Left-right symmetric superstring supergravity

M. V. Burova and K. A. Ter-Martirosyan

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A left-right (L–R) symmetric model of four-dimensional supergravity with $SO(10)$ gauge group, obtained as the low-energy limit in superstring theory, is considered. The spectrum of gauge fields and their interactions agree with the Weinberg-Salam theory and the model contains additional heavy bosons $W_R^{\pm}$ and Z'_{μ}. Besides the $N_g = 3$ generations of 16-plets the $SO(10)$ model includes "fragments" of such generations, which play the role of Higgs particles, and scalar chiral fields, whose number exceeds the number of generations by one. As a result each generation's neutrino acquires a stably small Majorana mass. It is shown that the scalar fields potential gives rise to spontaneous breaking of the $SU(2)_R$ group and the L–R symmetry, resulting in the appearance of the standard Weinberg-Salam theory in the low-energy region. However reasonable values for the mass M_x of the X boson and for $\sin^2\theta_W$ (θ_W being the Weinberg angle) can only be obtained in the model for a high mass scale $M_R \sim 10^{10}$–10^{12} GeV for the breaking of the right group $SU(2)_R$.

1. INTRODUCTION

The Lagrangian for low-energy supergravity (SUGRA) in four-dimensional space includes[1] three unknown functions of the scalar components of chiral superfields z^i: the Kähler function $G(z^i, z_i^+)$, the superpotential W and the gauge function $\chi(z^i)$, which determines the gauge constant $1/g_0^2 = \langle \mathrm{Re}\,\chi(z^i)\rangle_0$. These functions were found in the so-called no-scale theory in the work of the CERN group.[2] Later[3,4] it was shown that they can be obtained in a certain approximation in the low-energy limit from the ten-dimensional SUGRA,[5] which is the point-field limit of heterotic[6] superstrings. These functions have the form

$$G(z^i, z_i^+) = -\ln(s+s^+) - 3\ln(z+z^+ - {}^1/_3 y^i y_i^+) + \ln|\tilde{W}(z^i)|^2,$$
$$\tilde{W}(z^i) = [h_0 - \beta_0 \exp(-\gamma_0 s)] + W(y^i), \quad W(y^i) = h_{ijk} y^i y^j y^k, \quad (1)$$
$$\chi(z^i) \approx s.$$

The scalar fields $z^i = z^i(x)$ in Eq. (1) include: a) the fields in the gravitational sector $z(x) = Z(x)/M_P$, $s(x) = S(x)/M_P$, sometimes called dilaton and compacton, which have vacuum expectation values (vev) of the order of the Planck mass $M_P \approx 10^{19}$ GeV: $\langle s\rangle_0 \sim \langle z\rangle_0 \sim 1$; b) the fields $y^i(x) = Y^i(x)/M_P$ of physical particles, i.e., Higgs scalars and scalar components of quarks and leptons, for which $\langle Y^i(x)\rangle \sim M_W \ll M_P$, where $M_W \sim 10^2$ GeV is W-boson mass of the Weinberg-Salam theory.

In low-energy SUGRA one encounters several difficulties and unclear points. The mechanism by which the superpartner of the gauge field—the gaugino—acquires mass is unclear. The various types of gauginos: photino, gluino, Z^0- and W-ino all acquire at the scale $P \sim M_P$ the same mass[1)] $M_{1/2} \ll M_P$, which is vanishingly small compared to the Planck mass due to supersymmetry violation in the gravitational sector, while at the same time the gauge fields themselves remain massless. As a result supersymmetry violation is transferred to the sector of physical fields, where masses of all fields and vev's of all scalar Higgs fields are found to be proportional to the same quantity $M_{1/2}$.

In this manner, aside from the mass $M_{1/2}$ the low-energy SUGRA contains only dimensionless parameters: Yukawa constants h_{ikj} and the parameters h_0, β_0 and γ_0 of the superpotential $\tilde{W}$ in Eq. (1), which determine the vacuum values $s_0 = \langle s\rangle_0 = 1/g_0^2$ of the field $s(x)$ that minimize the corresponding potential $V(s) = |\partial\tilde{W}/\partial s|^2$.

So far we have no theory that describes compactification of superstrings and gives unique answers to questions about the gauge group and the structure of the matter fields after compactification in the $d = 4$ space at energies small compared to the Planck scale: $P \ll M_P$. It is not even clear whether one can do altogether without compactification and formulate[7] string theory directly in $d = 4$ space. Nevertheless in recent years a number of authors[8,9] discussed different versions of low-energy SUGRA models, compatible with current ideas about compactification and having direct relation to physical reality. Noteworthy among such models are the so-called left-right (L–R) symmetric theories (which were already studied in the seventies in the framework of grand unified theories[10]) with the following gauge groups:

1) the E_6 group,[8,9,10] broken by the Wilson-Hosotani loops[12] below the scale $M_X \lesssim M_P$ to the group

$$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{Y_L} \times U(1)_{Y_R}, \quad (2)$$

2) the simpler $SO(10)$ group,[8,13] broken below M_X to the symmetry

$$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}. \quad (3)$$

The breaking of the initial group proceeds in both cases without decrease in the rank $r = 6$ for E_6 and $r = 5$ for $SO(10)$.

The model with E_6 symmetry is more complicated, contains additional particles in each of the $N_g = 3$ generations (since the fundamental multiplet of matter fields in E_6 is a 27-plet) and results in too high values for M_X and $\sin^2\theta_W$. Below we consider the model based on $SO(10)$,[8,13] which is more economical and simpler.

All lepton and quark fields of one generation are united in this model in one 16-plet, which is the fundamental representation of the $SO(10)$ group. Beside the full $N_g = 3$ generations of 16-plets the theory may contain[13,14] additional fields of Higgs particles in the form of "fragments" of 16-

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TABLE I. The quantities I_{3L}, I_{3R} and Y_{B-L}.

	u^i	d^i	d^{ci}	u^{ci}	ν^i / N / $\overline{N^c}$	e^i / E / $\overline{E^c}$	e^{ci} / E^c / $\overline{E}$	ν^{ci} / N^c / $\overline{N}$	H_u^+	H_u^0	H_d^0	H_d^-
I_{3L}	$1/2$	$-1/2$	0	0	$1/2$	$-1/2$	0	0	$1/2$	$-1/2$	$1/2$	$-1/2$
I_{3R}	0	0	$1/2$	$-1/2$	0	0	$1/2$	$-1/2$	$1/2$	$1/2$	$-1/2$	$-1/2$
Y_{B-L}	$1/6$	$1/6$	$-1/6$	$-1/6$	$-1/2$	$-1/2$	$1/2$	$1/2$	0	0	0	0
Y^2_{B-L}	$1/36$	$1/36$	$1/36$	$1/36$	$1/4$	$1/4$	$1/4$	$1/4$	0	0	0	0

plets and "antifragments" of $\overline{16}$-plets, as well as in the form of fragments of the vector multiplet 10 containing the Higgs field $\mathscr{H}$. All these fields can be represented as follows:

$$16^i=\left[q_L{}^i=\begin{pmatrix}u\\d\end{pmatrix}_L^i,\ \left|l_L{}^i=\begin{pmatrix}\nu\\e\end{pmatrix}_L^i\right\}SU(2)_L,\ \underbrace{\begin{matrix}q_L^{ic}=(d^c,u^c)_L{}^i\\ l_L{}^{ic}=(e^c,\nu^c)_L{}^i\end{matrix}}_{SU(2)_R}\right],$$

$$16^0=\left[\begin{matrix}L_0=\begin{pmatrix}N\\E^-\end{pmatrix}\Big\}SU(2)_L\\ L_0{}^c=\underbrace{(E^cN^c)}_{SU(2)_R}\end{matrix}\right],$$

$$\overline{16}^0=\left[\begin{matrix}\overline{L}_0{}^c=\begin{pmatrix}\overline{E}^c\\\overline{N}^c\end{pmatrix}\Big\}SU(2)_L\\ \overline{L}_0=\underbrace{(\overline{N},\overline{E}^-)}_{SU(2)_R}\end{matrix}\right],\tag{4}$$

$$10^0=\left[\mathscr{H}=\underbrace{\begin{pmatrix}H_u{}^+H_d{}^0\\H_u{}^0H_d{}^-\end{pmatrix}}_{SU(2)_R}\Big\}SU(2)_L\right],$$

where the superscript i denotes the generation (with $i=0$ corresponding to the fragment generation) and the braces indicate the multiplets with respect to the $SU(2)_L$ and $SU(2)_R$ groups. The values of left and right isospin I_{3L}, I_{3R} of the corresponding $SU(2)_L$ and $SU(2)_R$ groups, and of the hypercharge Y_{B-L} of the $U(1)_{B-L}$ group for each of the particles in Eq. (4) are given in Table I. In this notation the electric charge in units of the proton charge e_p is given for each of the particles in Eq. (4) by

$$Q=(Y_{B-L}+I_{3R})+I_{3L}=Y+I_{3L},\tag{5}$$

where $Y=Y_{B-L}+I_{3R}$ is the usual hypercharge of the Weinberg-Salam model.

In SUGRA models based on the theory of compactified superstrings the breaking of the basic E_6 group [or $SO(10)$ group in the case considered here[4]] down to the symmetries (2), (3) proceeds at a high scale $P\sim M_X\lesssim M_P$ via the Hosotani mechanism. It is well-known[12,8,14] that in this process only those particles from the fragments (in our case $\underline{16}^0$ and $\overline{16}^0$, as well as $\underline{10}^0$) remain massless, i.e., do not acquire a mass $m\sim M_P$, whose fields are invariant under the action of the operator $\hat{W}$, generated by the Wilson-Hosotani loop. It is these fields only that remain[12] in the fragments (4). For this reason, in part, the fragments $\underline{16}^0$ and $\overline{16}^0$, do not contain the quark-like fields $\binom{U_0}{D_0}$ or (D_0^c, U_0^c), and the fragment $\underline{10}^0$ does not contain the color fields g_0 and g_0^c with charge $\pm 1/3$; after compactification these fields become heavy and fall out from the spectrum.

After breaking the $SO(10)$ symmetry we are left with the symmetry (3). Correspondingly the covariant derivative contains besides gluons and the fields of $W_{\mu L^a}$ and $W_{\mu R^a}$, corresponding to the $SU(2)_L$ and $SU(2)_R$ groups, one more neutral gauge field $B_\mu(x)$ of the $U(1)_{B-L}$ group:

$$iD_\mu=i\partial_\mu+\hat{\Lambda}_\mu,$$

$$\hat{\Lambda}_\mu=g_0{}'Y_{B-L}B_\mu+(g_2/2)(\tau_L{}^aW_{\mu L}{}^a+\tau_R{}^aW_{\mu R}{}^a),\tag{6}$$

where the gluon contribution is omitted and $I_{aL}=\tau_L^a/2$, $I_{aR}=\tau_R^a/2$, $a=1,2,3$ are the generators of $SU(2)_{L,R}$ ($\tau^a_{L,R}$ are Pauli matrices). The Lagrangian of the scalar sector of the model, i.e., of the scalar components $y_S^k(x)$ of the quarks, leptons, Higgs scalars, etc., will have the form

$$\mathscr{L}_S=\sum_k|iD_\mu y_S{}^k(x)|^2+V(y_S{}^k),\tag{7}$$

where $V(y_S^k)$ is the potential of the scalars; with the Kähler function $G(z^i,z_i^+)$ of the form (1) it is given by $V=|\partial W/\partial y_S^k|^2+V_D$, where V_D stands for the familiar D-terms of the potential (see Sec. 3).

Beside the fields (4), including the $N_g=3$ 16-plet generations, the $\underline{16}^0$ fragments, and $\overline{16}^0$ antifragments and fragments of the 10^0-plet of Higgs fields, the theory will need also neutral fields $\varphi^i(x)$–$SO(10)$ group scalars, one from each generation (including $i=0$). These fields are very important, since it is they that make possible Yukawa couplings not only in the form $h_{ij}16^i10^016^j$ of the 16^0 fragment, but also in the form $f_{ij}16^i\,\overline{16}^0\varphi^j$ of the $\overline{16}^0$ antifragment, where the corresponding $i=j=0$ term $f_0\,16^0\,\overline{16}^0\varphi^0$ is responsible for the large mass $m_0=f_0\langle\varphi^0\rangle\gg M_W$ of the experimentally unobserved fragment particles.

The superpotential $W(y^i)$ in (1) (with $y^i=16^i,16^0,\overline{16}^0,10^0,\varphi^i$) has the form

$$W(y^i)=h_{ij}16^i16^j10^0+f_{ij}16^i\overline{16}^0\varphi^j+{}^1/_2\zeta_{ij}\varphi^i\varphi^0\varphi^j,\tag{8}$$

where h_{ij}, f_{ij}, ζ_{ij} are Yukawa coupling constants and $i,j=0,1,2,3$. Upon decreasing the momentum P of the particles the fields of the scalars $\widetilde{N}^c$, $\widetilde{N}$, H_u^0, H_d^0, φ^0 acquire nonzero vev due to the couplings of the form $h_{00}16^016^010^0+f_{00}16^0\,\overline{16}^0\varphi^0+\frac{1}{2}\zeta_{00}(\varphi^0)^3$, with the physically interesting case being when

$$N_0{}^c\gg v_u\sim v_d\gg N_0,\tag{9}$$

where

$$N_0{}^c=\langle\widetilde{N}^c\rangle_0,\ N_0=\langle\widetilde{N}\rangle_0,\ v_u=\langle H_u{}^0\rangle,\ v_d=\langle H_d{}^0\rangle$$

and to simplify the notation the "tilde," denoting the scalar component of the superfield, and the brackets $\langle\ \dots\ \rangle_0$ for the vev, are omitted here and below. The scalar components of the antifragments also acquire vev

$$\bar{N}_0=N_0^c, \quad \bar{N}_0^c=N_0, \tag{10}$$

corresponding to the minimum of the contribution of V_D of the D-terms, with $V_D^{\min}=0$.

The mechanism for the appearance of such vev, proposed in the papers of Ref. 15, is connected with the form of the equations of the renormalization group (see Appendix, Sec. 1) and is described in detail in Sec. 3. As the particle momentum P decreases, first the group $SU(2)_R$ and the L-R symmetry of the theory are broken at $P\sim M_R \sim N_0^c$ ($M_R = M_{W_R}$ is the mass of the $W_{\mu R\pm}$ bosons), and one is left with the standard $SU(2)_L\times U(1)$ group, whose breaking at $P\sim M_{W_L}\sim 10^2$ GeV results from the fields H_u^0 and H_d^0 acquiring the vev's v_u, v_d: $v_u\sim v_d \sim M_W$.

In this fashion the set of fields (4) is minimal and for small $P\lesssim M_W$ gives rise to the standard "electroweak" Weinberg-Salam theory.

2. STRUCTURE OF THE *L-R* SYMMETRIC THEORY

2.1. The simplest and longest known structure[10] is that of the sector of charged W-bosons: $W_{\mu L\pm} = (W_{\mu L^1} \pm W_{\mu L^2})/\sqrt{2}$ and $W_{\mu R\pm}$. The mass matrix for these fields is obtained by extracting from the kinetic part of the Lagrangian $\mathscr{L}_S$, Eq. (7), the terms $\mathscr{L}_{M_{S\pm}}$ that are quadratic in these gauge fields and keeping in it just the vev of the neutral scalars (9). This gives

$$\mathscr{L}_{MS^\pm}=(g_2^2/4)\{[v^2+(N_0^c)^2](W_{\mu R}^i)^2+(v^2+N_0^2)(W_{\mu L}^i)^2 +2v_uv_dW_{\mu L}^iW_{\mu R}^i\},$$

where $i=1,2$ and $v^2=v_u^2+v_d^2$. For $N_0^c \gg v \gg N_0$ one of the two eigenvalues of the corresponding mass matrix (in which terms $\sim N_0 \ll v$ were ignored),

$$M_{1,2}^2=(g_2^2/4)\{(N_0^c)^2/2+v^2\mp[((N_0^c)^2/2)^2+4v_u^2v_d^2]^{1/2}\},$$

is very close to the mass of the W-boson of the Weinberg-Salam theory

$$M_1=M_{W_L}\approx(g_2v/2)[1-2v_u^2v_d^2/v^2(N_0^c)^2],$$

while the other,

$$M_2=M_{W_R}=g_2N_0^c/2+O(v),$$

is very large. The mixing angle χ_W of the $W_{\mu L\pm}$ and $W_{\mu R\pm}$ bosons is very small: $\sin 2\chi_W \approx 2\chi_W \approx 2v_uv_d/(N_0^c)^2 \ll 1$.

The sector of the neutral fields is not much more complicated.

2.2. Contribution of the B_μ, $W_{\mu L}^3$ and $W_{\mu R}^3$ fields to the covariant derivative (6):

$$\hat{\Lambda}_\mu^0=g_0'Y_{B-L}B_\mu+(g_2/\sqrt{2})[(I_{3L}+I_{3R})W_{\mu+}^0+(I_{3L}-I_{3R})W_{\mu-}^0],$$

where $W_{\mu\pm}^0=(W_{\mu L}^3 \pm W_{\mu R}^3)/\sqrt{2}$, can be put into the more convenient form

$$\hat{\Lambda}_\mu^0=eQA_\mu+(\bar{g}_0/\sqrt{2})[I_{3L}+I_{3R}-Qs_0^2)Z_{\mu 0} +c_0(I_{3L}-I_{3R})W_{\mu-}^0], \tag{11}$$

where $Y_{B-L}=Q-(I_{3L}+I_{3R})$ and

$$A_\mu(x)=c_0B_\mu+s_0W_{\mu+}^0,\quad Z_{\mu 0}=-s_0B_\mu+c_0W_{\mu+}^0$$

are the standard combinations of the B_μ and $W_{\mu+}^0$ fields, as in the Weinberg-Salam theory with

$$g_2=\bar{g}_0c_0,\quad \sqrt{2g_0}'=\bar{g}_0s_0,\quad \bar{g}_0=(g_2^2+2g_0'^2)^{1/2},$$

where $e=g_0'g_2/\bar{g}_0$ is the charge of the electron, $A_\mu(x)$ is the electromagnetic field potential and $e^{-2}=(g_0')^{-2}+2g_2^{-2}$.

The application of $\hat{\Lambda}_\mu^0$ to the scalar fields $\tilde{N}^c$, H_u^0 and H_d^0 possessing nonzero vev gives

$$\begin{aligned}-\langle\hat{\Lambda}_\mu^0\tilde{N}^c\rangle_0&=(\bar{g}_0/\sqrt{2})(Z_{\mu 0}-c_0W_{\mu-}^0)N_0^c,\\ -\langle\hat{\Lambda}_\mu^0H_u^0\rangle_0&=(\bar{g}_0/\sqrt{2})c_0W_{\mu-}^0v_u,\\ \langle\hat{\Lambda}_\mu^0H_d^0\rangle_0&=(\bar{g}_0/\sqrt{2})c_0W_{\mu-}^0v_d,\end{aligned} \tag{12}$$

where $\langle\tilde{N}^c\rangle_0=N_0^c$ and where the contribution of the field $\tilde{N}_0(x)$ was ignored since it is very small as a consequence of the relations (9). Substitution of these values into the part of the Lagrangian of the scalars that is quadratic in the gauge fields,

$$\mathscr{L}_{M_Z^0}=\sum_k\langle\hat{\Lambda}_\mu^0\tilde{y}_S^k\rangle_0^2,$$

shows that the field of the heavy Z_μ' boson consists of a linear combination $Z_{\mu 0}-c_0W_{\mu-}^0$, while the standard Z-boson is the combination $c_0Z_{\mu 0}+W_{\mu-}^0$, which is orthogonal to it. Therefore the fields of these bosons normalized to unity ($Z_\mu^2=Z_\mu'^2=1$) are

$$Z_\mu'=(-Z_{\mu 0}+c_0W_{\mu-}^0)(1+c_0^2)^{-1/2},\quad Z_\mu=(c_0Z_{\mu 0}+W_{\mu-}^0)(1+c_0^2)^{-1/2}.$$

In terms of these fields $\hat{\Lambda}_\mu^0$ is determined in a form analogous to that of the Weinberg-Salam theory:

$$\hat{\Lambda}_\mu^0=eQA_\mu+\bar{g}(I_{3L}-Qs^2)Z_\mu+g_2a_RZ_\mu', \tag{13}$$

where

$$a_R=\frac{(2c^2-1)^{1/2}}{c}I_{3R}-\frac{c}{(2c^2-1)^{1/2}}Y_{B-L}$$

does not contain I_{3L}; here

$$s=s_0/\sqrt{2}=g'/\bar{g},\quad c=[(1+c_0^2)/2]^{1/2}=g_2/\bar{g},$$

$$\bar{g}=(g'^2+g_2^2)^{1/2},\quad e=g'g_2/\bar{g},$$

therefore, in particular, $e^{-2}=(g')^{-2}+g_2^{-2}=(g_0')^{-2}+2g_2^{-2}$, i.e., $(g')^{-2}=(g_0')^{-2}+g_2^{-2}$.

Applying $\hat{\Lambda}_\mu^0$ in this form to the scalar fields $\tilde{N}^c$, H_u^0 and H_d^0 (or expressing the fields $Z_{\mu 0}$ and $W_{\mu-}^0$ in terms of Z_μ and Z_μ') we obtain

$$\langle\hat{\Lambda}_\mu^0\tilde{N}^c\rangle_0=-(\bar{g}/2c_0)cZ_\mu'N_0^c,\quad \langle\hat{\Lambda}_\mu^0H_u^0\rangle_0=-(\bar{g}/2)(Z_\mu-c_0Z_\mu')v_u,$$

$$\langle\hat{\Lambda}_\mu^0H_d^0\rangle_0=(\bar{g}/2)(Z_\mu-c_0Z_\mu')v_d,$$

with the sum of these squares, $\mathscr{L}_{M_{Z^0}}$, being the matrix of the squared masses of the fields Z_μ and Z_μ':

$$\mathscr{L}_{M_Z^0}=\frac{\bar{g}^2}{2}(Z_\mu, Z_\mu')\begin{pmatrix} v^2 & -c_0v^2\\ -c_0v^2 & (c^2N_0^c/c_0)^2+c_0^2v^2\end{pmatrix}\begin{pmatrix}Z_\mu\\ Z_\mu'\end{pmatrix},$$

where $v^2=v_u^2+v_d^2$ and where, in view of (9), the contribution due to $\langle\hat{\Lambda}_\mu\tilde{N}\rangle_0=(\bar{g}/2)(Z_\mu+s^2/c_0Z_\mu')N_0$ was omitted since the vev of the field satisfies $N_0\ll v$. The eigenvalues of this matrix

$$M_Z^2=\bar{g}^2v^2/4+O(v^4/(N_0^c)^2),\quad M_{Z'}^2=(\bar{g}^2/4)(c^2N_0^c/c_0)^2+O(v^2)$$

determine the masses of the physical fields Z_{phys} and Z'_{phys} and correspond to a very small angle $\chi^0_{Z_0} \approx c_0^3 v^2/c^4 (N_0^c)^2 \ll 1$ for the mixing of the Z'_μ and Z_μ fields.

3. THE SUPERPOTENTIAL, THE PROBLEM OF THE NEUTRINO MASS AND SPONTANEOUS BREAKING OF *L-R* SYMMETRY

3.1. The superpotential (8) contains many terms due to the presence in the theory of generations of particles, fragments of generations and singlets φ_i. Separating in (8) the terms with $i = j = 0$ we write it in the form

$$W = W_0 + \sum_{ij} W_{ij},$$

where

$$W_0 = h_0(L_0 \mathscr{H} L_0^c) + f_0(L_0 \bar{L}_0^c + L_0^c \bar{L}_0)\varphi^0 + {}^1/_3 \zeta_0 (\varphi^0)^3 + f_0'(\mathscr{H}\mathscr{H})\varphi^0,$$

$$W_{ij} = \tilde{h}_{ij}(q^i \mathscr{H} q^{jc}) + h_{ij}(l^i \mathscr{H} l^{jc}) + f_{ij}(l^i \bar{L}_0^c + l^{ic}\bar{L}_0)\varphi^j + {}^1/_2 \zeta_{ij}\varphi^i\varphi^j\varphi^0, \tag{14}$$

and $\zeta_0 = \frac{3}{2}\zeta_{00}$. Products of isotopic doublets are defined everywhere in invariant form, for example

$$L_0 \bar{L}_0^c = L_0^a \varepsilon_{ab} \bar{L}_0^{cb}, \quad \varepsilon_{ab} = -\varepsilon_{ba}, \quad L_0 \mathscr{H} L_0^c = L_0^a \varepsilon_{ab} \mathscr{H}^{bb'} \varepsilon_{b'a'} L_0^{ca'},$$

with $(\mathscr{H}\mathscr{H}) \equiv \frac{1}{2}\,\mathrm{Tr}(\mathscr{H}\varepsilon\mathscr{H}^+\varepsilon)$, i.e.,

$$\begin{aligned}
(L_0 \mathscr{H} L_0^c) &= (-EH_d^0 + NH_d^-)E^c + (EH_u^+ - NH_u^0)N^c,\\
(l^i \mathscr{H} l^{jc}) &= (-e^i H_d^0 + \nu^i H_d^-)e^{jc} + (e^i H_u^+ - \nu^i H_u^0)\nu^{jc},\\
(q^i \mathscr{H} q^{jc}) &= (-d^i H_d^0 + u^i H_d^-)d^{jc} + (d^i H_u^+ - u^i H_u^0)u^{jc},\\
l^i \bar{L}_0^c + l^{ic}\bar{L}_0 &= (e^i \bar{E}^c - \nu^i \bar{N}^c) + (e^{ic}\bar{E} - \nu^{ic}\bar{N}),\\
(\mathscr{H}\mathscr{H}) &= H_u^+ H_d^- - H_u^0 H_d^0, \quad L_0 \bar{L}_0^c = E\bar{E}^c - N\bar{N}^c,
\end{aligned} \tag{15}$$

and the product $L_0^c \bar{L}_0$ is defined analogously.

It is clear that the superpotential (14) gives rise to mass matrices for *d*- and *u*-quarks proportional to each other:

$$m_{ij}^d = -\tilde{h}_{ij} v_d, \quad m_{ij}^u = -\tilde{h}_{ij} v_u,$$

where $v_u = \langle H_u^0 \rangle_0$, $v_d = \langle H_d^0 \rangle_0$; analogously it gives for the electron and neutrino in each generation

$$\mu_{ij}^e = -h_{ij} v_d, \quad \mu_{ij}^\nu = -h_{ij} v_u.$$

For no choice of the constants $\tilde{h}_{ij}$ and h_{ij} do these matrices describe the physics of the mixing of quark generations (since they correspond to zero Kobayashi-Maskawa angles), to accomplish that one needs the more sophisticated approach of Ref. 16 utilizing, for example, Higgs scalars whose vev depend on the generation labels *i, j*. This problem is beyond the scope of the present work.

3.2. Let us consider a simpler problem: is it possible to obtain with the superpotential (14) a very small neutrino mass in each generation (for $v_u \sim v_d$) with a moderate (between MeV and GeV) mass of all electrons, i.e., electron, μ-meson and τ-meson?

We will show that the see-saw mechanism[17,18] indeed gives rise under spontaneous *L-R* symmetry breaking, i.e., under the conditions (9) $N_0^c \gg v_u \sim v_d \gg N_0$, to a stably small Majorana mass for all neutrinos.[2)]

Initially we confine ourselves to one generation and omit all indices *i, j*. The part of the superpotential (14) and (15) that contains the fields $\nu = \nu_L$, $\nu^c = \bar{\nu}_R$ and $\varphi = \varphi_L^j$:

$$W_\nu' = -hH_u^0 \nu\nu^c - f(\bar{N}^c \nu\varphi + \bar{N}\nu^c\varphi) + \zeta\varphi_0\varphi^2/2,$$

gives the following contribution to the fermionic Lagrangian of these fields:

$$\mathscr{L}_\nu' = -\frac{\mu}{2}(\nu\nu^c + \nu^c\nu) + \frac{m}{2}(\nu\varphi + \varphi\nu) + \frac{M}{2}(\nu^c\varphi + \varphi\nu^c) + \frac{m_\varphi}{2}\varphi^2 + \text{c.c.}$$

where

$$\frac{\mu}{2} = h\langle H_u^0\rangle_0 = hv_u, \quad \frac{M}{2} = -f\langle \bar{N}\rangle_0,$$

$$\frac{m}{2} = -f\langle \bar{N}^c\rangle_0, \quad \frac{m_\varphi}{2} = \zeta\langle\varphi^0\rangle_0.$$

For what follows only the magnitude of these masses matters (their sign is unimportant) and one should keep in mind, as is shown below, that $\langle \bar{N}_0\rangle_0 = N^c$ and $\langle \bar{N}^c\rangle_0 = N_0$, where $N_0^c \gg v_u \gg N_0$ and $\langle\varphi_0\rangle = \varphi_0 \sim N_0^c$. Consequently, $M \gg \mu \gg m$, $m_\varphi \sim M$, where $\mu \sim \mu^e$ with μ^e referring to the mass of the electron of the given generation.

Under these circumstances the quadratic form for $\mathscr{L}_\nu'$ is easily brought to principal axes[19,20]:

$$\mathscr{L}_\nu' = \frac{1}{2}(\nu, \nu^c, \varphi)\begin{pmatrix} 0 & -\mu & m \\ -\mu & 0 & M \\ m & M & m_\varphi \end{pmatrix}\begin{pmatrix} \nu \\ \nu^c \\ \varphi \end{pmatrix} + \text{c.c.}$$

$$= \frac{\mu m'}{M}\nu'^2 + M'\varphi_d'\bar{\varphi}_d' + \text{c.c.}, \tag{16}$$

where

$$m' \approx m + \mu m_\varphi/M', \quad M' = (M^2 + m_\varphi^2/4)^{1/2} + m_\varphi/2 \sim M,$$

here $\nu' = \nu + \eta\varphi + \lambda\nu^c$ is the field with left chirality, $\varphi_d' = \varphi_L' + \varphi_R'$ is the Dirac field, $\varphi_L' = \varphi - \eta\nu$, $\varphi_R' = \nu^c + \lambda\nu$, and $\eta = \mu/M'$, $\lambda = m/M' \ll 1$ are small parameters. It is seen that the neutrino acquires a small Majorana mass equal to $\mu_\nu^0 = \mu(m'/M)$, where $m'/M \approx \mu/M$ is a very small quantity.

The above discussion directly generalizes to the case of an arbitrary number N_g of generations: now μ, M, m_φ and m become matrices:

$$\mu_{ij} = 2h_{ij}v_u, \quad M_{ij} = -2f_{ij}N_0^c, \quad m_{\varphi ij} = 2\zeta_{ij}\langle\varphi^0\rangle_0, \quad m_{ij} = -2f_{ij}N_0,$$

and $\mathscr{L}_\nu'$ is brought to the form

$$\mathscr{L}_\nu' = \left(\frac{\mu m'}{M}\right)_{ij}\nu^{i'}\nu^{j'} + M_{ij}'\varphi_d^{i'}\bar{\varphi}_d^{j'} + \text{c.c.}\ ,$$

where $\nu^{i'}$ and $\varphi_d^{i'}$ are defined in the above indicated form through the matrices $\eta_{ij} = (\mu/M')_{ij} \ll 1$ and $\lambda_{ij} = (m/M')_{ij} \ll 1$ (for example, $\nu^{i'} = \nu^i + \eta_{ij}\varphi_L^j + \lambda_{ij}\nu^{jc}$). The Majorana masses of the neutrinos of all three generations may in principle be obtained by diagonalizing the matrix $\mu_{ij}^0 = (\mu m'/M)_{ij}$. They could increase with generation number *i* due to an increase in the quantities μ_{ij} and $m_{ij} \sim \mu_{ij}$ in the numerator, or instead decrease due to the more rapid increase of the denominator.

We note that a version of the theory is possible in which the neutrino has a Dirac, and not Majorana, mass: it is a bit more complicated and requires the introduction in each generation of not one chiral singlet $\varphi(x)$, but two: $\varphi = \varphi_L$ and $\varphi^c(x) = \overline{\varphi}_R$. The Lagrangian in the neutrino sector becomes in that case[19] in place of (16)

$$\mathscr{L}_\nu'' = -\mu\nu\nu^c + m\nu\varphi^c + M\nu^c\varphi + m_\varphi\varphi\varphi^c + \text{c.c.}$$

$$= (\nu, \varphi)\begin{pmatrix} -\mu & m \\ M & m_\varphi \end{pmatrix}\begin{pmatrix} \nu^c \\ \varphi^c \end{pmatrix} + \text{c.c.}$$

$$= -\left(m + \frac{\mu m_\varphi}{M}\right)\nu_d'\bar{\nu}_d' + M\varphi_d'\bar{\varphi}_d' + \text{c.c.},$$

where $\nu_d' = (\nu + \eta\varphi)_L - (\overline{\varphi}_c - \lambda'\overline{\nu}^c)_R$, $\varphi_d' = (\varphi - \eta\nu)_L + (\overline{\nu}^c + \lambda'\overline{\varphi}^c)_R$, $\lambda' = \lambda m_\varphi/m$, with η and λ having the above values. It is seen that after diagonalization the right neutrino has become almost pure $\overline{\varphi}^c$. This means, in part, that if there existed prior to diagonalization a transition of the type $\nu \to \overline{\nu}^c$ (for example via the neutrino magnetic moment), then its magnitude would be reduced upon diagonalization by a factor $\lambda = m/M \ll 1$. This remark applies also to the Majorana case (16).

3.3. Let us show how spontaneous breaking of L–R symmetry arises, i.e., how the inequalities $N_0^c \gg v \gg N_0$ (with $N_0^c \sim \varphi_0 \equiv \langle\varphi^0\rangle_0$) for the vev arise. To this end we keep in the superpotential (20) and (21) only fields with nonzero vev:

$$\langle W_0\rangle_0 = -h_0 N_0 H_u^0 N_0^c - f_0(N_0\overline{N}_0^c + N_0^c\overline{N}_0)\varphi_0 - f_0' H_u^0 H_d^0 - \zeta_0\varphi_0^3/3,$$

and, after constructing the potential of these scalar fields

$$V = \sum_i \left|\frac{\partial W_0}{\partial y_0^i}\right|^2 + V_3 + V_2 + V_D,$$

we find the size of their vev by minimizing V with respect to these vev. Here

$$V_D = {}^1/_2 D_a D^a = {}^1/_8 g_0'^2\{[(N_0^c)^2 - \overline{N}_0^2] + [(\overline{N}_0^c)^2 - N_0^2]\}^2 + {}^1/_8 g_2^2\{[\overline{N}_0^2 - (N_0^c)^2]^2 + [N_0^2 - (\overline{N}_0^c)^2]^2\}$$

are the D-terms of the potential,

$$V_2 = m_H^2(v_u^2 + v_d^2) + m_\varphi^2\varphi_0^2 + m_N^2[N_0^2 + (N_0^c)^2] + m_{\overline{N}}^2[(\overline{N}_0^c)^2 + \overline{N}_0^2]$$

are the mass terms, and

$$V_3 = 2h_0 A_h N_0 N_0^c v_u + 2f_0 A_f(N_0^c\overline{N}_0 + N_0\overline{N}_0^c)\varphi_0 + {}^2/_3 A_\zeta\zeta_0\varphi_0^3$$

are the cubic terms, which arise—like the mass terms—as a result of renormalization group evolution. Here the magnitude of the squared masses of the scalars satisfies $m_k^2 = m_k^2(P^2)$ and the coefficients $A_k = A_k(P)$ with the dimension of mass depend on the scale of the particle momentum $P^2 = -p^2$ and can be obtained by solving the equations of the renormalization group (see Appendix) with the boundary conditions $m_k^2|_{P=M_X} = A_k|_{P=M_X} = 0$, where $M_X \leqslant M_P$—the unification scale. The equations of the renormalization group determine the size of the derivatives dm_k^2/dl and dA_k/dl, where $l = \ln(M_X^2/P^2)$, and contain only one parameter with the dimensions of mass—the quantity $M_{1/2}$ (this parameter is of order 10^3–10^4 GeV and determines the mass scale of the superpartners; it is sometimes referred to as the supersymmetry-breaking scale, $M_{1/2} = M_{SS}$).

The contribution to these derivatives from the gauge interactions is positive, while that from the Yukawa interactions is negative, i.e., with increasing $l = \ln(M_X^2/P^2)$ the quantities m_k^2 and A_k start to decrease, and decrease faster the larger they are themselves and the larger the Yukawa coupling constants h_0, f_0, f_0', ζ_0, h_{ij}, etc. As a result, with decreasing P from $P = M_X$ the size of the squared masses of the scalars m_k^2 at first increases, but then begins to decrease, changing sign at some $P = P_k^0$, below which they become negative: $m_k^2 = -\mu_k^2(P)$ as is shown in Fig. 1 for the physically interesting case $h_0 \gg f_0 \sim \zeta_0$, when $\mu_\varphi^2 \gtrsim \mu_N^2 \gg \mu_H^2$, $P < P_k^0$. For negative $m_k^2 = -\mu_k^2$ the scalar fields acquire nonvanishing vev that are larger the larger the corresponding mass μ_k^2. The potential of the scalars is given in terms of these vev in the form

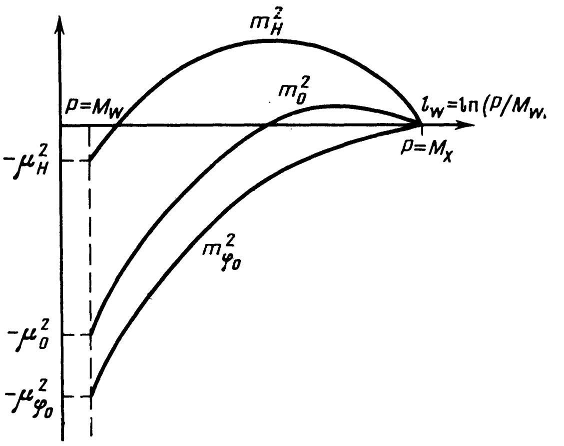

FIG. 1. The dependence on the momentum P, resulting from the equations of the renormalization group (see Appendix), of the squared masses of the scalar fields: m_H^2 (the fields H_d and H_u), $m_0^2 = (m_{N_0}^2 + m_{\overline{N}_0}^2)/2$ (the fields $\tilde{N}_0$ and $\tilde{N}_0^c$) and $m_{\varphi_0}^2$ (the field φ^0).

$$V = -\mu_H^2 v^2 - \mu_\varphi^2\varphi_0^2 - 2\mu_0^2 x + h_0 A_h v_u y + 2f_0(A_f x + \beta A_{f'} v_u v_d)\varphi_0 + {}^2/_3\gamma f_0 A_\zeta\varphi_0^3 + h_0^2({}^1/_4 y^2 + v_u^2 x) + h_0 f_0 v' y\varphi + f_0^2(x + \beta v_u v_d + \gamma\varphi_0^2)^2 + f_0^2(\beta^2 v_u^2 + 2x)\varphi_0^2, \quad (17)$$

where

$$2\mu_0^2 = \mu_N^2 + \mu_{\overline{N}}^2,$$

$$x = (N_0^c)^2 + N_0^2, \quad y = 2N_0^c N_0,$$

$$v^2 = v_u^2 + v_d^2, \quad v' = (\beta + 1)v_u + \beta v_d,$$

and the Yukawa constants ζ_0, f_0', enter (17) in the form

$$f_0' = \beta f_0, \quad \zeta_0 = \gamma f_0, \quad \beta \sim \gamma \sim 1.$$

Here we have taken into account that in the presence of D-terms the minimum of the potential with respect to $\overline{N}_0 = \langle\overline{N}\rangle_0$ and $\overline{N}_0^c = \langle\overline{N}^c\rangle_0$ is reached for values $\overline{N}_0 = N_0^c$, $\overline{N}_0^c = N_0$, for which $V_D = 0$. The vev values

$$N_0^c = \langle N^c\rangle, \quad N_0 = \langle N\rangle_0, \quad v_u = \langle H_u^0\rangle, \quad v_d = \langle H_d^0\rangle$$

are determined by the minimum of this potential and, in spite of its unwieldy form, are found quite simply by simultaneously solving the equations $\partial V/\partial y = \partial V/\partial x = \partial V/\partial\varphi_0^2 = 0$ in the form

$$f_0^2 x = f_0^2[(N_0^c)^2 + N_0^2] \approx \frac{(\gamma+1)\mu_\varphi^2/2 - \gamma^2\mu_0^2}{2\gamma+1} = \mu_1^2,$$
$$|h_0|N_0 N_0^c \approx v'\mu_\gamma, \quad f_0^2\langle\varphi_0\rangle^2 = \mu_\gamma^2 = \frac{(\gamma+1)\mu_0^2 - \mu_\varphi^2/2}{2\gamma+1}. \quad (18)$$

In the physically interesting case $\gamma \gg 1$, $f_0 \ll |h_0| \sim 1$, when $\mu_\varphi \gtrsim \mu_0 \gg v$, we obtain two solutions: with $N_0^c \gg N_0$ and with $N_0 \gg N_0^c$, and, as $P \ll M_X$ is decreased, the physical system of fields can fall into one of these states, corresponding to its minimum energy. The state with $N_0^c \gg N_0$ corresponds to the breaking for $P < N_0^c$ of the right group, i.e., to the Weinberg-Salam theory; for this state

$$N_0^c \approx \mu_1/f_0 > \mu_1, \quad N_0 \sim f_0 v'/|h_0| \ll v',$$

where $v' \sim M_W \approx 10^2$ GeV.

4. THE SIZE OF M_X AND $\sin^2\theta_W$

The main shortcoming of grand unified L–R symmetric theories is the fact that the change in the gauge constants $\alpha_n = g_n^2/4\pi$ $(n = 3,2)$, $\alpha' = g'^2/4\pi$ in the region $P < M_X$ leads, due to the presence of the two bosons $W_{\mu L}^{\pm}$ and $W_{\mu R}^{\pm}$, to rather large values of $M_X > M_P = 10^{19}$ GeV and $s^2 = \sin^2\theta_W \gtrsim 0.3$–$0.4$.[9] This is connected with the fact that the introduction into the theory of the additional $SU(2)_R$ symmetry leads to a decrease by six units of the coefficient $b = b' + b_{2L}$ in the variation

$$\alpha^{-1}(P) = (\alpha')^{-1} + \alpha_{2L}^{-1} = {}^8/_3\alpha_{GUT}^{-1} + (b/2\pi)\ln(M_X/P)$$

of the electromagnetic constant $\alpha = e^2/4\pi$.

Taking into account the fragment generations 16^0 and $\overline{16}^0$, from Eq. (4) drastically reduces M_X down to 10^{16}–10^{18} GeV, but $\sin^2\theta_W$ can be reduced to the experimental value ≈ 0.23 only by introducing new high mass scales (beside $M_W \approx 10^2$ GeV): $M_R \gg M_W$, $M_R/M_W \sim 10^{10}$–10^{12}. The supersymmetry-breaking mass scale $M_{SS} = M_{1/2}$, i.e., the size of the mass of the superpartners, has little effect on the above result.

In the $SO(10)$ case under consideration, i.e., the symmetry (3), an additional mass scale $M_R \gg M_W$ is possible. Here $(\alpha')^{-1} = (\alpha'_B)^{-1} + \alpha_2^{-1}$, where $\alpha'_B = g_0'^2/4\pi$ (see Sec. 2.2); further, for $P = M_X$ we have $\alpha_3 = \alpha_2 = \alpha_{GUT}$ but $\alpha' = \frac{5}{3}\alpha_{GUT}$, therefore $(\alpha'_B)^{-1} = \frac{2}{3}\alpha_{GUT}^{-1}$ for $P = M_X$.

For $n = 2,3$ we have the well-known relations

$$\alpha_n^{-1}(P) = \alpha_{GUT}^{-1} + b_n X(P), \quad X(P) = (1/2\pi)\ln(M_X/P), \quad (19)$$

where $b_3 = \frac{1}{2}K_3 - 9$, $b_2 = \frac{1}{2}K_2 - 6$, with K_3 and $K_2 = K_{2L}$, respectively, the number of color triplets of light particles (with $m_k < P$) and the number of $SU(2)_L$-doublets, and

$$(\alpha')^{-1}(P) = {}^5/_3\alpha_{GUT}^{-1} + b'X(P)$$

with $b' = b'_B + b_2$ and $b'_B = \Sigma_k (Y_{B-L})_k^2$, where the sum includes all particles with $m_k < P$. Expressing in these three relations (where $\alpha_2 = \alpha_{2L} = \alpha_{2R}$) the constant α_{GUT}^{-1} in terms of $\alpha_3^{-1}(P)$, taking into account that for $\alpha = e^2/4\pi$

$$\alpha^{-1}(P) = (\alpha')^{-1} + \alpha_2^{-1} = {}^8/_3\alpha_3^{-1}(P) + \tilde{b}X(P), \quad \tilde{b} = b' + b_2 - {}^8/_3 b_3$$

and that $\alpha_2^{-1} = \alpha^{-1}\sin^2\theta_W$, we obtain upon setting $P = M_W$,

$$X_0 = (1/2\pi)\ln(M_X/M_W) = (\alpha^{-1} - {}^8/_3\alpha_3^{-1})/\tilde{b},$$
$$\sin^2\theta_W = [\alpha_3^{-1} + (b_2 - b_3)X_0]/\alpha^{-1}, \quad (20)$$

where $\alpha^{-1} = \alpha^{-1}(M_W) = 128$ and $\alpha_3^{-1} = \alpha_3^{-1}(M_W) = 9$.

Here we have not taken into account the change in the coefficients $b_n \to b_{nR} = b_n - \Delta_n$ for $P < M_R$ due to the "dying off" of the contribution Δ_n of the heavy particles—the fragments 16^0, $\overline{16}^0$ and 10^0 with heavy mass[3)] $m_R^* \sim M_R$ (and possibly also superpartners if their mass is high: $M_{SS} \sim M_R$). For large $M_R \gg M_W$ taking this into account results in a noticeable effect changing (19) into

$$\alpha_n^{-1}(P) = \alpha_{GUT}^{-1} + \frac{b_n}{2\pi}\ln\frac{M_X}{M_R} + \frac{b_n - \Delta_n}{2\pi}\ln\frac{M_R}{P}$$
$$= \alpha_{GUT}^{-1} + b_n X(P) - \Delta_n Y(P),$$

where $Y(P) = (1/2\pi)\ln(M_R/P)$, and $n = 3,2$. For $(\alpha')^{-1}$ a similar equation is valid in which α_{GUT}^{-1} is replaced by $\frac{5}{3}\alpha_{GUT}^{-1}$, while b_n and $b_n - \Delta_n$ are replaced by b' and $b' - \Delta'_n$. Upon setting $P = M_W$ in these three equations we readily obtain analogously to (20)

$$X_0 = (1/2\pi)\ln(M_X/M_W) = (\alpha^{-1} - {}^8/_3\alpha_3^{-1} + \tilde{\Delta}Y_0)/\tilde{b},$$
$$\sin^2\theta_W = (\alpha_3^{-1} + b_{23}X_0 - \Delta_{23}Y_0)/\alpha^{-1}, \quad (21)$$

where $Y_0 = (1/2\pi)\ln(M_R/M_W)$, $\tilde{b}$ was defined above, $\tilde{\Delta} = \Delta' + \Delta_2 - \frac{8}{3}\Delta_3$, $b_{23} = b_2 - b_3$, and $\Delta_{23} = \Delta_2 - \Delta_3$.

Each generation's 16-plet (4) includes $K_3^0 = 4$ color triplets of q- and q^c-superfields and $K_2^0 = 4SU(2)_L$-doublets, and the fragments in (4) given in addition $\delta K_3 = 0$, $\delta K_2 = 4$. Therefore $b_3 = 6 - 9 = -3$, $b_2 = (6 - 6) + \frac{1}{2}\delta K_2 = 2$ for $N_g = 3$ generations of 16-plets.

According to Table I summing over the particles of the 16-plet gives

$$\sum_{k=1}^{(16)} (Y_{B-L})_k^2 = {}^4/_3,$$

while summing over the 16^0 fragments gives

$$\sum_{k=1}^{(16^0)} (Y_{B-L})_k^2 = 2$$

(here the Higgs particles from $\underline{10}^0$ do not contribute, since for them $Y_{B-L} = 0$), therefore $b_B = \frac{4}{3}N_g + 2 = 6$, i.e., $b' = b'_B + b_2 = 8$, or $(b_3, b_2, b') = (-3,2,8)$, $\tilde{b} = 18$.

Without taking into account the contribution of the fragments we would have instead $(b_3, b_2, b') = (-3,0,4)$, $\tilde{b} = 12$. Substitution into (20) without the fragment contribution produces wholly unreasonable numbers: $M_X/M_W = 4.5 \cdot 10^{23}$, $\sin^2\theta_W = 0.273$, while with the fragment contribution included we get $X_0 = \frac{52}{9}$ and $M_X \approx 0.5 \cdot 10^{18}$ GeV, but too high a value $\sin^2\theta_W = 0.296$, instead of $(\sin^2\theta_W)_{exp} = 0.230 \pm 0.005$.

Let us discuss now what happens for large M_R, i.e., when $M_R = 10^{n_R}$ GeV $\gg M_W$ [for $Y_0 = (n_R - 2)/2.73 > 1$] in two extreme limits.

a) $M_{SS} \approx M_R \gg M_W$, i.e., all superpartners are heavy. Here one needs to include in Δ_n contributions of the 16^0, $\overline{16}^0$, and 10^0 fragments, as well as of the superpartners of the 16-plet particles; for scalars it is obtained by multiplying the contribution of the superfield by $\frac{1}{3}$, while for fermions the multiplication factor is $\frac{2}{3}$. This yields

$$\Delta_3 = {}^1/_6 N_g K_3^0 = 2, \quad \Delta_2 = {}^1/_6 N_g K_2^0 + {}^1/_2\delta K_2 = 4.$$

In the region $P < M_R$ the group $SU(2)_R$ is broken so that here $b' = b'_Y$, where $b'_Y = \Sigma_k Y_k^2$ with $Y = Y_{B-L} + I_{3R}$ being the usual hypercharge of the Weinberg-Salam theory, and $\Delta' = b'_{P>M_R} - b'$ where $b'_{P>M_R} = 8$ was defined above. According to Table I,

$$\sum_k^{(16)} Y_k^2 = {}^{10}/_3$$

is the contribution from all the particles in the 16-plets, while the contribution from the fragments

$$\sum_k^{(16^0)} Y_k^2 = 3, \quad \sum_k^{(10^0)} Y_k^2 = 1$$

need not be considered because they are heavy. Therefore without the scalars (superpartners) the 16-plets contribute $b'_Y = (\frac{2}{3}) \cdot (\frac{10}{3}) N_g = \frac{20}{3}$, i.e., $\Delta' = 8 - \frac{20}{3} = \frac{4}{3}$, or

$$(\Delta_3, \Delta_2, \Delta') = (2, 4, {}^4/_3), \quad \bar{\Delta} = 0.$$

Equation (21) gives the same value $X_0 = \frac{52}{9}$, i.e., $M_X = 0.5 \cdot 10^{18}$ GeV, but now $\sin^2\theta_W = 0.296 - (n_R - 2)/175$. As can be seen, by increasing n_R one can get $\sin^2\theta_W$ to reach 0.230 only for $n_R - 2 \approx 10$–12, i.e., for a very high scale $M_R \approx 10^{12}$ GeV.

b) $M_R \gg M_W \sim M_{SS}$, i.e., fragments are heavy but superpartners are light. In that case $\Delta_3 = 0$ and $\Delta_2 = \frac{4}{3}$, since now only two doublets of heavy leptons from the fragments 16^0, $\overline{16}^0$ and two doublets of heavy higgsinos from 10^0 fail to contribute to b_2.

Omitting the contribution of these same particles to b'_Y we obtain $b'_Y = (10/3)N_g + (2/3)(3+1) = \frac{38}{3}$, whence $\Delta' = b' - b'_Y = 8 - \frac{38}{3} = -\frac{14}{3}$, or

$$(\Delta_3, \Delta_2, \Delta') = (0, {}^4/_3, -{}^{14}/_3), \quad \bar{\Delta} = -{}^{10}/_3.$$

Substitution of these results into Eq. (21) with $M_X = 10^{n_X}$ GeV gives

$$n_X = 17{,}8 - (n_R - 2)/5{,}4, \quad \sin^2\theta_W = 0{,}296 - (n_R - 2)/157,$$

i.e., $\sin^2\theta_W \approx 0.236$ even for $n_R - 2 = 8$ (and then $M_X \approx 2 \cdot 10^{16}$ GeV).

As is seen, in both cases (for $M_R \sim M_{SS}$ and for $M_R \gg M_{SS}$) the difference $0.296 - \sin^2\theta_W$ equals $(n_R - 2)/\eta_0$, where the number $\eta_0 \sim 150$–200 depends concretely on the masses of the various particles. In principle a version of the theory is possible for which η_0 is close to 10^2 so that the scale $M_R = 10^{n_R}$ GeV may be not very high: $M_R / M_W \approx 10^6$–10^7.

5. CONCLUSION

L–R symmetric SUGRA models with P-parity spontaneously broken in the low-energy region $P < M_R$ are esthetically very attractive. However in them unification of the interactions at $P = M_X$ gives too high a value of $\sin^2\theta_W \approx 0.296$. The experimental value of $\sin^2\theta_W \approx 0.23$ can here be achieved only for a very high $SU(2)_R$-symmetry-breaking scale $M_R \sim 10^{10}$–10^{12} GeV. Whether one may obtain values at the same time for the vev of the scalars H_u^0, H_d^0, of the electroweak theory of the order of $M_W \sim 10^2$ GeV by minimizing a potential of the form (18) remains unclear. In any event it is understood that this would require fine tuning of the Yukawa constants h_0, ζ_0, f_0 and f'_0 and of the masses μ_H^2, μ_φ^2, and μ_0^2, which are determined by these very same constants in the equations of the renormalization group (see Appendix).

Let us recall that the theory based on $SO(10)$, but not including the $SU(2)_R$ symmetry [i.e., having after $SO(10)$ breaking the rank 4 symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y$], automatically gives the experimental value of $\sin^2\theta_W$. Indeed, in such a theory there is no need for the 16^0 and $\overline{16}^0$ fragments which break the $SU(2)_R$ symmetry, and for N_g generations of 16-plets we have

$$(b_3, b_2, b') = (2N_g - 9, 2N_g - 5, {}^{10}/_3 N_g + 1), \quad \bar{b} = 20,$$

where

$$b' = \sum_k Y_k^2 N_g + 1 = {}^{10}/_3 N_g + 1,$$

which gives upon substitution into (21) the N_g-independent values $X_0 = 5.2$, $M_X = 1.3 \cdot 10^{16}$ GeV, and $\sin^2\theta_W = (9 + 4X_0)/\alpha^{-1} = 0.233$. The presence of fragments increases the coefficient b_2 by unity and gives rise to an unacceptable increase in $\sin^2\theta_W$ by $5.2/\alpha^{-1} = 0.04$. However, in theories of this type (to which the CERN E_6-model[2] is very close) there is no room in the superpotential (8), (14) for scalars φ^i and the term $f_{ij}(l^i \bar{L}_0^c + l^{ic}\bar{L}_0)\varphi^j$ is absent, making it impossible for the neutrino in each generation to have a stably small mass.

It may be that obtaining small vev $v_u = \langle H_u^0 \rangle \sim u_d = \langle H_d^0 \rangle \sim M_W$ simultaneously with large vev $N_0^c \sim M_R$ can ensure a superpotential that includes nonrenormalizable interactions, for example, of the form

$$(\lambda_0/M_P)(16^0\overline{16}^0)^2 = (\lambda_0/M_P)(L_0\bar{L}_0^c + L_0^c\bar{L}_0)^2,$$

which upon minimizing the potential would immediately yield $x_0 = (N_0^c)^2 + N_0^2 \approx \mu_0 M_P$, i.e., for $N_0^2 \gg N_0$ we would have $M_R \sim N_0^c \approx (\mu_0 M_P)^{1/2} \sim 10^{10}$–$10^{11}$ GeV. One is then still left with the problem of how to get small values for v_u and $v_d \sim M_W$ from a potential of the form (17) with the additional term $\lambda_0^2 x_0^3 / M_P^2$.

Aside from these shortcomings, specific to the L–R symmetric models, all SUGRA models contain in general unclear points, related to the choice of the basic functions of the form (1) and the choice of the supersymmetry-breaking scale $M_{1/2} = M_{SS}$. It is altogether unclear how this quantity, determined in the gravitational sector where the only available parameter is the Planck mass M_P, can have values tens of orders of magnitude smaller than M_P, which is precisely what is assumed in low-energy SUGRA.

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APPENDIX

Equations of the renormalization group in the *SO*(10) model

Below we give the renormalization group equations, obtained in the usual manner,[15] for the Yukawa constants and masses of the particles in the model with $SO(10)$ symmetry and a superpotential of the form (14)

$$W=W_0+W_{33},$$

$$W_0=h_0L_0\mathscr{H}L_0^c+f_0(L_0\bar{L}_0^c+L_0^c\bar{L}_0)\varphi^0+f_0'(\mathscr{H}\mathscr{H})\varphi^0+{}^1/_3\zeta_0(\varphi^0)^3,$$

$$W_{33}=\tilde{h}(q\mathscr{H}q^c)+h(l\mathscr{H}l^c)+f(l\bar{L}_0^c+l^c\bar{L}_0)\varphi+{}^1/_2\zeta\varphi^2\varphi^0,$$

where we take into account Yukawa coupling only of third-generation particles and

$$h_{33}=h,\quad \tilde{h}_{33}=\tilde{h},\quad f_{33}=f,\quad \zeta_{33}=\zeta,$$

$$q_3=q(x),\quad l_3=l(x),\quad \varphi_3=\varphi(x).$$

1) The equations for the normalized Yukawa constants

$$Y_{h_i}=(h_i(P)/4\pi)^2,\quad Y_{f_i}=(f_i(P)/4\pi)^2,$$

$$Y_{\zeta_0}=({}^1/_3\zeta_0/4\pi)^2,\quad Y_\zeta=({}^1/_2\zeta/4\pi)^2$$

have the form

$$\partial Y_{h_0}/\partial l=Y_{h_0}(6\tilde{\alpha}_2+\tilde{\alpha}_B'-5Y_{h_0}-2Y_{f_0}-Y_{f_0'}-Y_h-3Y_{\tilde{h}}),$$

$$\partial Y_{f_0}/dl=(3\tilde{\alpha}_2+\tilde{\alpha}_B'-2Y_{h_0}-6Y_{f_0}-2Y_{f_0'}-Y_{\zeta_0}-Y_f)Y_{f_0},$$

$$dY_{\zeta_0}/dl=-3(4Y_{f_0}+2Y_{f_0'}+Y_{\zeta_0})Y_{\zeta_0},$$

$$dY_{f_0'}/dl=(3\tilde{\alpha}_2-2Y_{h_0}-4Y_{f_0'}-4Y_{f_0}-Y_{\zeta_0}-3Y_{\tilde{h}}-2Y_h)Y_{f_0'},$$

$$dY_h/dl=Y_h(6\tilde{\alpha}_2+\tilde{\alpha}_B'-Y_{h_0}-Y_{f_0'}-5Y_h-3Y_{\tilde{h}}-2Y_f),$$

$$dY_{\tilde{h}}/dl=({}^{16}/_3\tilde{\alpha}_3+6\tilde{\alpha}_2+{}^1/_9\tilde{\alpha}_B'-Y_{h_0}-5Y_h-7Y_{\tilde{h}}-Y_{f_0})Y_{\tilde{h}},$$

$$dY_\zeta/dl=-(8Y_f+4Y_\zeta+4Y_{f_0}+2Y_{f_0'}+Y_{\zeta_0})Y_{\zeta_0},$$

where we used the notation $l=\ln(M_X^2/P^2)$, $\tilde{\alpha}_a=\alpha_a/4\pi=g_a^2/(4\pi)^2$. At $P=M_X$, i.e., $l=0$, these Yukawa constants satisfy the boundary condition $Y_i(0)=Y_i^0$ where Y_i^0 are dimensionless parameters of the theory.

2) The equations for the constants A_{h_i}, A_{f_i}, A_{ζ_i} with units of mass are

$$dA_{h_0}/dl=6\tilde{\alpha}_2M_2+\tilde{\alpha}_B'M_B-5Y_{h_0}A_{h_0}-2Y_{f_0}A_{f_0}$$
$$-Y_{f_0'}A_{f_0'}-Y_hA_h-3Y_{\tilde{h}}A_{\tilde{h}},$$

$$dA_{f_0}/dl=3\tilde{\alpha}_2M_2+\tilde{\alpha}_B'M_B-2Y_{h_0}A_{h_0}-6Y_{f_0}A_{f_0}$$
$$-2Y_{f_0'}A_{f_0'}-Y_{\zeta_0}A_{\zeta_0}-Y_fA_f,$$

$$dA_{\zeta_0}/dl=-3(4Y_{f_0}A_{f_0}+2Y_{f_0'}A_{f_0'}+Y_{\zeta_0}A_{\zeta_0}),$$

$$dA_{f_0'}/dl=3\tilde{\alpha}_2M_2-2Y_{h_0}A_{h_0}-4Y_{f_0'}A_{f_0'}-4A_{f_0}Y_{f_0}$$
$$-Y_{\zeta_0}A_{\zeta_0}-2Y_hA_h-3Y_{\tilde{h}}A_{\tilde{h}},$$

$$dA_h/dl=6\tilde{\alpha}_2M_2+\tilde{\alpha}_B'M_B-Y_{h_0}A_{h_0}-Y_{f_0'}A_{f_0'}-5Y_hA_h$$
$$-2Y_fA_f-3Y_{\tilde{h}}A_{\tilde{h}},$$

$$dA_{\tilde{h}}/dl={}^{16}/_3\tilde{\alpha}_3M_3+6\tilde{\alpha}_2M_2+{}^1/_9\tilde{\alpha}_B'M_B-Y_{h_0}A_{h_0}-5Y_hA_h$$
$$-7A_{\tilde{h}}Y_{\tilde{h}}-Y_{f_0}A_{f_0},$$

$$dA_\zeta/dl=-(8Y_fA_f+4Y_\zeta A_\zeta+4Y_{f_0}A_{f_0}+2Y_{f_0'}A_{f_0'}+A_{\zeta_0}Y_{\zeta_0}),$$

where

$$M_n(P)=M_{1/2}\tilde{\alpha}_n(P)/\alpha_n(M_X),\quad n=2,\ 3,\ B.$$

3) The equations for the squared masses $m_i^2(P)$ of the scalars:

$$dm_N^2/dl=3\tilde{\alpha}_2M_2^2+\tilde{\alpha}_B'M_B^2-2Y_{h_0}m_{h_0}^2-Y_{f_0}m_{f_0}^2,$$

$$dm_{\bar{N}}^2/dl=3\tilde{\alpha}_2M_2^2+\tilde{\alpha}_B'M_B^2-Y_{f_0}m_{f_0}^2-Y_fm_f^2,$$

$$dm_{\varphi^0}^2/dl=-4Y_{f_0}m_{f_0}^2-2Y_{f_0'}m_{f_0'}^2-Y_{\zeta_0}m_{\zeta_0}^2-Y_\zeta m_\zeta^2,$$

$$dm_H^2/dl=6\tilde{\alpha}_2M_2^2-Y_{h_0}m_{h_0}^2-Y_{f_0'}m_{f_0'}^2-3Y_{\tilde{h}}m_{\tilde{h}}^2-Y_hm_h^2,$$

$$dm_l^2/dl=3\tilde{\alpha}_2M_2^2+\tilde{\alpha}_B'M_B^2-3Y_hm_h^2-Y_fm_f^2,$$

$$dm_q^2/dl={}^{16}/_3\tilde{\alpha}_3M_3^2+3\tilde{\alpha}_2M_2^2+{}^1/_9\tilde{\alpha}_B'M_B^2-2Y_{\tilde{h}}m_{\tilde{h}}^2,$$

$$dm_\varphi/dl=-2Y_fm_f^2-Y_\zeta m_\zeta^2,$$

where we have introduced the notation

$$m_{h_0}^2=2m_N^2+m_H^2+A_{h_0}^2,\quad m_{f_0}^2=m_N^2+m_{\bar{N}}^2+m_{\varphi^0}^2+A_{f_0}^2,$$

$$m_{f_0'}^2=m_{\varphi^0}^2+2m_H^2+A_{f_0'}^2,\quad m_{\zeta_0}^2=3m_{\varphi^0}^2+A_{\zeta_0}^2,$$

$$m_h^2=2m_l^2+m_H^2+A_h^2,\quad m_{\tilde{h}}^2=2m_q^2+m_H^2+A_{\tilde{h}}^2,$$

$$m_f^2=m_l^2+m_{\bar{N}}^2+m_\varphi^2+A_f^2,\quad m_\zeta^2=2m_\varphi^2+m_{\varphi^0}^2+A_\zeta^2.$$

The boundary conditions at $P=M_X$ are $A_i^2(0)=m_i^2(0)=0$. Besides the "bare" Yukawa constants Y_i^0 the solutions depend only on one mass parameter $M_{1/2}$ (it is clear that all these constants should be chosen so that the $W_{\mu L}^{\pm}$ boson of the Weinberg-Salam theory has the correct value for its mass $M_W=83$ GeV).

1) The general formula[2] gives for the ratio $M_{1/2}/M_P$ a value equal to zero for $\chi(z^i)\approx s$, Eq. (1), while the mass of the gravitino ($s=3/2$) is $M_{3/2}\sim M_P$. It is the acquisition of such a mass by the gravitino that is responsible for supersymmetry breaking in the gravitational sector of the theory.

2) The authors are grateful to Z. G. Berezhiani, who called to their attention the "indirect" see-saw mechanism that is used here; a similar mechanism in a slightly simpler case was proposed by Witten[18] and used by Ellis, Hagelin and Nanopoulos.[19]

3) We assume for simplicity, to avoid the introduction of two limits, that the mass $m_R^*=f_0\langle\varphi^0\rangle_0\approx\mu_Y$ is of the same order as the threshold $M_{W_R}\approx N_0^c\approx\mu_1/f_0$ for the breaking of the $SU(2)_R$ group, i.e., that $M_R\sim m_R^*\sim M_{W_R}$. If $f_0\ll 1$ then this is possible for $\mu_1\ll\mu_Y$.

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