Interaction potential for two filaments and for an atom interacting with a filament

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A description is given of the van der Waals interaction between two thin filaments of condensed matter and of the interaction between an atom and a thin filament. A considerable difference between the interactions exhibited by insulator and metal filaments is due to a specific dispersion of one-dimensional electromagnetic excitations in the case of metal filaments. It is shown that in the case of strongly polar insulator filaments the nonadditive effects and the thermal contribution to the interaction are unusually large. In contrast to insulator filaments, the retardation and Casimir effects are negligible (even at low temperatures) for interacting metal filaments separated by a sufficiently large distance. The approximation of perfectly conducting filaments is invalid in this case, but the actual static conductivities are important. The results obtained by Nabutovskiĭ, Belosludov, and Korotkikh [Sov. Phys. JETP **50**, 352 (1979)] for the interaction of an atom with a thin filament apply only to an insulator. An analysis is made of the interaction of an atom with a metal filament. An earlier result of Zel'dovich [Zh. Eksp. Teor. Fiz. **5**, 22 (1935)] is found to be valid (with logarithmic precision) in a wide range of distances.

1. INTRODUCTION

Much work has been done on the van der Waals interaction between two thin parallel long cylindrical filaments and between an atom and a filament (see for example, Refs. 1–13). The reason why these problems are of interest can also be found in the cited papers. For example, the solution of these problems can be important in discussing some properties of quasi-one-dimensional systems[14] as well as in applications to biophysics[3,4,6,7] and to capillary phenomena.[13]

The published investigations can be divided in a natural manner into two groups. The results of one group are valid irrespective of the actual model microscopic description of the properties of the interacting bodies. In particular, the permittivities are not specified as a function of the frequency. However, this approach has been used only to deal with the interaction between thin insulator filaments.[3,4,6,7,13] When such results are applied to thin metal filaments, divergences are observed at low frequencies.[3,4,6,7,13]

In the other group of investigations the treatment is based right from the beginning on a relatively simple model describing the behavior of filaments in an electromagnetic field. This is the approach adopted in all (known to us) treatments of the interaction between thin metal filaments.[2,5,8–12] Several important differences have been found between the behavior of the interaction potentials of two thin insulator filaments and two thin metal filaments considered as a function of the distance R between them. For example, if we ignore the retardation effects and the influence of temperature we find that the energy (per unit length) of the interaction between insulator filaments $E_L(R)$ is proportional to R^{-5}. The same behavior is predicted also by an additive approximation and the collective effects influence only the coefficient of proportionality. On the other hand, a model description developed in Refs. 2, 8, 9, and 11 shows that under similar conditions the energy of the interaction between two thin metal filaments decreases much more slowly with the distance, $E_L(R) \propto R^{-2}$, which is accurate to within a logarithmic multiplier. This is due to the specific nature of an R-dependent dispersion of one-dimensional plasmons in interacting metal filaments.[2] Since interaction between metal filaments is governed primarily by the lf collective effect, the additive approximation cannot be used even in rough qualitative estimates.

A self-consistent description of the van der Waals interaction between two thin filaments and between an atom and a filament is developed below using a general theory of the van der Waals forces. The general results obtained apply to arbitrary thin filaments of condensed matter, which may be made of an insulator or a metal. This not only allows us to determine the ranges of validity of the expressions used earlier, but also to obtain new results. In Sec. 2 we shall give the initial general relationships and discuss the interaction between insulator filaments. We shall show that in the case of filaments formed from a strongly polar insulator the nonadditive effects and the thermal contribution to the interaction play an unusually important role at all distances. Corrections will be given to the expression for the interaction between thin insulator filaments as a result of the retardation effect. A description of the interaction between thin metal filaments is provided in Sec. 3. It is demonstrated there that the model treatment adopted in Refs. 2, 5, and 8–11 is valid only in a limited range of not-too-large distances. Expressions are obtained for the interaction of thin metal filaments separated by fairly large distances. It is interesting to note that in the case of this long-range interaction the influence of the retardation effect is always negligible. If metal filaments are sufficiently thin, the approximation of perfect conduction is invalid for any distance R. Expressions describing the interaction between insulator and metal filaments obtained in Sec. 4 are also new. This section is concerned also with the interaction between an atom and a thin filament.

The van der Waals interaction of bodies with thin condensed filaments is a rare example of a situation when not only the coefficients of proportionality but also exponents in the power laws describing the interaction may depend strongly on the actual properties of condensed bodies (even if we ignore the spatial dispersion of the permittivities of these bodies). We know of just one similar example and this is the interaction between bodies and thin condensed

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films.[11,15] In all other known cases the power exponents are independent of the actual properties of bodies. This is regarded as normal, so that in some cases the expressions representing in fact only the interaction of bodies with thin insulator filaments are presented by their authors without any qualifications as general expressions equally valid for any thin condensed filaments. This is often done even without mentioning that such expressions diverge in the lf range when applied to metal filaments. For example, this is true of Eqs. (23) and (24) in Ref. 13, which describe the interaction of an atom with a thin filament. Our treatment is in fact the first self-consistent analysis of the van der Waals interaction of an atom with a thin metal filament. In the simplest case the potential of the interaction of an atom with a thin metal filament is proportional (apart from a logarithmic factor R^{-4}) and this is valid in a fairly wide range of distances. It is remarkable that precisely this behavior of the interaction of an atom with a thin metal filament was established over 50 years ago by Zel'dovich [see Eq. (8) in Ref. 12; determination of the coefficient of proportionality was not part of the task undertaken in Ref. 12]. In comments on Zel'dovich's paper,[12] given in his collected works, it is pointed out that the results derived in Ref. 12 are valid only in the range of distances in which the spatial dispersion of the permittivity of a metal is important. However, it must be stressed that this applies only to a bulk (three-dimensional) metal. The behavior of the van der Waals interaction is affected directly not so much by the spatial dispersion of the permittivities of condensed bodies as by the dispersion of the eigenfrequencies of an electromagnetic field in the system, which depend on the distance between the bodies. In the case of thin metal filaments the specific dispersion of one-dimensional plasmons is responsible for the approximate validity of the Zel'dovich's results also in the range of distances where the spatial dispersion of the permittivities can be ignored.

2. GENERAL RELATIONSHIPS. INTERACTION BETWEEN TWO INSULATOR FILAMENTS

We shall consider two long parallel circular cylinders which are in a condensed medium. For simplicity, we shall assume that all the media are isotropic. We shall use $\varepsilon_{1,2}(\omega)$ and $a_{1,2}$ to denote the permittivities and radii of the first and second cylinders, respectively, and $\varepsilon_3(\omega)$ for the permittivity of the medium surrounding the cylinders.

The free energy of the van der Waals interaction between cylinders (per unit length in contact) can in this case be represented by (see, for example, Ref. 16)

$$F_L(R)=\frac{T}{\pi}\sum_{n=0}^{+\infty}{}'\int_0^{+\infty} dk\ln D(i\omega_n,k;R). \quad (1)$$

Here, $\omega_n=2\pi nT/\hbar$ and the prime of the summation symbol means that the term with $n=0$ is taken with half-weight. Integration in Eq. (1) is carried out with respect to a one-dimensional wave vector k, which is parallel to the cylinder axes. Zeros of the function $D(\omega,k;R)$ are all the eigenfrequencies (generally complex) of an electromagnetic field in the system, dependent on the distance R between the cylinder axes. Moreover, the function $D(\omega,k;R)$ is assumed to be normalized so that in the limit $R\to+\infty$ we have $D=1$.

In general, the function $D(i\omega_n,k;R)$ is very cumbersome. We shall consider only thin cylinders (filaments) with radii much less than the distances between them:

$$a_{1,2}\ll R. \quad (2)$$

The filaments are then regarded as essentially one-dimensional. In fact, in the case of the van der Waals interaction it is important to consider the range $k\lesssim R^{-1}$ and Eq. (2) shows that $ka_{1,2}\ll 1$. In this limiting case both plasma and other electromagnetic excitations in the filaments are of special one-dimensional nature. Consequently, in the case defined by Eq. (2) the collective effects have the greatest influence on the dependence of the van der Waals interaction on the distance. In particular, the additive approximation is altogether invalid in the case of metal filaments. It should be stressed that the condition of Eq. (2), which allows us to speak of thin (one-dimensional) filaments, imposes limitations only on the ratio $a_{1,2}/R$ and not on the thickness of the filaments as such. The thickness of the filaments may range from $a\gtrsim 5$ Å to fully macroscopic values such as $a\sim 10^3$ Å. The upper limit on $a_{1,2}$ is determined if we allow for Eq. (2) and the experimental facilities.

A standard electromagnetic calculation shows that if the condition (2) is obeyed, the function $D(i\omega_n,k;R)$ is of the form $D=D_1D_2$, where

$$D_1=1-AK_0^2(qR)-BK_1^2(qR)-CK_2^2(qR),$$
$$D_2=1-\tilde{A}K_0^2(qR)-\tilde{C}K_2^2(qR). \quad (3)$$

Here, $K_0(x)$, $K_1(x)$, and $K_2(x)$ are modified Bessel functions and $q^2(i\omega_n,k)=k^2+\omega_n^2\varepsilon_3(i\omega_n)/c^2$. The following notation is used in the system (3):

$$A=\frac{q^4a_1^2a_2^2(\varepsilon_1-\varepsilon_3)(\varepsilon_2-\varepsilon_3)}{4[\varepsilon_3+{}^1/_2\varepsilon_1(a_1q)^2K_0(a_1q)][\varepsilon_3+{}^1/_2\varepsilon_2(a_2q)^2K_0(a_2q)]}+\frac{1}{2}k^2\left(k^2-\frac{\omega_n^2}{c^2}\varepsilon_3\right)a_1^2a_2^2\frac{(\varepsilon_1-\varepsilon_3)(\varepsilon_2-\varepsilon_3)}{(\varepsilon_1+\varepsilon_3)(\varepsilon_2+\varepsilon_3)}, \quad (4)$$

$$B=\frac{1}{2}q^2k^2a_1^2a_2^2\left[\frac{\varepsilon_1-\varepsilon_3}{\varepsilon_1+\varepsilon_3}\,\frac{\varepsilon_2-\varepsilon_3}{\varepsilon_3+{}^1/_2\varepsilon_2(a_2q)^2K_0(a_2q)}+\frac{\varepsilon_1-\varepsilon_3}{\varepsilon_3+{}^1/_2\varepsilon_1(a_1q)^2K_0(a_1q)}\,\frac{\varepsilon_2-\varepsilon_3}{\varepsilon_2+\varepsilon_3}\right], \quad (5)$$

$$C=\frac{1}{2}q^2k^2a_1^2a_2^2\frac{(\varepsilon_1-\varepsilon_3)(\varepsilon_2-\varepsilon_3)}{(\varepsilon_1+\varepsilon_3)(\varepsilon_2+\varepsilon_3)},$$
$$\tilde{C}=\frac{1}{2}q^2\frac{\omega_n^2}{c^2}\varepsilon_3a_1^2a_2^2\frac{(\varepsilon_1-\varepsilon_3)(\varepsilon_2-\varepsilon_3)}{(\varepsilon_1+\varepsilon_3)(\varepsilon_2+\varepsilon_3)}, \quad (6)$$

$$\tilde{A}=\frac{1}{2}\frac{\omega_n^2}{c^2}\varepsilon_3\left(\frac{\omega_n^2}{c^2}\varepsilon_3-k^2\right)a_1^2a_2^2\frac{(\varepsilon_1-\varepsilon_3)(\varepsilon_2-\varepsilon_3)}{(\varepsilon_1+\varepsilon_3)(\varepsilon_2+\varepsilon_3)}. \quad (7)$$

All the permittivities in Eqs. (4)–(7) depend on the argument $i\omega_n$.

After substitution of the system (3) into Eq. (1) we find that an analysis of Eq. (1) shows that the behavior of the quantity $F_L(R)$ as a function of the distance depends strongly on the values of two dimensionless parameters:

$$\varkappa_{1,2}=\frac{\varepsilon_{1,2}(i\omega_c)}{2\varepsilon_3(i\omega_c)}\left(\frac{a_{1,2}}{R}\right)^2\ln\left(\frac{R}{a_{1,2}}\right), \quad (8)$$

where ω_c is the value of the frequency typical of the van der Waals interaction between filaments.

If $\varepsilon_{1,2}(i\omega_c)\lesssim\varepsilon_3(i\omega_c)$, we find that Eqs. (2) and (8) yield

$$\varkappa_{1,2} \ll 1. \tag{9}$$

In the case of insulator filaments the inequality of Eq. (9) is valid throughout the investigated range of distances defined by Eq. (2), with one exception which will be discussed at the end of the present section. The condition (9) allows us to neglect those terms in the denominators of Eqs. (4) and (5) which contain the function $K_0(a_{1,2}q)$. Consequently, it follows from Eqs. (1)–(9) that the following description of an interaction between two insulator filaments is possible in the simplest limiting cases.

The role of the characteristic frequency ω_c may be played by three dimensional parameters: ω_0, $c/\varepsilon_{30}^{1/2}R$, and $2\pi T/\hbar$. The frequency ω_0 is related, as usual, only to the absorption spectra of condensed media (compare with Refs. 17 and 18). When the condition $2\pi T/\hbar \ll \min(\omega_0, c/\varepsilon_{30}^{1/2}R)$ is satisfied, the influence of thermal fluctuations on the interaction can be ignored. Then, in the range of distances $a_{1,2} \ll R \ll c/\omega_0\varepsilon_{30}^{1/2}$, where $\omega_c = \omega_0$ and the retardation effect can be ignored, we find that the free energy per unit length is

$$F_L(R) = -\frac{9\hbar a_1^2 a_2^2}{1024 R^5}\int_0^\infty d\omega\left\{\frac{3}{4}\left(\frac{\varepsilon_1}{\varepsilon_3}-1\right)\left(\frac{\varepsilon_2}{\varepsilon_3}-1\right) + \frac{5}{2}\left[\left(\frac{\varepsilon_1}{\varepsilon_3}-1\right)\frac{\varepsilon_2-\varepsilon_3}{\varepsilon_2+\varepsilon_3} + \frac{\varepsilon_1-\varepsilon_3}{\varepsilon_1+\varepsilon_3}\left(\frac{\varepsilon_2}{\varepsilon_3}-1\right)\right] + 19\frac{\varepsilon_1-\varepsilon_3}{\varepsilon_1+\varepsilon_3}\frac{\varepsilon_2-\varepsilon_3}{\varepsilon_2+\varepsilon_3}\right\}. \tag{10}$$

All the permittivities depend now on the argument $i\omega$. Equation (10) agrees with the results of Refs. 3, 4, and 6.

Next, for distances $a_{1,2}, c/\omega_0\varepsilon_{30}^{1/2} \ll R \ll c\hbar/2\pi T$ we can assume that $\omega_c = c/\varepsilon_{30}^{1/2}R$. The retardation effect is then particularly important and the results can be represented as follows:

$$F_L(R) = -\frac{\hbar c a_1^2 a_2^2}{30\pi\varepsilon_{30}^{1/2}R^6}\left\{2\left(\frac{\varepsilon_{10}}{\varepsilon_{30}}-1\right)\left(\frac{\varepsilon_{20}}{\varepsilon_{30}}-1\right) + 3\left[\left(\frac{\varepsilon_{10}}{\varepsilon_{30}}-1\right)\frac{\varepsilon_{20}-\varepsilon_{30}}{\varepsilon_{20}+\varepsilon_{30}} + \frac{\varepsilon_{10}-\varepsilon_{30}}{\varepsilon_{10}+\varepsilon_{30}}\left(\frac{\varepsilon_{20}}{\varepsilon_{30}}-1\right)\right] + 26\frac{(\varepsilon_{10}-\varepsilon_{30})(\varepsilon_{20}-\varepsilon_{30})}{(\varepsilon_{10}+\varepsilon_{30})(\varepsilon_{20}+\varepsilon_{30})}\right\}. \tag{11}$$

The numerical factors in the above expression do not agree with those obtained in Ref. 6, which we regard as due to the inaccuracy of the calculations reported in Ref. 6. In the limiting case of low-density insulator filaments in vacuum, we find that Eq. (11) agrees—in contrast to the expression obtained in Ref. 6—with the results of summation of the Casimir–Polder pair interactions beween atoms in the filaments.

The contribution of thermal fluctuations can be described, in those cases when it is large, by a term with $n = 0$ in the sum of frequencies in Eq. (1). If the condition (9) is obeyed, we obtain

$$F_{L,n=0} = -\frac{9T\pi a_1^2 a_2^2}{1024 R^5}\left\{\frac{3}{4}\left(\frac{\varepsilon_{10}}{\varepsilon_{30}}-1\right)\left(\frac{\varepsilon_{20}}{\varepsilon_{30}}-1\right) + \frac{5}{2}\left[\left(\frac{\varepsilon_{10}}{\varepsilon_{30}}-1\right)\frac{\varepsilon_{20}-\varepsilon_{30}}{\varepsilon_{20}+\varepsilon_{30}} + \frac{\varepsilon_{10}-\varepsilon_{30}}{\varepsilon_{10}+\varepsilon_{30}}\left(\frac{\varepsilon_{20}}{\varepsilon_{30}}-1\right)\right] + 19\frac{(\varepsilon_{10}-\varepsilon_{30})(\varepsilon_{20}-\varepsilon_{30})}{(\varepsilon_{10}+\varepsilon_{30})(\varepsilon_{20}+\varepsilon_{30})}\right\}. \tag{12}$$

The role of thermal fluctuations may be important if, for example, the medium surrounding the filaments is a polar insulator (for example, water) and the filaments represent a nonpolar insulator. Then, the hf permittivities (on the upper imaginary frequency semiaxis) of polar and nonpolar insulators are frequently sufficiently close so that the quantity described by Eq. (10) is relatively small. There is also a considerable reduction in the value of the characteristic frequency ω_0. Equation (12) describes then a quantity which is usually of the same order of magnitude as that given by Eq. (10), but now at distances $R \ll c/\omega_0\varepsilon_{30}^{1/2}$. The free energy $F_L(R)$ can then be represented as a sum of the expressions (10) and (12).

However, the conditions in the interaction between thin filaments can be such that the thermal contribution to the interaction is large throughout the full range of distances even if there is no hf screening of the interaction between the filaments and the ambient medium. Let us consider, for example, filaments made from a polar insulator and located in vacuum or in a nonpolar insulator ($\varepsilon_{10} \sim \varepsilon_{20} \gg \varepsilon_{30}$). We must note that the thermal contribution of Eq. (12) to this interaction rises rapidly on increase in the ratios $\varepsilon_{10}/\varepsilon_{30}$ and $\varepsilon_{20}/\varepsilon_{30}$. On the other hand, the dispersion contribution of Eq. (10) hardly changes because of the small spectral interval of low frequencies where the permittivities of polar insulators are high. In the case of insulators the main contribution to the frequency integral in Eq. (10) comes from frequencies $\omega \sim 10^{16}\ \mathrm{s}^{-1}$. A considerable increase in the thermal contribution is a special feature of the interaction between thin strongly polar filaments. A similar behavior is exhibited only in the problem of the interaction of thin films. However, when bulk (three-dimensional) bodies of this kind interact, there is no increase in the thermal contribution.

When the ratios $\varepsilon_{10}/\varepsilon_{30}$ and $\varepsilon_{20}/\varepsilon_{30}$ are sufficiently long, the inequality of Eq. (9) can naturally prevail in a certain range of distances satisfying the condition of Eq. (2). Then, the thermal contribution to the interaction is described in general by a cumbersome equation which can be simplified only in the limit $\varkappa_{1,2} \gg 1$:

$$F_{L,n=0} = -\frac{\pi T}{8R\ln(R/a_1)\ln(R/a_2)}. \tag{13}$$

The condition $\varkappa_{1,2} \gg 1$ together with the inequality (2) impose very serious restrictions. For example, in the case of identical filaments when $R = 10a$ it follows form $\varkappa \gg 1$ that $\varepsilon_{10} = \varepsilon_{20} \gg 87\varepsilon_{30}$. We shall be interested in the case when $\varkappa \sim 1$ ($\varepsilon_{10} = \varepsilon_{20} \approx 87\varepsilon_{30}$, $a_1 = a_2 = a$). The corresponding thermal contribution is within an interval set by the limits that follow from Eqs. (12) and (13). At room temperature, we find from Eq. (12) that $F_{L,n=0}(R) = 7\times 10^{-16}$ erg/R. Then, Eq. (13) and the same conditions as before yield $F_{L,n=0}(R) = 30\times 10^{-16}$ erg/R. The disperse contribution can be estimated by substituting $\varepsilon_3 = 1$ into Eq. (10):

$$\varepsilon_1(i\omega) = \varepsilon_2(i\omega) = 1 + \Omega^2/(\omega^2+\omega_0^2).$$

Next, integrating with respect to the frequency and assuming that $\omega_0 \approx 2\times 10^{16}\ \mathrm{s}^{-1}$ and $\Omega^2 \approx 1.4\omega_0$, we obtain $F_L(R) \approx 1.3\times 10^{-16}$ erg/R for the case under discussion ($R = 10a$). It is therefore clear that if the filaments consist of a strongly polar insulator, then because of the nonadditive effects the room-temperature thermal contribution to the interaction is at least comparable with the dispersion contribution and can exceed the latter throughout the full range of distances $R \gg a_{1,2}$.

The expression (13) for the thermal contribution had been found earlier in Ref. 19 (compare also with Refs. 8 and 10) using a model analysis of the interaction between metal filaments. Consequently, the result in question is attributed in these treatments to lf fluctuations in conducting media. In fact, the form of Eq. (13) is unrelated to any specific lf properties of thin metal filaments. This expression describes the thermal (entropy) contribution to the interaction if the static permittivities of the filaments are sufficiently high and the condition $\varkappa_{1,2} \gg 1$ is satisfied at a distance that obeys the inequality of Eq. (2).

3. INTERACTION OF TWO METAL FILAMENTS

In the case of metal filaments we find that Eqs. (10)–(12) give infinite values because of unrestricted rise of the permittivity of a metal on reduction in the frequency. Consequently, in the case of metal filaments Eq. (9) is not obeyed at any distance R. It also follows from Eqs. (2) and (8) that in the range of characteristic imaginary frequencies the permittivity of metal filaments assumes very large values, as expected at moderately high frequencies. A self-consistent analysis of the general expressions (1)–(7) confirms this.

The dependence of the van der Waals interaction between metal filaments on the distance between them is more complex and it differs considerably from the corresponding dependence in the case of insulator filaments. It is determined by the specific form of the frequency dispersion of the permittivities of metals at moderately high frequencies. We shall describe this dispersion by the simple and familiar expression

$$\varepsilon_{1,2}(i\omega) \approx \frac{\Omega_{1,2}^2}{\omega(\omega+\nu_{1,2})} = \frac{4\pi\sigma_{1,2}\nu_{1,2}}{\omega(\omega+\nu_{1,2})}. \qquad (14)$$

Moreover, we shall assume that metal filaments are surrounded by an insulating medium and that in the characteristic frequency range we can assume approximately that the permittivity $\varepsilon_3(i\omega)$ assumes its static value ε_{30}.

In addition to the dimensional parameters $2\pi T/\hbar$ and $c/\varepsilon_{30}^{1/2}R$ the following characteristic frequencies are encountered:

$$\tilde{\omega}_{1,2}(R) = \min\left\{\frac{a_{1,2}\Omega_{1,2}}{(2\varepsilon_{30})^{1/2}R}\ln^{1/2}\left(\frac{R}{a_{1,2}}\right); \left(\frac{a_{1,2}}{R}\right)^2 \frac{2\pi\sigma_{1,2}}{\varepsilon_{30}}\ln\left(\frac{R}{a_{1,2}}\right)\right\}. \qquad (15)$$

It is important to note that the frequencies $\tilde{\omega}_{1,2}$ associated with characteristics of metal filaments depend also on the distance R between the filaments. This is the main qualitative difference from the case of two insulator filaments when the characteristic frequency ω_0 associated with the properties of the filaments is independent of R (Sec. 2). This circumstance is closely related to the specific nature of the dispersion of long-wavelength electromagnetic excitations in one-dimensional metals. For example, the first expression in the braces in Eq. (15) represents exactly the frequencies of one-dimensional plasmons

$$\omega_{1,2}(k) = ka_{1,2}\Omega_{1,2}K_0^{1/2}(ka_{1,2})/(2\varepsilon_{30})^{1/2}$$

in each of the filaments on condition that $\omega_{1,2} \gg \nu_{1,2}$ when the wave vectors are $k \sim R^{-1}$. The second expression represents strongly damped excitations in the frequency range $\omega \ll \nu_{1,2}$ when $k \sim R^{-1}$.

The characteristic frequencies $\tilde{\omega}_{1,2}$ are small compared with the corresponding frequency ω_0 in the case of insulator filaments (on condition that the medium surrounding the insulator filaments does not screen too greatly their interaction). In fact, in rough estimates we can usually assume that $\omega_0 \sim \Omega_{1,2}$ and it follows from Eqs. (2) and (15) that $\tilde{\omega}_{1,2} \ll \Omega_{1,2}$.

We then find from Eqs. (14) and (15) that

$$\varepsilon_{1,2}(i\tilde{\omega}_{1,2}) \approx \left(\frac{R}{a_{1,2}}\right)^2 \frac{\varepsilon_{30}}{\ln(R/a_{1,2})} \gg \varepsilon_{30}, \qquad (16)$$

which—subject to Eq. (8)—yields

$$\varkappa_{1,2}(i\tilde{\omega}_{1,2}) \sim 1 \qquad (17)$$

throughout the investigated range of distances. The inequality (16) allows us to simplify Eq. (3) by neglecting the contribution of the function D_2, the last two terms in the expression for D_1, and the second term in Eq. (4) for the coefficient A. It follows from Eq. (17) that the denominator of Eq. (4) should include terms containing $K_0(a_{1,2}q)$. Consequently, Eqs. (1)–(4), (8), (16), and (17) yield the free energy of the interaction of two metal filaments per unit length in contact:

$$F_L(R) = -\frac{a_1^2 a_2^2 T}{4\pi}\sum_{n=0}^{\infty}{}' \int_0^{+\infty} \frac{q^4\varepsilon_1(i\omega_n)\varepsilon_2(i\omega_n)K_0^2(qR)\,dk}{[\varepsilon_{30}+{}^1/_2\varepsilon_1(i\omega_n)(a_1q)^2K_0(a_1q)][\varepsilon_{30}+{}^1/_2\varepsilon_2(i\omega_n)(a_2q)^2K_0(a_2q)]}. \qquad (18)$$

Equation (18) simplifies in several limiting cases of interest. For example, in the range of distances

$$a_{1,2} \ll R \ll \frac{a_{1,2}\Omega_{1,2}}{\nu_{1,2}(2\varepsilon_{30})^{1/2}}\ln^{1/2}\left(\frac{R}{a_{1,2}}\right) \qquad (19)$$

we can ignore the influence of collisions. The contribution of thermal fluctuations is small at temperatures

$$T \ll \min\left\{\left(\frac{a_{1,2}}{R}\right)\frac{\hbar\Omega_{1,2}}{2\pi(2\varepsilon_{30})^{1/2}}\ln^{1/2}\left(\frac{R}{a_{1,2}}\right); \frac{c\hbar}{2\pi\varepsilon_{30}^{1/2}R}\right\}. \qquad (20)$$

When the conditions (19) and (20) are satisfied, we obtain approximately from Eq. (18)

$$F_L(R) = -\frac{\hbar\overline{(\Omega a)}}{8\pi(2\varepsilon_{30})^{1/2}R^2\ln^{1/2}(R/a_1)\ln^{1/2}(R/a_2)}, \qquad (21)$$

where

$$\overline{(\Omega a)} = \frac{2^{1/2}cx_1x_2(1+x_1x_2)}{(x_1+x_2)\ln^{1/2}(R/a_1)\ln^{1/2}(R/a_2)}, \qquad (22)$$

$$x_{1,2}=\frac{a_{1,2}\Omega_{1,2}}{2^{1/2}c}\ln^{1/2}\left(\frac{R}{a_{1,2}}\right)\left[1+\frac{a_{1,2}^2\Omega_{1,2}^2}{2c^2}\ln\left(\frac{R}{a_{1,2}}\right)\right]^{-1/2} \quad (23)$$

The role of the retardation effect is described by the following dimensionless parameter:

$$\gamma_{1,2}=\frac{a_{1,2}\Omega_{1,2}}{2^{1/2}c}\ln^{1/2}\left(\frac{R}{a_{1,2}}\right). \quad (24)$$

In the case of sufficiently thin filaments with radii $a_{1,2}\lesssim 20$ Å we find that simple estimates show that the inequality $\gamma_{1,2}\ll 1$ is obeyed practically throughout the range of distances defined by Eq. (19). If $\gamma_{1,2}\ll 1$, we can ignore the retardation effect and then Eqs. (22) and (23) yield

$$\overline{(\Omega a)}=\frac{a_1a_2\Omega_1\Omega_2}{a_1\Omega_1\ln^{1/2}(R/a_1)+a_2\Omega_2\ln^{1/2}(R/a_2)}. \quad (25)$$

In the case of identical filaments, Eqs. (21) and (25) give

$$F_L(R)=-\frac{\hbar\Omega a}{16\pi(2\varepsilon_{30})^{1/2}R^2\ln^{3/2}(R/a)}. \quad (26)$$

This simplest example can be used conveniently to identify the nature of the approximations used throughout in calculation of the interaction between metal filaments.

Equations (18)–(20) yield for the free energy of the interaction between identical filaments an expression which is more accurate than Eq. (26):

$$F_L(R)=-\frac{\hbar\Omega a}{8\pi(2\varepsilon_{30})^{1/2}R^2}\int_0^\infty\frac{dx\,xK_0^2(x)}{K_0^{3/2}(ax/R)}. \quad (27)$$

If $\varepsilon_{30}=1$, the above expression is identical with the corresponding results reported in Refs. 2 and 11. The integral in Eq. (27) is dominated by the contribution from the range $x\lesssim 1$. Therefore, using the condition of Eq. (2), we find that the function in the denominator of the integral in Eq. (27) can be described by an asymptotic expression valid at low values of the argument:

$$K_0^{3/2}(ax/R)\approx\ln^{3/2}(R/ax).$$

Equation (26) is "logarithmically" correct, compared with Eq. (27). The former is obtained if $x=1$ is substituted in the argument of the logarithmic function and this function is taken outside the integral. It should be noted that in Ref. 2 a similar procedure is used and $x=1/2$ is substituted. The integral occurring in Eq. (27) is interpolated in Ref. 11 in the range of distances considered there and this is done using $(a/R)^{0.8}$.

If $\gamma_{1,2}\gg 1$, when the retardation is particularly important, it follows from Eqs. (21) and (22) that

$$F_L(R)=-\frac{c\hbar}{8\pi\varepsilon_{30}^{1/2}R^2\ln(R/a_1)\ln(R/a_2)}. \quad (28)$$

This expression describes the Casimir effect in the case of two perfectly conducting filaments. Under real conditions the limiting case $\gamma\gg 1$ is difficult to achieve but the condition $\gamma\sim 1$ can be satisfied (for example, this condition is satisfied if $a=100$ Å, $R=10^3$ Å and $\Omega\approx 2\times 10^{16}$ s^{-1}). Similarly, it is difficult to investigate experimentally the interaction of metal filaments when the inequality opposite to that of Eq. (20) is satisfied and Eq. (13) is valid. However, we can satisfy the relation

$$T\sim(a/R)\hbar\Omega\ln^{1/2}(R/a)/2\pi(2\varepsilon_{30})^{1/2}$$

(for example, this condition can be satisfied if $R=30a\ll 10^{-4}$ cm, $\Omega\sim 2\times 10^{16}$ s^{-1}, and $T\approx 300$ K). The interaction then is described approximately by the sum of Eqs. (21) and (13).

In the case of distances large compared with those defined in Eq. (19), i.e., when

$$R\gg a_{1,2},\ \frac{a_{1,2}\Omega_{1,2}}{v_{1,2}(2\varepsilon_{30})^{1/2}}\ln^{1/2}\left(\frac{R}{a_{1,2}}\right), \quad (29)$$

we have $\tilde{\omega}_{1,2}\ll v_{1,2}$ and, therefore, the interaction between metal filaments is due to their lf conducting properties. The role of thermal fluctuations is then small if

$$T\ll\min\left\{\left(\frac{a_{1,2}}{R}\right)^2\frac{\hbar\sigma_{1,2}}{\varepsilon_{30}}\ln\left(\frac{R}{a_{1,2}}\right);\ \frac{c\hbar}{2\pi\varepsilon_{30}^{1/2}R}\right\}. \quad (30)$$

It is interesting to note that the retardation effect can be significant only if the distances are not too large,

$$R\lesssim\frac{2\pi\sigma_{1,2}a_{1,2}^2}{\varepsilon_{30}^{1/2}c}\ln\left(\frac{R}{a_{1,2}}\right), \quad (31)$$

provided that $\gamma_{1,2}\gg 1$, which ensures compatibility of Eqs. (29) and (31). Therefore, if $\gamma_{1,2}\ll 1$, the retardation effect has no influence whatever on the interaction between metal filaments for any distance between them. Moreover, in the case of distances

$$R\gg 2\pi\sigma_{1,2}a_{1,2}^2\ln(R/a_{1,2})/c\varepsilon_{30}^{1/2}$$

satisfying also Eqs. (29) and (30), we find from Eq. (18) that

$$F_L(R)\approx-\frac{\pi\hbar\sigma_1\sigma_2a_1^2a_2^2}{32\varepsilon_{30}R^3}\left[\sigma_2a_2^2\ln\left(\frac{R}{a_2}\right)-\sigma_1a_1^2\ln\left(\frac{R}{a_1}\right)\right]^{-1}\times\ln\left[\frac{\sigma_2a_2^2\ln(R/a_2)}{\sigma_1a_1^2\ln(R/a_1)}\right] \quad (32)$$

for arbitrary $\gamma_{1,2}$.

If the filaments are identical, we obtain from the above expressions

$$F_L(R)=-\frac{\pi\hbar\sigma a^2}{32\varepsilon_{30}R^3\ln(R/a)}. \quad (33)$$

The conditions under which Eqs. (32) and (33) are valid should be supplemented by the requirement which allows us to neglect the anomalous skin effect. In the simplest case this results in an inequality

$$(R/a)\gg 2\pi\sigma\lambda\ln^{1/2}(R/a)/c\varepsilon_{30}^{1/2},$$

where λ is the mean free path of electrons.

4. INTERACTION OF AN ATOM WITH A FILAMENT

The general expressions for the interaction of an impurity atom in a liquid with a thin filament located at a distance $R\gg a$ in the same liquid can be obtained from Eqs. (1)–(6) which describe the interaction of two filaments in a liquid. It is sufficient to regard one of the filaments as a dilute solution of impurity atoms by assuming that

$$\varepsilon_2\approx\varepsilon_3+N(\partial\varepsilon_p/\partial N)_{N=0},$$

where N is a low volume density of the number of impurity atoms. We then have

$$F_L(R) \approx N_L U(R),$$

where $U(R)$ is the free energy (potential) of the interaction of one impurity atom with a filament and $N_L = \pi a_2^2 N$ is the linear density of impurity atoms. This is in many respects analogous to the conclusion reached by Pitaevskiĭ in Ref. 20 for the interaction of two impurity atoms in a homogeneous liquid. For brevity we shall consider only the case when an atom and a filament are in vacuum and we shall use the simplest expressions describing the interaction of two thin filaments, one of which is an insulator.

The expressions for the interaction of an atom with a thin insulator filament follow directly from Eqs. (10)–(12):

$$U(R) = -\frac{9\hbar a^2}{128R^5} \int_0^{+\infty} d\omega\, \alpha(i\omega) \frac{(\varepsilon(i\omega)-1)(\varepsilon(i\omega)+7)}{\varepsilon(i\omega)+1},$$

$$a \ll R \ll \lambda_0; \qquad (34)$$

$$U(R) = -\frac{\hbar c \alpha_0 a^2 (\varepsilon_0 - 1)(7\varepsilon_0 + 39)}{15\pi(\varepsilon_0+1)R^6}, \quad R \gg \lambda_0; \qquad (35)$$

$$U_{n=0}(R) = -\frac{9\pi a^2 T}{128R^5} \frac{\alpha_0(\varepsilon_0-1)(\varepsilon_0+7)}{\varepsilon_0+1}. \qquad (36)$$

A different method was used to obtain the same expressions in Ref. 13. The discrepancy between the numerical coefficients in Eq. (35) given above and Eq. (23) in Ref. 13 is to the best of our knowledge due to a misprint in Ref. 13. It should be stressed that Eqs. (34)–(36) apply only to an insulator filament and that in the case of metals these equations yield infinite values.

Before we determine the interaction of an atom with a metal filament, we shall consider the interaction of an insulator filament with a metal one. The relevant calculation is in many respects analogous to that given in the preceding section. In the range of distances defined by Eq. (19) and at temperatures described by Eq. (20) we find [in this case Eqs. (19) and (20) should apply naturally only to a metal filament]

$$F_L = -\frac{\hbar\Omega_1 a_1 a_2^2 (\varepsilon_{20}-\varepsilon_{30})\{\varepsilon_{20}+\varepsilon_{30}[5-4\gamma_1((1+\gamma_1^2)^{1/2}-\gamma_1)]\}}{24\sqrt{2}\,\pi\varepsilon_{30}^{3/2}(\varepsilon_{20}+\varepsilon_{30})(1+\gamma_1^2)^{1/2}R^4 \ln^{1/2}(R/a_1)}. \qquad (37)$$

The dimensionless parameter γ_1 is given by Eq. (24).

Next, in the case when Eqs. (29) and (30) and the inequality $R \gg 2\pi\sigma_1 a_1^2 \ln[(R/a_1)/c\varepsilon_{30}^{1/2}]$ are obeyed, we find that

$$F_L(R) = -\frac{9\pi\hbar\sigma_1 a_1^2 a_2^2 (\varepsilon_{20}-\varepsilon_{30})}{1024\varepsilon_{30}^2(\varepsilon_{20}+\varepsilon_{30})R^5}\Big[(3\varepsilon_{20}+13\varepsilon_{30}) \times \ln\Big(\frac{c\varepsilon_{30}^{1/2}R}{\pi\sigma_1 a_1^2 \ln(R/a_1)}\Big) - 10\varepsilon_{30}\Big]. \qquad (38)$$

Finally the thermal contribution to the interaction of insulator and metal filaments is described by the expression

$$F_{L,n=0}(R) = -\frac{T\pi a_2^2(\varepsilon_{20}-\varepsilon_{30})(\varepsilon_{20}+7\varepsilon_{30})}{128\varepsilon_{30}(\varepsilon_{20}+\varepsilon_{30})R^3 \ln(R/a_1)}. \qquad (39)$$

We shall now substitute $\varepsilon_3 = 1$ in Eqs. (37)–(39), consider the limit of a low-density insulator filament ($\varepsilon_{20} \approx 1 + 4\pi N\alpha_0$), and use the relationship

$$F_L = N_L U = \pi a_2^2 N U.$$

Consequently, we find from Eqs. (37)–(39) the following expressions for the interaction of an atom with a metal filament:

$$U(R) = -\frac{\hbar\Omega a\alpha_0[3-2\gamma((1+\gamma^2)^{1/2}-\gamma)]}{6\sqrt{2}\,\pi(1+\gamma^2)^{1/2}R^4 \ln^{1/2}(R/a)}, \qquad (40)$$

$$U(R) = -\frac{9\pi\hbar\sigma\alpha_0 a^2}{256R^5}\Big[8\ln\Big(\frac{cR}{\pi\sigma a^2 \ln(R/a)}\Big) - 5\Big], \qquad (41)$$

$$U_{n=0}(R) = -\frac{\pi\alpha_0 T}{8R^3 \ln(R/a)}. \qquad (42)$$

The interaction of an atom with a metal filament is characterized by the same main features as the interaction between two metal filaments. Worth pointing out, in particular, are a slow fall of the interaction with distance [compared with the case of an insulator filament—see Eqs. (34)–(36)], a very small role of the retardation effect throughout the investigated range of distances on condition that $\gamma \ll 1$, and a possible significant thermal contribution to the interaction even at relatively short distances. In the limit $\gamma \ll 1$, when the retardation effect can be ignored, we find from Eq. (40) that

$$U(R) = -\frac{\hbar\Omega a\alpha_0}{2^{3/2}\pi R^4 \ln^{1/2}(R/a)}. \qquad (43)$$

As mentioned above, the $U(R) \propto R^{-4}$ behavior of the interaction of an atom with a metal filament under similar conditions had long ago been predicted by Zel'dovich.[12] In the other limit of $\gamma \gg 1$, we find from Eqs. (40) and (24) that

$$U(R) = -\frac{\hbar c\alpha_0}{3\pi R^4 \ln(R/a)}, \qquad (44)$$

which corresponds to the Casimir force between an atom and a perfectly conducting filament. It should be stressed in this connection that at sufficiently large distances and low temperatures the interaction of an atom with a metal filament is described by Eq. (41) and we cannot regard a filament as perfectly conducting.

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