Magnetic superfluidity in He^3

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The region of stable magnetic superfluid transport in the A and B phases of He^3 is determined. The stability criterion is chosen to be the condition that the energy of small static fluctuations be positive-definite near states with magnetic-moment component flows. This criterion is a generalization of the Landau criterion in superfluidity theory. The critical gradients are determined for cases when the transport is determined by the gradient of the order-parameter rotation angle in spin space (spin transport) or by the gradient of the phase of the magnetic-moment precession (precession transport), as well as for the general case of gradients of both angle variables. Spin-dynamic equations are derived, averaged over the fast changes of two angle variables: the rotation angle of the order parameter and the precession phase. The solution obtained for these equations corresponds to the regime of dynamic suppression of the magnetic moment, in which spin pumping by a rotating magnetic field maintains the magnetic moment far below the equilibrium. When such a regime is produced in a chamber with He^3, a magnetic superflow should be produced in a channel leading out of this chamber.

1. INTRODUCTION

"Magnetic superfluidity" is defined in the present paper as nondissipative macroscopic spin transport resulting in a deformed state of a magnetically ordered medium with spatial rotation (twisting) of the order parameter in spin space. The spin flow is similar in this case to mass superflow (nondecaying flow) in a superfluid, and also to elastic flow of the torque in a twisted solid.[1] Similar phenomena are possible in an ordered medium with a definite topology of the order-parameter space. The justification for its existence is the presence of a soft (i.e., almost-Goldstone) mode described by a pair of canonically conjugate variables of the particle-number–phase (or moment-angle) type. The energy can include in this case small terms invariant to the phase shift (rotation) and violating the particle-number (moment) conservation law. Although these terms alter substantially the character of the nondissipative transport (they make states with flows inhomogeneous and reduce them in the limit to chains of isolated solitons or domain walls), they admit of the existence and observation of the phenomenon if they are small enough.[1]

As applied to superfluid He^3, magnetic superfludity was discussed by us starting with Refs. 2 and 3, and with allowance for spin nonconserving processes starting with Ref. 4. The role of these processes was analyzed earlier for magnetically ordered states of a solid.[5,6] Magnetic superfluidity in superfluid He^3 was initially discussed mainly for the A-phase in connection with experiments on longitudinal magnetic relaxation,[3] which were interpreted with the aid of this phenomenon. Recently, however experimental[7,8] and theoretical[9] studies were made of superfluid transport of the magnetic moment in the B phase. This phenomenon was first observed directly in experiment in a continuous regime, and not only the flux and phase, but also the phase slip were observed. The magnetic superfluidity observed in the B phase has a number of properties that distinguish it from the phenomenon investigated earlier in the A phase. What was considered in the A phase was a longitudinal mode connected with longitudinal NMR, but in the B phase was investigated a transverse mode (connected with transverse NMR), the angular variable for which is not the order-parameter rotation angle, but the precession phase, i.e., the rotation angle of the transverse component of the magnetic moment. Spatial twisting of this phase gives rise to a flux of a canonically conjugate quantity (different from the longitudinal component of the spin), called in Ref. 10 the precession angular momentum. An investigation of the critical gradients for nondissipative transport of the precession angular momentum has shown that, in contrast to transport of the longitudinal component of the spin moment, it is possible only for large enough deviations from equilibrium (the angle between the precessing angular momentum and the magnetic-field vector must exceed 104°).[10].

The present paper is devoted to further study of magnetic superfluidity within the framework of the Leggett-Takagi macroscopic spin dynamics eveloped for the superfluid phase of He^3 (see Refs. 12–14 and the bibliography therein). The principal assumption of this theory, valid so long as the significant scales are large compared with the microscopic scales (of exchange origin) over which the order-parameter structure is established, is that the entire motion reduces to rotations in three-dimensional spin space while the order parameter itself retains a rigid structure. Since this theory is universal for magnetically ordered media,[15] the present investigation of magnetic superfluidity in He^3 carried out within its framework can be easily extended to include other magnetically ordered media.

The structure of states with magnetic supercurrents is analyzed by investigating the corresponding trajectories in the space in which the order parameter varies—the space of three-dimensional rotations. Besides the previously investigated states with twisting of only the order-parameter rotation angle or only the precession phase, we consider also the more general case of twisting of both angle variables. The stability of the magnetic supercurrents is investigated on the basis of a generalized Landau criterion, by checking the positiveness of the quadratic form that determines the energy of small static fluctuations. The critical precession flux gradi-

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ents previously determined in this manner for a homogeneous long channel[10] increase jumpwise, at a spin-moment inclination angle 104°, from zero to value of the order of the reciprocal dipole length. In experiment,[7,8], however, after the angle 104° was reached, a rather smooth rise of the critical gradient was observed. A possible cause of this discrepancy was assumed in Ref. 10 to be the difference between the value of the experimentally determined deflection angles in the chamber where the spin is pumped and in the channel where the magnetic supercurrent is observed. This explanation is qualitatively confirmed by the more detailed theory developed in the present paper.

The method of producing magnetic supercurrents of various types is assumed in this paper to be continuous spin pumping by a rotating RF field in a homogeneous liquid contained in a chamber from which the spin moment is extracted through some channel (such a geometry was used in the experiments of Refs. 7 and 8, but the liquid was not homogeneous because the applied constant magnetic field was not uniform). To this end, we derive in the Appendix dynamics equations that take into account dissipation and averaging over fast oscillations of the angle variables. They differ from Fomin's analogous equations[12,13] in that account is taken of the contribution of the precession-phase fast oscillations, and also of the nutation oscillations (tipping of the chosen physical axis away from the precessing spin moment). On the basis of the derived equations we demonstrate the possibility of dynamically suppressing the moment in both the B and A phase: by applying an RF field of required intensity it is possible to maintain a state in which the magnetic moment is significantly lower than the equilibrium value.

2. EQUATIONS OF MACROSCOPIC SPIN DYNAMICS

All the phenomena considered in the present paper evolve over scales considerably exceeding the exchange scales over which is established the tensor structure of the order parameter (the coherence length). This permits the use of a dynamic theory in which the entire evolution of the order parameter reduces to rotations in three-dimensional spin space. A discussion of a theory of this type, which is well known in magnetism and is based in the idea of spontaneous symmetry breaking, can be found in the review by Andreev and Marchenko[15] and in the literature cited there. The dynamic behavior of the system is described by three pairs of canonically conjugate moment and angle variables ($M_i - \varphi_i$, $i = 1,2,3$):

$$\frac{\partial M_i}{\partial t} = -\gamma\frac{\delta F}{\delta \varphi_i} - \frac{\partial f}{\partial(\delta F/\delta M_i)}$$

$$\frac{\partial \varphi_i}{\partial t} = \gamma\frac{\delta F}{\delta M_i} - \frac{\partial f}{\partial(\delta F/\delta \varphi_i)}, \qquad (1)$$

where γ is the gyromagnetic ratio, F the free energy, and f a dissipative function which is a quadratric form of the functional derivatives $\delta F/\delta M_i$ and $\delta F/\delta \varphi_i$. As the angle variables, following Fomin,[12–14] we introduce the angles α, $\Phi = \alpha + \gamma$ and β (α, β and γ are the Euler angles). The moments canonically conjugate to them are respectively: $P = M_z - M_\xi$, M_ξ and M_β, where M_z is the projection of the moment $\mathbf{M}$ on the z axis of the immobile coordinate frame, M_ξ is the projection of $\mathbf{M}$ on the ξ axis of the moving coordinate frame, and M_β is the projection of $\mathbf{M}$ on the axis perpendicular to the axes z and ξ. We shall consider the case of a strong magnetic field $\mathbf{H}$ along the z axis, when the components of the total free energy $F = F_0 + F_\perp + G$, viz., the energy of interaction with the alternating magnetic field $\mathbf{H}_\perp$ (having projections $H_\perp \cos\varphi$ and $H_\perp \sin\varphi$ along the x and y axes),

$$F_\perp = -\mathbf{M}\mathbf{H}_\perp = -[M_\xi(1-\cos\beta) - P\cos\beta]\frac{\sin(\alpha-\varphi)}{\sin\beta} - M_\beta\cos(\alpha-\varphi) \qquad (2)$$

and the ordering energy $G = F_B = V$, which includes the gradient energy F_0 (Ref. 16):

$$F_0 = \frac{M^2}{2\chi} - \mathbf{MH} = \frac{1}{\chi}\left(\frac{M_\xi^2 + PM_\xi}{1+\cos\beta} + \frac{P^2}{2\sin^2\beta}\right) - (M_\xi + P)H. \qquad (3)$$

Here χ is the magnetic susceptibility (in the case of the A phase—the susceptibility in a direction perpendicular to the spin vector $\mathbf{d}$). If only the energy F_0 is retained in the free energy and the dissipation is neglected, the motion of the order-parameter spin space reduces to superposition of two rotations: rotation around the moment vector $\mathbf{M}$ with angular velocity $\omega_M = \gamma M/\chi$ in a moving coordinate frame that rotates (precesses) in turn around the z axis at the Larmor frequency $\omega_L = \gamma H$ (Ref. 17). Since the energy F_0 is independent of the angles α and Φ, the moments conjugate to them P and M_ξ are integrals of the motion. On the other hand, the angle β and the moment M_β can execute oscillatory motion corresponding to nutation. Only two angles, α and Φ, can be "fast" variables,[13] i.e., can vary rapidly and monotonically in space and in time (become twisted), executing a large number of complete 2π rotations. Of particular importance to us is motion without nutation, when the directions of the axis ξ and of the moment $\mathbf{M}$ coincide, in which case $\beta = \text{const}$, $M_\beta = 0$, $M_\xi = M$, $P = M(\cos\beta - 1)$, $\partial\alpha/\partial t = -\omega_L$, $\partial\Phi/\partial t = \omega_M - \omega_L$. The angle Φ which characterizes the resultant rotation of the order-parameter spin space in the labortory frame, and in the limit $\beta \to 0$ it becomes the angle of rotation about the z axis. If the corrections due to the ordering energy G are taken into account in nutation-free motion, the moments P and M_ξ cease to be integrals of the motion, and oscillations of the angles α and Φ as well as nutation of the angle β appear against the background of the constant fast twisting of α and Φ. Both the oscillations of α and Φ and the nutation of β are small in terms of the parameter Ω/ω_L (Ω is the longitudinal-NMR frequency). A special case is the homogeneous B phase, for which the dipole energy is independent of the angle α (of the precession phase), so that the moment P conjugate to α (called the precession moment in Ref. 10) remains an exact integral of the motion for any ratio Ω/ω_L.[14] An entire class of exact solutions in an arbitrary magnetic field was therefore obtained for the homogeneous B phase.[18–21]. We shall consider hereafter states with spatial gradients of the two fast variables α and Φ, in both the A and B phases.[13] For these states, motions with twisting of α and Φ, obtained without allowance for the ordering energy and for dissipation, can serve as the starting point for the construction of a perturbation theory in Ω/ω_L, which uses them as a zeroth approximation. Averaging here over the oscillation periods of the fast variables, we obtain equations for the averaged values of α and Φ, and also of the moments P and M. These equations are derived in the Appendix. Taking into account the spin-pumping interaction with the rotating transverse magnetic field, the equations take the form

$$\frac{\partial\alpha}{\partial t}=-\omega_L+\frac{\gamma}{M}\frac{\partial\bar{G}}{\partial u}+\gamma H_\perp \operatorname{ctg}\beta\sin(\alpha-\varphi), \quad (4)$$

$$\frac{\partial P}{\partial t}=-\nabla \mathbf{j}_P+\gamma MH_\perp\sin\beta\cos(\alpha-\varphi)-D_P, \quad (5)$$

$$\frac{\partial\Phi}{\partial t}=\omega_M-\omega_L+\frac{\gamma(1-u)}{M}\frac{\partial\bar{G}}{\partial u}-\frac{1-\cos\beta}{\sin\beta}\gamma H_\perp\sin(\alpha-\varphi), \quad (6)$$

$$\frac{\partial M}{\partial t}=-\nabla \mathbf{j}_M-D_M, \quad (7)$$

where $u=\cos\beta$, D_P, and D_M are the dissipative terms, while $H_\perp$ and φ are the amplitude and angle of the rotating transverse magnetic field ($H_x=H_\perp\cos\varphi, H_y=H_\perp\sin\varphi$). The fluxes of the moment P and of the precession moment P

$$\mathbf{j}_P=-\gamma\frac{\partial F_B}{\partial\nabla\alpha},\quad \mathbf{j}_M=-\gamma\frac{\partial F_B}{\partial\nabla\Phi}, \quad (8)$$

are determined by the expression for the averaged gradient energy, in which only the fast-variable gradients have been retained:

$$F_B=A(u)\frac{(\nabla\alpha)^2}{2}+B(u)\nabla\alpha\nabla\Phi+C(u)\frac{(\nabla\Phi)^2}{2}, \quad (9)$$

where the rigidity coefficients A, B, and C depend on the precession angle β and on the orientation of the gradients relative to the z axis (i.e., are generally speaking tensors, see Eqs. (A.14) and (A.15), as well as Ref. 14). In this set of equations, the dissipative terms determined by Eqs. (A.12) and (A.13) are retained only in the equations for the moments. In the equations for the angles, these terms appear in first order in the energy G and vanish after averaging. The main difference between Eqs. (4)–(7) and the previously derived average equations is that the later were derived with averaging over the oscillation periods of the two fast angle variables (cf. Refs. 12 and 22).

The procedure for averaging the spin-dynamics equations can be carried out also if the angle oscillations are not small against the background of their monotonic variation. In this case the almost linear change of the angles in time and space goes over into a sequence of abrupt steplike changes of the angle (solitons or domain walls), which separate constant-angle regions. The averaged equations describing the motion of such a soliton chain in terms of its density and velocity (the hydrodynamic theory of a soliton chain) were derived and investigated in Refs. 23 and 24 for the longitudinal mode of ^{3}He-A.

3. STRUCTURE OF STATES WITH MAGNETIC NONDISSIPATIVE FLUXES, AND THEIR STABILITY

In the absence of dissipation and spin pumping, Eqs. (4)–(7) have stationary solutions with linear variation (twisting) of the angles α and Φ in time and space at constant velocity. We introduce the order-parameter space and trace the trajectories in this space under conditions of this twisting. Since the entire motion of the order parameter in the considered dynamic model reduces to rotations, it is represented fully enough in the space of three-dimensional rotations introduced by the well-known method.[25] To each rotation through an angle θ about the axis specified by the unit vector $\mathbf{n}$ there corresponds a point on a sphere of radius π, with a radius vector directed along $\mathbf{n}$ and with a length equal to the angle θ. Diametrically opposite points on this sphere correspond to one state. The angle θ and the components of the vector $\mathbf{n}$ are connected with the Euler angles by the relations

$$\cos\theta={}^1/_2(\cos\Phi\cos\beta+\cos\Phi+\cos\beta-1), \quad (10)$$

$$n_x=\frac{\sin(\beta/2)\cos(\alpha-\Phi/2)}{(1-\cos^2(\Phi/2)\cos^2(\beta/2))^{1/2}},$$
$$n_y=\frac{\sin(\beta/2)\sin(\alpha-\Phi/2)}{(1-\cos^2(\Phi/2)\cos^2(\beta/2))^{1/2}}, \quad (11)$$
$$n_z=\frac{\cos(\beta/2)\sin(\Phi/2)}{(1-\cos^2(\Phi/2)\cos^2(\beta/2))^{1/2}}.$$

The space of three-dimensional rotations is isomorphous to the total space of the B-phase order-parameter which is a rotation matrix with directrix $\mathbf{n}$. On the other hand, for simple antiferromagnetic ordering with one vector $\mathbf{d}$, just as in an A-phase with fixed orbital vector $\mathbf{l}$, the true order-parameter space is narrower than the rotation space, since the group of rotations around the vector $\mathbf{d}$ does not alter the state of the liquid. Therefore all the changes of the vector $\mathbf{d}$ are exhaustively mapped by any diametral section of the rotation-space sphere, a section ismorphous to the surface of the three-dimensional sphere traced by the end point of an arbitrarily directed vector $\mathbf{d}$. The order-parameter variation space is shown in Fig. 1. According to (10) and (11), when α is twisted the angle θ does not change, and the directrix $\mathbf{n}$ rotates around the z axis. The trajectories of this motion are circles of arbitrary radius with centers on the sphere diameter and located in planes perpendicular to the z axis (Fig. 1a). If only the angle Φ is twisted, with $\alpha=$ const, the trajectory "pierces" the sphere many times, returning each time to a diametrally opposite point (Fig. 1b). Figure 1c shows an example of a trajectory with simultaneous twisting of α and Φ.

The fact that nondissipative transport of the spin-moment components takes place in states with twisting of the angles α and Φ follows directly from Eqs. (5) and (7) for the balance of P and M and from expressions (8) for the fluxes. The dissipative process that eliminate the twisting is the phase loss, in orther words the phase slipping, i.e., a change of the phase advance on some section of the channel length by 2π or by a multiple of 2π. In a channel with macroscopic transverse dimensions, the phase slipping is effected by motion of the vortices across the channel.[1] The nondissipative transport of the spin moment is stable, meaning also observable, if the phase slipping is accompanied by a surmounting of a sufficiently high activation barrier. Such a barrier can be of topological origin if the contour for the trajectory with the twisting cannot be contracted by continuous deformation (homotopy) into a point.[25] It is seen from Fig. 1, however, that all the trajectories with twisting of α and Φ have no topological stability. The twisting of α (Fig. 1a) is always eliminated by contracting the contour into a point. On the

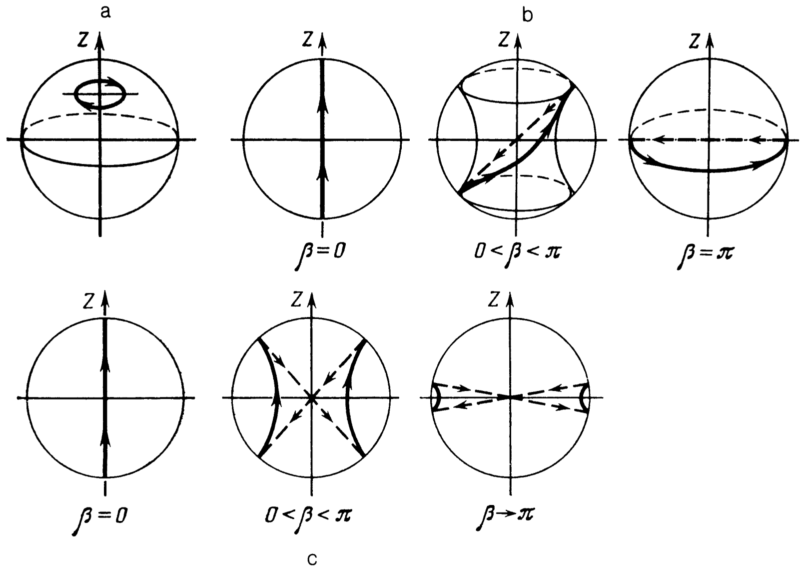

FIG. 1. Twist trajectories (thick lines with arrowheads) of the precession phase α and of the rotation angle Φ of the order parameter in the order-parameter space (three-dimensional-rotation space). The single-cavity hyperboloids are the surfaces $\beta=$ const: a) twisting of α; b) twisting of Φ for various β; c) twisting of α and Φ, $\Phi=2\alpha$ for various β.

other hand, twisting of Φ with repeated "piercing" of the sphere is either eliminated completely by homotopy if the number of piercings is even (the process of contraction of a contour with two piercings by a $\beta\to\pi$ transition is clear from Fig. 1c), or is transformed by homotopy into a contour that pierces the sphere once if the number of piercings was initially odd. We, however, regard dissipative transport as a macroscopic phenomenon, whereby in a channel of macroscopic length the angle is twisted a large number of turns. Yet twisting by only one turn (one piercing of the sphere) over a macroscopic length corresponds to a vanishingly small gradient (transport velocity) and is of no interest to us. Such a topological analysis, however, is valid only in the presence of complete degeneracy in the order-parameter space considered by us. There exist energies that lift this degeneracy and delimit a narrower space in which certain twist trajectories become topologically stable. Thus, magnetic-anisotropy energy in the A phase stabilizes the spin supercurrent in longitudinal geometry with twisting of Φ and with a moment $\mathbf{M}$ directed along the z axis ($\beta=0$).[1,4] There can exist also barriers of nontopological origin,[1,25] for example in the case of mass superflow in the A phase.[26] In either case, the necessary condition for stability is that the state with the nondissipative flow correspond to the minimum of a suitably chosen thermodynamic potential. This reduces to the condition that the quadratic form for the energy of the static fluctuations be positive-definite in the vicinity of the state with the flow. This condition has already been used earlier,[1,26] it is a generalization of the known Landau criterion, and will be used later on in this paper. In this approach to the stability problem, the state with the flow is treated as a quasi-equilibrium one. It is actually a stationary dynamic state, in which dissipative loss of spin is compensated for by external pumping. This loss, however, is noticeable only in a scale on the order of the macroscopic length of the channel (see the discussion in Ref. 1). The method employed is therefore a check on the stability to local phase slipping, which takes place over a scale that is small compared with the channel length. If the stability condition is violated, the phase-slipping processes not suppressed by a high activation barrier lead to an exponential damping of the spin flow with increase of the distance from the channel end through which the spin is pumped.

4. NONDISSIPATIVE PRECESSION TRANSPORT

An example of magnetic superflow is precession-moment transport in the B phase, investigated in Refs. 7–10. In experiments on nonlinear transverse NMR, the spin precession is excited by a pulse that rotates the moment $\mathbf{M}$ through a definite angle without noticeably changing its absolute value. According to the theory,[12–14] the ensuing relaxation is such that there are likewise no noticeable deviations of M from its equilibrium value χH. The reason is that the dipole energy singles out those trajectories for which the angle Φ ceases to be a rapidly changing variable ($\partial\Phi/\partial t=0$) and $\omega_M\sim\omega_L$ ($M\sim_\chi H$). The stationary value of Φ is determined from the non-averaged equation that leads to a zero change of the moment $M_\xi\approx M$. The resultant dynamics has one degree of freedom corresponding to the angle—moment variable pair α and $P=M_z-M=M(u-1)$ Eqs. [(4) and (5)]. The spatial twisting in the fast variable, in the precession phase α, corresponds to nondissipative transport of the precession moment P.

Following Ref. 10, we determine the critical value of the gradient $\nabla\alpha=h_\alpha$ at which twisting in α becomes unstable. The initial equation for the free energy is

$$F=F_0+F_B+V=\frac{M^2}{2\chi}-(M+P)H+{}^1/_2A(u)(\nabla\alpha)^2+V(u)$$
$$=\chi\gamma^{-2}[-{}^1/_2\omega_L{}^2-\omega_L{}^2(u-1)]+{}^1/_2A(u)(\nabla\alpha)^2+V(u). \quad (12)$$

It is recognized here that

$$P=M(u-1)=\chi H(u-1)=(\chi/\gamma)\omega_L(u-1),$$

and the only gradient remaining in Eq. (9) is $\nabla\alpha$ with respect to the fast variable. For the angle Φ in the dipole energy for the B phase,

$$V(u,\Phi)=\frac{2}{15}\frac{\chi\Omega^2}{\gamma^2}\left[(u+1)\cos\Phi+u-\frac{1}{2}\right]^2, \quad (13)$$

we must choose its stationary value determined, with dissipation neglected, by minimizing V with respect to Φ, i.e., by the condition $\partial v/\partial\Phi = 0$. It follows hence that

$$(u>-{}^1/_4):\ \cos\Phi=\frac{{}^1/_2-u}{1+u},\quad V(u)=0,\qquad \text{for } \beta<104^\circ$$

$$(u<-{}^1/_4):\ \Phi=0,\quad V(u)=\frac{8}{15}\frac{\chi\Omega^2}{\gamma^2}\left(u+\frac{1}{4}\right)^2\quad \text{for } \beta>104^\circ. \tag{14}$$

To check on the stability one must choose a thermodynamic potential

$$\tilde F=F+\frac{\omega_P}{\gamma}P+\frac{1}{\gamma}\mathbf{j}_P\nabla\alpha, \tag{15}$$

that has a minimum at the specified values of $P=M(u_0-1)$ and $\nabla\alpha=h_\alpha$. This is done by a suitable choice of the Lagrangian multipliers $\omega_P=-\gamma\partial F/\partial P$ (precession rate) and $\mathbf{j}_P=-\gamma\partial F/\partial\nabla\alpha=A(u_0)\mathbf{h}_\alpha$ (precession-moment flux). It is easy to verify that the fluctuations of M and Φ always increase $\tilde F$, so that it sufficies to retain in the fluctuation energy only the terms quadratic in the small deviations $u=u-u_0$ and $\nabla\alpha'=\nabla\alpha-\mathbf{h}_\alpha$ (we omit hereafter the zero subscript of u_0);

$$\delta F={}^1/_2A(u)(\nabla\alpha')^2+A'(u)\mathbf{h}_\alpha\nabla\alpha'u' + {}^1/_2[{}^1/_2A''(u)h_\alpha^2+V''(u)]u'^2. \tag{16}$$

The fluctuation energy is positive-definite, i.e., the twisting in α is stable so long as h_α does not exceed the critival value

$$h_c=[A(u)V''(u)/(A'^2(u)-A''(u)A(u)/2)]^{1/2}. \tag{17}$$

Since the dipole energy and all its derivatives vanish at $\beta<104°$, we have $h_c=0$ and the nondissipative precession transport is unstable. For $\beta<104°$, using (14) and expressions (A.14) and (A.15) for the rigidity constant $A(u)$, we find from (17) that

$$h_c=\frac{4\Omega}{3\sqrt5}\left\{\frac{c_\parallel^2(1-u)^2+c_\perp^2(1-u^2)}{[c_\parallel^2+(c_\perp^2-c_\parallel^2)u]^2+c_\perp^4/3}\right\}^{1/2},\quad \text{for } \nabla\alpha\perp\hat z$$

$$h_c=\frac{4}{3}\sqrt{\frac{2}{5}}\,\Omega\left\{\frac{c_\parallel^2u(1-u)+c_\perp^2(1-u)^2}{[(c_\parallel^2+2(c_\perp^2-c_\parallel^2)(1-u)]^2+c_\parallel^4/3}\right\}^{1/2} \tag{18}$$

for $\nabla\alpha\|\hat z$.

A plot of $h_c(u)$ for the case $\nabla\alpha\perp\hat z$ is shown in Fig. 2. At $u=-1/4(\beta=104°)$ the critical gradient increases jumpwise from zero to a value on the order of the reciprocal dipole length $\xi_D^{-1}\sim\Omega/c$. But we have obtained the dependence of h_c on the value of $u=\cos\beta$ in a channel in which superfluid transport takes place. Experiment,[7,8] on the other hand, yielded the dependence of h_c on the difference $\omega_P-\omega_L$. This turned out to be a square-root dependence with the values of h_c everywhere lower than ξ_D^{-1}. As indicated in Ref. 10, to explain this behavior, account must be taken of the difference between the value of $u=\cos\beta$ in terms of which h_c was defined in Eqs. (17) and (18) for the channel, and the value of $u_v=\cos\beta_v$ which determines the difference $\omega_P-\omega_L$, in the volume in which the spin is pumped. The spatial distribution of β is determined from the condition that the precession rate ω_P be constant in space for a stationary flow ($\nabla\alpha$ does not vary with time), exactly as the chemical potential remains unchanged in stationary superflow. This condition yields a connection analogous to the Bernoulli equation in hydrodynamics, between the local values of $\nabla\alpha$ and $u=\cos\beta$ in different points of space. Comparing the expressions for $\omega_P=-\gamma M^{-1}(\partial F/\partial u)$ in the channel, where $\nabla\alpha\neq0$, and in the chamber, where $\nabla\alpha=0$, and using (12) and (14), we get

$$-\omega_P=-\omega_L+\frac{16}{15}\frac{\Omega^2}{\omega_L}\left(u_v+\frac14\right)$$
$$=-\omega_L+\frac12A'(u)\frac{\gamma^2}{\chi\omega_L}(\nabla\alpha)^2+\frac{16}{15}\frac{\Omega^2}{\omega_L}\left(u+\frac14\right). \tag{19}$$

These relations can be used to determine the dependence of h_c on $u_v=\cos\beta_v$ directly connected with the difference

$$\omega_L-\omega_P=\frac{16}{15}\frac{\Omega^2}{\omega_L}\left(u_v+\frac14\right),$$

from the previously obtained dependence of h_c on $u=\cos\beta$ in the channel [Eqs. (17) and (18)]. The $b_c(u_v)$ curve is drawn on the same Fig. 2 as the $h_c(u)$ curve. The section with the vertical slope on the $h_c(u)$ curve goes over into a section with a finite slope on the $b_c(u_v)$ curve. The expression for h_c on this section is obtained by substituting $u=-1/4$ and $\nabla\alpha=h_c$ in (19):

$$h_c=\left[\frac{32}{15}\frac{\chi\Omega^2(u_v+{}^1/_4)}{\gamma^2A'(-{}^1/_4)}\right]^{1/2}=\left[\frac{2\chi\omega_L(\omega_L-\omega_P)}{\gamma^2A'(-{}^1/_4)}\right]^{1/2}. \tag{20}$$

Using expressions (A.14) and (A.15) for $A(u)$, we obtain

$$h_c=[4(\omega_P-\omega_L)\omega_L/(5c_\parallel^2-c_\perp^2)]^{1/2},\quad \text{for } \nabla\alpha\perp\hat z$$

$$h_c=[2(\omega_P-\omega_L)\omega_L/(5c_\perp^2-3c_\parallel^2)]^{1/2}\quad \text{for } \nabla\alpha\|\hat z\ |. \tag{21}$$

Expression (21) coincides in the case $\nabla\alpha\perp\hat z$ with the expression obtained for the critical gradient α_{c1} by Fomin.[9]

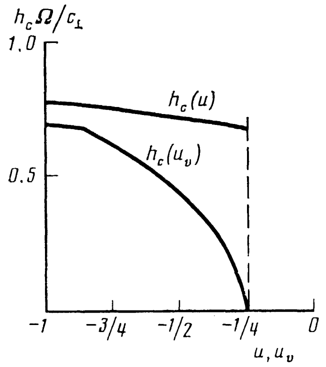

FIG. 2. Critical gradient h_c for precession transport. The figure shows the dependences on $u=\cos\beta$ in the channel with the precession flux, and on the quantity

$$u_v=\cos\beta_v = -{}^1/_4-{}^{15}/_{16}\omega_L(\omega_P-\omega_L)/\Omega^2$$

in the chamber where the spin pumping takes place. In the case of a nonuniform magnetic field, ω_L is the Larmor frequency at that point of the chamber having the same value of the field as in the channel.

He, however, assumed that α_{c1} is the critical gradient for a specified flux through the channel, whereas a given precession phase difference was assumed in our above calculation of h_c. Were the flux rather than the phase difference given, the term $\gamma^{-1}\mathbf{j}_P \nabla\alpha = -A(u_0)\mathbf{h}_\alpha \nabla\alpha$ in Eq. (15) for the thermodynamic potential $\widetilde{F}$, would be replaced by the term $-A(u)\mathbf{h}_\alpha \nabla\alpha$, which would contribute to the fluctuation energy, in view of the $u-u_0$ fluctuation, and this would influence the calculation result substantially. According to Fomin, at a given phase difference the critical gradient is another quantity, α_{c2}, which exceeds α_{c1} (i.e., our h_c), but is of the same order of magnitude. This other value was determined by a calculation showing that at $\nabla\alpha > \alpha_{c2}$ there are no stationary solutions with a specified difference $\omega_P - \omega_L$. In a state with $\nabla\alpha = \alpha_{c2}$, however, the angle β is zero, and according to Ref. 10 the instability in the present paper should set in earlier.

An additional constraint on the possibility of observing the nondissipative precession transport is imposed by the very condition that a regime with a fixed angle Φ exist. As indicated at the beginning of this section, the angle Φ is determined from the condition $\partial M_\xi/\partial t = 0$. If spin diffusion is significant, this condition reduces to an equation more complicated than $\alpha V/\partial\Phi = 0$ used above[14]:

$$-\gamma\partial V/\partial\Phi - DM(1-u^2)(\nabla\alpha)^2 = 0, \tag{22}$$

where D is the spin-diffusion coefficient. This equation has a solution for Φ so long as the gradient $\nabla\alpha$ does not exceed a value on the order of $\sim\Omega/(D\omega_L)^{1/2}$. This constraint may become significant on going to fields stronger than those used so far in experiment.

5. NONDISSIPATIVE TRANSPORT WITH TWISTING OF TWO ANGLES

Besides the simpler cases of magnetic superfluid transport, the averaged spin-dynamic equations (14)–(17) have also stationary states with simultaneous twisting of the two fast angle variables Φ and α. Let us examine the stability of such states. The thermodynamic potential F used to calculate the stability must include, besides the free energy, terms with Lagrangian multipliers that ensure a minimum of the potential given by the mean values of M, $P = M(u-1)$, $\nabla\Phi = \mathbf{h}_\Phi$ and $\nabla\alpha\mathbf{h}_\alpha$. We shall not give in detail the derivation of the quadratic form for the fluctuation energy, since it is similar to the derivation given in the preceding section. A substantial contribution to this form is made only by the gradient and dipole energies; the fluctuations of $\mathbf{M}$ are dicarded since they obviously increase the energy. Ultimately, the fluctuation energy is

$$\begin{aligned}\delta F = {}&{}^1/_2A(u)(\nabla\alpha')^2 + B(u)\nabla\alpha'\nabla\Phi' + {}^1/_2C(u)(\nabla\Phi')^2\\ &+(A'(u)\mathbf{h}_\alpha + B'(u)\mathbf{h}_\Phi)\nabla\alpha' u'\\ &+(B'(u)\mathbf{h}_\alpha + C'(u)\mathbf{h}_\Phi)\nabla\Phi' u' + {}^1/_2({}^1/_2A''(u)h_\alpha^2\\ &+B''(u)\mathbf{h}_\alpha\mathbf{h}_\Phi + {}^1/_2C''(u)h_\Phi^2 + V''(u))u'^2,\end{aligned} \tag{23}$$

where $A(u)$, $B(u)$, and $C(u)$ are the rigidity constants in Eq. (9) for the gradient energy, and are defined by (A.14) and (A.15). The dipole energy determined from (13) for the B phase is averaged over Φ, which is now the fast variable:

$$V(u) = \frac{\chi\Omega^2}{5\gamma^2}\left(u^2 + \frac{1}{2}\right). \tag{24}$$

For the energy δF to be positive-definite, all the coefficients of the diagonal terms must be positive. According to expressions (A.14) and (A.15) for the rigidity constants A, B, and C, and according to the condition $V'' > 0$, which will be shown below to be necessary if δF is to be positive, this restricts only the gradient h_α, if the gradients are directed along z. In this case $A'' < 0$ ($B'' = C'' = 0$ in both cases), and the requirement for stability is

$$h_\alpha < (2V''(u)/(-A''(u)))^{1/2}. \tag{25}$$

We can next minimize δF with respect to u' (the condition that the coefficient of u'^2 be positive ensures that the extremum with respect to u' is a minimum), after which we obtain a quadratic form of the fluctuations of the two gradients, and the condition that it be positive-definite yields the following inequalities for the gradients h_α and h_Φ:

for $\mathbf{h}_\alpha$, $\mathbf{h}_\Phi \perp \hat{z}$:

$$\begin{aligned}&[2K^2(1+u) - KK_2(1+u)^2 + {}^1/_4K_2^2(1+3u^2)]h_\alpha^2\\ &\quad -[2K^2(1+u) + 2K_2Ku]h_\alpha h_\Phi + K^2h_\Phi^2\\ &\quad < \chi\Omega^2\gamma^{-2}(K - {}^1/_2K_2)(1-u^2),\end{aligned} \tag{26}$$

for $\mathbf{h}_\alpha$, $\mathbf{h}_\Phi \| \hat{z}$:

$$\begin{aligned}&[16K_1^2(1+u) + 8K_1K_2(1+u)^2 + 2K_2^2(1+3u^2)]h_\alpha^2\\ &\quad -[16K_1^2(1+u) + 16K_1K_2u]h_\alpha h_\Phi + 8K_1^2h_\Phi^2\\ &\quad < \chi\Omega^2\gamma^{-2}(4K_1 + K_2)(1-u^2).\end{aligned} \tag{27}$$

Here $K = 2K_1 + K_2$, $K_1 = (\chi/4\gamma^2)(2c_\perp^2 - C_\|^2)$, $K_2 = (\chi/\gamma^2)(C_\|^2 - C_\perp^2$. As $u\to 1$ ($\beta\to 0$), the stability region contracts. The reason is that the order-parameter space contour around which the trajectory is "wound" in the case of α twisting shrinks strongly in this limit and contracts to a point. If it is assumed, however, that the phase slipping of the α precession proceeds without difficulty, there still remains Φ twisting due to the multiple "piercing" of the sphere. One can visualize a state in which, as a result of the slipping, the energy takes on the lowest value for the given twisting. Such a state, as will be now shown, remains stable also as $\beta\to 0$. Minimizing the gradient energy (9) with respect to $\nabla\alpha$ for a given $\nabla\Phi$, we get

$$\nabla\alpha = -\frac{B}{A}\nabla\Phi, \quad F_B = {}^1/_2(C - B^2/A)(\nabla\Phi)^2. \tag{28}$$

The rest of the stability-condition derivation is similar to that used before, and we find as a result that the considered state is stable until the gradient $\nabla\Phi$ reaches the following critical values:

for $\nabla\Phi$, $\nabla\alpha \perp \hat{z}$:

$$h_c = \left(\frac{2(1+u)}{5}\right)^{1/2} \frac{\Omega[c_\perp^2(1+u) - c_\|^2 u]}{c_\|^2 c_\perp}, \tag{29}$$

for $\nabla\Phi$, $\nabla\alpha \| \hat{z}$:

$$h_c = \left(\frac{1+u}{5}\right)^{1/2} \frac{\Omega[c_\|^2 u + c_\perp^2(1-u)]}{(c_\perp^2 - c_\|^2/2)c_\|}. \tag{30}$$

Equations (29) and (30) determine the critical gradients for a supercurrent of the same type (due to Φ twisting) in the B phase as considered in Refs. 1 and 4 for the A phase. In the B phase, however, the stability of such a transport is maintained by the averaged dipole energy. In the A phase, the dipole energy does not stabilize the supercurrent, since $V''(u)<0$ there; this is offset, however, by a stronger stability factor—the anisotropy energy of the magnetic susceptibility, which makes the magnetic superfluid transport stable at gradients exceeding the reciprocal dipole length.

6. OBSERVATION OF MAGNETIC SUPERFLUID TRANSPORT

The concept of magnetic superfluid transport was first invoked for the interpretation of experiments on longitudinal-magnetization relaxation in the A phase,[2,3] but this interpretation was not unique. Closely connected with the magnetic superfluidity phenomenon were experiments on magnetic-soliton propagation in the A phase.[27] A chain of such solitons is the texture, twisted in Φ, discussed in Sec. 3. In the experiments mentioned there, the spin was pumped by a pulse that rotated the spin through 180° at one end of the channels formed by a stack of parallel plates. This excited a packet of solitons propagating along the channels and transporting spin. The solitons were recorded at the other end of the channels by an NMR procedure. However, a large number of distinctive features of magnetic superfluidity can be observed only if the spins are continuously pumped. It was just this procedure which was first realized in the B-phase experiments.[7,8]

To pump the spin in a cw regime it is natural to use a rotating magnetic field. In experiments on precession transport,[7,8] they applied in addition a constant-magnetic-field gradient, as a result of which there was produced in the chamber an inhomogeneous texture with a dynamic domain.[14] We discuss here the possibility of uniformly pumping the spin. It is possible to use for this purpose relations (A.5) and (A.7) of the Appendix. We begin with the B phase. To determine the dissipative terms in the Leggett-Takagi mechanism it is necessary to calculate certain mean values of the derivatives with respect to the dipole energy [see Eq. (13)]:

$$\left\langle\left(\frac{\partial V}{\partial\beta}\right)^2\right\rangle=\frac{2\chi^2\Omega^4}{225\gamma^4}\sin^2\beta\,(35\cos^2\beta+10\cos\beta+2),$$
$$\left\langle\left(\frac{\partial V}{\partial\Phi}\right)^2\right\rangle=\frac{2\chi^2\Omega^4}{225\gamma^4}(1+\cos\beta)^2(5\cos^2\beta-2\cos\beta+2). \quad (31)$$

We seek homogeneous stationary states with low values of β and M. In this case (5) and (7) can be written in the form

$$\frac{\partial P}{\partial t}=-\gamma M\tilde{H}_\perp\beta+\frac{94}{225}\frac{\varkappa\chi^3\Omega^4}{\gamma^4 M}\beta^2,$$
$$\frac{\partial M}{\partial t}=-\frac{94}{225}\frac{\varkappa\chi^3\Omega^4}{\gamma^4 M}+\frac{8}{45}\frac{\varkappa\chi^2\Omega^4}{\gamma^4 H}, \quad (32)$$

where $\tilde{H}_\perp=-H_\perp\cos(\alpha-\varphi)$ is the component, perpendicular to the precessing moment, of the rotating transverse magnetic field $\mathbf{H}_\perp$. The angle $\alpha-\varphi$ is determined from (4), and it is assumed here that the moment precesses at the same rate as the angular velocity ω of the field $H_\perp$, i.e., $\partial(\alpha-\varphi)/\partial t=0$. If the frequency ω satisfies the transverse NMR condition, then $\alpha-\varphi=180°$: and $\tilde{H}_\perp=H_\perp$.

The stationary-state parameters are determined from the condition $\partial P/\partial t=\partial M/\partial t=0$:

$$M=\frac{47}{20}\beta^2\chi H,\quad \tilde{H}_\perp=\frac{32}{423}\frac{\varkappa\chi\Omega^4}{\beta^3\gamma^5H^2},$$
$$\dot{Q}=\gamma M\tilde{H}_\perp H_\beta=\frac{8}{45}\frac{\varkappa\chi^2\Omega^4}{\gamma^4}, \quad (33)$$

where $\dot{Q}$ is the best power released per unit volume in this spin-pumping method.

A similar stationary state maintained by a rotating magnetic field can be produced also in the A phase, the dipole energy for which is given by

$$V=-\frac{\chi\Omega^2}{\gamma^2}\left[\cos\Phi\frac{1+u}{2}+\cos(\Phi-2\alpha)\frac{1-u}{2}\right]^2, \quad (34)$$

where Ω is now the frequency of the longitudinal NMR in the A phase. In the calculations for the A phase, the averagings are over both fast variables α and Φ. For the stationary-state parameters we obtain

$$M=2\beta^2\chi H,\quad \tilde{H}_\perp=\frac{1}{16}\frac{\varkappa\chi\Omega^4}{\beta^3\gamma^5H^2},\quad \dot{Q}=\frac{1}{8}\frac{\varkappa\chi^2\Omega^4}{\gamma^4}. \quad (35)$$

Solving the problem of small-oscillations about the obtained stationary state, we can verify that this state is stable.

Thus, in both the A and B phases it is possible to achieve dynamic suppression of the magnetic moment if the value of the latter under continuous spin pumping is much lower than the equilibrium value. If such a regime is realized in a chamber having an exit channel through which the liquid can flow into a region where the spin pumping less intense or zero, magnetic superfluid transport will be effected in such a channel.

7. CONCLUSION

The region of existence of stable magnetic superfluid transport was determined in the present paper from the condition that the small-fluctuation energy be positive near a state with magnetic nondissipative flux. This condition can be regarded as a generalization of the well known Landau criterion in superfluidity theory. The use of this criterion points to the feasibility of stable superfluid spin transport in the A phase, and also of superfluid spin and precession transport in the B phase, determined in the general case by twisting of two "fast" angle variables: the rotation angle in the order-parameter spin state and the precession phase. For the experimentally observed precession transport in the B phase, the Landau criterion yields the dependence of the critical gradient on the angular velocity of the precession, and this dependence agrees qualitatively with experiment. It must be remembered, however, that the Landau criterion can give only the upper bound of the critical gradient, and actually the superflow dissipation, i.e., the phase slipping, always sets in earlier as a result of activated formation of vortices and of their motion across the field.

The spin-dynamics equations derived in the present paper, averaged over the two fast angle variables, point to the existence of a regime of dynamic suppression of the magnetic moment, in both the A and B phases; this regime an be used to effect stationary magnetic nondissipative transport.

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APPENDIX

Derivation of averaged spin-dynamics equations for the fast angle variables and their conjugate moments

In the strong-field limit, small oscillatory increments to the free precession of the moment **M** determine the rate of dissipation of the noequilibrium part of the moment **M**. Let the angle $\beta=\beta_0$ satisfy the adiabaticity condition $\partial F/\partial\beta=0$, which is exact for free precession of **M** without nutation (the ξ axis coincides with the **M** axis). This condition is a quadratic equation for P, and its solution, under the assumption that the ordering energy G is low, yields in lieu of the relation $P=M_\xi(\cos\beta-1)$ for free precession without nutation (see Sec. 2) the more accurate relation

$$P=M_\xi(\cos\beta_0-1)-\frac{\sin\beta_0}{M_\xi}\frac{\partial G}{\partial\beta}. \tag{A.1}$$

We introduce next as the dynamic variable, in place of β, the small angle $\beta'=\beta-\beta_0$ that determines the nutation oscillations. In this case β_0 is regarded as a function of the moments P and M_ξ and is defined by (A.1). Since the nutation oscillations of β' are small, we can expand the free energy in powers of β':

$$F=\frac{M_\xi^2}{2\chi}-(M_\xi+P)H+G(\beta_0)+F_\perp(\beta_0)+\frac{\partial F}{\partial\beta}\beta'+\frac{1}{2}\frac{\partial^2F}{\partial\beta^2}\beta'^2+\frac{M_\beta^2}{2\chi}. \tag{A.2}$$

We retain in this expansion the term with $\partial F/\partial\beta$, although it is equal to zero at $\beta=\beta_0$. The point is that the derivation of the equations of motion calls for derivatives of this term, which are no longer equal to zero and yield non-Hamiltonian terms. The equations of motion (1) assume the form (the non-Hamiltonian terms, which do not contribute to the dissipation, are contained in the square brackets):

$$\frac{\partial P}{\partial t}=-\gamma\frac{\delta G}{\delta\alpha}\gamma M_\xi H_\perp\sin\beta\cos(\alpha-\varphi)-\frac{\partial f}{\partial(\delta F/\delta P)}, \tag{A.3}$$

$$\frac{\partial\alpha}{\partial t}=\gamma H_\perp\operatorname{ctg}\beta\sin(\alpha-\varphi)+\gamma\frac{\partial G}{\partial\beta}\frac{\partial\beta_0}{\partial P}+\left[\gamma\frac{\partial^2F}{\partial P\,\partial\beta}\beta'\right]-\frac{\partial f}{\partial(\delta F/\delta\alpha)};$$

$$\frac{\partial M_\xi}{\partial t}=-\gamma\frac{\delta G}{\delta\Phi}-\frac{\partial f}{\partial(\delta F/\delta M_\xi)},\qquad \frac{\partial\Phi}{\partial t}=\frac{\gamma M}{\chi}-\omega_L+\gamma\frac{\partial G}{\partial\beta}\frac{\partial\beta_0}{\partial M_\xi}+\left[\gamma\frac{\partial^2F}{\partial M_\xi\,\partial\beta}\beta'\right]-\frac{\partial f}{\partial(\delta F/\delta\Phi)}; \tag{A.4}$$

$$\frac{\partial M_\beta}{\partial t}=-\gamma\frac{\partial^2F}{\partial\beta^2}\beta'-\frac{\partial f}{\partial(\delta F/\delta M_\beta)},$$

$$\frac{\partial\beta'}{\partial t}=-\left[\frac{\partial\beta_0}{\partial t}\right]+\frac{\gamma M_\beta}{\chi}-\frac{\partial f}{\partial(\delta F/\delta\beta)}. \tag{A.5}$$

The rotating magnetic field $\mathbf{H}_\perp$ is assumed weak, and is therefore retained only in Eqs. (A.3), where it is important for the balance of the moment P and for matching the rates of change of the angle φ and of the precession phase α. The partial derivative in the non-Hamiltonian terms of the equations can be calculated in the $F\approx F_0$ approximation (we put hereafter $M_\xi=M$):

$$\frac{\partial^2F}{\partial\beta^2}=\frac{M^2}{2\chi},\qquad\frac{\partial^2F}{\partial P\,\partial\beta}=\frac{M}{\chi\sin\beta_0},\qquad\frac{\partial^2F}{\partial M\,\partial\beta}=\frac{M\sin\beta_0}{\chi(1+\cos\beta_0)}. \tag{A.6}$$

We solve in operator form (i.e., using Green's functions) the last pair of equations in (A.5):

$$\beta'=\frac{1}{\omega_M^2+\hat\partial_t^2}\left[-\frac{\partial^2\beta_0}{\partial t^2}-\frac{\partial}{\partial t}\frac{\partial f}{\partial(\delta F/\delta\beta)}-\frac{\gamma}{\chi}\frac{\partial f}{\partial(\delta F/\delta M_\beta)}\right], \tag{A.7}$$

$$M_\beta=\frac{\omega_M^2}{\omega_M^2+\hat\partial_t^2}\frac{\chi}{\gamma}\left[\frac{\partial\beta_0}{\partial t}+\frac{\partial f}{\partial(\delta F/\delta\beta)}\right]-\frac{1}{\omega_M^2+\hat\partial_t^2}\hat\partial_t\left(\frac{\partial f}{\partial(\delta F/\delta M_\beta)}\right), \tag{A.8}$$

where $\omega_M=\gamma M/\chi$ and we have introduced the differentiation operator $\hat\partial_t=\partial/\partial t$. Since β_0 is a function of P and M, defined implicitly by the condition $\partial F/\partial\beta=0$, we have

$$\frac{\partial\beta_0}{\partial t}=-\left(\frac{\partial^2F}{\partial\beta^2}\right)^{-1}\left[\frac{\partial^2F}{\partial P\,\partial\beta}\frac{\partial P}{\partial t}+\frac{\partial^2F}{\partial M\,\partial\beta}\frac{\partial M}{\partial t}\right]=\frac{\gamma}{M\sin\beta_0}\left[\frac{\partial G}{\partial\alpha}+(1-\cos\beta_0)\frac{\partial G}{\partial\Phi}\right]. \tag{A.9}$$

We find next the periodic increments to the linear-in-time contributions to α and Φ. We retain here only the dissipative terms, since only they are significant in the final averaged equations. According to (A.3) and (A.4), we have

$$\alpha'=-\hat\partial_t^{-1}\left\{\frac{\gamma M}{\chi\sin\beta_0}\frac{1}{\omega_M^2+\hat\partial_t^2}\left[\hat\partial_t\frac{\partial f}{\partial(\delta F/\delta\beta)}+\frac{\gamma}{\chi}\frac{\partial f}{\partial(\delta F/\delta M_\beta)}\right]+\frac{\partial f}{\partial(\delta F/\delta\alpha)}\right\}, \tag{A.10}$$

$$\Phi'=-\hat\partial_t^{-1}\left\{\frac{\gamma M\sin\beta}{\chi(1+\cos\beta)}\frac{1}{\omega_M^2+\hat\partial_t^2}\left[\hat\partial_t\frac{\partial f}{\partial(\delta F/\delta\beta)}+\frac{\gamma}{\chi}\frac{\partial f}{\partial(\delta F/\delta M_\beta)}\right]+\frac{\partial f}{\partial(\delta F/\delta\Phi)}\right\}. \tag{A.11}$$

These small oscillatory increments must be taken into account when the equations for P and M are averaged [see (A.3) and (A.4)]. The equations for α and Φ can be averaged with the oscillatory increments neglected. Ultimately, after averaging Eqs. (A.3) and (A.4), we obtain Eqs. (4)–(7) with the following expressions for the dissipative terms:

$$D_P=\frac{\partial f}{\partial(\delta F/\delta P)}+\gamma\left\langle\frac{\partial^2G}{\partial\alpha^2}\alpha'\right\rangle+\gamma\left\langle\frac{\partial^2G}{\partial\alpha\,\partial\Phi}\Phi'\right\rangle, \tag{A.12}$$

$$D_M=\frac{\partial f}{\partial(\delta F/\delta M)}+\gamma\left\langle\frac{\partial^2G}{\partial\alpha\,\partial\Phi}\alpha'\right\rangle+\gamma\left\langle\frac{\partial^2G}{\partial\Phi^2}\Phi'\right\rangle. \tag{A.13}$$

Expression (9) for the gradient energy is obtained by averaging over a rather unwieldy initial equation for the gradient energy,[14] in which one can neglect the gradient of the angle β, since it is not a fast variable. We finally get for the B phase

$$\nabla\alpha,\quad\nabla\Phi\|\hat z$$

$$A=\frac{2\chi}{\gamma^2}[c_\perp^2(1-u)^2+c_\|^2u(1-u)],$$

$$B=-\frac{2\chi}{\gamma}(c_\perp^2-c_\parallel^2/2)(1-u),$$

$$C=\frac{2\chi}{\gamma}(c_\perp^2-c_\parallel^2/2). \tag{A.14}$$

$\nabla\alpha,\quad \nabla\Phi\perp\hat{z}$:

$$A=\frac{\chi}{\gamma^2}[c_\perp^2(1-u^2)+c_\parallel^2(1-u)^2],$$

$$B=-\frac{\chi}{\gamma}c_\parallel^2(1-u),\qquad C=\frac{\chi}{\gamma^2}c_\parallel^2, \tag{A.15}$$

where $c_\parallel$ and $c_\perp$ are the velocities of the longitudinal and transverse spin waves.

If the dissipation is determined by the Leggett-Takagi mechanism, the dissipative function is equal to

$$f=\frac{\varkappa}{2}\left(\frac{\partial F}{\partial \boldsymbol{\omega}}-\left[\mathbf{M},\frac{\partial F}{\partial \mathbf{M}}\right]\right)^2, \tag{A.16}$$

where $\boldsymbol{\omega}$ is the vector of a small total rotation in spin space. Changing to the Euler angles we have

$$f=\frac{\varkappa}{2}\left\{\left(\frac{\partial F}{\partial\beta}\bigg|_{\mathbf{M}}\right)^2+\frac{1}{\sin^2\beta}\left(\frac{\partial F}{\partial\alpha}\bigg|_{\mathbf{M}}\right)^2+\frac{2}{1+\cos\beta}\left[\left(\frac{\partial F}{\partial\Phi}\bigg|_{\mathbf{M}}\right)^2+\frac{\partial F}{\partial\Phi}\bigg|_{\mathbf{M}}\frac{\partial F}{\partial\alpha}\bigg|_{\mathbf{M}}\right]\right\}, \tag{A.17}$$

where the derivatives with respect to the angles are taken for a fixed vector $\mathbf{M}$. Since the moments that are conjugate to the Euler angles are the components of $\mathbf{M}$ along moving axes, they vary if the Euler angles change and $\mathbf{M}$ remains fixed. It follows hence that

$$\frac{\partial F}{\partial\Phi}\bigg|_{\mathbf{M}}=\frac{\partial F}{\partial\Phi},\quad \frac{\partial F}{\partial\beta}\bigg|_{\mathbf{M}}=\frac{\partial F}{\partial\beta}+\left(\frac{\partial F}{\partial M_\xi}-\frac{\partial F}{\partial P}\right)\frac{M_\xi(\cos\beta-1)}{\sin\beta},$$

$$\frac{\partial F}{\partial\alpha}\bigg|_{\mathbf{M}}=\frac{\partial F}{\partial\alpha}+\left(\frac{\partial F}{\partial M_\xi}-\frac{\partial F}{\partial P}\right)M_\beta\sin\beta+\frac{\partial F}{\partial M_\beta}\frac{P\cos\beta-M_\xi(1-\cos\beta)}{\sin\beta}. \tag{A.18}$$

The differences between the derivatives $(\partial F/\partial\alpha)$ and $(\partial F/\partial\beta)$ and the derivatives $\partial F/\partial\alpha$ and $\partial F/\partial\beta$ at fixed components P, M_ξ, and M_β are small in the limit of a strong field (fast precession), but they must be taken into account even in this limit, for otherwise the dissipative terms will be lost from the equations for $\partial P/\partial t$, $\partial M_\xi/\partial t$ and $\partial M_\beta/\partial t$.

For a homogeneous B phase, the dissipation is determined by the Leggett-Takagi mechanism, and the ordering energy reduces to the dipole energy, which is independent of α. Using expression (A.17) for the dissipative function, and the fact that only the dipole energy depends on Φ and β, we obtain expressions for the dissipative terms in the balance equations for P and M:

$$D_P=-\frac{\varkappa\chi}{M}\left\langle\left(\frac{\partial V}{\partial\beta}\right)^2\right\rangle-\frac{\varkappa\gamma}{1+\cos\beta}\left[\sin\beta\left\langle\frac{\partial V}{\partial\beta}\frac{\hat\partial_t}{\omega_M^2+\hat\partial_t^2}\frac{\partial V}{\partial\Phi}\right\rangle+(1-\cos\beta)\left\langle\frac{\partial V}{\partial\Phi}\frac{\omega_M}{\omega_M^2+\hat\partial_t^2}\frac{\partial V}{\partial\Phi}\right\rangle\right], \tag{A.19}$$

$$D_M=\frac{2\varkappa\chi}{(1+\cos\beta)(M-\chi H)}\left\langle\left(\frac{\partial V}{\partial\Phi}\right)^2\right\rangle+\frac{\varkappa\chi}{M}\left\langle\left(\frac{\partial V}{\partial\beta}\right)^2\right\rangle-\frac{\varkappa\gamma\chi H}{(1+\cos\beta)(M-\chi H)}\left[\sin\beta\left\langle\frac{\partial V}{\partial\beta}\frac{\hat\partial_t}{\omega_M^2+\hat\partial_t^2}\frac{\partial V}{\partial\Phi}\right\rangle+(1-\cos\beta)\left\langle\frac{\partial V}{\partial\Phi}\frac{\omega_M}{\omega_M^2+\hat\partial_t^2}\frac{\partial V}{\partial\Phi}\right\rangle\right]. \tag{A.20}$$

The terms containing the operator $(\omega_M^2+\hat\partial_t^2)^{-1}$, are due to nutation oscillations and vanish in the limit as $\beta\to 0$. They were therefore disregarded in the discussion of the dynamic suppression of the moment in Sec. 6.

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