Quasiparticle current in superconductor-semiconductor-superconductor junctions

A. V. Tartakovskiĭ and M. V. Fistul'

Moscow Institute of Steel and Alloys

(Submitted 6 April 1988)
Zh. Eksp. Teor. Fiz. **94**, 353–367 (September 1988)

We show that the quasi-particle current in a superconductor-semiconductor-superconductor junction can increase considerably due to the resonant passage of quasiparticles along special trajectories from periodically arranged localized centers. It follows from the developed theory that when the resistivity of the junction increases one observes a transition from a current excess to a current deficit in the current-voltage characteristic (at high voltages). We also study the effect of the transparency of the boundaries on the resonant tunneling in such junctions.

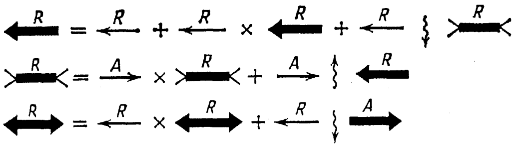
I. INTRODUCTION

Superconducting junctions with semiconductor layers (S-Sm-S) belong to the most interesting work couplings: the Josephson effect is combined in them with a unusual mechanism of current flow. The properties of such systems can differ strongly from the properties of tunnel junctions and depend on the density of localized centers (LC) in the semiconductor. A theoretical treatment of the superconducting current flow in S-Sm-S junctions was given in the papers by Aslamazov and one of the present authors.[1–4] In particular, the elucidated the important rôle played by the fluctuations produced in the large-scale potential by inhomogeneities in the arrangement of charged LC and they analyzed the possibilities for resonance tunneling. It turned out that in a wide temperature range the superconducting current of an S-Sm-S junction is determined by the resonance passage of Cooper pairs through the semiconductor layer along special trajectories from the periodically arranged LC (Lifshitz's resonance-percolation trajectories[5]). However, although the probability for forming such trajectories is small, when the density of LC increases, and even well in advance of the onset of degeneracy, the resonance mechanism for current transmission turns out to be preferable to the usual tunneling, thanks to the smallness of the damping of the coherent electrons.

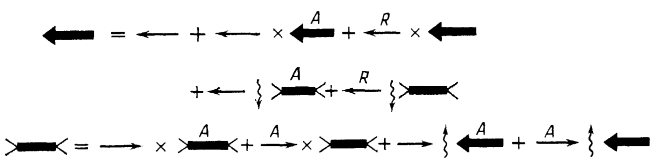
A large number of papers have been devoted to an experimental study of the properties of S-Sm-S junctions, and recently particular attention has been paid to junctions with a layer of amorphous semiconductors.[6–11] On the basis of a comparison with the theory of the dependence of the Josephson current on the temperature and on the thickness of the semiconductor layer, it was concluded in Ref. 6 that a resonance mechanism causes its transmission. At the same time the assumption was expressed in the literature[6,7] that the so far theoretically unexplained peculiarities in the current-voltage characteristics (CVC) of S-Sm-S junctions, as current excess of deficit at large voltages, which are characteristic for structures with a direct conductivity, also arise thanks to the resonance mechanism of current transmission. To explain these and a number of other features we find in the present paper the quasiparticle current of an S-Sm-S junction when there is resonance tuneling of the electrons.

2. GENERAL EXPRESSION FOR THE TOTAL CURRENT THROUGH A JUNCTION

The current density can be expressed in terms of the Keldysh Green function $G(\mathbf{r},t;\ \mathbf{r}',t')$ of the system; to find these we use the Gor'kov equations written in integral form, using Keldysh's method[12] (we neglect the interelectron interaction and the interaction between the electrons and other quasiparticles):



(1a)



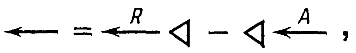
(1b)

The heavy lines correspond here to the exact Green functions G, F^+, and F (the latter two are anomalous ones) of a system with superconducting order parameter $\Delta(\mathbf{r},t)$ and external perturbation operator $\hat{U}(\mathbf{r},t)$ (in the case of interest to us this is simply the potential of the applied electric field); the thin lines correspond to the normal unperturbed system; the lines with the symbols $R(A)$ denote retarded (advanced) functions, and the lines without symbols denote Keldysh functions; the incoming and outgoing wavy lines correspond to factors $i\Delta$ and $-i\Delta^*$; the cross corresponds to $-i\hat{U}$.

In an unperturbed stationary system the thermodynamic identity,

$$G_\omega(\mathbf{r},\mathbf{r}')=\operatorname{th}\frac{\omega}{2T}\{G_\omega^R(\mathbf{r},\mathbf{r}')-G_\omega^A(\mathbf{r},\mathbf{r}')\} \qquad (2)$$

is satisfied. We rewrite (2) in diagram form:



(3)

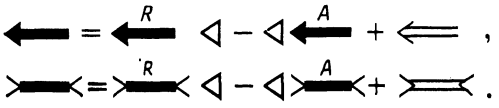
where the triangle correspond to the function

$$f_0(t,t')=\int \operatorname{th}\frac{\omega}{2T}\exp[-i\omega(t-t')]\frac{d\omega}{2\pi}.$$

When $\hat{U}(\mathbf{r},t)$ and $\Delta(\mathbf{r},t)$ are switched on adiabatically the system becomes, generally speaking, nonstationary and nonequilibrium so that Eq. (3) for the exact Green functions does no longer hold. To express the Keldysh functions in terms of the retarded and advanced ones we introduce new functions (they correspond to the so-called anomalous parts

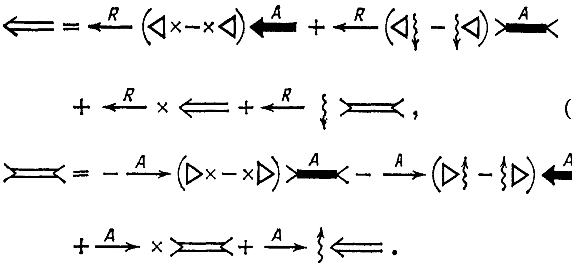
 0038-5646/88/091935-08$04.00

of the Keldysh functions[13]) in accordance with the equations



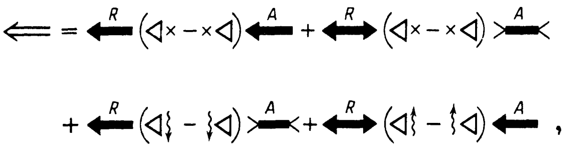
(4)

Substituting (3) and (4) into (1b) and using (1a) we get

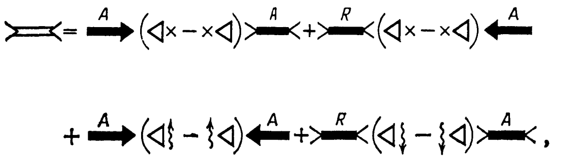


(5)

Equations (5) uniquely determine the functions we introduced. Using (1a) and the identity $f_0(t,t') = -f_0(t',t)$ (a consequence of the fact that $\tanh(\omega/2T)$ is an odd function) we can easily check by direct substitution that the formulae



(6a)



(6b)

give a solution of the set (5). It is no longer possible to change in these formulae directly to an electric field which is constant in time: the terms containing $f_0(t_1,t_2)[U(t_2) - U(t_1)]$ deviate significantly from zero for any arbitrarily slow switching-on of $U(\mathbf{r},t)$, but they vanish when we substitute $U(\mathbf{r},t) = U(\mathbf{r})$. Nonetheless one can transform Eqs. (4), (6a) to a form in which this substitution is legitimate:

$$G(\mathbf{r},t;\mathbf{r}',t')=\int dt_1[G^R(\mathbf{r},t;\mathbf{r}',t_1)f^+(\mathbf{r}',t_1,t') - f^+(\mathbf{r},t,t_1)G^A(\mathbf{r},t_1;\mathbf{r}',t')] + \frac{1}{2m}\int d^3r_1\,dt_1\,dt_2\{[\nabla_{r_1}f^+(\mathbf{r}_1,t_1,t_2)] \times\{G^R(\mathbf{r},t;\mathbf{r}_1,t_1)\nabla_{r_1}G^A(\mathbf{r}_1,t_2;\mathbf{r}',t') - G^A(\mathbf{r}_1,t_2;\mathbf{r}',t')\nabla_{r_1}G^R(\mathbf{r},t;\mathbf{r}_1,t_1)\}+[\nabla_{r_1}f^-(\mathbf{r}_1,t_1,t_2)] \times\{F^R(\mathbf{r},t;\mathbf{r}_1,t_1)\nabla_{r_1}F^{+A}(\mathbf{r}_1,t_2;\mathbf{r}',t') - (\nabla_{r_1}F^R(\mathbf{r},t;\mathbf{r}_1,t_1))F^{+A}(\mathbf{r}_1,t_2;\mathbf{r}',t')\}\}, \quad (7)$$

where

$$f^\pm(\mathbf{r},t_1,t_2)=f_0(t_1,t_2)\exp\left[\pm i\int_{t_1}^{t_2} U(\mathbf{r},t)\,dt\right].$$

To check that Eq. (7) is identical with Eqs. (4), (6a), it is sufficient to integrate the integral over $\mathbf{r}_1$ by parts, to use the Gor'kov equations to get expressions for the Lablacians which appear, and to integrate the terms with derivatives with respect to t_1 and t_2 by parts over the time variable. We have dropped in (7) the terms which depended explicitly on $\Delta(\mathbf{r},t)$, since they are identically equal to zero due to the Josephson time dependence of the phase of the order parameter [the fact that only $U(\mathbf{r},t)$ is the physical perturbation manifests itself here].

We denote the region occupied by the barrier by B and its boundary with the banks by σ_1 and σ_2. The integration over $\mathbf{r}_1$ in (7) is practically only over B, where there is an electric field, and hence, $\nabla f^{\pm} \neq 0$. Putting $U(\mathbf{r},t) = U(\mathbf{r})$, Fourier transforming over the time difference, and using the identity

$$\nabla T[x\nabla y - y\nabla x] = \mathrm{div}\,\{T[x\nabla y - y\nabla x]\} - T[x\nabla^2 y - y\nabla^2 x],$$

we can easily transform Eq. (7) into

$$G(\mathbf{r},t;\mathbf{r}',t')=\int\frac{d\omega}{2\pi}\exp[-i\omega(t-t')]\Big\{\sum_{i=1,2}\frac{1}{2m}\int_{r_1\in\sigma_i} d\mathbf{S} \times\Big[\mathrm{th}\frac{\omega-U_i}{2T}[G_\omega^R(\mathbf{r},t;\mathbf{r}_1)\nabla_{r_1}G_\omega^A(\mathbf{r}_1;\mathbf{r}',t') - G_\omega^A(\mathbf{r}_1;\mathbf{r}',t')\nabla_{r_1}G_\omega^R(\mathbf{r},t;\mathbf{r}_1)] + \mathrm{th}\frac{\omega+U_i}{2T}[F_\omega^R(\mathbf{r},t;\mathbf{r}_1)\nabla_{r_1}F_\omega^{+A}(\mathbf{r}_1;\mathbf{r}',t') - F_\omega^{+A}(\mathbf{r}_1;\mathbf{r}',t')\nabla_{r_1}F_\omega^R(\mathbf{r},t;\mathbf{r}_1)]\Big] - \frac{1}{2m}\int_B d^3r_1\Big[\mathrm{th}\frac{\omega-U(\mathbf{r}_1)}{2T}[G_\omega^R(\mathbf{r},t;\mathbf{r}_1)\nabla_{r_1}^2G_\omega^A(\mathbf{r}_1;\mathbf{r}',t') - (\nabla_{r_1}^2G_\omega^R(\mathbf{r},t;\mathbf{r}_1))G_\omega^A(\mathbf{r}_1;\mathbf{r}',t')] + \mathrm{th}\frac{\omega+U(\mathbf{r}_1)}{2T}[F_\omega^R(\mathbf{r},t;\mathbf{r}_1) \times\nabla_{r_1}^2F_\omega^{+A}(\mathbf{r}_1;\mathbf{r}',t') - F_\omega^{+A}(\mathbf{r}_1;\mathbf{r}',t')\nabla_{r_1}^2F_\omega^R(\mathbf{r},t;\mathbf{r}_1)]\Big] + \Big[G_\omega^R(\mathbf{r},t;\mathbf{r}')\,\mathrm{th}\frac{\omega-U(\mathbf{r}')}{2T} - \mathrm{th}\frac{\omega-U(\mathbf{r})}{2T}G_\omega^A(\mathbf{r};\mathbf{r}',t')\Big]\Big\}, \quad (8)$$

where U_1, U_2 are the values of $U(\mathbf{r})$ at the banks of the junction, so that the voltage $U = U_1 - U_2$; the vector $d\mathbf{S}$ is directed outwards from B.

In what follows we shall assume that the points $\mathbf{r},\mathbf{r}'$ lie in the region B. Using the Gor'kov equations and assuming, since the electron-phonon interaction constant is small, that the order parameter vanishes in the layer we can easily prove the validity of the equations

$$G_\omega^R(\mathbf{r},t;\mathbf{r}_1)\nabla_{r_1}^2G_\omega^A(\mathbf{r}_1;\mathbf{r}',t') - (\nabla_{r_1}^2G_\omega^R(\mathbf{r},t;\mathbf{r}_1))G_\omega^A(\mathbf{r}_1;\mathbf{r}',t') = 2m[G_\omega^R(\mathbf{r},t;\mathbf{r}')\delta(\mathbf{r}_1-\mathbf{r}') - \delta(\mathbf{r}-\mathbf{r}_1)G_\omega^A(\mathbf{r};\mathbf{r}',t')], \quad (9a)$$

$$F_\omega^R(\mathbf{r},t;\mathbf{r}_1)\nabla_{r_1}^2F_\omega^{+A}(\mathbf{r}_1;\mathbf{r}',t') - F_\omega^{+A}(\mathbf{r}_1;\mathbf{r}',t')\nabla_{r_1}^2F_\omega^R(\mathbf{r},t;\mathbf{r}_1)=0. \quad (9b)$$

Substituting (9) into (8) we discover that the last three terms in (8) must be dropped. Using now the expression for the current density

$$\mathbf{j}(\mathbf{r},t) = \frac{e}{2m}(\nabla_{r'} - \nabla_r)G(\mathbf{r},t;\mathbf{r}',t')|_{r'=r}, \quad (10)$$

we get the following general expression for the total current J through the junctions:

$$J=\frac{e}{4m^2}\int\frac{d\omega}{2\pi}\int_{r\in\sigma}d\sigma_\alpha\left\{\int_{r'\in\sigma_1}dS_\beta\left[\operatorname{th}\frac{\omega}{2T}-\operatorname{th}\frac{\omega-U}{2T}\right]\right.\times\left(\frac{\partial}{\partial r_4^\alpha}-\frac{\partial}{\partial r_1^\alpha}\right)$$

$$\times\left(\frac{\partial}{\partial r_3^\beta}-\frac{\partial}{\partial r_2^\beta}\right)[G_\omega^R(\mathbf{r}_1,t;\mathbf{r}_2)G_\omega^A(\mathbf{r}_3;\mathbf{r}_4,t)$$

$$-F_\omega^{+R}(\mathbf{r}_1,t;\mathbf{r}_2)F_\omega^A(\mathbf{r}_3;\mathbf{r}_4,t)]\,|_{\substack{r_1=r_4=r\\ r_2=r_3=r'}}+\operatorname{th}\frac{\omega}{2T}\left(\frac{\partial}{\partial r_4^\alpha}-\frac{\partial}{\partial r_1^\alpha}\right)$$

$$\left.[G_\omega^R(\mathbf{r}_1,t;\mathbf{r}_4)-G_\omega^A(\mathbf{r}_1;\mathbf{r}_4,t)]\,|_{r_1=r_4=r}\right\} \qquad (11)$$

(σ is an arbitrary surface passing through the whole of the layer). In deriving (11) we changed the sign of ω in the anomalous functions and at the same time we swapped the indexes $4\leftrightarrow1$, $3\leftrightarrow2$, and we used the identity $F_{-\omega}^{R(+A)}(\mathbf{r},t;\mathbf{r}')=F_\omega^{A(+R)}(\mathbf{r}';\mathbf{r},t)$; we also used Eqs. (9), integrated over B with a change to integrals over the surfaces σ_1 and σ_2. In (11) the henceforth $U_1=U, U_2=0$.

Equation (11) is very convenient for finding the current in an S-Sm-S junction as it enables us to take easily into account the explicit coordinate dependence of the Green functions. Because of the complicated form of the potential barrier this makes our method preferable for this problem as compared to the method of Green functions integrated over the energy variables.[13] We note that Eq. (11) is applicable to any kind of weak coupling.

3. CURRENT IN A NORMAL JUNCTION WITH A SEMICONDUCTOR LAYER

We demonstrate the method of calculating Green functions and the current using Eq. (11) first of all in the simplest case of a normal junction, and more so as many expressions obtained can then be used with small changes also for the case of a superconducting junction.

For the sake of simplicity we shall assume in what follows that the layer is a plane layer of thickness L. We find the Green functions of such a junction with a chain of N localized centers (LC) of energy E_D in the points $\mathbf{a}_1,\ldots,\mathbf{a}_N$ of the layer, where $\mathbf{a}_1$ is positioned close to the plane σ_1 at a distance y_1, and $\mathbf{a}_N$ close to σ_2 at a distance y_2 from it. We choose the potential of a single LC in the form $v(r)=-\beta, |\mathbf{r}-\mathbf{a}_i|\leqslant r_0$; $v(\mathbf{r})=0, |\mathbf{r}-\mathbf{a}_i|>r_0$, with $\varkappa_0 r_0\ll1$, where the sub-barrier momentum is

$$\varkappa_0=[2m(V-\mu-E_D)]^{1/2}$$

($V-\mu$ is the height of the barrier in the semiconductor). We then have

$$G_{\omega\,imp}^R(\mathbf{r},\mathbf{r}')=G_\omega^R(\mathbf{r},\mathbf{r}')-\sum_i\beta\int_{|\mathbf{r}_1-\mathbf{a}_i|<r_0}d^3\mathbf{r}_1G_\omega^R(\mathbf{r},\mathbf{r}_1)G_{\omega\,imp}^R(\mathbf{r}_1,\mathbf{r}'), \qquad (12)$$

where $G_{\omega\mathrm{imp}}^R, G_\omega^R$ are, respectively, the Green functions of a junction with and without an LC. Putting in (12) $|\mathbf{r},\mathbf{r}'-\mathbf{a}_i|\gg r_0$ we get

$$G_{\omega\,imp}(\mathbf{r},\mathbf{r}')=G_\omega^R(\mathbf{r},\mathbf{r}')-\sum_i\beta\frac{4}{3}\pi r_0^3G_\omega^R(\mathbf{r},\mathbf{a}_i)G_{\omega\,imp}^R(\mathbf{a}_i,\mathbf{r}'). \qquad (13)$$

If, however, we put in (12) $\mathbf{r}=\mathbf{a}_i$ $(i=1,\ldots,N)$, $|\mathbf{r}'-\mathbf{a}_i|\gg r_0$ we get a set of linear equations to determine $G_{\omega\mathrm{imp}}^R(\mathbf{a}_i,\mathbf{r}')$:

$$\left[1+\beta\int_{|\mathbf{r}_1-\mathbf{a}_i|<r_0}d^3\mathbf{r}_1G_\omega^R(\mathbf{a}_i,\mathbf{r}_1)\right]G_{\omega\,imp}^R(\mathbf{a}_i,\mathbf{r}')$$

$$+\sum_{i\neq j}\beta\frac{4}{3}\pi r_0^3G_\omega^R(\mathbf{a}_i,\mathbf{a}_j)G_{\omega\,imp}^R(\mathbf{a}_j,\mathbf{r}')=G_\omega^R(\mathbf{a}_i,\mathbf{r}'). \qquad (14)$$

Solving it and performing in (13), (14) a renormalization of the LC potential through the introduction of a finite scattering length for scattering by the LC

$$g(\omega)=\frac{2\pi}{m(\varkappa-\varkappa_0)} \qquad (15)$$

(for details see the Appendix) we get

$$G_{\omega\,imp}^R(\mathbf{r},\mathbf{r}')=G_\omega^R(\mathbf{r},\mathbf{r}')-\sum_{i,j}\frac{\mathscr{D}_{ji}^R}{\mathscr{D}^R}G_\omega^R(\mathbf{r},\mathbf{a}_i)G_\omega^R(\mathbf{a}_j,\mathbf{r}'), \qquad (16)$$

where $\mathscr{D}^R$ is the determinant and the $\mathscr{D}_{ij}^R$ are the algebraic cofactors of the elements with indexes i,j of the matrix $\hat{\mathscr{D}}^R$,

$$\hat{\mathscr{D}}^R=\begin{vmatrix} g^{-1}(\omega)+G_1^R(\omega-U) & G(\mathbf{a}_1,\mathbf{a}_2) & & & 0\\ G(\mathbf{a}_1,\mathbf{a}_2) & g^{-1}(\omega) & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & g^{-1}(\omega) & G(\mathbf{a}_{N-1},\mathbf{a}_N)\\ 0 & & & G(\mathbf{a}_{N-1},\mathbf{a}_N) & g^{-1}(\omega)+G_2^R(\omega) \end{vmatrix}. \qquad (17)$$

In Eq. (17) we have used the following notation:

$$G(\mathbf{r}_1,\mathbf{r}_2)=-\frac{m\varkappa}{2\pi}h(|\mathbf{r}_1-\mathbf{r}_2|),\qquad h(r)=\frac{1}{\varkappa r}\exp(-\varkappa r), \qquad (18)$$

$$G_{1,2}^R(\omega)=-\frac{m\varkappa}{2\pi}g_{1,2}^R(\omega)D_{1,2}h(2y_{1,2}),$$

$$g_{1,2}^R(\omega)=\frac{2k_{1,2}\varkappa}{k_{1,2}^2+\varkappa^2}in_{1,2}^R(\omega)+\frac{\varkappa^2-k_{1,2}^2}{k_{1,2}^2+\varkappa^2}, \qquad (19)$$

while in a normal junction simply $n_{1,2}^R=1$ (the meaning of these symbols is that it is easy to change over to the case of a superconducting junction, when $\mathrm{Re}\,n_{ij}^R$ has the meaning of the density of states of the quasiparticles in the banks), $k_{1,2}$

are the Fermi momenta in the banks, and $D_{1,2}$ the transparencies of the boundaries σ_1 and σ_2.

We put in Eq. (11) $\sigma=\sigma_2$ and introduce the operator

$$\hat{L}_\sigma(\mathbf{r}_1,\mathbf{r}_2)=\int_{r_1\in\sigma} dS_\alpha \frac{1}{2m}\left(\frac{\partial}{\partial r_2{}^\alpha}-\frac{\partial}{\partial r_1{}^\alpha}\right)\Big|_{r_2=r_1} ; \qquad (20)$$

equation (11) can then be rewritten in the case of a normal junction in the form

$$J=e\int\frac{d\omega}{2\pi}\left[\operatorname{th}\frac{\omega}{2T}-\operatorname{th}\frac{\omega-U}{2T}\right]\hat{L}_{\sigma_1}(\mathbf{r}_2,\mathbf{r}_3)\hat{L}_{\sigma_2}(\mathbf{r}_1,\mathbf{r}_4)$$
$$\times\{G^R_{\omega\, imp}(\mathbf{r}_1,\mathbf{r}_2)G^A_{\omega\, imp}(\mathbf{r}_3,\mathbf{r}_4)\} \qquad (21)$$

[the last term in (11) does not contribute to the current]. Splitting off in (16) the main terms, we get for the Green functions in (21),

$$G^R_{\omega\, imp}(\mathbf{r}_1,\mathbf{r}_2)=-\frac{\mathscr{D}_{1N}{}^R}{\mathscr{D}^R}G_\omega{}^R(\mathbf{r}_1,\mathbf{a}_N)G_\omega{}^R(\mathbf{a}_1,\mathbf{r}_2),$$
$$G^A_{\omega\, imp}(\mathbf{r}_3,\mathbf{r}_4)=-\frac{\mathscr{D}_{N1}{}^A}{\mathscr{D}^A}G_\omega{}^A(\mathbf{r}_3,\mathbf{a}_1)G_\omega{}^A(\mathbf{a}_N,\mathbf{r}_4). \qquad (22)$$

To evaluate the current by using Eqs. (21) and (22) it is necessary to use the equations

$$-(+)\hat{L}_{\sigma_{1(2)}}(\mathbf{r}_1,\mathbf{r}_2)\{G_\omega{}^R(\mathbf{r},\mathbf{r}_1)G_\omega{}^A(\mathbf{r}_2,\mathbf{r}')\}$$
$$=G_\omega{}^R(\mathbf{r},\mathbf{r}')-G_\omega{}^A(\mathbf{r},\mathbf{r}'), \qquad (23)$$

which are valid if $\mathbf{r}_1,\mathbf{r}'$ are close to one of the boundaries σ_1,σ_2. This equation can easily be obtained if we integrate Eq. (10) over the region B and afterwards change to an integration over the surfaces σ_1 and σ_2 (one of these integrals will be exponentially small and we must drop it). It is necessary to note here that although the three-dimensional Green functions $G^{R(A)}_\omega(\mathbf{r},\mathbf{r}')$ become infinite when $\mathbf{r}=\mathbf{r}'$, this divergence is contained only in their real part, while their imaginary part is finite (see Appendix)

$$2i\operatorname{Im}G_\omega{}^R(\mathbf{a}_{1(N)},\mathbf{a}_{1(N)})=G^R_{1(2)}(\omega-U_{1(2)})-G^A_{1(2)}(\omega-U_{1(2)}). \qquad (24)$$

From Eq. (21), through (22)–(24) and using the fact that $\mathscr{D}^R_{1N}=\mathscr{D}^A_{N1}=G(\mathbf{a}_1,\mathbf{a}_2)...G(\mathbf{a}_{N-1},\mathbf{a}_N)$, we get the following expression:

$$J=4e\int\frac{d\omega}{2\pi}\left[\operatorname{th}\frac{\omega}{2T}-\operatorname{th}\frac{\omega-U}{2T}\right]\operatorname{Im}G_1{}^R(\omega-U)\operatorname{Im}G_2{}^R(\omega)$$
$$\times|\mathscr{D}^R|^{-2}G^2(\mathbf{a}_1,\mathbf{a}_2)\dots G^2(\mathbf{a}_{N-1},\mathbf{a}_N). \qquad (25)$$

As in Refs. 3 and 4 we shall assume that the points $\mathbf{a}_1,\dots,\mathbf{a}_N$ form a resonance trajectory with maximum statistical weight, i.e., $|\mathbf{a}_1-\mathbf{a}_2|\approx\cdots\approx|\mathbf{a}_{N-1}-\mathbf{a}_N|\approx 2y$; we then find

$$J=4e\int\frac{d\omega}{2\pi}\left[\operatorname{th}\frac{\omega}{2T}-\operatorname{th}\frac{\omega-U}{2T}\right]$$
$$\times\frac{\tilde{D}_1\tilde{D}_2\operatorname{Im}g_1{}^R(\omega-U)\operatorname{Im}g_2{}^R(\omega)}{|Q^R|^2}, \qquad (26)$$

where the matrix Q^R is obtained from $\hat{\mathscr{D}}^R$ by dividing all its elements by $G(2y)$ while $\tilde{D}_{1,2}=D_{1,2}h(2y_{1,2})/h(2y)$. Noting that

$$\begin{vmatrix} 2x & 1 & & & 0 \\ 1 & 2x & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & 2x & 1 \\ 0 & & & 1 & 2x \end{vmatrix}_{N\times N}=Q_N(x),$$

where $Q_N(x)=\sin(N+1)\theta/\sin\theta$, $\cos\theta=x$ is a second rank Chebyshev polynomial, and introducing the width B of the resonance zone

$$B=\frac{V-\mu}{\varkappa y}\exp(-2\varkappa y), \qquad (27)$$

we can easily show that

$$Q^R=Q_N\left(\frac{\omega-E_D}{2B}\right)+[\tilde{D}_1g_1{}^R(\omega-U)+\tilde{D}_2g_2{}^R(\omega)]Q_{N-1}\left(\frac{\omega-E_D}{2B}\right)$$
$$+\tilde{D}_1\tilde{D}_2g_1{}^R(\omega-U)g_2{}^R(\omega)Q_{N-2}\left(\frac{\omega-E_D}{2B}\right). \qquad (28)$$

It is clear from (26), (28) that electrons with frequencies $|\omega-E_D|\leqslant 2B$ enter into resonance. The levels E_D are spread out by the large-scale fluctuations of the potential over the band gap of the semiconductor. One must thus average Eq. (26) over E_D, i.e., in fact, over the position of the trajectory along the area of the junction[3]:

$$I=\int dE_D\mathscr{F}(E_D)J(E_D)=\mathscr{F}(\mu)\int dE_DJ(E_D), \qquad (29)$$

where $\mathscr{F}(E)$ is the distribution function of the random potential. The calculation for this integration are considerably simplified in the case $\varkappa=k_1=k_2$, when $g^R_{1,2}=in^R_{1,2}$. We rewrite the integral in Eq. (29):

$$\int\frac{dE_D}{Q^RQ^A}=\fint\frac{dE_D}{(Q^R-Q^A)Q^A}-\fint\frac{dE_D}{(Q^R-Q^A)Q^R}, \qquad (30)$$

where the integrals on the right-hand side of (30) are taken in the sense of principal value. In view of the analyticity of the corresponding Green functions Q^A (Q^R) as functions of E, there are no zeroes in the upper (lower) half-plane. Hence, assuming $n^R_{1,2}(\omega)$ to be purely real functions, i.e., $n^R_i=n^A_i=n_i$ we can conclude that Eq. (30) is determined by the residues in the zeros of the function Q^R-Q^A, and, in fact, of the polynomial Q_{N-1}. As a result we get

$$I=\mathscr{F}(\mu)2eB\int d\omega\left[\operatorname{th}\frac{\omega}{2T}-\operatorname{th}\frac{\omega-U}{2T}\right]$$
$$\times\frac{\tilde{D}_1\tilde{D}_2n_1(\omega-U)n_2(\omega)}{[\tilde{D}_1n_1(\omega-U)+\tilde{D}_2n_2(\omega)][1+\tilde{D}_1\tilde{D}_2n_1(\omega-U)n_2(\omega)]}. \qquad (31)$$

Equation (31) has a simple physical meaning. In the case of resonance tunneling one adds (rather than multiplies as in the case of the usual tunneling) the resistivity of the boundaries $R_1\propto(D_1n_1)^{-1}$, $R_2\propto(D_2n_2)^{-1}$ and the resistivity of the resonance zone $R_B\propto(n^*\tau/m^*)^{-1}$ where the effective electron density $n^*\propto B$ and the mass $m^*\propto B^{-1}$, while the time τ is connected with the departure of the electron from the band into the banks and is determined by the penetrability of the boundaries, $\tau\propto(D_1n_1+D_2n_2)^{-1}$. Adding all resistivities we get the resistivity of the junction corresponding to Eq. (31).

As in Ref. 5, one must average Eq. (31) over y and the angle θ which is determined by the bending of the trajectory, and furthermore over y_1, y_2. For the probability for the formation of the trajectory we have[5]

$$W(y, \theta) = \exp[N \ln(y^2\theta^2 ac) - \pi c N y^3], \qquad (32)$$

where c is the density of the LC in the semiconductor and $a = \varkappa_0^{-1}$ the radius of the LC.

We average Eq. (31) (using the fact that $n_1 = n_2 = 1$) over y_1, y_2 for the case where the boundaries are very transparent $D_1, D_2 \gtrsim h(2y)$, and as a result we get for the conductivity of a single resonance trajectory (in units of $e^2/\hbar$)

$$\rho^{-1} = B\mathscr{F}(\mu) \qquad (33)$$

(the condition $\hat{\mathscr{D}}_1 \sim \hat{\mathscr{D}}_2 \sim 1$ is satisfied for characteristic y_1, y_2). Averaging over y, θ we finally get with exponential accuracy the dependence of the junction conductivity on the thickness L of the semiconductor layer:

$$R^{-1} \propto \exp\left[-2\left(\frac{L}{a}\left|\ln cLa^2\right|\right)^{1/2}\right],$$

$$L \gg a\frac{\ln^2(\min\{D_1, D_2\})}{|\ln(ca^3)|} \qquad (34)$$

(the optimal value is $2y_0 \simeq (La|\ln cLa^2|)^{1/2}$). It is clear from Eq. (31) that resonantly tunneling electrons do not "feel" the barriers at the boundary provided the condition $D_1, D_2 \gtrsim h(2y)$ is satisfied (it is natural to call such boundaries clean ones). As a result Eq. (34) is the same as the corresponding expression obtained in Ref. 5 under the assumption that there were no barriers at the boundary.

We consider the opposite case: to fix the ideas, let us have $D_1 = \min\{D_1, D_2\} < h(2y_0)$. When the condition $\hat{\mathscr{D}}_1, \hat{\mathscr{D}}_2 < 1$ is satisfied, the conductivity of the resonance trajectory averaged over E_D equals

$$\rho^{-1} = (V-\mu)\mathscr{F}(\mu)\frac{D_1 D_2 h(2y_1) h(2y_2)}{D_1 h(2y_1) + D_2 h(2y_2)} \qquad (35)$$

and is independent of y. The optimal value of y is thus determined from the condition that $W(y,\theta)$ is a maximum under the restriction that $\hat{\mathscr{D}}_1, \hat{\mathscr{D}}_2 < 1$. Starting from that condition and Eq. (35) one can easily show that the optimal y_1, y_2 are determined from the conditions $h(2y_1) \sim 1$, $h(2y_2) = D_1/D_2$, and y from the condition $h(2y) = D_1$, i.e., $2y_1' \approx a|\ln(\min\{D_1, D_2\})|$; in that case the optimal conductivity of the trajectory equals

$$\rho^{-1} = (V-\mu)\mathscr{F}(\mu)\min\{D_1, D_2\}, \qquad (36)$$

and using the probability for the formation of a trajectory the resistivity is determined by the expression

$$R^{-1} \sim \exp\left\{-\frac{L}{a}\frac{|\ln(cLa^2)|}{|\ln(\min\{D_1, D_2\})|}\right\},$$

$$L \ll a\frac{\ln^2(\min\{D_1, D_2\})}{|\ln(ca^3)|} \qquad (37)$$

(when this condition is satisfied the trajectories with $\hat{\mathscr{D}}_1, \hat{\mathscr{D}}_2 \gg 1$ give a small contribution which also guarantees the self-consistency of the calculation). We note that although in contrast to the preceding case the conductivity of the optimal trajectory is determined by the boundary barriers for the optimal trajectories we have as before $\hat{\mathscr{D}}_1 \simeq \hat{\mathscr{D}}_2 \approx 1$.

4. QUASIPARTICLE CURRENT IN A SUPERCONDUCTING S-Sm-S JUNCTION

We first of all elucidate the time dependence of the Green functions which occur in Eq. (11). To do this we regard the superconducting order parameter $\Delta(\mathbf{r},t) = \Delta(\mathbf{r})\exp[-2iU(\mathbf{r})t]$ as a perturbation. Writing out the corresponding diagram series and bearing in mind that $\Delta(\mathbf{r}) = 0$ in the semiconductor layer we easily understand that

$$G^R(\mathbf{r}, t; \mathbf{r}', t') = \int \frac{d\omega}{2\pi} \sum_k G^R(\mathbf{r}, \omega_k; \mathbf{r}', \omega) e^{-i[\omega_k t + \omega t']},$$

$$F^{(+)R}(\mathbf{r}, t; \mathbf{r}', t') = \int \frac{d\omega}{2\pi} \sum_k F^{(+)R}(\mathbf{r}, \omega_k; \mathbf{r}', \omega) e^{-i[\omega_k t + \omega t']}, \qquad (38)$$

$$\omega_k = \omega + 2kU.$$

Using (38) we get from (11), if we take (20) into account, an expression for the quasiparticle current:

$$J_{\mathrm{qu}} = \frac{e}{4m^2}\int \frac{d\omega}{2\pi}\left[\mathrm{th}\frac{\omega}{2T} - \mathrm{th}\frac{\omega - U}{2T}\right]\hat{L}_\sigma(\mathbf{r}_1, \mathbf{r}_4)$$

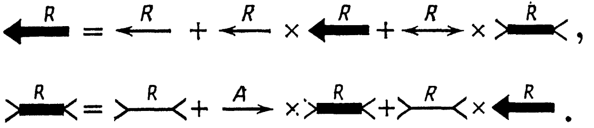
$$\times \hat{L}_{\sigma_1}(\mathbf{r}_2, \mathbf{r}_3) \sum_k \{G^R(\mathbf{r}_1, \omega_k; \mathbf{r}_2, \omega) G^A(\mathbf{r}_3, \omega; \mathbf{r}_4, \omega_k)$$

$$- F^{+R}(\mathbf{r}_1, \omega_k; \mathbf{r}_2, \omega) F^A(\mathbf{r}_3, \omega; \mathbf{r}_4, \omega_k)\} \qquad (39)$$

[the last term in (11) does not contribute to the constant current].

From among all the Green functions occurring in (39), in a junction without LC, the G-functions with $k = 0$ and the F-functions with $k = 0, -1$ have no assured smallness, and in the series corresponding to them we must drop, as being exponentially small, the diagrams which contain "tunnel" lines, i.e., lines connecting vertices in which the integration goes over difficult sides of the junction. However, using the Gor'kov equations we see that all functions necessary for evaluating the current are obtained from the corresponding Green functions of a stationary junction without an external field through a simple substitution of the frequency: $\omega \to \omega - U_1$ or $\omega \to \omega - U_2$, depending on which bank the two coordinates are close to.

In a junction with LC the tunnel lines introduce no smallness because of the possibility of resonance tunneling and, generally speaking, all Green functions in (39) are important. To determine them it is necessary to solve a set of equations which is analogous to (12),



(40)

We have written Eqs. (40) in a coordinate-time representation; the thin lines correspond to junctions without LC and we consider the LC potential as a perturbation. Substituting Eq. (38) into (40), and solving the infinite set of equations which we obtain by the method developed in the preceding section, we get in matrix block form

$$G^R_{imp}(\omega_k, \mathbf{r}; \mathbf{r}', \omega) = G^R(\omega, \mathbf{r}; \mathbf{r}', \omega)\,\delta_{k,0}$$

$$-\hat{g}_+{}^R(k, \mathbf{r})\,\hat{g}^R_{imp}(k) - \hat{f}_1{}^R(k-1, \mathbf{r})\,\hat{f}^{+R}_{imp}(k-1) - \hat{f}_2{}^R(k, \mathbf{r})\,\hat{f}^{+R}_{imp}(k),$$

$$F^{+R}_{imp}(\omega_k, \mathbf{r}; \mathbf{r}', \omega) = F^{+R}(\omega_k, \mathbf{r}; \mathbf{r}', \omega)\,(\delta_{k,0} + \delta_{k,-1})$$

$$-\hat{g}_-{}^A(k, \mathbf{r})\,\hat{f}^{+R}_{imp}(k) - \hat{f}_1{}^{+R}(k, \mathbf{r})\,\hat{g}^R_{imp}(k+1) - \hat{f}_2{}^{+R}(k+1, \mathbf{r})\,\hat{g}^R_{imp}(k), \qquad (41)$$

$$\begin{Vmatrix} \ddots & & & & & \\ & \hat{\mathscr{D}}^R(-1) & \hat{F}_2^R(-1) & & 0 & \\ & \hat{F}_2^{+R}(-1) & \hat{\mathscr{D}}_-^A(-1) & \hat{F}_1^{+R}(0) & & \\ & & \hat{F}_1^R(0) & \hat{\mathscr{D}}^R(0) & \hat{F}_2^R(0) & \\ & & & \hat{F}_2^{+R}(0) & \hat{\mathscr{D}}_-^A(0) & \hat{F}_1^{+R}(+1) \\ & 0 & & & \hat{F}_1^R(+1) & \hat{\mathscr{D}}^R(+1) \ \ddots \\ & & & & & \ddots \end{Vmatrix} \cdot \begin{Vmatrix} \vdots \\ \hat{g}^R_{imp}(-1) \\ \hat{f}^{+R}_{imp}(-1) \\ \hat{g}^R_{imp}(0) \\ \hat{f}^{+R}_{imp}(0) \\ \hat{g}^R_{imp}(+1) \\ \vdots \end{Vmatrix} = \left(\ldots, 0, \hat{f}_1^{+R}(-1, \mathbf{r}'), \hat{g}^R(0, \mathbf{r}'), \hat{f}_2^{+R}(0, \mathbf{r}'), 0, \ldots\right)^T, \qquad (42)$$

$$\hat{\mathscr{D}}^R(k) = \hat{\mathscr{D}}^R(\omega_k), \quad \hat{\mathscr{D}}_-{}^A(k) = \hat{\mathscr{D}}^A(-\omega_k),$$

$$\hat{g}_\pm{}^R(k, \mathbf{r}) = (G^R(\pm\omega_k, \mathbf{r}; \mathbf{a}_1, \pm\omega_k), \ldots, G^R(\pm\omega_k, \mathbf{r}; \mathbf{a}_N, \pm\omega_k)),$$

$$\hat{f}_1^{(+)R}(k, \mathbf{r}) = (F^{(+)R}_{\omega_k}(\mathbf{r}, \mathbf{a}_1), 0, \ldots, 0), \quad \hat{f}_2^{(+)R}(k, \mathbf{r}) = (0, \ldots, 0, F^{(+)R}_{\omega_k}(\mathbf{r}, \mathbf{a}_N)),$$

$$\hat{g}^R(\hat{f}^{+R})_{imp}(k) = \begin{pmatrix} g_1{}^R(f_1^{+R})_{imp}(k) \\ \vdots \\ g_N{}^R(f_N^{+R})_{imp}(k) \end{pmatrix}.$$

We exchanged the positions of $\mathbf{a}_i$ and $\mathbf{r}'$ at the same time as transposing in (42); the matrix $\hat{\mathscr{D}}^R(\omega)$ is given by Eqs. (17)-(19),

$$\hat{F}_1^{(+)R}(k) = \begin{Vmatrix} F_1^{(+)R}(\omega_k) & 0 & \cdots \\ 0 & & \\ \vdots & & 0 \end{Vmatrix}_{N\times N},$$

$$\hat{F}_2^{(+)R}(k) = \begin{Vmatrix} & & \vdots \\ & 0 & 0 \\ \cdots & 0 & F_2^{(+)R}(\omega_k) \end{Vmatrix}_{N\times N}$$

$$F^{(+)R}_{1,2}(\omega) = F^{(+)R}_\omega(\mathbf{a}_{1(N)}, \mathbf{a}_{1(N)}) = -\frac{m\varkappa}{2\pi} f^{(+)R}_{1,2}(\omega) D_{1,2} h(2y_{1,2}),$$

$$f^{(+)R}_{1,2}(\omega) = \frac{2k_{1,2}\varkappa}{k^2_{1,2} + \varkappa^2}\, i p^{(+)R}_{1,2}(\omega), \qquad (43)$$

where (see Appendix) in (19),(43)

$$n^R_{1,2}(\omega) = \frac{\omega}{[(\omega + i\delta)^2 - |\Delta_{1,2}|^2]^{1/2}}, \qquad (44)$$

$$p^{(+)R}_{1,2}(\omega) = (-)\frac{\Delta^{(*)}_{1,2}}{[(\omega + i\delta)^2 - |\Delta_{1,2}|^2]^{1/2}}.$$

Equations (39),(41),(42) take into account the possibility of resonance tunneling, both the usual one and also when we take into account processes such as Andreev reflection. However, the second possibility can be realized only when at least one of the frequencies $-\omega$ or $-(\omega - 2U)$ together with ω falls into the resonance zone. In that case, clearly, there appears a limitation on the LC energy: $E_D \leqslant 2B$ or $|E_D - 2U| \leqslant 2B$. If the characteristic value $B \ll \Delta_{1,2}$ the contribution to the current from resonance tunneling taking Andreev reflection into account will be proportional to B^2/Δ and we can neglect it in comparison with the contribution from the usual resonance tunneling which is proportional to B. We may thus assume, when $B \ll \Delta_{1,2}$, that only the frequency ω falls into the resonance zone and as a result Eqs. (39), (41), and (42) can be considerably simplified: in (39) there remains only the G-functions with $k = 0$ where

$$G^R_{\omega\, imp}(\mathbf{r}, \mathbf{r}') = G_\omega{}^R(\mathbf{r}, \mathbf{r}') - \hat{g}_+{}^R(0, \mathbf{r})\,\hat{g}^R_{imp}(0),$$

$$\hat{\mathscr{D}}^R(\omega)\,\hat{g}^R_{imp}(0) = \hat{g}^R(0, \mathbf{r}')^T.$$

Noting that these relations are practically the same as Eqs. (16) and (17) (the only difference being the actual form of the functions $n^R_{1,2}$) we can at once write down Eqs. (25),(26) for the quasiparticle current and also Eq. (31) in which

$$n_{1,2}(\omega) = \operatorname{Re} n^R_{1,2}(\omega) = \frac{|\omega|}{[\omega^2 - |\Delta_{1,2}|^2]^{1/2}}\,\theta(|\omega| - |\Delta_{1,2}|) \qquad (45)$$

[$\theta(\omega)$ is the Heaviside function]. Averaging, as in the previous section, Eq. (31), taking into account (45), we get in the case of "clean" boundaries

$$I_{qu}=(2R)^{-1}\int d\omega\left[\operatorname{th}\frac{\omega}{2T}-\operatorname{th}\frac{\omega-U}{2T}\right]$$

$$\times\theta(|\omega-U|-|\Delta_1|)\theta(|\omega|-|\Delta_2|),\quad(46)$$

where R is given by Eq. (34). In the case of high barriers at the boundary the quasiparticle current is described by the formula

$$I_{qu}=(2R)^{-1}\int d\omega\left[\operatorname{th}\frac{\omega}{2T}-\operatorname{th}\frac{\omega-U}{2T}\right]$$

$$\times\frac{(D_1+D_2)n_1(\omega-U)n_2(\omega)}{D_1n_1(\omega-U)+D_2n_2(\omega)},\quad(47)$$

and R is given by Eq. (37).

It is clear from Eqs. (46) and (47) that for large voltages $U\gg\Delta_{1,2}$ we must observe on the CVC a current deficit;

$$I_{qu}=\frac{U}{R}-I_{ins}\operatorname{th}\frac{U}{2T},\quad I_{ins}=\frac{\alpha_1\Delta_1+\alpha_2\Delta_2}{R},\quad\alpha_{1,2}\sim1.\quad(48)$$

We consider now the case $B\gg\Delta$ and we restrict ourselves to the range of high voltages $U\gg\Delta$. It is clear that Eq. (39) differs from Eq. (21) for a normal junction only in the frequency ranges $|\omega-U|\sim\Delta_1$ and $|\omega|\sim\Delta_2$. We consider the range $|\omega|\sim\Delta_2$; for such frequencies all anomalous functions except $F_2^{(+)R}(\omega)$ in Eq. (42) are small in the parameter Δ/U. Bearing in mind that the frequencies ω and $-\omega$ fall simultaneously in the resonance zone (provided $E_D\lesssim B$), we choose in Eq. (39) $\sigma=\sigma_2$ for the G functions and $\sigma=\sigma_1$ for the F functions [the possibility of such a separate choice is provided by Eq. (9b)]. In the main approximation we get then from (41) and (42)

$$G^R_{imp}(\mathbf{r}_1,\omega;\mathbf{r}_2,\omega)$$

$$=-\frac{1}{\tilde{\mathscr{D}}^{R2}}[\tilde{\mathscr{D}}^{R2}_{1,N}G_\omega{}^R(\mathbf{r}_1,\mathbf{a}_N)+\tilde{\mathscr{D}}^{R2}_{1,2N}F_\omega{}^R(\mathbf{r}_1,\mathbf{a}_N)]G^R_{\omega-U}(\mathbf{a}_1,\mathbf{r}_2);\quad(49)$$

$$F^{+R}_{imp}(\mathbf{r}_1,\omega;\mathbf{r}_2,\omega)=-\frac{\tilde{\mathscr{D}}^{R2}_{1,N+1}}{\tilde{\mathscr{D}}^{R2}}G_{-(\omega+U)}(\mathbf{r}_1,\mathbf{a}_1)G_{\omega+U}(\mathbf{a}_1,\mathbf{r}_2),$$

where $\hat{\mathscr{D}}^{R2}$ defines the matrices

$$\hat{\mathscr{D}}^{R2}=\begin{pmatrix}\hat{\mathscr{D}}^R(\omega) & F_2{}^R(\omega)\\ \hat{F}_2{}^+(\omega) & \hat{\mathscr{D}}^A(-\omega)\end{pmatrix}.\quad(50)$$

Expressing the algebraic cofactors in (49) in terms of the minors of the matrices $\hat{\mathscr{D}}^R,\hat{\mathscr{D}}^A$ and using the analogous formulae for G^A,F^A and also formulae like (23) (with different combinations of G and F functions), we get for the result of the action of the operators $\hat{L}_\sigma\cdot\hat{L}_{\sigma_1}$ on the Green functions in (39) the following expression:

$$X(\Delta_2)=4|\tilde{\mathscr{D}}^{R2}|^{-2}\operatorname{Im}[G_1{}^R(\omega-U)]G^2(\mathbf{a}_1,\mathbf{a}_2)$$

$$\ldots G^2(\mathbf{a}_{N-1},\mathbf{a}_N)\{\operatorname{Im}[G_2{}^R(\omega)]\mathscr{D}^A(-\omega)\mathscr{D}^R(-\omega)$$

$$-\operatorname{Im}[F_2{}^R(\omega)F_2{}^{+R}(\omega)\mathscr{D}_{11}{}^A(-\omega)\mathscr{D}^R(-\omega)]$$

$$+F_2{}^A(\omega)F_2{}^{+R}(\omega)\operatorname{Im}[G_1{}^A(-(\omega+U))]$$

$$\times G^2(\mathbf{a}_1,\mathbf{a}_2)\ldots G^2(\mathbf{a}_{N-1},\mathbf{a}_N)\},\quad(51)$$

$$\tilde{\mathscr{D}}^{R2}=\mathscr{D}^R(\omega)\mathscr{D}^A(-\omega)-F_2{}^R(\omega)F_2{}^{+R}(\omega)\mathscr{D}_{11}{}^R(\omega)\mathscr{D}_{11}{}^A(\omega).$$

Considering the optimal trajectories which we determine above we put

$$\sin\varphi\sin\psi=\omega/2B,\quad\cos\varphi\cos\psi=E_D/2B,$$

$$-\pi/2<\varphi<\pi/2,\quad0<\psi<\pi;$$

in that case

$$X(\Delta_2)=\frac{4}{|\tilde{Q}^{R2}|^2}\{|1+n_2{}^R|^2+|p_2{}^R|^2+4\operatorname{Re}n_2{}^R\operatorname{ctg}^2(\varphi-\psi)$$

$$\sin^2N(\varphi-\psi)\},$$

$$\tilde{Q}^{R2}=2(\cos^2\varphi-\cos^2\psi)^{-1}\{\cos2N\varphi\cos^2\varphi-\cos2N\psi\cos^2\psi$$

$$+n_2{}^R(\omega)[\cos2N\varphi\sin^2\psi-\cos2N\psi\sin^2\varphi]$$

$$-i(1+n_2{}^R(\omega))[\sin2N\varphi\cos\varphi\sin\psi-\sin2N\psi\sin\varphi\cos\psi]\}\quad(52)$$

(for the sake of simplicity of exposition we put $\varkappa=k_1=k_2$). As $\omega\sim\Delta_2\ll B$ we have $\sin\varphi\approx0$, $\cos\varphi\approx1$. When $E_D\ll B$ we can put $\cos\psi\approx0$, $\sin\psi\approx1$ and then

$$X(\Delta_2)=1+\frac{|p_2{}^R(\omega)|^2}{|1+n_2{}^R(\omega)|^2}.\quad(53)$$

To study Eq. (52) when $E_D\sim B$ we use the following fact. In (52) $X(\Delta_2)$ as function of φ,ψ has beats with a characteristic period $\pi/2N$. When $2N\varphi/\pi$ and $2N\psi/\pi$ are integers with the same parity, $X(\Delta_2)$ is, as before, the same as expression (53) (and for any $\varkappa,k_{1,2}$), but in the intervals lying between these values it does not exceed it (when $\varkappa\ll k$ it is even small $\sim\varkappa/k$), but everywhere $X(\Delta_2)-X(\Delta_2=0)>0$. Hence it follows that Eq. (53) is applicable for any $E_D\lesssim B$, if we are only interested in the difference of the CVC from Ohm's law for high voltages.

Combining (53) with the analogous expression which is valid in the region $|\omega-U|\sim\Delta_1$ we finally get

$$I_{qu}=(2R)^{-1}\int d\omega\left[\operatorname{th}\frac{\omega}{2T}-\operatorname{th}\frac{\omega-U}{2T}\right]$$

$$\times\left(1+\frac{|p_1{}^2(\omega-U)|^2+|p_2{}^R(\omega)|^2}{|n_1{}^R(\omega-U)+n_2(\omega)|^2}\right)\quad(54)$$

Expression (54) is the same as the formula obtained for clean microbridges in Ref. 14; it follows from it that in the CVC when $U\gg\Delta_{1,2}$ we must observe (under the condition $B\gg\Delta_{1,2}$) an excess current:

$$I_{KB}=\frac{U}{R}+I_{ex}\operatorname{th}\frac{U}{2T},\quad I_{ex}=\alpha\frac{\Delta_1+\Delta_2}{R}.\quad(55)$$

When we evaluate the integral in (54) exactly we find $\alpha=\frac{4}{3}$, but in view of the approximations made we can state only that $\alpha\sim1$.

5. DISCUSSION OF THE RESULTS

The results obtained show that the quasiparticle current is in a broad range of parameters of the semiconductor layer determined by resonance tunneling which is caused by the presence in the semiconductor of trajectories of LC which are arranged at approximately equal distances $2y$ from one another. As a result a narrow zone of width B is formed [Eq. (27)] in which the electron current occurs without damping.

It turns out that resonance tunneling is sensitive to the presence of barriers at the boundaries of the semiconductor.

If the barriers are small (clean boundaries) so that the condition

$$\min\{D_1, D_2\} \gg \exp\left[-\left(\frac{L}{a} |\ln cLa^2| \right)^{1/2} \right], \quad (56)$$

is fulfilled, the main contribution to the current comes from trajectories with distances between the LC $2y_0 \sim (La|\ln(cLa^2)|)^{1/2}$ which is independent of the magnitude of the barriers. A comparison of the tunneling and resonance exponents in the expression for the current shows that the resonance tunneling becomes the main one for LC densities satisfying the relation

$$c \gg a^{-3}(L/a)^{-1} \exp(-L/a). \quad (57)$$

It is necessary to note that the restriction on the density is much weaker than for a Josephson junction.[3,4] This is connected with the fact that the quasiparticle current is proportional to B and not to B^2 as in the case of Cooper pairs.

When condition (56) is not satisfied (large barriers) resonance tunneling turns out to be most favorable along chains of LC arranged at distances $2y_0' \sim a|\ln \min\{D_1,D_2\}|$. The current is in that case determined by Eq. (47). In that case, the way the junction resistivity depends on the thickness of the semiconductor, Eq. (37), changes strongly when we change the transparency of the boundaries. We can use Eq. (37) to explain the change in the slope of that function, observed in Ref. 8, when the boundaries are oxidated at the junctions with amorphous silicon. It is necessary to note that in contrast to SIS junctions for which the CVC starts from $U = \Delta$,[15] in S-Sm-S junctions even when there is a barrier present on one of the boundaries the CVC starts from $U = 2\Delta$. This is connected with the possibility of resonance tunneling through the whole of the junction (together with the barrier).

The magnitude of the quasiparticle current for high voltages depends strongly on the relation between B and Δ. For small thicknesses of the Sm layer

$$L < a \frac{\ln^2[\Delta/[(V-\mu)]}{|\ln ca^3|} \quad (58)$$

in the clean case, and when the condition

$$\min\{D_1, D_2\} > \frac{\Delta}{V-\mu} \quad (59)$$

is satisfied the zone B is sufficiently broad in the case of high barriers ($B \gg \Delta$) for Andreev reflection to lead to an excess current [Eq. (55)]. It is clear that when $B \gg \Delta$ the S-Sm-S junction occupies an intermediate position between a superconducting structure and direct conductivity and an SIS junction: there is an excess current and an exponential dependence of the resistivity on the thickness of the semiconductor. In the opposite case ($B \ll \Delta$) and Andreev reflection is unimportant, since its contribution is proportional to B^2/Δ rather than to B. This leads to a current deficit for high voltages [Eq. (48)]. Indeed, in Ref. 9 a transition was observed from current excess to deficit in the CVC when the junction resistivity was increased (with an increase in the thickness or a change in the composition of the Sm layer). Only a current excess was observed in Refs. 6 and 7, since the weak condition (58) was satisfied for all junction thicknesses used. However, with junctions with oxidated boundaries[10,11] a current deficit was observed in the CVC which is connected with the impossibility to satisfy the rigid condition (59).

The authors express their gratitude to A. A. Abrikosov, A. I. Glazman, A. I. Larkin, and K. A. Matveev for useful discussions of the results obtained.

APPENDIX

Using the method of Ref. 2 we can find the Green functions of the one-dimensional stationary junction without LC:

$$G_\omega^R(y,y') = -\frac{m}{\varkappa}[e^{-\varkappa|y-y'|} + g_1^R(\omega) D_1 e^{-\varkappa(y+y')}],$$

$$F_\omega^{(+)R}(y,y') = -\frac{m}{\varkappa} f_1^{(+)R}(\omega) D_1 e^{-\varkappa(y+y')}, \quad (A1)$$

where the coordinates y,y' are inside the barrier, and to fix the idea close to the side S_1 (y,y' are reckoned from σ_1). Bearing in mind the uniformity of the junction we go over to the three-dimensional functions

$$G_{\omega,\mu}(\mathbf{r},\mathbf{r}') = \int \frac{d^2\mathbf{p}}{(2\pi)^2} G_{\omega,\mu-p^2/2m}(y,y') e^{i\mathbf{p}(\rho-\rho')}, \quad (A2)$$

where $\mathbf{r} = (y,\rho)$, and μ is the chemical potential. We then get Eqs. (18),(43) directly. In (14) we get

$$1+\beta \int_{|\mathbf{r}_1-\mathbf{a}_1|<r_0} d^3\mathbf{r}_1\, G_\omega^R(\mathbf{a}_1,\mathbf{r}_1) = \frac{4}{3}\pi r_0^3 \beta \{g^{-1}(\omega) + G_1^R(\omega)\}, \quad (A3)$$

where $\varkappa_0$ from Eq. (15) satisfies the relation

$$\beta = \frac{1}{mr_0^2} + \frac{2}{3}\frac{\varkappa_0}{mr_0}. \quad (A4)$$

In the limit as $r_0 \to 0$, $\beta \to \infty$, if (A4) is satisfied, we get Eqs. (16) and (17).

[1] L. G. Aslamazov and M. B. Fistul', Pis'ma Zh. Eksp. Teor. Fiz. **30**, 233 (1979) [JETP Lett. **30**, 213 (1979)].
[2] L. G. Aslamazov and M. B. Fistul', Zh. Eksp. Teor. Fiz. **81**, 382 (1981) [Sov. Phys. JETP **54**, 206 (1981)].
[3] L. G. Aslamazov and M. B. Fistul', Zh. Eksp. Teor. Fiz. **83**, 1170 (1982) [Sov. Phys. JETP **56**, 666 (1982)].
[4] L. G. Aslamazov and M. B. Fistul', Zh. Eksp. Teor. Fiz. **86**, 1516 (1984) [Sov. Phys. JETP **59**, 887 (1984)].
[5] I. M. Lifshitz and V. Ya. Kirpichenov, Zh. Eksp. Teor. Fiz. **77**, 989 (1979) [Sov. Phys. JETP **50**, 499 (1979)].
[6] V. N. Gubankov, S. A. Kovtonyuk, and V. P. Koshelets, Zh. Eksp. Teor. Fiz. **89**, 1335 (1985) [Sov. Phys. JETP **62**, 773 (1985)].
[7] L. É. Amatuni, V. P. Koshelets, G. A. Ovsyannikov, and I. L. Serpuchenko, Zh. Tekh. Fiz. **56**, 1363 (1986) [Sov. Phys. Tech. Phys. **31**, 800 (1986)].
[8] S. J. Bending and M. R. Beasley, Phys. Rev. Lett. **55**, 324 (1985).
[9] E. L. Hu, L. D. Jackel, A. R. Strand, *et al.*, Appl. Phys. Lett. **32**, 584 (1978).
[10] D. A. Rudman, R. E. Howard, D. F. Moore, *et al.*, IEEE Trans. Magn. **MAG-15**, 582 (1979).
[11] D. A. Rudman and M. R. Beasley, Appl. Phys. Lett. **36**, 1010 (1980).
[12] L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys. JETP **20**, 1018 (1965)].
[13] A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **68**, 1927 [Sov. Phys. JETP **41**, 966 (1975)].
[14] A. V. Zaĭtsev, Zh. Eksp. Teor. Fiz. **78**, 221 (1980) [Sov. Phys. JETP **51**, 111 (1980)].
[15] A. D. Zaikin and G. F. Zharkov, Zh. Eksp. Teor. Fiz. **81**, 1781 (1981) [Sov. Phys. JETP **54**, 944 (1981)].

Translated by D. ter Haar