Superconducting fluctuations in tunnel junctions below the critical temperature

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Corrections to the quasiparticle component of the current in superconducting tunnel junctions of various types for fluctuations of the modulus and phase of the order parameter are calculated for temperatures below the critical temperature. The corresponding increments in the current-voltage characteristic are very nonlinear. They are manifested even in first order in the transparency of the barrier. They increase logarithmically toward T_c and are seen particularly strongly if the fluctuating electrode is a thin amorphous film.

1. INTRODUCTION

Electron-electron coupling is known to have an important effect on the current-voltage characteristics of tunnel and Josephson structures with amorphous electrodes. For example, coupling in the diffusion channel gives rise to well-known anomalies at a zero voltage, which are manifested at the scale of τ^{-1} (Ref. 1; τ is the electron elastic relaxation time). If the electrodes of the junction (or at least one of them) are superconductors, superconducting fluctuations (the electron-electron coupling in the Cooper channel) in them at temperatures slightly above T_c give rise to maxima and minima which alternate in an extremely peculiar way. These extrema are seen even at a far lower voltage scale, $eV \sim T - T_c, T_c$ (Ref. 2).

In analyzing the electron-electron coupling in the superconducting phase, we can no longer separate this coupling into independent diffusion and Cooper channels. In this case the interactions of quasiparticles above the condensate with large and small momentum transfers can no longer be analyzed without consideration of the condensate of Cooper pairs in the system. As a result it turns out that fluctuation processes which do not conserve the number of quasiparticles above the condensate, and which have no analog above T_c, can occur in such a system. All of these processes can ultimately be reduced to fluctuations of the module and phase of the order parameter and the scalar potential and an interference of the two.[3–6]

The influence of all these processes on the Josephson component of the current in a tunnel junction was studied in detail in Ref. 6. In the present paper we use the formalism developed in Ref. 6 to study the effect of the electron-electron coupling on the quasiparticle current of an S_1–I–S_2 junction at temperatures below the critical temperature. We are primarily interested in fluctuations of the phase and modulus of the order parameter, which give rise to increments in the tunneling current which are singular near T_c. As we will show below, incorporating fluctuation effects of this type even in first order in the barrier transparency (rather than in second order, as has been the approach in several studies[7–10]) can significantly affect the shape of the current-voltage characteristics at the typical voltages, $eV \sim \Delta, \tau_s^{-1}$, where τ_s is the time scale of the electron relaxation with spin flip. The nature and presence of singularities are determined to a large extent by the temperature and superconducting properties of the electrodes which make up the junction. We will discuss three types of film junctions: 1) a symmetric junction of two identical gapless superconductors; 2) a superconductor-(normal metal) junction; and 3) a junction between two different superconductors, one of which is gapless, while the other has no magnetic impurities. In all cases the fluctuation correction to the tunneling current has a temperature dependence which is logarithmic in $T - T_c$. This behavior is analogous to that which has been seen previously[2] for temperatures above the critical temperature. In addition to that temperature dependence, the corrections to the tunneling current at $T < T_c$ depend nontrivially on the linear dimensions of the superconducting film which forms the junction. This dependence stems from the fluctuations of the phase of the order parameter in the superconducting electrode along the tunneling barrier.[6]

2. FEYNMAN DIAGRAMS FOR THE TUNNELING CURRENT

As was shown in Ref. 6, the quasiparticle current which flows through a tunnel junction can be written as follows in the formalism of a temperature Feynman-diagram technique:

$$I_{qp}(V) = -2e\,\mathrm{Im}\,[K_0^R(V) + K_3^R(V)], \tag{1}$$

where V is the voltage applied to the junction, and the quantities $K_i^R(V)$ are the correlation functions of the one-electron Green's functions of the electrodes continued analytically into the upper half-plane of the complex frequency $(\omega_\nu - i\omega)$,

$$K_i(\omega_\nu) = \frac{1}{2}\,\mathrm{Sp}\,T\sum_{\varepsilon_n}\sum_{\mathbf{pk}}|T_{\mathbf{pk}}|^2\{\hat{\sigma}_i\hat{G}_{\mathrm{I}}(\mathbf{p},\varepsilon_n+\omega_\nu)\hat{\sigma}_i\hat{G}_{\mathrm{II}}(\mathbf{k},\varepsilon_n)\} \tag{2}$$

with the subsequent replacement $\omega = eV$.

The quantity $T_{\mathbf{pk}}$ in (2) is the matrix element of the tunneling Hamiltonian; ε_n and ω_ν are the fermion and boson frequencies; the subscripts I and II distinguish between the "left-hand" and "right-hand" electrodes; and $\hat{G}(\mathbf{k}, \varepsilon_n)$ is the one-electron Green's function of a superconductor containing impurities in the Nambu matrix formalism:

$$\hat{G}(\mathbf{k},\varepsilon_n) = \mu_l(\mathbf{k},\varepsilon_n)\hat{\sigma}_l = (-i\tilde{\varepsilon}_n\hat{\sigma}_0 + \tilde{\Delta}_n\hat{\sigma}_1 - \xi_{\mathbf{k}}\hat{\sigma}_3)(\tilde{\varepsilon}_n^2 + \tilde{\Delta}_n^2 + \xi_{\mathbf{k}}^2)^{-1}.$$

Here

$$\tilde{\varepsilon}_n = \varepsilon_n + \tilde{\varepsilon}_n/2\tau_1(\tilde{\varepsilon}_n^2 + \tilde{\Delta}_n^2)^{1/2}, \quad \tilde{\Delta}_n = \Delta + \tilde{\Delta}_n/2\tau_2(\tilde{\varepsilon}_n^2 + \tilde{\Delta}_n^2)^{1/2},$$

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Δ is the order parameter, $\xi_{\mathbf{k}} = E(\mathbf{k}) - E_F$ is the electron energy, measured from the Fermi level, and $\hat{\sigma}_i$ are the Pauli matrices, given by

$$\hat{\sigma}_0=\begin{pmatrix}1&0\\0&1\end{pmatrix},\quad \hat{\sigma}_1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad \hat{\sigma}_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\quad \hat{\sigma}_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}.$$

Here and below, a repeated index means a summation [except in (2), where no summation is made over the index i], and $l = 0, 1, 2, 3$. The relaxation times τ_1 and τ_2 are defined by

$$\tau_{1,2}^{-1}=\frac{1}{\pi}nmp_F\left[\,|u_1|^2\pm\frac{1}{4}s(s+1)\,|u_2|^2\right],$$

where $u_{1,2}$ are the amplitudes for scattering without and with a flipping of the electron spin, n is the impurity concentration, s is the spin of the impurity atom, and p_F is the Fermi momentum.

The Feynman diagram corresponding to the correlation functions $K_i(\omega_\nu)$ is shown in Fig. 1. Since the matrix elements $T_{\mathbf{pk}}$ depend only weakly on the energy near the Fermi surface, expression (2) can be rewritten with the help of the Green's functions of each of the electrodes, integrated over the energy[6]

$$K_i(\omega_\nu)=\frac{1}{8\pi e^2R_n}\operatorname{Sp}T\sum_{\varepsilon_n}\{\sigma_i\hat{g}_{\rm I}(\varepsilon_n+\omega_n)\hat{\sigma}_i\hat{g}_{\rm II}(\varepsilon_n)\},\quad (3)$$

where

$$\hat{g}(\varepsilon_n)=\frac{1}{N(0)}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\hat{G}(\mathbf{p},\varepsilon_n),\quad (4)$$

R_n is the resistance of the junction in the normal state, and $N(0)$ is the density of electron states at the Fermi level.

To first order in the electron-electron coupling, the correction to the tunneling current is represented by the two diagrams in Fig. 2a. Of the second-order corrections, only the first is of interest. The corresponding diagram is shown in Fig. 2b. As was found in Ref. 2, at $T > T_c$ it leads to a nonlinearity of the current-voltage characteristic at low voltages ($eV \sim T - T_c$), and the corresponding contribution may turn out to be on the order of (or even greater than) the contributions from the diagrams in Fig. 2a.

The question of averaging diagrams of this sort for the tunneling current over the impurity positions was examined in detail in Refs. 6 and 11. Here we need to carry out an average in the gapless superconductor; i.e., we need to consider the electron scattering with spin flip. The matrix impurity vertex function found in Refs. 6 and 11 takes the following form in this case:

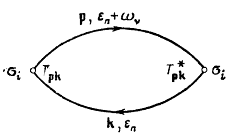

FIG. 1. Feynman diagram for the correlation function of the exact one-electron Green's functions. This diagram determines the total current which flow through the tunnel junction. The circles represent the matrix elements of the tunneling Hamiltonian, which sends an electron from one electrode to the other.

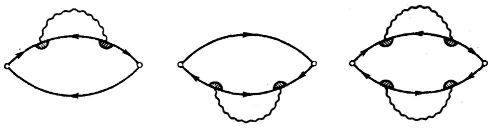

FIG. 2. Corrections of (a) first and (b) second orders in the electron-electron coupling in the electrodes.

$$T(\mathbf{q},\omega_1,\omega_2)=\left(1-\frac{1}{2\pi}S(\mathbf{q},\omega_1,\omega_2)w\right)^{-1},\quad (5)$$

where

$$w=\begin{pmatrix}\tau_1^{-1}&0&0&0\\0&-\tau_2^{-1}&0&0\\0&0&-\tau_2^{-1}&0\\0&0&0&-\tau_1^{-1}\end{pmatrix},$$

and $S(\mathbf{q}, \omega_1, \omega_2)$ was calculated in Refs. 11 and 6. It is a loop of two Green's functions with σ matrices at the vertices:

$$S_{lm}(\mathbf{q},\omega_1,\omega_2)=\frac{1}{2}\operatorname{Sp}\int d\xi_{\mathbf{p}}\int\frac{d\Omega_{\mathbf{p}}}{4\pi}\{\hat{G}(\mathbf{p},\omega_1)\hat{\sigma}_l\hat{G}(\mathbf{p}-\mathbf{q},\omega_2)\hat{\sigma}_m\}.$$

The vertices in the diagrams in Fig. 2 which contain matrix elements of the tunneling Hamiltonian, on the other hand, are not renormalized by the impurities, since such a renormalization would correspond to a scattering of electrons from different electrodes by the same scattering center.

3. SYMMETRIC JUNCTION

In this section of the paper we examine the effect of electron-electron coupling on the quasiparticle current which flows through a symmetric film tunnel junction between two gapless ($\Delta \ll \tau_s^{-1} \ll T$) superconductors at $T < T_c$ ($T_c - T \ll T_c$). We assume that the films are quite dirty ($T\tau \ll 1$) and that the film thickness d is small in comparison with the correlation length $\xi(T)$, so that the fluctuations in the system are two-dimensional.

We begin with the first-order correction, which is determined by the sum of the two diagrams in Fig. 2a (their contributions are obviously equal in this case):

$$2(K_0^{(1)}(\omega_\nu)+K_3^{(1)}(\omega_\nu))$$
$$=\frac{1}{4\pi e^2R_n}\operatorname{Sp}T\sum_{\varepsilon_n}[\hat{g}_{\rm I}(\varepsilon_n+\omega_\nu)+\hat{\sigma}_3\hat{g}_{\rm I}(\varepsilon_n+\omega_\nu)\hat{\sigma}_3]\delta\hat{g}_{\rm II}(\varepsilon_n).\quad (6)$$

The function $\hat{g}_I(\varepsilon_n + \omega_\nu)$ in this expression is

$$\hat{g}_{\rm I}(\varepsilon_n+\omega_\nu)=[-i\,\widetilde{(\varepsilon_n+\omega_\nu)}\,\hat{\sigma}_0+\tilde{\Delta}_{n+\nu}\hat{\sigma}_1]$$
$$\times[\widetilde{\varepsilon_n+\omega_\nu)^2}+\tilde{\Delta}^2_{n+\nu}]^{-1/2},$$

and $\delta\hat{g}_{\rm II}(\varepsilon_n)$ is the correction to the Green's function of first order in the electron-electron coupling. Using the expansion (which is valid at $\Delta\tau_s \ll 1$)

$$\frac{\tilde{\varepsilon}_n}{\tilde{\Delta}_n}\approx\operatorname{Sgn}\varepsilon_n\left[\frac{1+|\varepsilon_n|\tau_s}{\Delta\tau_s}-\frac{1}{2}\frac{\Delta\tau_s}{(1+|\varepsilon_n|\tau_s)^2}\right],$$

and also introducing the notation

$$P^{(i)}(\varepsilon_n)=\frac{1}{2}\,\mathrm{Sp}\{\delta\hat{g}(\varepsilon_n)\hat{\sigma}_i\},$$

we can rewrite (6) as

$$2[K_0^{(1)}(\omega_\nu)+K_s^{(1)}(\omega_\nu)]=-\frac{i}{\pi e^2R_n}T\sum_{\varepsilon_n}\mathrm{Sgn}(\varepsilon_n+\omega_\nu)\times\left[1-\frac{1}{2}\frac{(\Delta\tau_s)^2}{(1+|\varepsilon_n+\omega_\nu|\tau_s)^2}\right]P^{(0)}(\varepsilon_n). \quad (7)$$

The following expression was found for $P^{(0)}(\varepsilon_n)$ in Ref. 6:

$$P^{(0)}(\varepsilon_n)=-i\pi\tau^2\,\mathrm{Sgn}\,\varepsilon_nT\sum_{\Omega_k}\int(d\mathbf{q})\{T_{11}^2(\mathbf{q},\varepsilon_n,\varepsilon_n-\Omega_k)\times L_{11}(\mathbf{q},\Omega_k)+T_{22}^2(\mathbf{q},\varepsilon_n,\varepsilon_n-\Omega_k)L_{22}(\mathbf{q},\Omega_k)-T_{33}^2(\mathbf{q},\varepsilon_n,\varepsilon_n-\Omega_k)L_{33}(\mathbf{q},\Omega_k)\}. \quad (8)$$

Ω_k is the boson frequency, and $L_{ik}(\mathbf{q},\Omega_k)$ are the components of the matrix fluctuation propagator (the vertex part of the electron-electron coupling).[6] The terms with $L_{11}(\mathbf{q},\Omega_k)$ and $L_{22}(\mathbf{q},\Omega_k)$ describe fluctuations of the modulus and phase, respectively, of the order parameter; the term with $L_{33}(\mathbf{q},\Omega_k)$ corresponds to fluctuations of the scalar potential. We will not consider the last term in (8) below since its temperature dependence is not singular near the critical temperature. Substituting in the explicit expressions for $T_{11}(\mathbf{q},\omega_1,\omega_2)$ and $T_{22}(\mathbf{q},\omega_1,\omega_2)$ (within $\Delta\tau_s\ll1$),

$$T_{11}(\mathbf{q},\omega_1,\omega_2)=T_{22}(\mathbf{q},\omega_1,\omega_2)=\frac{\theta(\omega_1\omega_2)}{\tau(|\omega_1|+|\omega_2|+D\mathbf{q}^2+\Gamma)},$$

where $D=(\frac{1}{3})v_F{}^2\tau$ is the electron diffusion coefficient, and $\Gamma=2\tau_s{}^{-1}$, we find the following result for the correction to the current in first order in the electron-electron coupling:

$$\delta I_{qp}^{(1)}(V)=\frac{2\pi}{eR_n}\mathrm{Im}\left\{T\sum_{\varepsilon_n}\int(d\mathbf{q})\,\mathrm{Sgn}\,\varepsilon_n\,\mathrm{Sgn}(\varepsilon_n+\omega_\nu)\times\left[1-\frac{2\Delta^2}{(\Gamma+2|\varepsilon_n+\omega_\nu|)^2}\right]C(\mathbf{q},\varepsilon_n)\right\}^R_{i\omega_\nu\to eV},$$

$$C(\mathbf{q},\varepsilon_n)=T\sum_{\Omega_k}\frac{\theta[\varepsilon_n(\varepsilon_n-\Omega_k)]}{(|\varepsilon_n|+|\varepsilon_n-\Omega_k|+D\mathbf{q}^2+\Gamma)^2}\times[L_{11}(\mathbf{q},\Omega_k)+L_{22}(\mathbf{q},\Omega_k)]. \quad (9)$$

We transform the sum over ε_n in (9) into a contour integral in the complex plane, using the customary rule

$$T\sum_{\varepsilon_n}(\ldots)\to\frac{1}{4\pi i}\oint d\varepsilon\,\mathrm{th}\frac{\varepsilon}{2T}(\ldots)$$

(the integration contour is shown in Fig. 3).

As a result we find, for $eV\ll T$,

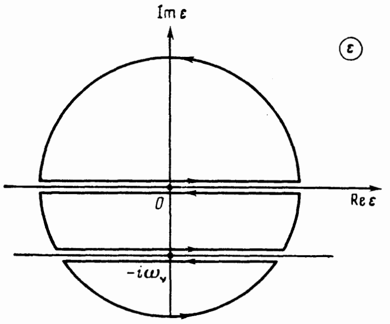

FIG. 3. Integration contour for Eq. (9).

$$\delta I_{qp}^{(1)}(V)=-\frac{V}{R_n}\frac{1}{2T}\int(d\mathbf{q})\int_{-\infty}^{+\infty}d\varepsilon\frac{1}{\mathrm{ch}^2(\varepsilon/2T)}\times\mathrm{Re}\,C^A(\mathbf{q},\varepsilon)\left\{1-2\Delta^2\,\mathrm{Re}\frac{1}{[\Gamma-2i(\varepsilon+eV)]^2}\right\}. \quad (10)$$

The quantities $C^{R(A)}(\mathbf{q},\varepsilon)$ are the analytic continuations of the function $C(\mathbf{q},\varepsilon_n)$ into the upper and lower, respectively, half-planes of the complex frequency. The function $C^{R(A)}(\mathbf{q},\varepsilon)$ is calculated in the Appendix. Integrating over the energy and momentum, we find, to logarithmic accuracy,

$$\delta I_{qp}^{(1)}(V)=-\frac{V}{R_n}\frac{21\zeta(3)}{\pi^2}\frac{1}{p_F{}^2ld}\left[2\ln\frac{L}{\max\{L_T,d\}}+\ln\frac{T_c}{T_c-T}\right]-\frac{V}{R_n}\frac{3\pi}{4}\frac{1}{p_F{}^2ld}\frac{T\Delta^2}{\Gamma^3}\ln\frac{\Gamma^2L^2}{D(T_c-T)}\varphi_1\left(\frac{eV}{\Gamma}\right),$$

$$\times\ \varphi_1(x)=\frac{3x^2-1}{(1+x^2)^3}. \quad (11)$$

Here L is a characteristic dimension of the electrode along the barrier, $L_T\sim(D/T)^{1/2}$ is the thermal length, and $\zeta(x)$ is the Riemann ζ-function. The question of a cutoff of the divergence of the integral over momentum in (10) was discussed in detail in Ref. 6.

The first-order correction can thus be split into two terms: linear and nonlinear in the voltage. The nature of the correction will be discussed in detail below; here we would simply like to point out that the first term is analogous to the contribution from the corresponding diagrams calculated for the case $T>T_c$. The important difference is the appearance of the logarithm of the disruption of long-range order in the lowered-dimensionality system because of fluctuations in the phase of the order parameter. The second term in (11) has no analog above T_c.

We turn now to the calculation of the second-order correction (the corresponding diagram is in Fig. 2b). After some straightforward manipulations, the analytic expression corresponding to this diagram can be written

$$\delta I_{pq}^{(2)}(V)=-\frac{1}{\pi eR_n}\mathrm{Im}\left\{T\sum_{\varepsilon_n}P^{(0)}(\varepsilon_n+\omega_\nu)\,P^{(0)}(\varepsilon_n)\right\}^R_{i\omega_\nu\to eV}. \quad (12)$$

Carrying out an analytic continuation, and again retaining only the leading term in eV/T, we find

$$\delta I_{qp}^{(2)}(V)=\frac{V}{R_n}\frac{1}{4T}\int(d\mathbf{q}_1)\int(d\mathbf{q}_2)\int_{-\infty}^{+\infty}d\varepsilon\,\mathrm{Re}\,C^R(\mathbf{q}_1,\varepsilon)$$

$$\times\ \mathrm{Re}\,C^R(\mathbf{q}_2,\varepsilon+eV). \tag{13}$$

Integrating over the energy in (13), we find

$$\delta I_{qp}^{(2)}(V)=-\frac{V}{R_n}\frac{9\pi T^3}{4(p_F{}^2ld)^2}\sum_{i,j=1}^{2}\Phi_{ij},$$

$$\Phi_{ij}=\mathrm{Re}\frac{\partial^2}{\partial(eV)^2}\int_a^\infty dz_1\int_a^\infty dz_2\frac{1}{(z_1+b_i)(z_2+b_j)}\frac{1}{z_1+z_2+\Gamma-ieV},$$

$$a=DL^{-2},\quad b_1=b=\frac{16}{\pi}(T_c-T),\quad b_2=0. \tag{14}$$

Using the relations $a\ll b\ll\Gamma$, we find the following result for the second-order correction:

$$\delta I_{qp}^{(2)}(V)=\frac{V}{R_n}\frac{9\pi}{2(p_F{}^2ld)^2}\left(\frac{T}{\Gamma}\right)^3\chi\left(\frac{eV}{\Gamma}\right), \tag{15}$$

$$\chi(x)=-\varphi_1(x)\ln^2\frac{\Gamma^2L^2}{D(T_c-T)}-\varphi_2(x)\ln\frac{\Gamma^2L^2}{D(T_c-T)},$$

$$\varphi_2(x)=2\varphi_1(x)\ln(1+x^2)+\frac{x(2x^2-3)}{(1+x^2)^3}\,\mathrm{arctg}\,x. \tag{16}$$

The fluctuation correction to the tunneling current in the case of a symmetric junction is thus

$$\delta I_{qp}(V)=-\frac{V}{R_n}\frac{1}{p_F{}^2ld}\left\{\frac{21\zeta(3)}{\pi^2}\ln\frac{T_c{}^2L^2}{D(T_c-T)}\right.$$

$$\frac{3\pi}{4}\left(\frac{T}{\Gamma}\right)^3\ln\frac{\Gamma^2L^2}{D(T_c-T)}\left[\left(\frac{\Delta^2}{T^2}+\frac{6}{p_F{}^2ld}\ln\frac{\Gamma^2L^2}{D(T_c-T)}\right)\right.$$

$$\left.\left.\times\ \varphi_1\left(\frac{eV}{\Gamma}\right)+\frac{6}{p_F{}^2ld}\varphi_2\left(\frac{eV}{\Gamma}\right)\right]\right\}. \tag{17}$$

In analyzing the contribution of δR_{fl} as a function of the differential resistance of the junction as a function of the voltage, $R_d(V)$, we also need to consider another correction, δR_{pm}, which stems from the presence of paramagnetic impurities in the electrodes of this junction[12]:

$$\frac{\delta R_{pm}}{R_n}=\frac{7\zeta(3)}{4\pi^2}\frac{\Delta^2}{T^2}-\frac{\pi}{8}\frac{\Delta^4}{T\Gamma^3}\frac{9x^4-14x^2+1}{(1+x^2)^4}\bigg|_{x=eV/\Gamma}.$$

The behavior of δR_{fl} and that of δR_{pm} are extremely similar in the two-dimensional case [see Fig. 4, which shows both contributions for the case $p_F{}^2ld=1000$, $(T_c-T)/T_c=0.06$]. It can be seen from Fig. 4 that for sufficiently thin films the fluctuation contribution is predominant near T_c. With distance from the critical temperature, and with increasing thickness of the films, the role of the fluctuations is weakened, the contributions initially becomes comparable in magnitude, and then the contribution from δR_{pm} becomes the leading contribution.

4. JUNCTION BETWEEN A SUPERCONDUCTOR AND A NORMAL METAL

We now consider an asymmetric tunnel junction, one of whose electrodes is a normal metal, while the other is a superconductor, at a temperature slightly below the critical temperature. (The opposite situation, of a junction between a deeply cooled superconductor, with $T_{c1}\gg T$, and a fluctuating superconductor, with $T-T_{c2}\ll T_{c2}$, was analyzed by Baramidze and Cheĭshvili.[13]) The electron-electron coupling in the normal electrode is assumed here to be inconsequential. The correction to the tunneling current of such a junction will in this case be represented by the single diagram in Fig. 2a. The corresponding analytic expression is given by (6), where $\hat{g}_1(\varepsilon_n+\omega_\nu)$ is now to be understood as the Green's function of the normal metal

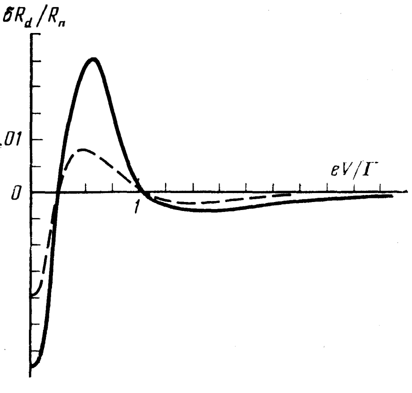

FIG. 4. Corrections to the differential resistance of a symmetric junction versus the voltage applied to the junction. Solid line—The fluctuation correction; dashed line—the component which stems from the presence of paramagnetic impurities in the fluctuating electrode $p_F{}^2ld=1000$, $\tau=(T_c-T)/T_c=0.06$.

$$\hat{g}_1(\varepsilon_n+\omega_\nu)=-i\pi\,\mathrm{Sgn}(\varepsilon_n+\omega_\nu)\hat{\sigma}_0.$$

After analytic continuation we find

$$\delta I_{q.p}(V)=-\frac{V}{R_n}\frac{1}{4\pi T}\int_{-\infty}^{\infty}d\varepsilon\,\mathrm{Im}\,P^R(\varepsilon)\frac{1}{\mathrm{ch}^2(\varepsilon/2T)}. \tag{18}$$

The function $-\pi^{-1}\mathrm{Im}P^R(\varepsilon)$ is the normalized correction to the electron state density, $\delta N(\varepsilon)/N(0)$. Since the total number of states in the band is determined exclusively by the number of unit cells in the sample, we clearly have

$$\int_{-\infty}^{\infty}\delta N(\varepsilon)\,d\varepsilon=0. \tag{19}$$

Electron-electron coupling in the Cooper channel gives rise to a correction to the state density. As a measure of the energy dependence of this correction we can use either the gap in the spectrum of elementary excitations (in the absence of a pair-rupture mechanism) or the reciprocal of the time scale for electron relaxation with spin flip, $\tau_s{}^{-1}$ (in a gapless superconductor). When we consider magnetic-impurity concentrations which are not too high ($\tau_s{}^{-1}\ll T$), we find that the integral (19) is determined by energies which are low in comparison with the temperature. Returning to the integral on the right side of (18), we see that the integration over the region $\varepsilon\ll T$ [where $\cosh^2(\varepsilon/2T\simeq 1]$ gives us zero, and values $\varepsilon\sim T$ are important. We can thus return to expression (8) for the calculation of $P^R(\varepsilon)$, and assuming that the frequencies are Matsubara frequencies, and retaining only the term with $k=0$ in the sum over Ω_k, as the term which is most singular in terms of the proximity to T_c, we can then carry out the analytic continuation directly. Although this procedure is in principle incorrect, and at $\varepsilon \ll T$ it may lead to an incorrect functional dependence $P^R(\varepsilon)$, the function $P^R(\varepsilon)$ calculated in this method agrees at $\varepsilon \sim T$ with the exact function, so it can be used in place of the latter in evaluating the integral in (18). As a result we find an expression for the correction to the tunneling current which is the same, to within a factor of 1/2 (since only one of the two diagrams in Fig. 2 is being taken into consideration), as the term which is linear in the voltage on the right side of (17):

$$\delta I_{qp}(V) = -\frac{V}{R_n}\frac{21\zeta(3)}{2\pi^2}\frac{1}{p_F^2 ld}\ln\frac{T_c^2L^2}{D(T_c-T)}. \qquad (20)$$

We wish to emphasize that this result [in contrast with (17)] applies to both gapless superconductors and superconductors which do have a gap in the excitation spectrum. The only condition involved here is that the typical energies of the electron-electron interaction in the Cooper channel be small in comparison with the temperature.[1)]

5. ASYMMETRIC JUNCTION

In this section of the paper we consider a junction between two superconductors, one of which is gapless and near its critical temperature, while the other has a gap in its excitation spectrum, and fluctuations in it are suppressed either by the three-dimensional nature of the electrode or because the temperature is sufficiently far from the critical temperature. The correction to the tunneling current is again represented by the single diagram in Fig. 2a. In the corresponding analytic expression, (6), $\hat{g}_I(\varepsilon_n+\omega_\nu)$ is now the Green's function of a superconductor which does not contain magnetic impurities:

$$\hat{g}_I(\varepsilon_n+\omega_\nu) = \pi(-i(\varepsilon_n+\omega_\nu)\hat{\sigma}_0+\Delta_I\hat{\sigma}_1)((\varepsilon_n+\omega_\nu)^2+\Delta_I^2)^{-1/2},$$

where Δ_I is the gap in the excitation spectrum. In place of (9) we now find

$$\delta I_{qp}(V) = \frac{\pi}{eR_n}\operatorname{Im}\left\{\int (d\mathbf{q})\, T \times \sum_{\varepsilon_n}\operatorname{Sgn}\varepsilon_n \frac{\varepsilon_n+\omega_\nu}{[(\varepsilon_n+\omega_\nu)^2+\Delta_I^2]^{1/2}}C(\mathbf{q},\varepsilon_n)\right\}^R_{i\omega_\nu\to eV}. \qquad (21)$$

Carrying out the analytic continuation, we find the expression

$$\delta I_{qp}(V) = -\frac{1}{4eR_n}\operatorname{Re}\int (d\mathbf{q})\int_{\Delta_I}^{\infty}\frac{\varepsilon\, d\varepsilon}{(\varepsilon^2-\Delta_I^2)^{1/2}} \times\left\{-\operatorname{th}\frac{\varepsilon+eV}{2T}[C^A(-\varepsilon-eV,\mathbf{q})+C^R(-\varepsilon-eV,\mathbf{q})]\right.$$
$$+\operatorname{th}\frac{\varepsilon-eV}{2T}[C^A(\varepsilon-eV,\mathbf{q})+C^R(\varepsilon-eV,\mathbf{q})]$$
$$\left.+2\operatorname{th}\frac{\varepsilon}{2T}[C^A(-\varepsilon-eV,\mathbf{q})-C^A(\varepsilon-eV,\mathbf{q})]\right\}. \qquad (22)$$

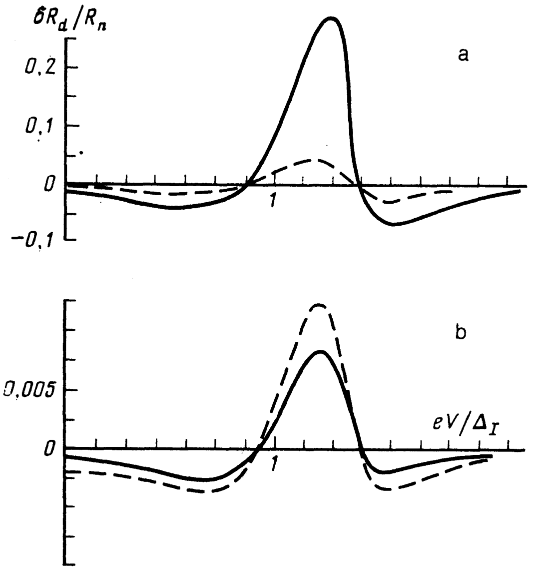

FIG. 5. Corrections to the differential resistance of an asymmetric junction between two superconductors versus the voltage applied to the junction. Solid line—Fluctuation correction; dashed line—component stemming from the presence of paramagnetic impurities in the fluctuating electrode. a) $p_F^2ld = 500$, $(T_c-T)/T_c = 0.04$, $\Gamma/\Delta_I = 0.05$; b) $p_F^2ld = 7500$, $(T_c-T)/T_c = 0.08$, $\Gamma/\Delta_I = 0.1$

From this point on the calculations are carried out for two limiting cases: 1) The critical temperature of electrode I, T_{cI}, is approximately equal to T_{cII}, so that under the condition $T_{cII}-T \ll T_{cII}$ the relation $\Delta_I/T \ll 1$ holds simultaneously; 2) $T_{cI} \gg T_{cII}$ and thus $\Delta_I/T \gg 1$.

In the first case, low voltages, $eV \ll T$, are of interest. The hyperbolic tangents in the integrand in (22) can then be approximated by their arguments. As a result, (22) becomes

$$\delta I_{qp}(V) = -\frac{V}{R_n}\frac{3\pi}{(2)^{1/2}p_F^2 ld}T\Delta_I^2\frac{\partial}{\partial(\Delta_I^2)} \times \operatorname{Re}\int_a^{\infty} dx\left(\frac{1}{x}+\frac{1}{x+b}\right)[4(\Delta_I^2-(eV)^2)+(2x+\Gamma)^2$$
$$+4ieV(2x+\Gamma)^2]^{-1/2}. \qquad (23)$$

The integral in (23) can be expressed in terms of elementary functions, but we will not reproduce the entire result here because of its length. Figure 5 shows corresponding diagrams for various relations among the parameters DL^{-2}, T_c-T, Γ, and Δ. We can also write a few asymptotic expressions for the most interesting case, $\Delta_I/\Gamma \gg 1$:

$$\delta I_{gp}(V) = \frac{V}{R_n}\frac{3\pi}{4p_F^2ld}\begin{cases}\dfrac{T\Delta_I^2}{[\Delta_I^2-(eV)^2]^{3/2}}\ln\dfrac{[\Delta_I^2-(eV)^2]^2L^2}{\Delta_I^2D(T_c-T)}, & eV<\Delta_I;\ \Delta_I^2-(eV)^2\gg\Gamma\Delta_I,\\[2ex] -\dfrac{1}{\sqrt{2}}\dfrac{T}{\Gamma}\left(\dfrac{\Delta_I}{\Gamma}\right)^{1/2}\ln\dfrac{\Gamma^2L^2}{D(T_c-T)}, & eV\sim\Delta_I;\ |(eV)^2-\Delta_I^2|\ll\Gamma\Delta_I,\\[2ex] -\dfrac{\pi T\Delta_I^2}{(eV)^3}-\dfrac{3T\Delta_I^2\Gamma}{2(eV)^4}\ln\dfrac{(eV)^2L^2}{D(T_c-T)}, & eV\gg\Delta_I.\end{cases} \qquad (24)$$

In the opposite limit, $\Delta_I \gg T$, we use the approximation $\tanh(x/T \approx \mathrm{Sgn}\, x$. After some lengthy but elementary calculations we find the following asymptotic expressions [the current-voltage characteristic begins with $eV = \Delta_I$ if we ignore the terms which contain a factor of $\exp(-\Delta_I/T)$]:

a) For $(eV)^2 - \Delta_I^2 \ll TeV$,

$$\delta I_{qp}(V) = \frac{V}{R_n}\frac{3}{p_F^2 ld}\begin{cases} \dfrac{\pi(2)^{1/2}T^2}{\Gamma^{3/2}(eV)^{1/2}}\ln\dfrac{\Gamma^2 L^2}{D(T_c-T)}, & (eV)^2-\Delta_I^2 \ll \Gamma eV, \\ \dfrac{2T^2 eV}{[(eV)^2-\Delta_I^2]^{3/2}}\ln\dfrac{(eV)^2-\Delta_I^2}{\Gamma eV}\ln\dfrac{\Gamma[(eV)^2-\Delta_I^2]L^2}{D(T_c-T)eV}, & \\ \qquad (eV)^2-\Delta_I^2 \gg \Gamma eV. & \end{cases} \tag{25}$$

b) For $TeV \ll (eV)^2 - \Delta_I^2 \ll \Delta_I^2$,

$$\delta I_{qp}(V) = \frac{V}{R_n}\frac{6}{p_F^2 ld}\frac{T^2 eV}{[(eV)^2-\Delta_I^2]^{3/2}}\ln\frac{(eV)^2-\Delta_I^2}{\Gamma eV}\ln\frac{T^2L^2}{D(T_c-T)}. \tag{26}$$

6. DISCUSSION OF RESULTS

These results show that superconducting fluctuations give rise, even in first order in the barrier transparency, to corrections to the tunneling current which are very nonlinear functions of the voltage applied to the junction. In analyzing these corrections one should note that in the absence of electron-electron coupling the current-voltage characteristic of a tunnel junction would have deviations from Ohm's law.

The origin of all these nonlinearities can be explained in the following way. The representation which we used for the tunneling current [see (1), (2)] is equivalent to a representation of this current through a convolution of the state densities of the two electrodes of the junction and a Fermi distribution function[12]:

$$I \sim \int N_1(\varepsilon)N_2(\varepsilon+eV)\left[\mathrm{th}\frac{\varepsilon+eV}{2T} - \mathrm{th}\frac{\varepsilon}{2T}\right]d\varepsilon.$$

Condition (19), imposed on possible corrections to the state density, means that these corrections reduce to the formation of regions of relatively close spacing and regions of relatively wide spacing in the equidistant ladder of levels of a normal metal [corresponding to $N(\varepsilon) = \mathrm{const}$], at a fixed total number of levels. The correction to Ohm's law in the case of the junction between a superconductor and a normal metal stems from the circumstance that when there is a slight shift of the levels, on the average, up the energy scale the population of the levels (which is determined by a Fermi distribution function) decreases, so the total current also decreases. The scale value for the manifestation of this effect is evidently $eV \sim T$. When, on the other hand, the state density is not constant in either of the electrodes, the tunneling current is influenced by an "interference" effect in addition to the effect described above. If the region of relatively close spacing of levels in one of the electrodes corresponds to a region of relatively wide spacing in the other when the origin of the energy scale is shifted by eV, the effective number of quasiparticles participating in the charge transport decreases, with the result that the total current decreases. When the voltage is changed, the picture may reverse. This effect is responsible for the nonlinearity of the current-voltage characteristic on voltage scales much smaller than T.

As usual, fluctuation effects can be observed only in thin and dirty electrodes, at temperatures close to the critical temperature for at least one of them.

We have been able to take account of how the fluctuations in the phase and modulus of the order parameter influence the quasiparticle component of the current at temperatures below T_c for S_1–N–S_2 junctions only in the case in which the fluctuations occur in an electrode which is in a gapless state. Accordingly, the positions of the extrema on its current-voltage characteristic are determined by the parameters τ_s^{-1} and Δ_I, and they do not depend on the proximity of the temperature to T_{c2}. As we mentioned earlier, the fluctuation corrections which we have calculated should actually be compared with the current-voltage characteristic corresponding to a superconducting tunnel junction "spoiled" by a sufficiently high concentration of paramagnetic impurities in this electrode. The deviation of this current-voltage characteristic from Ohm's law (we will call it the "paramagnetic contribution" to the tunneling current) turns out to be exceedingly similar to the deviation caused by fluctuation corrections. However, while the fluctuation component of the tunneling current increases in magnitude as T_c is approached, the paramagnetic component decreases. The results of corresponding numerical calculations of both components, for several values of the parameters $p_F^2 ld$, Γ, and $(T_c - T)/T_c$, for symmetric and asymmetric junctions, are shown in Figs. 4 and 5.

It can be seen from these figures that near T_c the fluctuation component of the tunneling current dominates in sufficiently thin films. As the distance from the transition point increases, and also with increasing thickness of the electrode, the fluctuation component gradually fades away, to the point that it is indistinguishable against the background of the paramagnetic component. This striking similarity between the fluctuation and paramagnetic corrections of Ohm's law is of course not simply fortuitous. To explain it we can invoke the following arguments: In a superconductor with paramagnetic impurities, the Bose condensate of Cooper pairs does not include all of the Cooper pairs; there is some distribution of these pairs with respect to binding energy.[14] The situation is exactly the same when we take fluctuations into account; near T_c they smear out the energy distribution of Cooper pairs in the superconductor, as paramagnetic impurities do.

The analogy between the effect of superconducting fluctuations and that of paramagnetic impurities on the properties of a superconductor can be pursued. For example, it can be seen from the results of Refs. 4 and 6 that these two factors shift the critical temperature in the same way [for fluctuations, one should take the phase relaxation time to be $\tau_\varphi^{-1} \sim (T/p_F^2 ld)\ln p_F^2 ld$, as is verified by self-consistent estimates of this quantity at temperatures below the critical temperature]. The superconducting fluctuations themselves—again in a manner reminiscent of the effect of paramagnetic impurities—give rise to a finite phase relaxation time τ_φ. This circumstance makes it unnecessary to artificially introduce a pair rupture mechanism in order to cut off the divergence of the anomalous Maki-Thompson contribution to the conductivity in the case of low-dimensionality systems.

Finally, we can apparently assert that, as in the case with paramagnetic impurities and also when fluctuations of the electromagnetic field are taken into account,[15] there is always a gapless superconductivity slightly below the critical temperature (but outside the critical region) due to the fluctuational spreading of the density of one-electron states along the energy scale.

We are deeply indebted to A. A. Abrikosov, B. L. Al'tshuler, A. G. Aronov, and M. Yu. Reĭzer for numerous discussions from which emerged ideas regarding the effect of superconducting fluctuations on a one-electron phase and the nature of the superconductivity itself near the critical temperature.

APPENDIX

To calculate the functions $C^{R(A)}(q, \varepsilon)$, we transform the sum over Ω_k in (9) into a contour integral in accordance with the rule for boson frequencies:

$$T\sum_{\Omega_k} f(\Omega_k) \to \frac{1}{4\pi i}\oint d\omega \,\mathrm{cth}\frac{\omega}{2T} f(-i\omega).$$

We find

$$C(\mathbf{q}, \varepsilon_n) = T\sum_{\Omega_k}\frac{\theta[\varepsilon_n(\varepsilon_n-\Omega_k)]}{(|\varepsilon_n|+|\varepsilon_n-\Omega_k|+D\mathbf{q}^2+\Gamma)^2} \times [L_{11}(\mathbf{q},\Omega_k)+L_{22}(\mathbf{q},\Omega_k)]$$

$$= \frac{1}{4\pi i}\theta(\varepsilon_n)\left\{\int_{-\infty}^{+\infty}\mathrm{cth}\frac{z}{2T}dz\frac{1}{(2\varepsilon_n+iz+D\mathbf{q}^2+\Gamma)^2} \times [L_{11}^R(\mathbf{q},z)+L_{22}^R(\mathbf{q},z) - L_{11}^A(\mathbf{q},z)-L_{22}^A(\mathbf{q},z)] + \int_{+\infty+i\varepsilon_n}^{-\infty+i\varepsilon_n}\mathrm{cth}\frac{z}{2T}dz\frac{1}{(2\varepsilon_n+iz+D\mathbf{q}^2+\Gamma)^2}[L_{11}^R(\mathbf{q},z) + L_{22}^R(\mathbf{q},z)]\right\} + \frac{1}{4\pi i}\theta(-\varepsilon_n)$$

$$\times\left\{\int_{-\infty}^{+\infty}\mathrm{cth}\frac{z}{2T}dz\frac{1}{(-2\varepsilon_n-iz+D\mathbf{q}^2+\Gamma)^2} \times [L_{11}^R(\mathbf{q},z)+L_{22}^R(\mathbf{q},z)-L_{11}^A(\mathbf{q},z)-L_{22}^A(\mathbf{q},z)] + \int_{-\infty-i\varepsilon_n}^{+\infty-i\varepsilon_n}\mathrm{cth}\frac{z}{2T}dz \times\frac{1}{(-2\varepsilon_n-iz+D\mathbf{q}^2+\Gamma)^2}[L_{11}^A(\mathbf{q},z)+L_{22}^A(\mathbf{q},z)]\right\}. \quad (A1)$$

The second terms in braces are small in comparison with the first and can be ignored. Adopting the assumption (confirmed below) that the integral over z is dominated by the region $z \ll T$, and making use of the odd parity in z of the functions $L_{11}^R(z) - L_{11}^A(z)$ and $L_{22}^R(z) - L_{22}^A(z)$, we find the following results for the analytic continuations of $C^R(\mathbf{q}, \varepsilon)$ and $C^A(\mathbf{q}, \varepsilon)$:

$$C^{R(A)}(\mathbf{q},\varepsilon) = -\frac{\partial}{\partial(D\mathbf{q}^2+\Gamma)}\frac{T}{2\pi i}\int_{-\infty}^{+\infty}dz\frac{D\mathbf{q}^2+\Gamma\mp 2i\varepsilon}{(D\mathbf{q}^2+\Gamma\mp 2i\varepsilon)^2+z^2} \times\frac{L^R(\mathbf{q},z)-L^A(\mathbf{q},z)}{z}, \quad (A2)$$

where

$$L^{R(A)}(\mathbf{q},z) = L_{11}^{R(A)}(\mathbf{q},z)+L_{22}^{R(A)}(\mathbf{q},z).$$

We need explicit expressions for the analytic continuations of $L_{11}^{R(A)}(\mathbf{q},z)$ and $L_{22}^{R(A)}(\mathbf{q},z)$. The results of Ref. 6 cannot be applied directly here, since the matrix fluctuation propagator calculated there in terms of Matsubara frequencies contains a Kronecker delta $\delta_{\Omega_k,0}$, and because of this the analytic continuation from imaginary frequencies is not single-valued. An analytic continuation of a matrix fluctuation operator was derived in Ref. 5, but the representation used for it was not the best. The correspondence between the representations used in Refs. 5 and 6 can be established with the help of the rotation matrix

$$S = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ -i & 0 & 0 & -i \\ 0 & -1 & -1 & 0 \end{pmatrix}.$$

For the matrix components $L_{ij}^{R(A)}(\mathbf{q},z)$ of interest here we find

$$L_{11}^{R(A)}(\mathbf{q},z) = -\frac{8T}{\pi N(0)}\left\{\frac{7\zeta(3)\Delta^2}{4\pi^2T^2} + D\mathbf{q}^2\mp iz\left(1+\frac{\gamma}{D\mathbf{q}^2\mp iz}\right)\right\}^{-1},$$

$$L_{22}^{R(A)}(\mathbf{q},z) = -\frac{8T}{\pi N(0)}\left\{D\mathbf{q}^2\mp iz\left[\frac{1}{B}+\frac{\mp iz\pi^2\gamma\rho}{D\mathbf{q}^2(\mp iz+D\mathbf{q}^2)}\right]^{-1}\right\}^{-1},$$

$$B = \frac{2}{\pi^2}\left[1+\frac{2\gamma}{\mp iz+D\mathbf{q}^2}\right]^{-1/2}\psi'\left(\frac{1}{2}+\rho\left[1+\frac{2\gamma}{\mp iz+D\mathbf{q}^2}\right]^{1/2}\right). \quad (A3)$$

Here we have introduced $\gamma = \Delta^2\tau_s$, $\rho = 1/2\pi T\tau_s$, and

$\psi'(x)$; the latter is the second logarithmic derivative of the Γ function. Since the integrand in (A2) falls off rapidly as $z\to\infty$, the integration contour can be closed by an infinitely remote semicircle in the upper half-plane, and we can make use of the theorem of residues:

$$C^{R(A)}(\mathbf{q},\varepsilon)=-\frac{\partial}{\partial(D\mathbf{q}^2+\Gamma)}\frac{T}{2\pi i}\Big\{2\pi i\sum_k \operatorname{res}\Big[-\frac{L^A(\mathbf{q},z)}{z}\times\frac{D\mathbf{q}^2+\Gamma\mp 2i\varepsilon}{(D\mathbf{q}^2+\Gamma\mp 2i\varepsilon)^2+z^2},z_k\Big]+J_1\Big\}, \quad \text{(A4)}$$

where the quantities z_k are the poles of the integrand in the upper half-plane, and $\operatorname{res}[f(z),z_k]$ is the residue of the function $f(z)$ at the point z_k. The quantity J_1 is the sum of the integrals along those boundaries of the region of analyticity of the integrand which are not part of the real axis.

A calculation from (A4) yields

$$C^{R(A)}(\mathbf{q},\varepsilon)=-\frac{8T}{\pi N(0)}\frac{T}{(\Gamma+2D\mathbf{q}^2\mp 2i\varepsilon)^2}\Big\{\frac{1}{b+D\mathbf{q}^2}+\frac{1}{D\mathbf{q}^2}\Big\}. \quad \text{(A5)}$$

[1]We would like to take this opportunity to correct an error in Ref. 2. The calculation method which was used there, in which the summation over Ω_k was limited to the term with $k=0$, yielded the correct form of the entire current-voltage characteristic, with its maxima and minima, and also the correct asymptotic behavior at $eV\gtrsim T-T_c$, in accordance with the arguments presented above. For $eV\ll T-T_c$, however, a logarithmic divergence arose and was cut off through the introduction of a pair rupture mechanism. Actually, the correct analytic continuation automatically leads to a finite value of $\delta R_{fl}^{(2)}(0)$, and the asymptotic behavior at small V is

$$\frac{\delta R_{fl}^{(2)}(V)}{R_n}=\frac{0.496}{(p_F^{\,2}ld)^2}\left(\frac{T_c}{T-T_c}\right)^3\left[-1+2.78\left(\frac{eV}{T-T_c}\right)^2\right].$$

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