Dynamic solitons in ferromagnetic films

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It is established that magnetization solitons can exist in thin ferromagnetic films, where account must be taken of the magnetodipole interaction. A nonlinear integro-differential equation is obtained for the description of the magnetization dynamics in the film. A solution for low-amplitude magnetic solitons is obtained for frequencies close to homogeneous resonance. It is shown that allowance for the magnetodipole interaction leads to a decrease of the degree of localization of the degree of excitation in the soliton produced only if the number of magnons bound in it exceeds a finite critical value. Soliton dynamics at low magnetization precession frequencies is investigated within the framework of the adiabatic approximation. Soliton properties in film samples and in bulk magnets are compared.

Attempts are made at present in a number of laboratories to observe experimentally specific localized excitations of magnetically ordered media—magnetic solitons.[1,2,3] In contrast to other well known localized excitations in magnets, such as domain walls, the existence and stability of magnetic solitons is determined by their internal dynamics and by the precession of the magnetization vector in them.[4] No general qualitative theory of such excitation has been fully developed to this day.[4,5] The theoretically obtained soliton properties that admit in principle of comparison with experiment are the dynamic structure factor,[6,7] absorption of a high-frequency field,[8] and the contribution of solitons to the thermodynamics of magnets.[6,9] The greater part of the theoretical results, however, was obtained for the one-dimensional case and without allowance for the magnetodipole interaction. Experiments, on the other hand, are performed mainly on high-Q iron-garnet film samples,[1,10] or on yttrium iron garnet (YIG) slabs, in which soliton dynamics in domain walls is studied.[2,11] In such a geometry, allowance for the demagnetizing fields becomes important, and in a number of cases decisive. This calls for the development of a theory of magnetic solitons in samples of finite size, particularly in magnetic films.

Consider an easy-axis magnet film of thickness h, with the z axis normal to the film. We confine ourselves to the case when the magnetization distribution depends only on one coordinate and varies along the x axis. This geometry corresponds to the experimental conditions of Refs. 1, 2, and 11, and also to many studies of spin-wave generation by strip antennas in magnetic films (see, e.g., Ref. 10). To be specific, we assume that the anisotropy axis lies in the film plane (this simulates the situation in a YIG) and is directed along the y axis. We shall show below how the characteristics of low-amplitude solitons are changed by a different choice of anisotropy direction: along the x axis (corresponding to the experimental conditions in Refs. 2 and 11), or perpendicular to the film (as in magnetic bubble domains). The geometry of the problem and the orientations of the axes are shown in Fig. 1.

The ferromagnet energy was chosen in the simplest form

$$E=\int dV\left\{\frac{\alpha}{2}(\nabla\mathbf{M})^2-\frac{\beta}{2}M_y^2-\frac{1}{2}\mathbf{M}\mathbf{H}^{(m)}\right\},\qquad(1)$$

where α and β are the exchange and anisotropy constants ($\beta>0$ for the easy-axis case), $\mathbf{M}$ is the magnetization, and $\mathbf{H}^{(m)}$ is the intrinsic magnetic field.

We choose the boundary condition on the film surface in the form

$$\left.\frac{\partial\mathbf{M}}{\partial z}\right|_s=0,$$

which admits of a magnetization distribution that is homogeneous over the film thickness. The dependence of the magnetization on the coordinate z can be neglected if the characteristic dimension Δ of the inhomogeneity distribution of $\mathbf{M}$ along the x axis is much larger than the film thickness h. It can be easily shown that the magnetization distribution over the film thickness is of the form $\mathbf{M}(z)\propto z^2k^{3}3h$, where $k=1/\Delta$, which leads to the inequality $kh\ll 1$. For domain walls of width Δ on the order of the magnetic length $l_0=(\alpha/\beta)^{1/2}$, the foregoing inequality reduces to $h\ll l_0$. So strong an inequality is difficult to attain in contemporary magnetic films with low damping, although the relation $h\lesssim l_0$ can be satisfied in submicron magnetic-bubble films.

Homogeneity of the magnetization over the film thickness is easier to achieve for dynamic solitons. It will be shown below that this condition reduces to satisfaction of the inequality $h(1-\omega/\omega_0)^{1/2}\ll l_0$. We have introduced here the symbol $\omega_0=\tilde{\omega}(1+Q)^{1/2}$ for the homogeneous ferromagnetic resonance frequency. Without allowance for the magnetodipole interaction, the corresponding frequency is $\tilde{\omega}=2\beta M_0\mu_0/\hbar$. To facilitate a comparison of the results with the equations of Ref. 19, we use the notation $Q=4\pi/\beta$. (To avoid misunderstadings, we note that Q usually denotes

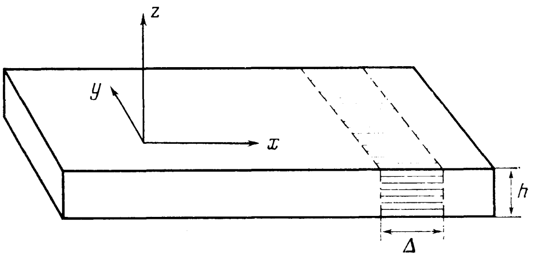

FIG. 1.

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the reciprocal quantity, called the quantity $\beta/4\pi$, called the quality factor of the magnetic film.)

The inequality $h(1-\omega/\omega_0)^{1/2} \ll l_0$ can be satisfied even for thick films with $h > l_0$ if the precession frequency ω_0 of the vector $\mathbf{M}$ in the soliton is close to the resonance frequency.

Neglecting damping, the system of dynamic equations includes the Landau-Lifshitz equations and the magnetostatics equations:

$$\frac{\hbar}{2\mu_0}\frac{\partial \mathbf{M}}{\partial t} + \alpha\left[\mathbf{M}\frac{\partial^2 \mathbf{M}}{\partial x^2}\right] + \beta[\mathbf{M}\mathbf{n}_y]M_y + [\mathbf{M}\mathbf{H}^{(m)}] = 0, \quad (2)$$

$$\operatorname{rot}\mathbf{H}^{(m)} = 0, \quad \operatorname{div}(\mathbf{H}^{(m)} + 4\pi\mathbf{M}) = 0, \quad (3)$$

where $\mathbf{n}_y$ is a unit vector along the anisotropy axis y.

Account is taken in Eq. (2) of the fact that in the principal approximation the vector $\mathbf{M}$ depends only on the coordinate x. Using the Green's function of Eqs. (3), it is easy to express the intrinsic magnetic fields $\mathbf{H}_{(m)}$ in terms of the distribution of the magnetization $\mathbf{M}$. If the magnetization field gradients are small, $kh \ll 1$, the solution of the magnetostatic problem takes the form

$$H_x^{(m)} = 2\pi h \frac{\partial}{\partial x}\hat{g}M_x, \quad H_y^{(m)} = 0,$$
$$H_z^{(m)} = -4\pi M_z - 2\pi h \frac{\partial}{\partial x}\hat{g}M_z, \quad (4)$$

where the standard notation is introduced for the Hilbert transform

$$\hat{g}u(x) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{u(x')\,dx'}{x'-x}. \quad (5)$$

Measuring the time in units of $1/\tilde{\omega}$ and the coordinates in units of l_0 we obtain for the magnetization unit vector $\mathbf{m} = \mathbf{M}/M_0$ the integro-differential equation

$$\frac{\partial}{\partial t}\mathbf{m} + \left[\mathbf{m}\frac{\partial^2}{\partial x^2}\mathbf{m}\right] + [\mathbf{m}\hat{J}\mathbf{m}] = 0,$$
$$\hat{J} = \operatorname{diag}\left(\gamma\frac{\partial}{\partial x}\hat{g},\ 1,\ -Q-\gamma\frac{\partial}{\partial x}\hat{g}\right), \quad (6)$$

where $\gamma = Qh/2l_0$.

Let us find the dispersion law for linear spin waves in a film ferromagnet. Introducing the complex function $\Psi = m_z + im_x$, we linearize Eq. (6) in terms of the small deviations m_z and m_x of the magnetization from its ground state $m = (0,1,0)$

$$i\frac{\partial\Psi}{\partial t} - \frac{\partial^2\Psi}{\partial x^2} + \Psi + \frac{Q}{2}(\Psi + \Psi^*) + \gamma\frac{\partial}{\partial x}\hat{g}\Psi^* = 0. \quad (7)$$

In view of the simplicity of Hilbert transforms of geometric functions, it is easy to obtain the spin-wave dispersion law:

$$\omega^2 = (1 + k^2 + \gamma|k|)(\omega_0^2 + k^2 - \gamma|k|). \quad (8)$$

This expression coincides with the dispersion law obtained for ferromagnetic films in Refs. 12 and 13 in the limit $kh \ll l_0$. Allowance for the magnetodipole interaction and for the finite film thickness leads to linearity of $\omega = \omega(k)$ for long waves: $\omega^2 = \omega_0^2 + \gamma Q(k)$ in contrast to the dispersion

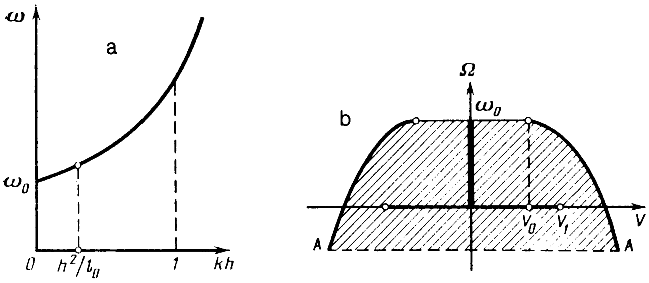

FIG. 2.

law for magnons in an unbounded sample.[4] At the ferromagnetic-resonance frequency the $\omega = \omega_0$ the spin-wave group velocity is $V_0 = \gamma Q/2\omega_0$. The finite group velocity for $k = 0$ is a consequence of neglecting retardation effects in the magnetostatics equations (3). The obtained dispersion law (8) is plotted in Fig. 2a. Note that the function $\omega = \omega(k)$ is linear in the interval $kh \lesssim (h/l_0)^2 l_0$, and therefore the inequality $kh \ll l_0$ used by us for thin films with $h \ll l_0$ is valid both in the linearity region $\omega - \omega_0 \sim |k|$ and in the region where the dispersion law is essentially nonlinear.

It is convenient to represent the obtained dispersion law in a coordinate frame moving at the spin-wave group velocity $V = \partial\omega/\partial k$. Introducing the magnetization-vector precession frequency in a coordinate frame moving with group velocity $\Omega = \omega - kV$, the $\Omega = \Omega(V)$ dependence takes the form shown in Fig. 2b. The spin waves correspond to the parabolas AA, and the shaded region under these parabolas corresponds to two-parameter magnetic solitons, i.e., localized magnet excitations that propagate with velocity V and are characterized by a magnetization precession of frequency Ω. It is interesting to note that the soliton existence region is bounded not only by the spin-wave dispersion-law lines, but also by a horizontal segment ($\omega = \omega_0$, $|V| < V_0$). This distinguishes the case considered here from all the hertofore known situations, and we do not know the character of the corresponding solutions. The velocity-axis segment $|V| < V_1$ in Fig. 2b corresponds to the domain walls. Their dynamics is the subject of an extensive literature[14] and will not be considered here.

We confine ourselves to immobile soliton excitations ($V = 0$) with positive magnetization-vector precession frequency ($\omega > 0$). Corresponding to such solitons is the frequency-axis segment $0 \leqslant \omega \leqslant \omega_0$ in Fig. 2b. Unfortunately, Eq. (6) can be solved only approximately in particular cases of low precession frequencies ($\omega \ll \omega_0$) and of a small difference between ω and the frequency ω_0 of the homogeneous ferromagnetic resonance.

In the limit $\omega_0 - \omega \ll \omega_0$ the soliton amplitude is small, and an asymptotic solution procedure[15] can be used, choosing as the small expansion parameter the quantity $\varepsilon = (1 - \omega/\omega_0)^{1/2} \ll 1$. The magnetization components m_x and m_z can then be represented by the following double series

$$m_x = \sum_{n=1}^{\infty}{}' A_n \sin n\omega t, \qquad \begin{Bmatrix} A_n \\ B_n \end{Bmatrix} = \sum_{s=0}^{\infty}{}' \begin{Bmatrix} a_{ns} \\ b_{ns} \end{Bmatrix} \varepsilon^{n+2s}, \quad (9)$$
$$m_z = \sum_{n=1}^{\infty}{}' B_n \cos n\omega t,$$

where Σ' denotes summation over odd numbers.

Accurate to ε^3, the coefficients in the temporal fundamental yield equations for the functions $A \equiv A_1$ and $B \equiv = B_1$:

$$\frac{d^2A}{dx^2} - A + \omega B + \frac{A}{8}(3A^2+B^2) + \gamma \frac{d}{dx}\hat{g}A = 0,$$

$$\frac{d^2B}{dx^2} - (1+Q)B + \omega A + \frac{1}{8}(1+Q)B(3B^2+A^2) - \gamma \frac{d}{dx}\hat{g}B = 0. \quad (10)$$

Substituting in (10) the expansions of the functions A and B in powers of the small parameter ε and expressing the frequency ω in terms of $\varepsilon(\omega = \omega_0 - \omega_0\varepsilon^2)$, we obtain equations for a_{1s} and b_{1s}. Owing to the two-component character of the considered system, the asymptotic procedure of obtaining the solution differs somewhat from that described in Ref. 15. The equations of first order in specify the connection between the functions a_{11} and b_{11}, viz., $a_{11} = \omega_0 b_{11}$. The equations of third order in ε contain besides the functions a_{11} and b_{11} the coefficients a_{12} and b_{12}. The condition for the compatibility of the resultant two equations is set by the relation $a_{12} = \omega_0 b_{12} + F\{b_{12}\}$ and by the equation for the function $b \equiv b_{11}$:

$$(1+\omega_0^2)\frac{d^2B}{dx^2} + Q\gamma\hat{g}\frac{db}{dx} - \varepsilon^2\omega_0^2\left\{2 - (1+\omega_0^2)\frac{b^2}{2}\right\}b = 0. \quad (11)$$

Comparing the first terms of (11), two different cases can be discerned: $db/dx \ll b\gamma Q/(2+Q)$ and $db/dx \sim b\gamma Q/(2+Q)$. In the first case the term with the second derivative in (11) can be discarded; this corresponds to the so-called exchange-free interaction

$$b(x) = \frac{2}{(1+\omega_0^2)^{1/2}} f(\xi), \quad \xi = \frac{2\omega_0^2}{Q\gamma}\varepsilon^2 x, \quad (12)$$

and the function $f(\xi)$ satisfies the equation

$$f - f^3 - \frac{d}{d\xi}\hat{g}f = 0. \quad (13)$$

This equation plays apparently an important role not only in the considered magnetic problem, but also in the solution of a large group of nonlinear problems with planar geometry and volume interaction (see, e.g., Ref. 16). It is known that for local interaction, in the one-dimensional case, study of low-amplitude excitations leads frequently to the Korteweg–de Vries (KdV) equation or to a modified KdV equation,[17] depending on the degree of nonlinearity. For quadratic nonlinearity in planar problems with volume interaction, the KdV equation is replaced by the Benjamin-Ono nonlinear integrodifferential equation, which is completely integrable. For self-similar solutions, the Benjamin-Ono equation reduces to Eq. (13) in which the term f^3 is replaced by f^2. In the case of a more symmetric, cubic nonlinearity, low-amplitude waves on a plane can be considered within the framework of Eq. (13). In particular, Eq. (13) is arrived at by consideration of magnetic solitons even for a different choice of the anisotropy-axis direction, and only the definition (12) of the function f and of the coordinate ξ changes somewhat. If the anisotropy axis is perpendicular to the film plane (along the z axis), then

$$b = 2^{1/2} f(\xi), \quad \xi = \frac{2\omega_0^2}{\gamma}\varepsilon^2 x,$$

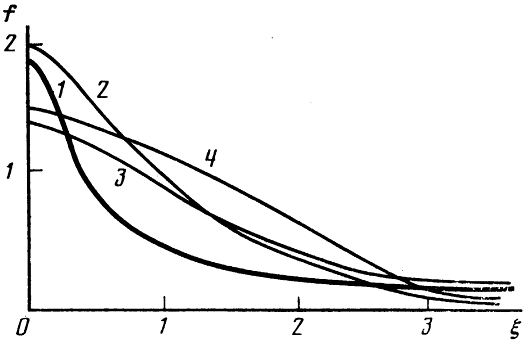

FIG. 3.

and if the axis is directed along x we have

$$b = \frac{2\omega_0}{(1+\omega_0^2)^{1/2}}, \quad \xi = -\frac{2\omega_0^2}{\gamma}\varepsilon^2 x.$$

The soliton solution of (13) was obtained by a numerical method. A plot of $f = f(\xi)$ is shown for it in Fig. 3 (curve 1). For comparison, the same figure shows a self-similar solution of the Benjamin-Ono equation, corresponding to the substitution $f^3 \to f^2$ in Eq. (13) (curve 2). Curves 3 and 4 show the solutions of the corresponding local equation for the one-dimensional problems

$$f - f^n - d^2f/d\xi^2 = 0$$

with $n = 3$ (curve 3) and $n = 2$ (curve 4).

The main feature of the considered soliton is the nonexponential (power-law) behavior of the asymptote of this solution at large distances ($f(\xi) \propto \xi^{-2}$ as $\xi \to \pm\infty$) and the unusual, for low-amplitude solitons, dependence of the localization-region size Δ on the amplitude ε. In our case $\Delta \sim \varepsilon^{-2}$, i.e., the soliton is more weakly localized than in the one-dimensional magnetic problems, where $\Delta \sim \varepsilon^{-1}$ (Ref. 4). At the same time, comparison of curves 1 and 3 of Fig. 3 shows that in a film soliton-one can distinguish between two regions: slowly decreasing asymptotes and a compact soliton core (more compact than in magnetic solitons without allowance for magnetodipole interaction). Note that a narrow core is separated also in domain walls in ferromagnetic films.[14]

If a magnetic soliton is regarded as a bound state of a large number of magnons, a weak localization of solitons in a film attests to a weaker coupling between them. It is interesting to note that in one-dimensional problems one encounters sometimes also solitons with power-law asymptotes (algebraic solitons),[4,5] but only at selected values of the soliton parameters. In these limiting situations the quasiparticle binding energy in the soliton is zero in any case.

The final expression for the principal approximation of the solution for a magnetic soliton in a film can be rewritten in the form

$$\begin{Bmatrix} m_x \\ m_z \end{Bmatrix} \approx \frac{2\varepsilon}{(1+\omega_0^2)^{1/2}} f\left[\frac{2(1+Q)}{Q\gamma}\varepsilon^2 x\right]\begin{Bmatrix} \omega_0 \sin\omega t \\ \cos\omega t \end{Bmatrix}. \quad (14)$$

It follows from (14) that the inequality $\Delta \gg h$ is satisfied for $1 - \omega/\omega_0 \ll (Q/2\omega_0)^2$, and the inequality $db/dx \ll bQ\gamma/(2+Q)$ for $1 - \omega/\omega_0 \ll (h/l_0)^2 Q^4/8\omega_0^2(2+Q)$. Which of these inequalities is stronger is determined by the parameter $hQ/l_0(2+Q)$, the ratio of which to unity can in principle be arbitrary. If, however, the frequency ω is close enough to ω_0 it is possible to satisfy both inequalities, which ensure correctness of our approximations.

Let us find the number N of magnons bound in a soliton. To this end it is necessary to calculate in the semiclassical approximation the adiabatic integral $I = N/\hbar$ (Refs. 4 and 5):

$$I=(\hbar/2\mu_0)h\int dx\frac{1}{2\pi}\int_0^T dt(M_0-M_y)\frac{d}{dt}\operatorname{arctg}\frac{M_x}{M_z}. \quad (15)$$

Substitution of (14) in (15) shows readily that as the soliton amplitude tends to zero ($\omega\to\omega_0$) its adiabatic invariant, and hence also the number of bound magnons, remains finite:.

$$N(\omega_0)\approx\frac{M_0}{\mu_0}h^2\frac{Q^2}{4(2+Q)(1+Q)^{1/2}}\int_{-\infty}^{\infty}d\xi f^2(\xi). \quad (16)$$

Computer calculations yielded $\int f^2 d\xi \approx 2.49$. Thus, the final expression for the number of bound magnons per unit length along the y axis takes the form

$$N(\omega_0)\approx 0{,}62\frac{M_0}{\mu_0}h^2\frac{Q^2}{(2+Q)(1+Q)^{1/2}}. \quad (17)$$

The presence of a limiting value of N as $\omega\to\omega_0$ is reminiscent of the situation for two-dimensional dynamic solitons.[4,5] In our case, however, expression (17) contains a factor Q^2, therefore the difference between the limiting value of N and zero is due exclusively to allowance for the magnetodipole interaction. Moreover, as seen from (17), the role of the characteristic volume occupied by the magnons in a soliton is assumed by h^2. (In two-dimensional solitons, for which $N(\omega_0)\approx 5.8\ (M_0/\mu_0)l_0^2$, an analogous role is assumed[4] by l_0^2.

The existence of a threshold value $N(\omega_0)$ seems to complicate the task of exciting magnetic solitons in films.

To assess the stablity of the obtained solitons we must know the sign of the derivative $dN/d\omega$ as $\omega\to\omega_0$. It can be determined if a solution exists for a low-amplitude soliton for one more frequency close to ω_0.

Equation (11) was found to have an exact solution at the frequency

$$\omega_*=\omega_0\{1-\gamma^2Q^2/6(1+Q)(2+Q)\}. \quad (18)$$

It is easy to verify that for $\omega=\omega_*$ (i.e., for $\varepsilon = Q\gamma/[6(1+Q)(2+Q)]^{1/2}$) this solution leads to the following expression for the magnetization in the soliton:

$$\begin{Bmatrix} m_x \\ m_z \end{Bmatrix}=\frac{4\gamma Q/[3(2+Q)(1+Q)^{1/2}]}{1+[\gamma Qx/3(2+Q)]^2}\begin{Bmatrix}\omega_0\sin\omega_* t\\ \cos\omega_* t\end{Bmatrix}. \quad (19)$$

Of course, this solution is meaningful only if the inequality $1-\omega_*/\omega_0 \ll 1$ is satisfied, i.e., for $\varepsilon \ll 1$ (the condition that expansion in terms of a small parameter be valid). This leads to the requirement $\gamma \ll 1$ or $h/l_0 \ll 1/Q^2$. For films that are not too thin, this relation can be satisfied only in the case of strong anisotropy $\beta \gg 4\pi$ ($Q \ll 1$). If the condition $h/l_0 \ll 1/Q$ is simultaneously met, we arrive at the inequality $\Delta \gg h$ which was assumed by us initially (the latter can be easily verified by recognizing that the soliton width is $\Delta \sim l_0^2/hQ^2$).

Quasiclassical quantization of the solution (19) leads to the value of the number of magnons bound in the soliton:

$$N(\omega_*)\approx\frac{\pi}{3}\frac{M_0}{\mu_0}h^2\frac{Q^2}{(2+Q)(1+Q)^{1/2}}. \quad (20)$$

Comparng (20) with (17) we get $N(\omega_0)\approx 0.6N(\omega_*)$ and hence $dN/d\omega<0$ near the frequency ω_0. Although the proof of soliton stability requires a separate analysis in each specific case, in all the previously considered problems[4,5] such an inequality sign attested (for positive precession frequency) to stability of the dynamic solitons.

We consider now the inverse limiting case-immobile low-frequency solitons. If $\omega \ll \omega_0$, dynamic solitons in magnets can be regarded as bound states of two domain walls of opposite sign.[4,5] In a magnetic film, even in the case of uniaxial anisotropy, allowance for the magnetodipole interaction leads to effective biaxiality of the crystal. It is known[4] that domain walls bound in a biaxial ferromagnet into a dynamic soliton oscillate relative to each other. At low soliton frequency the distance between the domain walls exceeds for the greater part of the time the magnetic length, and individual domain walls preserve their individuality in the course of the interaction. At $\omega \ll \omega_0$, therefore, the problem of a soliton in a magnetic film can be solved in the adiabatic limit. It was indicated at the beginning of the article that the dependence of the domain-wall magnetization on the film thickness can be neglected only for extremely thin films with $h \ll l_0$, a condition not yet met for YIG films. At low frequencies, however, the damping becomes less significant, and when dealing with such solitons one can consider only films made of a material with large damping. In particular, the inequality $h \ll l_0$ is fully attainable for permalloy films,[20] in which bound states of domain walls were observed in experiment.[20,21]

In the case of thin ferromagnetic films with $h \ll l_0$ we can use the smallness of the parameter γ and regard the terms in Eq. (6) which are proportional to γ as a perturbation. We rewrite this equation in the form

$$\frac{\partial \mathbf{m}}{\partial t}+\left[\mathbf{m}\frac{\partial^2\mathbf{m}}{\partial x^2}\right]+[\mathbf{m}\hat{J}_0\mathbf{m}]=\gamma\mathbf{R}(\mathbf{m}),$$
$$\mathbf{R}=[\mathbf{m}\hat{J}_1\mathbf{m}], \quad (21)$$

where

$$\hat{J}_0=\operatorname{diag}(0,1,-Q),\qquad \hat{J}_1=\operatorname{diag}\left(\frac{\partial}{\partial x}\hat{g},0,-\frac{\partial}{\partial x}\hat{g}\right).$$

Equation (21) describes, in the limit $\gamma=0$, the dynamics of magnetization in a biaxial unbounded ferromagnet. In this limit, the solution corresponding to a moving domain wall, using the angle variables (θ,φ)

$$\mathbf{m}=(\sin\theta\cdot\sin\varphi,\ \cos\theta,\ \sin\theta\cdot\cos\varphi) \quad (22)$$

can be represented in the form[4,5]

$$\begin{Bmatrix} m_z^0 \\ m_x^0 \end{Bmatrix}=\begin{Bmatrix}\cos\varphi_0\\ \sin\varphi_0\end{Bmatrix}\sin\theta_0,\qquad m_y^0=\cos\theta_0, \quad (23)$$

where $\varphi_0 = \text{const}$ and

$$\theta_0 = s\cdot 2\,\mathrm{arctg}\exp[\varkappa(x-x_0(t))],\quad \varkappa=(1+Q\cos^2\varphi_0)^{1/2} \qquad (24)$$

(s is the topological charge of the domain wall). A domain-wall center with coordinate $x_0 = V(\varphi_0)t + \delta$ moves with velocity

$$\frac{dx_0}{dt} = V(\varphi_0) = -sQ\cos\varphi_0\sin\varphi_0/\varkappa. \qquad (25)$$

The solution (23)–(25) is characterized by two arbitrary parameters, φ_0 and δ. Allowance for the magnetodipole interaction (i.e., of the right-hand side of Eq. (21)) leads to a change of the structure and dynamics of the domain walls. For a small perturbation ($\gamma \ll 1$) this change can be taken into account in the adiabatic approximation. It reduces to the assumption that in response to a small perturbation the solution (23)–(25) retains its functional form, but φ and δ become slowly varying functions of time. In this case the role of the perturbation is played by the action exerted on the given domain wall by the second wall bound with the given one into a soliton pair. Inasmuch as in the domain wall (23)–(25) the deflection of the magnetization vector from its basic state decreases exponentially as the distance from its center increases, we can regard, in the principal approximation, two domain walls with centers separated by a distance L that exceeds greatly the width $\Delta \sim 1/\varkappa \sim 1$ of each of them (in dimensional units $L \gg l_0$), as practically noninteracting. At large distances the weak interaction between the walls is determined mainly by a nonlocal magnetodipole interaction with a power-law decrease. It will be shown below that this interaction is decisive up to distances $L \sim l_0 \ln(Qh/l_0)$. The interaction between the domain walls leads to formation of a weakly bound state of two walls with topological charges $s = \pm 1$ (low-frequency soliton), in which the walls retain their individuality and oscillate about the common centroid at a frequency $\omega \ll \omega_0$.

Let us examine the simplest case of a low-frequency soliton with an immobile centroid. The magnetization in this soliton precesses in phase at all points of the sample, i.e., φ is independent of coordinate.[4,5] If the center of the soliton is at the origin ($x = 0$), the soliton solution takes in the adiabatic approximation the form of two domain walls of type (23)–(25) with $s = \pm 1$, symmetric about the origin at the points $\pm x_0$, i.e., the soliton size is $L = 2x_0$):

$$\mathbf{m}_{ad} = \overset{0}{\mathbf{m}}_{s=1}\{x-x_0(t),\varphi(t)\} + \overset{0}{\mathbf{m}}_{s=-1}\{x+x_0(t),\varphi(t)\}. \qquad (26)$$

In Eq. (26), $x_0(t) = V\{\varphi(t)\}t + \delta(t)$ and

$$V\{\varphi\} = -sQ\sin\varphi\cos\varphi/\varkappa(\varphi). \qquad (27)$$

The slowly varying functions $\delta(t)$ and $\varphi(t)$ are determined from the so-called adiabatic equations. To find them, we consider small increments to the adiabatic approximation: $\mathbf{m} = \mathbf{m}_{ad} + \delta\mathbf{m}$. The function $\delta\mathbf{m}$ is small to the extent that the parameter γ is small and satisfies the linearized equation

$$\hat{L}\{\mathbf{m}_{ad}\}\delta\mathbf{m} = \mathbf{\Gamma}(\mathbf{m}_{ad}, \dot{\varphi}, \dot{\delta}), \qquad (28)$$

in which the smallness of the right-hand side is due to γ and to the slow evolution of the adiabatic parameters φ and δ. The evolution equations for them are obtained from the condition that the right-hand side of the inhomogeneous equation (28) be orthogonal to the eigenfunctions of the homogeneous equation $\hat{L}\mu = 0$ (Ref. 22).

We rewrite Eq. (21) in terms of the angle variables (22):

$$\frac{\partial^2\theta}{\partial x^2} - \left[1 + \left(\frac{\partial\varphi}{\partial x}\right)^2 + Q\cos^2\varphi\right] \times \sin\theta\cos\theta + \sin\theta\frac{\partial\varphi}{\partial t} = \gamma\frac{\delta W}{\delta\theta},$$
$$\frac{\partial}{\partial x}\left(\sin^2\theta\frac{\partial\varphi}{\partial x}\right) + Q\sin^2\theta\cos\varphi\sin\varphi - \sin\theta\frac{\partial\theta}{\partial t} = \gamma\frac{\delta W}{\delta\varphi}, \qquad (29)$$

where the nonlocal term W is given by

$$W = -\frac{1}{2\pi}\int dx \oint \frac{dx'}{x'-x}\left\{\frac{\partial m_z(x)}{\partial x} m_z(x') - \frac{\partial m_x(x)}{\partial x} m_x(x')\right\}. \qquad (30)$$

Since the distance between the domain walls L is assumed to be large ($L \gg \Delta$), the overlap of the fields of the individual walls is exponentially small in the principal approximation. We can then consider in Eqs. (29) separately the solutions for each of the domain walls, taking the presence of the other wall into account only in the nonlocal term (30).

We seek the solution for each of the walls in the form

$$\theta = \theta_0 + \theta_1,\quad \varphi = \varphi_0 + \varphi_1,\quad \theta_1\varphi_1 \sim \gamma \ll 1. \qquad (31)$$

Substituting these equations in (29) and linearizing them in the small increments θ_1 and φ_1, we obtain a system of two linear differential equations. It is convenient to rewrite them in the form of one vector equation

$$\hat{L}\Phi = \Gamma, \qquad (32)$$

for the column vector $\Phi = \mathrm{col}(\theta_1\varphi_1)$. In Eq. (32) the row vector Γ is of the form

$$\Gamma = \left(\gamma\frac{\delta W}{\delta\theta} - \sin\theta_0\cdot\dot{\varphi},\quad \gamma\frac{\delta W}{\delta\varphi} - \sin\theta_0\frac{\partial\theta_0}{\partial x}\dot{\delta}\right), \qquad (33)$$

and $\hat{L}$ is a linear differential operator whose elements are functions of θ_0 and φ_0:

$$L_{11} = \frac{\partial^2}{\partial x^2} - \cos 2\theta_0(1+Q\cos^2\varphi_0),$$
$$L_{12} = \sin\theta_0\left(Q\cos\theta_0\sin 2\varphi_0 + \frac{\partial}{\partial t}\right),$$
$$L_{21} = \sin\theta_0\left(\frac{1}{2}Q\cos\theta_0\sin 2\varphi_0 - \frac{\partial}{\partial t}\right), \qquad (34)$$
$$L_{22} = Q\sin^2\theta_0\cos 2\varphi_0.$$

The homogeneous equation $\hat{L}\Phi = 0$ has the following two linearly independent solutions: $\Phi_1 = \mathrm{col}(\partial\theta_0/\partial x, 0)$ and $\Phi_2 = \mathrm{col}(\partial\theta_0/\partial\varphi, 0)$. The physical meaning of these solutions is clear. The first (Φ_1) corresponds to a small shift of the domain-wall center (i.e., to a shear mode) and the second (Φ_2) to a small change of the azimuthal angle (i.e., of the domain-wall velocity). It follows from the solution (23)–(25) that $\partial\theta_0/\partial x = \varkappa\,\mathrm{sech}\,\varkappa(x - x_0)$ and $\partial\theta_0/$

$\partial\varphi = (Vx + Q\cos 2\varphi t)\operatorname{sech}\varkappa(x - x_0)$.

In the case of an immobile soliton centroid, φ is independent of the coordinate and we have for a domain wall with center at the point $x = x_0$

$$\frac{\delta W}{\delta\theta} = \cos 2\varphi \cos\theta_0 \frac{\partial}{\partial x}\hat{g}\sin\theta,$$
$$\frac{\delta W}{\delta\varphi} = -\sin 2\varphi \sin\theta_0 \frac{\partial}{\partial x}\hat{g}\sin\theta, \qquad (35)$$

where $\theta_0 = \theta_0(x - x_0)$ is the solution (23), (24) for a wall at the point $x = x_0$, while $\theta_0(x - x_0) + \theta_0(x + x_0)$ is the adiabatic approximation (26), (27) of the solution for two domain walls with coordinates x_0 and x_0. The orthogonality conditions $(\Phi_1 \cdot \Gamma) = (\Phi_2 \cdot \Gamma) = 0$ yield adiabatic equations for φ and δ. Taking the obvious relation $\int_{-\infty}^{\infty} \sin\theta_0(\partial\theta_0/\partial x)dx = 2$ into account, they can be represented in the form

$$\frac{d\varphi}{dt} = \frac{1}{2}\gamma\int\frac{\delta W}{\delta\theta}\frac{\partial\theta_0}{\partial x}dx, \qquad (36)$$

$$2\frac{d\delta}{dt} = \gamma\int\frac{\delta W}{\delta\varphi}dx + \gamma V\int\frac{\delta W}{\delta\theta}\frac{(x-x_0)\,dx}{\operatorname{ch}\varkappa(x-x_0)}. \qquad (37)$$

We confine ourselves to the simplest case of a strongly anisotropic magnet with $Q \ll 1$. In this limit we have $\varkappa \approx 1$ and $V \sim Q \ll 1$. The second term in the right-hand side of (37) can therefore be omitted. We introduce the variable $\zeta = x - x_0$, reckoned from the center of one of the domain walls. If the latter is chosen to be a wall with topological charge $s = \pm 1$, i.e., the left-hand side in the soliton, we have for it $x_0 < 0$. In this case the soliton dimension is $L = -2x_0 = -2Vt - 2\delta$. We take into account the symmetry of the function $\theta_0(\zeta)$ and rewrite Eqs. (36) and (37) in the form

$$\frac{d\varphi}{dt} = \frac{\gamma}{2}\cos 2\varphi\int_{-\infty}^{\infty} d\zeta\frac{\partial\theta_0}{\partial\zeta}\cos\theta_0(\zeta)\frac{\partial}{\partial\zeta}\hat{g}\sin[\theta_0(\zeta-L)],$$
$$\frac{dL}{dt} = \tilde{Q}\sin 2\varphi + \gamma\sin 2\varphi\int_{-\infty}^{\infty}\sin\theta_0(\zeta)\frac{\partial}{\partial\zeta}\hat{g}\sin[\theta_0(\zeta-L)], \qquad (38)$$

where

$$\tilde{Q} = Q + \frac{\gamma}{\pi}\int_{-\infty}^{\infty} d\eta \operatorname{sech}\eta\frac{\partial}{\partial\eta}\int_{-\infty}^{\infty} d\eta'\frac{\operatorname{sech}\eta'}{\eta'-\eta}. \qquad (39)$$

The renormalization of the constant Q takes into account the weak change of the domain-wall structure by self-action via the magnetic field.

To estimate the integrals in (38) for $L \gg 1$ ($L \gg l_0$ in dimensional units) we can replace approximately $\sin\{\theta_0(\zeta - L)\} = \operatorname{sech}\varkappa(\zeta - L)$ by $\pi\delta(\zeta - L)$ and obtain a final expression for the adiabatic equations:

$$\frac{d\varphi}{dt} = -\pi\gamma\frac{1}{L^3}\cos 2\varphi, \qquad (40)$$

$$\frac{dL}{dt} = \left(\tilde{Q} + \pi\gamma\frac{1}{L^2}\right)\sin 2\varphi. \qquad (41)$$

To investigate the dynamics of a magnetic soliton there is no need to solve this system of equations. It is easy to verify that (40) and (41) are Euler equations for the following effective Lagrangian (written in the initial dimensional units):

$$\mathcal{L}_\cdot = 2M_0^2(\alpha\beta)^{1/2}h\left\{\frac{1}{\bar{\omega}l_0}L\frac{\partial\varphi}{\partial t} - \frac{1}{2}\cos 2\varphi\left[\tilde{Q} + \pi\gamma\left(\frac{l_0}{L}\right)^2\right]\right\}. \qquad (42)$$

The effective Lagrangian (42) can be obtained also directly from the exact Lagrangian of the initial system (see Ref. 4):

$$\mathcal{L} = \int dV\left\{\frac{\hbar}{2\mu_0}(M_0 - M_y)\frac{\partial\varphi}{\partial t}\right\} - E \qquad (43)$$

with an energy E given by Eq. (1). The transition from (43) to (42) corresponds to the known Slonczewski procedure.[23] The individual terms in the effective Lagrangian (42) have a clear physical meaning. The "kinetic" term is proportional to $L(\partial\varphi/\partial t)$ and is obtained from the first term of (43) if it is recognized that the length of the remagnetized region, in which the difference $M_0 - M_y \approx 2M_0$ differs from zero is equal to L. The quantity $hM_0^2(\alpha\beta)^{1/2}\tilde{Q}\cos 2\varphi$ describes the energy change of the two domain walls in a biaxial ferromagnet when the azimuthal angle φ is varied (i.e., when the Bloch domain walls change into Néel walls). In this case the additional magnetic anisotropy in the *xz* plane, characterized by the parameter $\tilde{Q}$, is induced by the magnetodipole interaction. The "difficult" axis is z, as is customary for thin films with $h \ll l_0$, in which the minimum energy is possessed by a Néel wall with the magnetization rotated in the plane of the film.[14] Finally, the last term in (42), proportional to $(l_0/L)^2$, describes domain-wall interaction via their intrinsic magnetic fields. Owing to the long-range character of this interaction, the dynamics of the domain walls bound in the soliton differs from that in an unlimited biaxial ferromagnet.[4]

Since the system is conservative, its total energy is constant, being an integral of the motion. An expression for the energy is easily obtained by the usual rules from the effective Lagrangian (42):

$$E = 2E_0(\varphi)h + M_0^2(\alpha\beta)^{1/2}h\cos 2\varphi\cdot\pi\gamma(l_0/L)^2$$
$$\approx (4+\tilde{Q})M_0^2(\alpha\beta)^{1/2}h + M_0^2(\alpha\beta)^{1/2}h\cos 2\varphi\cdot\left[\tilde{Q} + \pi\gamma\left(\frac{l_0}{L}\right)^2\right], \qquad (44)$$

where $E_0 = \sigma(1 + \tilde{Q}\cos^2\varphi)^{1/2}$ is the domain-wall energy in the biaxial ferromagnet and $\sigma = 2M_0^2(\alpha\beta)^{1/2}$. Equation (44) differs substantially from the expression for the energy of a magnetic soliton in a biaxial-ferromagnet slab of thicknesss h, without allowance for the magnetodipole interaction[4]:

$$E = 2E_0(\varphi)h - h\sigma\exp(-L/l_0). \qquad (45)$$

It is seen from a comparison of (44) and (45) that the domain wall interaction via the magnetic fields, which decreases following a power law, becomes comparable with their interaction via overlap of the exponential asymptotes (without allowance for the magnetodipole term) at distances $L \sim L_* = l_0|\ln(\tilde{Q}h/l_0)|$. If the magnet has strong anisotropy ($\tilde{Q} \ll 1$) and its thickness is small ($h \ll l_0$) this dimension exceeds substantially the magnetic length l_0. It is seen from (44) that the magnetodipole interaction of domain walls depends strongly on the angle φ. For in-phase rotation of the magnetization, Bloch walls with $\varphi = 0$ repel one another, while Néel walls (with $\varphi = \pm\pi/2$) attract. Corresponding to low-frequency solitons are coupled domain walls that are separated in the course of the oscillations by large distances. The total energy of such solitons differs therefore little from the energy of two solitary Néel walls: $0 < 2\sigma h - E \ll E$. In place of the energy E it is convenient to parametrize the degree of soliton excitation by the values L_m of the maximum distance between the oscillating domain walls. This parameter is connected with the energy by the relation $E = 2\sigma h(1 - \gamma\pi l_0^2/4L_m^2)$. From (44) follows also a connection betwen L and φ:

$$\cos 2\varphi = -(1+\lambda^2/L_m^2)/(1+\lambda^2/L^2), \tag{46}$$

where $\lambda^2 = \pi\gamma l_0^2/\tilde{Q} = (\lambda/2)hl_0$. In the case of thin films with $h \ll l_0$, the resultant characteristic parameter with the dimension of length is $\lambda \ll l_0$. For large distances between the walls, $L \gg l_0$, Eq. (46) simplifies to $L/L_m \approx [1 + 2L_m^2 \cdot (\pi/2 - \varphi)^2/\lambda^2]^{-1/2}$. Thus, when L changes from L_m to the value L_* up to which an analysis within the framework of Eqs. (40) and (41) is valid, the angle φ changes little near $\varphi = \pi/2$, and deviates from $\pi/2$ by an amount $\sim (h/l_0)^{1/2}/\ln(\tilde{Q}h/l_0) \ll 1$. At smaller distances $L < L_*$ the rotation of the magnetization continues and reaches the angle $\varphi = 0$ at the closest distance $L = L_0$ between the domain walls, at which both are pure Bloch walls. The value of L_0 can be easily estimated. For $L < L_*$ we can use expression (45), from which we find, for $E \approx 2\sigma h$ (at $\varphi = 0$), that $L_0 \sim l_0 \ln(1/\tilde{Q})$, i.e., for thin films with $h \ll l_0$ we have indeed $L_0 \ll L_*$.

Thus, the minimum size of a low-frequency soliton in a thin film is the same as in a bulk sample, but the maximum size differs substantially, and this leads to specific relations for the number of magnons bound in the soliton and for its frequency. The difference manifests itself at low frequencies, when $L_m \gg L_*$ (the corresponding estimate for the frequencies will be given below). In this limit, the main contribution to the integral characteristics of the soliton (such as the number N of magnons and the period T) yields that part of the oscillation period in which the distance betwen the walls is large: $L \sim L_m$ and Eqs. (40) and (41) can be used.

We carry out a semiclassical quantization of low-frequency solitons and calculate the dependence of their frequency on the number N of bound magnons. The adiabatic invariant (15) can be approximately represented in the form

$$I = [(\hbar M_0 h)/2\pi\mu_0] \int_0^T dt\, L \frac{d\varphi}{dt}$$

and the number of the bound nucleons is then

$$N = \frac{4hM_0}{\pi\mu_0} \int_{L_0}^{L_m} L\left(\frac{d\varphi}{dt} \Big/ \frac{dL}{dt}\right) dL. \tag{47}$$

Substituting in (47) expressions (40) and (41) for $d\varphi/dt$ and dL/dt and using (46), the resultant integral is easily evaluated and the relation $N = N(L_m)$ obtained:

$$N \approx \frac{2}{\pi^{1/2}} \frac{M_0}{\mu_0} h(hl_0)^{1/2} \ln(L_m/l_0). \tag{48}$$

Since Eqs. (40) and (41) can be used only for $L > L_*$, the result (48) is valid for $L_m \gg L_*^2/L_0$, i.e., for $L_m \gg l_0[\ln(Qh/l_0)]^2/|\ln Q|$.

The domain-wall oscillation frequency corresponding to a specified energy or a specified value L_m can be easily found by calculating the oscillation period $T = 4\int_{L_0}^{L_m} dL/(dL/dt)$. Substituting expression (41) for dL/dt in dimensional units into the integral, we obtain

$$T \approx \frac{4}{\pi^{1/2} Q\omega_0} \frac{L_m^2}{l_0(hl_0)^{1/2}}. \tag{49}$$

It follows hence that the soliton oscillation frequency is

$$\frac{\omega}{\omega_0} = \frac{\pi^{3/2}}{2} Q \frac{l_0(hl_0)^{1/2}}{L_m^2}. \tag{50}$$

It follows from the foregoing estimates for the allowable values of L_m and from (50) that the results are valid at very low frequencies

$$\omega/\omega_0 \ll Q(h/l_0)^{1/2}(\ln Q)^2/[\ln(Qh/l_0)]^4.$$

For $Q \sim 1$ and $h \sim l_0$, however, they can be used as estimates if the weaker inequality $\omega \ll \omega_0$ is satisfied. Equation (50) differs substantially from the corresponding dependence for an unbounded biaxial ferromagnet, in which $\omega/\omega_0 \sim Q^{1/2}\exp(-L_m/l_0)$ (Ref. 4).

Comparing Eqs. (48) and (50) we can reconstruct the sought dependence of the number of magnons bound in a soliton on its frequency:

$$N \sim \frac{M_0}{\mu_0} h(hl_0)^{1/2} \ln\left[\frac{\omega_0}{\omega} Q\left(\frac{h}{l_0}\right)^{1/2}\right]. \tag{51}$$

We note first of all that as $\omega \to 0$ the number N of bound magnons increases without limit, in contrast to a soliton in an unbounded biaxial ferromagnet, where it is finite as $\omega \to 0$ and is equal (recalculated to the thickness h) to $(M_0/\mu_0) \times hl_0 \ln Q$ (Ref. 4). On the other hand, relation (51) is close to the corresponding relation for an unbounded uniaxial magnet, for which $N_0 \sim (M_0/\mu_0) \times hl_0|\ln(\omega/\omega_0)|$. These values, however differ by a factor $(h/l_0)^{1/2}$, i.e., $N \ll N_0$.

The asymptotic values of the function $N = N(\omega)$ as $\omega \to 0$ and $\omega \to \omega_0$ make it possible to determine qualitatively the function $N(\omega)$ in the entire frequency range $0 < \omega < \omega_0$. It is shown by curve 1 of Fig. 4. For comparison, curves 2 and 3 are plots of $N = N(\omega)$ for solitons in an unbounded easy-axis and biaxial ferromagnet. Curve 4 corresponds to radially symmetric two-dimensional solitons without allowance for the magnetodipole interaction.

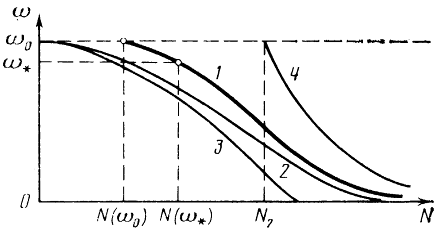

FIG. 4.

We can draw thus the following conclusions. Magnetic films, just as in unbounded samples, retain an ability to contain dynamic magnetic solitons. In films, however, solitons have a number of peculiarities: they are less localized and are formed by binding of a finite number of magnons; this complicates the problem of exciting such solitons.

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