Dynamics of impurity magnetic moments in a dilute AuV alloy

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An equation for the frequency dependence of the magnetic susceptibility in the Kondo alloy AuV is derived by conformal field theory. The low-temperature behavior of this alloy is described by the Wess-Zumino-Novikov-Witten theory on the $SU(2)$ group.

1. INTRODUCTION

A recent paper[1] contains convincing experimental proof of low-temperature scaling in the dilute magnetic alloy AuV. The paper cites the results of measurements of the impurity part of the magnetic susceptibility $\chi(T)$ in a zero field and of the impurity part of the magnetic moment $M(H)$ for $\mu_B H \gg kT$. At temperatures and magnetic fields much lower than the Kondo temperature, these quantities have power-law dependences;

$$\chi(T)=\frac{\alpha_1}{T}\left(\frac{T}{T}\right)^{\beta_1},\quad M(H)=\alpha_2\left(\frac{H}{T_K}\right)^{\beta_2}. \tag{1}$$

The fact that scaling can be realized in the Kondo effect was first shown by Nozieres and Blandin.[2] They have shown that scaling sets in when an electron scattered by an impurity has more degrees of freedom than the impurity. Their treatment was qualitative. A more exact solution was obtained later[3,4] for a model that describes such a scattering of conduction electrons by a magnetic impurity.

The Hamiltonian of this model, proposed in Ref.2, is

$$H=\sum_{k,m,\sigma}\varepsilon(k)a^+_{km\sigma}a_{km\sigma}+\frac{1}{N}J\sum_{m=1}^{n}\sum_{\sigma,\sigma'=\pm 1/2}\sum_{k,p}a_{km\sigma}(\sigma\mathbf{S})_{\sigma\sigma'}a_{pm\sigma'}. \tag{2}$$

An impurity with spin S scatters conduction electrons in a state with an orbital angular momentum $l(2l+1=n)$, k is the modulus of their momentum, m is the orbital-momentum projection, and σ is the spin projection. The orbital momentum of the impurity shell is zero (orbital singlet). The interaction is therefore diagonal in the projection of the electron orbital momentum. Scaling takes place in the model (2) at $n>2S$ (Refs. 2–4).

An orbital singlet is the ground state for the configuration $3d^5$ (Mn), and also for the configurations $3d^3(S=3/2)$ and $3d^8(S=1)$ in the presence of a strong crystal field of cubic symmetry.[2] The crystal field prevents the electrons of the impurity shell from having orbital-momentum projections $m=\pm 1$, and holes from having $m=0$ and ± 2. The configuration $3d^3$ corresponds to the vanadium ion.

The presence of the crystal field should cause conduction electrons belonging to different representations of the cubic symmetry group (E and T) to correspond to different interaction constants J_e and J_t. If $J_e \ll J_t$, then $J_e \to 0$ at low temperatures, interacting with the impurity will be only the electrons that are transformed in accordance with the T representation (there are exactly three of them), and the situation $n=2S$ is effectively realized. In the dilute AuV solution, however, $J_e \approx J_t$ and a different fixed point is realized.

This statement is confirmed also by the correspondence of the theoretical values of the indices[3,4]

$$\beta_1=4/(n+2),\quad \beta_2=2/n \tag{3}$$

and by the experimental data for AuV ($n=5$).[1]

The model (2) with $n>2S$ is remarkable because one can calculate in it a number of dynamic quantities; this had not been possible heretofore for the usual Kondo effect. The model (2) just as any model describing scattering by a single spherically symmetric potential, is one-dimensional. What distinguishes it from the one-dimensional theories, however, is the presence of scaling, a fact equivalent in the 1 + 1-dimensional theories to the presence of conformal symmetry.[5] The procedure of finding correlation functions in conformal theories has by now been well developed (see Ref. 6 in connection with our case).

We present below the result of calculations of the electric resistance and of the dynamic and magnetic susceptibilities.

The model(2) with $n>2S$ belongs to the universality class of the Weiss-Zumino-Novikov-Witten (WZNW) model on the $SU(2)$ group with a central charge,[4,6]

$$C=3n/(n+2). \tag{4}$$

This model was investigated from the standpoint of conformal field theory in Ref. 6. It remains for us only to ascertain which operators in the model (2) correspond to definite conformal fields of the WZNW model, and use the result of Ref. 6 to describe the low-temperature dynamics of the model.

It is obvious from Eqs. (1) and (3) that the dimensionality of the impurity spin operator S_a is $\Delta_S=2/(n+2)$. It coincides with the dimensionality of the primary field $\Phi(1)$ in the WZNW model.[6]

Let us recall what the term "primary field" means and how fields are classified in the WZNW model.

Conformal theory states[5] that in any model having conformal symmetry there exists a set of fields $\Phi_\Delta(z,\bar z)$, called primary, whose two-point correlation functions are transformed most simply under conformal transformations:

$$G(z,\bar z;z',\bar z')=\left(\frac{\partial w}{\partial z}\right)^{\Delta}\left(\frac{\partial w'}{\partial z'}\right)^{\Delta}\left(\frac{\partial \bar w}{\partial \bar z}\right)^{\bar\Delta}\left(\frac{\partial \bar w'}{\partial \bar z'}\right)^{\bar\Delta}\times G(w(z),\bar w(\bar z);w(z'),\bar w(\bar z')). \tag{5}$$

Two-point correlators on an infinite complex z plane

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($z = x/v + it$, v is the excitation velocity) are power-law functions. Introduction of a finite temperature into the theory corresponds to a transition from consideration of the model on an infinite plane to consideration on an infinitely long strip of width $1/T$. The transition from the plane to the strip is by the conformal transformation

$$w(z)=\exp(2\pi zT). \qquad (6)$$

Since all two-point functions are power-law at $T=0$, it follows that by substituting (6) in (5) we find that at a finite temperature the correlation functions of the primary fields of the conformal theory take the form

$$G(z,\bar{z};z',\bar{z}')=(\pi T/\mathrm{sh}\,\pi T(z-z'))^{2\Delta} \times(\pi T/\mathrm{sh}\,\pi T(\bar{z}-\bar{z}'))^{2\bar{\Delta}}. \qquad (7)$$

In the WZNW theory the primary fields $\Phi_{m,m}^{(l,\bar{l})}$ are tensors with respect to the right-hand and left-hand actions of the $SU(2)$ group. The numbers l and $\bar{l}$ are those of the representations (the values of the angular momentum). In particular, the operator $\Phi^{(1,1)}$ is transformed in accordance with the representation $l=\bar{l}=1$, i.e., the associated $SU(2)$ group representation. It can thus be described also as a vector: $S^a = \varepsilon^{abc}\Phi_{bc}^{(1,1)}$—the impurity spin of the *s-d* model.

The difference between the general case considered in conformal theory and the case of the *s-d* model is that the correlation functions of the impurity spin depend only on one coordinate—the Matsubara time. Equation (7) applied to the impurity-spin operator takes the form

$$\langle S^a(t)S^b(0)\rangle=A\delta_{ab}[(T/T_K)/\sin\pi T|t|]^{4/(n+2)}, \qquad (8)$$

A is a number that depends only on n and $2S$.

The Fourier transform of the function (8) yields, upon continuation to the real frequency axis, the following equation for the dynamic magnetic susceptibility:

$$\chi(\omega)=\frac{\alpha_1}{T}\left(\frac{T}{T_K}\right)^{4/(n+2)} \times\mathrm{Re}\left\{\frac{\Gamma(2/(n+2)-i\omega/2\pi T)}{\Gamma(n/(n+2)-i\omega/2\pi T)}\right\}\frac{\Gamma(n/(n+2))}{\Gamma(2/(n+2))}, \qquad (9)$$

where $\Gamma(x)$ is the gamma function.

Another measurable quantity is electric resistance. Its low-temperature behavior is easily established. It is known that at $T=0$ the electric resistance in the *s-d* model is determined by the magnetic moment of the impurity:

$$R(H)=R_0\cos^2\left[\frac{\pi}{2S}\mu_{imp}(H)\right], \qquad (10)$$

R_0 is the electric resistance corresponding to the unitary limit. According to Eqs. (1) and (3) we have for $H\ll T_K$

$$\frac{R(H,T)}{R_0}=\begin{cases}1-\mathrm{const}(H/T_K)^{2/n}, & H\gg T\\ 1-\mathrm{const}(T/T_K)^{n/(n+2)}, & H<T\end{cases}. \qquad (11)$$

From (11) follows apparently also the more general equation

$$R(\omega,T)/R_0=1-\mathrm{const}\,T\chi(\omega,T). \qquad (12)$$

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