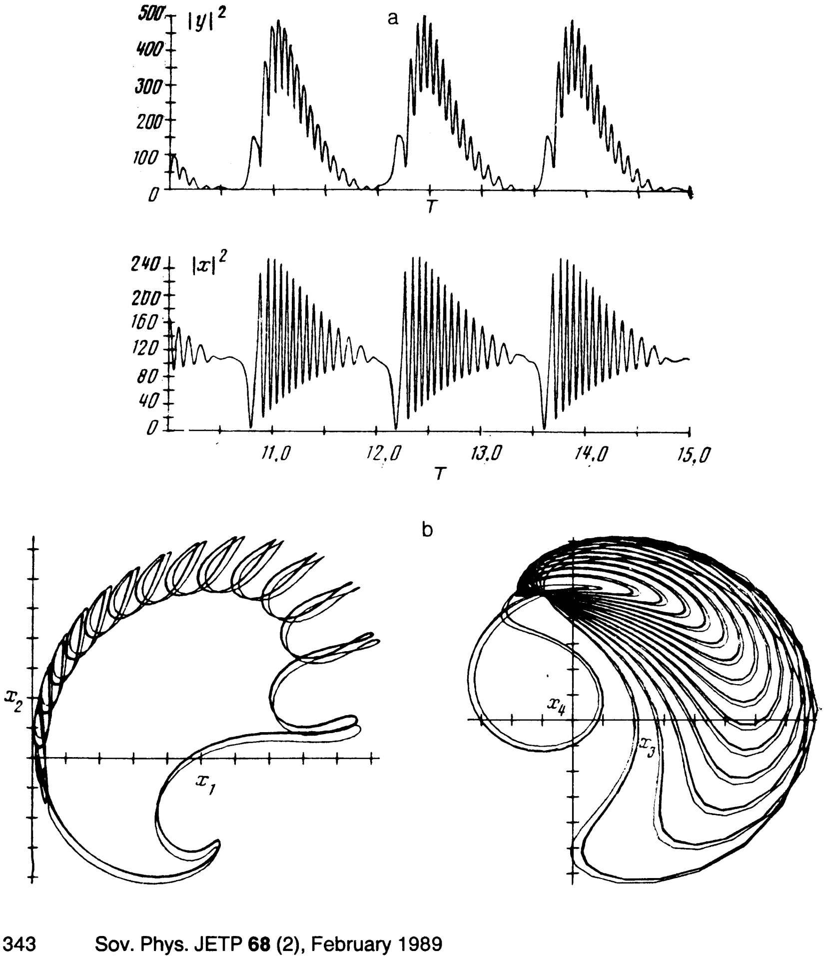

FIG. 5. Time evolution of $|x|^2$ and $|y|^2$; projections of phase velocities onto a plane in a regime in which bistability appears, with $\sigma = 10$, $\alpha = 23.6$, $\delta = 110$ ($\delta > \delta_2 = 98$), and $P = 245$.

crystal is $n_m \sim 10^{19}$ cm^{-3}. Our numerical estimates thus lead to the conclusion that there is a real possibility of observing self-oscillations in the exciton part of the spectrum during intense excitation of a crystal.

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