Hydrodynamic fluctuations in two-dimensional antiferromagnets

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The dynamical properties of layer quasi-two-dimensional antiferromagnets are considered in the framework of a macroscopic approach. The spectra of the hydrodynamic modes are studied in an isotropic antiferromagnet and in the case of anisotropy of the "easy plane" type. The corrections to the sound attenuation that arise from interaction with the spin waves are found.

INTRODUCTION

Layer magnetic systems, in which the exchange integral within a layer is several orders of magnitude greater than the interlayer exchange integral, can be accurately approximated as two-dimensional. It is known that in such systems there are weakly damped spin waves, the interaction of which has a substantial effect on the formation of the spectrum, the dynamical susceptibility, etc. In the hydrodynamic region the spin waves correspond to fluctuations of the classical order parameter, which describes, in particular, the dynamics on large scales. Two-dimensional ferromagnetic systems were investigated earlier in Refs. 1 and 2. An important effect in this case is the presence of fluctuational spin-wave damping, proportional to the square of the wave vector k; in the hydrodynamic region this damping exceeds the bare damping ($\propto k^4$) due to the standard kinetic terms in the equations of the macroscopic dynamics. The fluctuations turn out to be important precisely because of the two-dimensionality of the space.

Two-dimensional systems with antiferromagnetic order within a layer were considered in Ref. 3 with the use of a macroscopic approach and in Ref. 4 with the application of a microscopic technique. The results of these papers differ from each other, because the analysis in Ref. 3 is of an antiferromagnetic model in which the macroscopic order parameter is an element of the rotation group in spin space, while, on the other hand, the microscopic description given in Ref. 4 corresponds, in the classical limit, to a system whose order parameter is a unit vector.

In the present paper we shall consider an antiferromagnet with a vector order parameter, using a macroscopic approach analogous to that which was applied in Ref. 3. The results obtained contain substantial differences from the properties of the model considered in Ref. 3. One of our results—namely, the correction to the spin-wave velocity, coincides with the expression obtained in Ref. 4 in the framework of a microscopic theory. The results are presented of a calculation of the fluctuation corrections to the spin-wave spectrum in an isotropic antiferromagnet and in the case of anisotropy of the "easy plane" type. We also study the spectrum of the diffusion mode and the corrections to the sound-wave spectrum that arise from the interaction with the spin waves.

2. HYDRODYNAMIC ACTION FOR THE ANTIFERROMAGNET

The starting point for the construction of the action describing the hydrodynamic fluctuations is the Hamiltonian in the exchange approximation, written in terms of the spin density **s** and the order parameter **n**, which is a unit vector:

$$H=\mathbf{s}^2/2\chi+{}^1/{}_2\chi b(\nabla\mathbf{n})^2. \qquad (1)$$

The paramagnetic susceptibility χ is introduced for correct normalization, and the coefficient b determines the velocity of the spin waves. The expression (1) assumes isotropy in the plane of the layer. This assumption simplifies the analysis considerably but does not affect the final results. A more general case would involve the presence in (1) of the combination $(\mathbf{sn})^2/2\chi_\parallel + [\mathbf{s}^2-(\mathbf{sn})^2]/2\chi_\perp$ instead of the first term. However, we shall see below that the addition of terms proportional to $(\mathbf{n}\cdot\mathbf{s}^2)$ to the Hamiltonian does not change the nondissipative parts of the equations of motion, and leads only to an unimportant change of the kinetic terms. We shall therefore use the expression (1) in what follows, making no distinction between $\chi_\parallel$ and $\chi_\perp$.

In accordance with the fact that the vector **n** transforms according to the adjoint representation of the rotation group in spin space, the generators of which are the components of the spin density **s**, the Poisson brackets for these variables have the standard form[5]:

$$\begin{aligned}\{s_a(\mathbf{r}_1),\ s_b(\mathbf{r}_2)\}&=-\varepsilon_{abc}s_c(\mathbf{r}_1)\delta(\mathbf{r}_1-\mathbf{r}_2),\\ \{s_a(\mathbf{r}_1),\ n_b(\mathbf{r}_2)\}&=-\varepsilon_{abc}n_c(\mathbf{r}_1)\delta(\mathbf{r}_1-\mathbf{r}_2),\\ \{n_a(\mathbf{r}_1),\ n_b(\mathbf{r}_2)\}&=0.\end{aligned} \qquad (2)$$

On the basis of (2), we obtain from (1) the equations of motion

$$\partial\mathbf{n}/\partial t=\chi^{-1}[\mathbf{sn}]+2\eta[\nabla^2\mathbf{n}+\mathbf{n}(\nabla\mathbf{n})^2]+\mathbf{f}_n, \qquad (3a)$$

$$\partial\mathbf{s}/\partial t=b\chi[\mathbf{n}\nabla^2\mathbf{n}]+\eta'\nabla^2\mathbf{s}+\mathbf{f}_s. \qquad (3b)$$

We have included in the right-hand sides of Eqs. (3) the standard kinetic terms and random forces $\mathbf{f}_n$ and $\mathbf{f}_s$. The form of the kinetic terms is determined by the fact that **s** is a conserved quantity in the exchange approximation, while the fluctuations of **n** correspond to two Goldstone modes. We denote the corresponding kinetic coefficients by η and η'.

In the linear approximation Eqs. (3) describe the spin waves associated with oscillations of the direction of the unit vector **n**, and the diffusion mode of the fluctuations of the longitudinal (with respect to **n**) component of the vector **s** (we denote this component by $\sigma=\mathbf{n}\cdot\mathbf{s}\chi^{-1}$). The diffusive character of this mode follows in a natural way from the fact that the variable σ is an integral of the nondissipative equations of motion, the conservation of this integral being violated by the kinetic terms.

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For the following it is convenient to eliminate **s** from Eq. (3a), as a result of which we obtain an equation for **n** of the form

$$\left[\mathbf{n}\frac{\partial^2\mathbf{n}}{\partial t^2}\right]-\left(b+2\eta\frac{\partial}{\partial t}\right)[\mathbf{n}\nabla^2\mathbf{n}]-\eta'\nabla^2\left(\left[\mathbf{n}\frac{\partial\mathbf{n}}{\partial t}\right]+\mathbf{n}\sigma\right.$$
$$\left.-2[\mathbf{n}\nabla^2\mathbf{n}]\right)+\frac{\partial}{\partial t}(\mathbf{n}\sigma)=\left(\eta'\nabla^2+\frac{\partial}{\partial t}\right)[\mathbf{n}\mathbf{f}_n]+\frac{1}{\chi}\mathbf{f}_s. \tag{4}$$

If we simply set $\sigma\equiv 0$, the nondissipative part of Eq. (4) can be obtained from the well known Lagrangian given in Ref. 6:

$$L={}^1/_2\chi[(\partial\mathbf{n}/\partial t)^2-b(\nabla\mathbf{n})^2].$$

At the same time, allowance for the fluctuations of σ can turn out to be important (see Sec. 4), and, therefore, in the following we shall study the effect of the diffusion mode corresponding to σ.

We can apply to Eq. (4) the general procedure described in detail in Ref. 7 for the construction of the hydrodynamic action. The correlators of the random forces have the form

$$\langle f_n^a(\mathbf{r}_1,t_1)f_n^b(\mathbf{r}_2,t_2)\rangle=4g\eta(\delta_{ab}-n_an_b)\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(t_1-t_2),$$
$$\langle f_s^a(\mathbf{r}_1,t_1)f_s^b(\mathbf{r}_2,t_2)\rangle=-2\eta'T\chi\nabla^2\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(t_1-t_2), \tag{5}$$

where $g=T/b\chi$. Averaging over the random forces, we obtain for the hydrodynamic Lagrangian the following expression:

$$\mathscr{L}=\mathbf{pF}+2ig\eta\left(\left[\mathbf{n}\frac{\partial\mathbf{p}}{\partial t}\right]+\eta'[\mathbf{n}\nabla^2\mathbf{p}]\right)^2+igb\eta'(\nabla\mathbf{p})^2. \tag{6}$$

Here **p** is an auxiliary vector field and **F** denotes the left-hand side of Eq. (4). The fermion regulators ensuring normalization of the distribution function

$$W\propto\exp\left(i\int\mathscr{L}d^2r\,dt\right)$$

are omitted, since taking them into account corresponds only to choosing a definite regularization of the frequency integrals in the expressions for the fluctuation corrections.[7]

We now choose a definite parametrization of the vectors **n** and **p**. With the aim of preserving the rotational invariance, we shall use the representation given in Ref. 8:

$$\mathbf{n}=R\mathbf{n}_0,\quad \mathbf{p}=R(\mathbf{y}+\pi\mathbf{n}_0),\quad R=\exp(x_+S_--x_-S_+). \tag{7}$$

The vector $\mathbf{n}_0$ is in the direction of the chosen axis z, and the two-component vector **y** is orthogonal to $\mathbf{n}_0$. The operators S_a are the generators of the algebra $so(3)$ in the adjoint representation: $(S_a)_{bc}=-i\varepsilon_{abc}$. The spin-rotation matrix R, which depends on the two parameters $x_\pm$, describes elements of a homogeneous space: $S^2=SO(3)/SO(2)$. The Lagrangian (6) takes an invariant form in terms of the currents

$$A_a=R^T\nabla_aR=iA_a^3S_3+A_a^+S_--A_a^-S_+, \tag{8}$$

which are elements of the algebra $so(3)$. In the formula (8), $\nabla_a\equiv(\partial/\partial t;\nabla_\mu)$; here and below the Greek indices run over the spatial values $\mu=1,2$. Introducing the notation

$$D_aA_a^\pm=\nabla_aA_a^\pm\mp iA_a^3A_a^\pm,$$
$$\mathscr{D}_ay_\pm=\nabla_ay_\pm\mp iA_a^3y_\pm+\pi A_a^\pm,$$
$$\mathscr{D}_a\pi=\nabla_a\pi-y_-A_a^+-y_+A_a^-, \tag{9}$$

we rewrite (6) in the form

$$\mathscr{L}=iy_+(D_0A_0^+-bD_\mu A_\mu^+)+2i\eta\mathscr{D}_0y_-D_\mu A_\mu^+$$
$$+2i\eta\eta'D_\mu A_\mu^+\mathscr{D}_\nu^2y_--i\eta'A_0^+\mathscr{D}_\mu^2y_-+\text{h.c.}$$
$$-\sigma(\mathscr{D}_0\pi+\eta'\mathscr{D}_\mu^2\pi)+4ig\eta(\mathscr{D}_0y_-+\eta'\mathscr{D}_\mu^2y_-)$$
$$\cdot(\mathscr{D}_0y_++\eta'\mathscr{D}_\nu^2y_+)+igb\eta'[2\mathscr{D}_\mu y_-\mathscr{D}_\mu y_++(\mathscr{D}_\mu\pi)^2]. \tag{10}$$

The derivatives $\mathscr{D}_a$ in (10), by definition, commute with multiplication by $A_a^\pm$. We draw attention to the fact that, unlike the hydrodynamic Lagrangian of Ref. 3, (10) contains covariant derivatives of the currents (8), this being connected with the fact that the dynamics is analyzed on a homogeneous space and not on the rotation group.

The general form of fluctuational corrections to (10) for small deviations from the equilibrium ground state is as follows:

$$\delta\mathscr{L}_{fl}=iy_-\Sigma_{-+}x_++iy_+\Sigma_{+-}x_-$$
$$+iy_-\Pi_{-+}y_++i\pi\Sigma_d\sigma+{}^1/_2i\pi\Pi_d\pi. \tag{11}$$

The part of (10) quadratic in the deviations from equilibrium gives, when (11) is taken into account, the nonzero pair correlators

$$\langle x_+y_-\rangle=-\langle x_-y_+\rangle=iG(\omega,\mathbf{k})=-[(\omega+i\Delta)^2-\varepsilon^2]^{-1}, \tag{12}$$

$$\langle x_+x_-\rangle=-\mathscr{D}(\omega,\mathbf{k})=-G(\omega,\mathbf{k})G(-\omega,-\mathbf{k})$$
$$\cdot[\Pi_{-+}(\omega,\mathbf{k})+4g\eta(\omega^2+\eta'^2k^4)+2\eta'gbk^2], \tag{13}$$

$$\langle\sigma\pi\rangle=iG_\parallel(\omega,\mathbf{k})=-(\omega+i\eta'k^2-\Sigma_d)^{-1}, \tag{14}$$

$$\langle\sigma\sigma\rangle=-\mathscr{D}_\parallel(\omega,\mathbf{k})$$
$$=G_\parallel(\omega,\mathbf{k})G_\parallel(-\omega,-\mathbf{k})[\Pi_d(\omega,\mathbf{k})+2g\eta'k^2], \tag{15}$$

where

$$\Delta=(\eta+{}^1/_2\eta')k^2-{}^1/_2\omega^{-1}\,\mathrm{Im}\,\Sigma_{-+},\quad \varepsilon^2=bk^2+2\eta\eta'k^4-\Delta^2.$$

The poles of the G-functions (12) and (14) determine the spectra of the spin waves and the diffusion mode with allowance for both the bare dissipative terms and the fluctuation corrections:

$$\omega_{sw}=\pm\varepsilon-i\Delta, \tag{16}$$

$$\omega_{dif}=-i\eta'k^2+\Sigma_d. \tag{17}$$

3. THE SPIN-WAVE SPECTRUM

To obtain corrections to the spectrum (16) we integrate the distribution function

$$W\propto\exp\left(i\int\mathscr{L}\,d^2r\,dt\right)$$

over the fast degrees of freedom, which we separate out in the following invariant manner[8]:

$$R\to Rh,\quad \mathbf{y}\to\hat{\mathbf{y}}+\mathbf{y}. \tag{18}$$

Here **y** and $h(x_\pm)$ describe the fast degrees of freedom with wave vectors q in the interval $k<q<\Lambda$ (Λ is the upper boundary of the hydrodynamic region and k is the characteristic wave vector of the slow variables R and $\hat{\mathbf{y}}$). Strictly speaking, the region of large momenta $q \sim \Lambda \gg k$ gives rise to corrections only to the real part of the spectrum, but, as one can convince oneself, the expression that arises for the fluctuation contribution to the damping as a result of application of the procedure (18) is valid for arbitrary relative magnitudes of the external momentum k and the loop momenta over which the integration is performed.

The fields σ and π appear quadratically in (10), and, therefore, in the analysis of the corrections to the spin-wave part of $\mathscr{L}$ we can perform explicit functional integration of W over these fields. As a result, a large number of additional nonlocal interaction vertices of the fields $y_\pm$ and $x_\pm$ arise. These effective interactions are of the same degree in the gradients ∇_a as the part of $\mathscr{L}$ that does not contain σ or π, but their expansion in $x_\pm$ and $y_\pm$ starts from quartic terms. In each such vertex there is a kinetic coefficient η or η', and therefore they can be regarded as new dissipative terms in the dynamics of the variables $x_\pm$ and $y_\pm$. When all the dissipative terms (both the bare terms and those which arise after elimination of σ and π) are taken into account the spin-wave Lagrangian takes an extremely cumbersome form. However, it turns out that the first fluctuation correction to this Lagrangian, which arises when only the reactive vertices are taken into account, makes a nonanalytic contribution, linear in k, to the damping. In the calculation of this contribution in leading order in the bare values of η and η' the dissipative terms can be omitted, and in (10) it is sufficient to confine oneself to the first two terms. Moreover, the effect of the dissipative terms on the corrections to the spin-wave velocity also turns out to be unimportant. In the one-loop approximation the pairings, corresponding to Figs. 1 and 2, of the fast degrees of freedom contribute to the spin-wave part of $\mathscr{L}$ a correction of the form

$$\delta_1\mathscr{L} = -\frac{ig}{4\pi}\ln\left(\frac{\Lambda}{k}\right)[\hat{y}_- D_0 A_0^+ - (b\hat{y}_- - 2\eta D_0 \hat{y}_-) D_\mu A_\mu^+] + \text{h.c.} \tag{19}$$

It follows from (19) that neither b nor η acquires one-loop corrections. This indicates an important difference between the properties of the system under consideration and those of the model used in Ref. 3 (we note that in Ref. 3 η' was set equal to zero from the outset). It follows from (19) that in the one-loop approximation the fields $y_\pm$ acquire a multiplicative Z-factor

$$Z_y = 1 - (g/4\pi)\ln(\Lambda/k).$$

Because of the absence of a one-loop correction to the term $4ig\eta\mathscr{D}_0 y_+ + \mathscr{D}_0 y_-$ in (10), this means that the charge g is also renormalized:

$$g_R = Z_g g, \quad Z_g = Z_y^{-2} = 1 + (g/2\pi)\ln(\Lambda/k).$$

As we should have expected, the renormalization of g follows the usual law for the $O(3)$ σ-model, since in the static limit g_R is the effective temperature of the fluctuations.

FIG. 1.

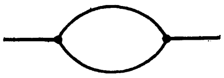

FIG. 2.

The two-loop contributions to $\mathscr{L}$, corresponding to the diagram in Fig. 3, also do not lead to the appearance of corrections to the spin-wave velocity. This is connected with the fact that the subtraction of the quadratic divergences from the corresponding expressions leaves only those pairings of the fast degrees of freedom that do not contain derivatives ∇_a on internal lines, and, consequently, the corrections to the first two terms in (10) are equal to each other. The same argument is valid for corrections of the normal-ordering type (see Figs. 1 and 3) of any order, this being connected with the obvious "relativistic covariance" of the reactive terms in (10). The first important contributions to $\mathscr{L}$ arise from the diagrams of Figs. 4 and 5. Here a solid line denotes the correlator $\langle x_+ x_- \rangle$, a mixed (solid and dashed) line denotes $\langle x_+ y_- \rangle$, a triangle denotes the background current $A_a^\pm$, and a dashed external line denotes $\hat{y}_\mp$. In the calculation of the correction corresponding to the diagram of Fig. 4 the resulting expression must be expanded in the frequency and wave vector of the slow variables, to the linear terms that give rise to logarithmic integrals. Ultimately, we obtain

$$\delta_{2a}\mathscr{L} = \frac{i}{(4\pi)^2} g^2\left(\ln\frac{\Lambda}{k}\right)^2 \hat{y}_-\left[3\frac{\partial}{\partial t}A_0^+ + b\nabla_\mu A_\mu^+\right] + \text{h.c.} \tag{20}$$

It follows from (20) that b acquires a correction

$$\delta b = -b\left(\frac{g}{2\pi}\ln\frac{\Lambda}{k}\right)^2. \tag{21}$$

The presence of the contribution (21) implies that, even in the two-loop approximation, the dynamical Lagrangian (10) is not renormalizable, since there is no cancellation of the terms quadratic in $\ln(\Lambda/k)$ (or, equivalently, of terms $\propto \varepsilon^{-2}$ in the calculation in a space of $d = 2 - \varepsilon$ dimensions). We emphasize that, despite the nonrenormalizability of the dynamical theory as a whole, the behavior of the static charge g under renormalization has the usual character for statics.[9] It also follows from this that the relation $g = T/b\chi$ is valid only for the bare values of g and b, which subsequently acquire independent fluctuational corrections.

To determine the fluctuational imaginary part of the spectrum we shall find the correction to $\mathscr{L}$ of the form $iy_-\Pi_{-+}y_+$, which is given by the diagram of Fig. 5. Representing the expression found above for the correction $\delta_{2a}\mathscr{L}$ (before the expansion in the gradients of the slow variables) in the case of small deviations from equilibrium in the form $iy_-\Sigma_{-+}x_+ + iy_+\Sigma_{+-}x_-$, we can convince ourselves that $\Sigma_{\mp\pm}(\omega,\mathbf{k})$ and $\Pi_{-+}(\omega,\mathbf{k})$ are connected by the relation

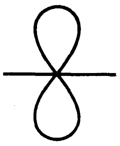

FIG. 3.

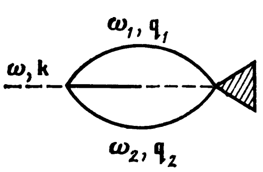

FIG. 4.

$$\mathrm{Im}\,\Sigma_{\mp\pm}(\omega, \mathbf{k}) = \mp(\omega/2gb)\Pi_{-+}(\pm\omega, \mathbf{k}), \tag{22}$$

which follows from the fluctuation-dissipation theorem.

In the calculation of Π_{-+} it turns out that, when the imaginary part of the spectrum is disregarded in the $\mathscr{D}$-functions, the expression corresponding to the diagram of Fig. 5 contains kinematic singularities at the value 0 or π of the angle between the loop momenta q_1 and q_2. In addition, in leading order in the bare dissipative terms, momenta $q_{1,2} \sim \Lambda$ do not make a contribution to Π_{-+}, so that the expansion in the frequency ω and wave vector k (see Fig. 5 for the notation) gives only corrections having the structure of the bare self-energy part: $\delta\Pi_{-+} \propto g^3\eta(bk^2, \omega^2)$, which vanishes for $\eta \to 0$. The fluctuation contribution, which does not depend on η or η', comes from the region $q_{1,2} \sim k$. Assuming this to be the main contribution, we can obtain a self-consistent equation for Π_{-+} (the region of applicability of this approximation is discussed in Sec. 5).

Analysis of the expression corresponding to the diagram of Fig. 5 shows that the dependence of the function $\Pi_{-+}(\omega, \mathbf{k})$ on ω and k is smooth in the neighborhood of the unperturbed mass shell $\omega^2 = bk^2$, i.e., Π_{-+} changes only by an amount on the order of itself when we move away from the mass shell by an amount $\omega^2 - bk^2 \sim bk^2$. In this case the corrections to the spectrum are determined in the standard way by the values of Π_{-+} on the mass shell. Confining ourselves to the dependence of Π_{-+} on k and assuming that $\omega^2 = bk^2$, from the condition of homogeneity in k we obtain that the solution of the self-consistency equation is $\Pi_{-+} = 4gb\gamma k$, with a dimensionless kinetic coefficient γ. We note that the appearance of fluctuational contributions of the same degree in k for both parts of the spectrum is entirely analogous to the situation familiar in the case of a ferromagnet.[1,2] It should also be noted that a contribution to Π_{-+} that is nonanalytic in k cannot be obtained as a result of the usual renormalization-group expansion (correspondingly, γ does not contain $\ln\Lambda$).

The self-consistent calculation of γ is performed more conveniently in the coordinate representation. The function $\mathscr{D}(\mathbf{r}, t)$ with $\Pi_{-+} = 4gb\gamma k$ has the form

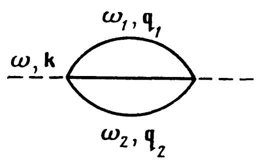

FIG. 5.

$$\mathscr{D}(\mathbf{r}, t) = \frac{g}{2\pi}\left\{\ln r + \mathrm{Re}\left[\frac{b^{1/2}}{\lambda}\ln\frac{i\lambda|t| + [r^2 + (i\lambda|t|)^2]^{1/2}}{r}\right]\right\} + \mathrm{const}, \tag{23}$$

where $\lambda = (b - \gamma^2)^{1/2} - i\gamma$. The presence in $\mathscr{D}$ of the constant term expresses the dependence on the method of infrared regularization. Nevertheless, by shifting derivatives one can bring the expression for Π_{-+} to a form containing only well defined first derivatives of $\mathscr{D}$-functions:

$$\Pi_{-+}(\omega, \mathbf{k}) = 4\int_0^\infty dt \int d^2r \exp(i\mathbf{kr}) \cdot \left[-\omega\frac{\partial\mathscr{D}}{\partial t}\sin\omega t + ik_\mu\nabla_\mu\mathscr{D}\cos\omega t\right]\left[\left(\frac{\partial\mathscr{D}}{\partial t}\right)^2 - b(\nabla_\mu\mathscr{D})^2\right]. \tag{24}$$

The kinematic singularity in the coordinate representation arises as a logarithmic divergence on the "light cone", i.e., at $r = b^{1/2}t$.

In the general case the solution of the self-consistency equation (24) cannot be obtained in explicit form. We shall give its solution in the case when $\ln(b^{1/2}/\gamma)$ is a large number. Then, separating out the part containing this factor against the background of contributions of order unity, we obtain

$$\Pi_{-+} = (1/2\pi)g^3kb^{3/2}\ln(b^{1/2}/\gamma).$$

With the same accuracy we find from this the solution of the self-consistency equation:

$$\gamma = (1/8\pi)b^{1/2}g^2\ln(1/g^2). \tag{25}$$

Thus, the two-loop contributions lead to a spin-wave spectrum of the form

$$\omega_{sw} = [(b+\delta b)^{1/2} - i\gamma]k$$

with δb from (21) and γ from (25). The expression (21) for δb is valid only for $[g\ln(\Lambda/k)^2] \ll 1$; otherwise, the loop expansion for the corrections is not applicable. This condition is considerably more stringent than the condition $g_R \ll 1$. Because of the absence of the usual renormalization-group behavior of the charge b, allowance for the corrections to it that arise in higher orders does not reduce to the replacement of g by g_R in (21). In Sec. 5 we shall discuss the physical conditions under which we can confine ourselves to the first correction (21) to δb.

4. SPECTRUM OF THE DIFFUSION MODE

In the preceding section we considered the interactions due to the spin-wave part of the Lagrangian (10). For completeness of the description it is necessary also to study effects associated with the diffusion mode. Indeed, if, e.g., Π_d were to acquire corrections proportional to k^α, with $\alpha < 1$, the induced spin-wave interactions arising after the elimination of σ and π would substantially modify the dynamics in the hydrodynamic region. Below we shall convince ourselves that this does not occur, and, consequently, the preceding treatment is correct. The fields σ and π appear quadratically in the dynamical Lagrangian (10), and therefore, in leading order in g, the corrections to Σ_d and Π_d come from the diagram of Fig. 2, in which the internal lines correspond to the G- and $\mathscr{D}$-functions of the spin waves. The vertices are given by the part of (10) linear in σ or π. In this case the dissipative terms containing η' lead only to a one-loop renormalization of this kinetic coefficient, giving a contribution to Σ_d of the form

$$\delta\Sigma_d \propto g\eta' k^2 \ln(\Lambda/k).$$

By virtue of the logarithmic character of the renormalization such corrections are small in proportion to the smallness of the bare value η'. This fact corresponds to the one-loop renormalizability of the Lagrangian (10)—a circumstance expressed in the absence of one-loop counterterms with an operator structure different from (10). Therefore, we shall consider the vertices not containing η' that could in principle lead to a contribution to Π_d with a lower power in k than the bare contribution $\sim k^2$ (but also not containing $\ln\Lambda$ by virtue of the one-loop renormalizability discussed above).

The finite correction parts arising from the diagrams of the type depicted in Fig. 2 are connected by the relation

$$\Pi_d = -2(T/\chi)\,\mathrm{Im}\,\Sigma_d. \tag{26}$$

Direct calculation of Π_d in leading order in the bare value of η gives

$$\Pi_d = (1/3\pi) g^2\eta b k^2. \tag{27}$$

Thus, the fluctuation contribution made to the spectrum of the diffusion mode by the interaction with the spin waves has the same dependence on k as the bare contribution. Consequently, the correlator $\langle\sigma\sigma\rangle$ preserves the diffusion properties: It has frequency width $\sim k^2$ and its integral over the frequency is $\sim$const. A diffusion peak in neutron scattering could be observed, in principle, for neutron beams propagating in the plane of the magnetic layers.

We also draw attention to the difference between this situation and the behavior, considered in Ref. 8, of the bare diffusion mode of the fluctuations of the modulus of the spin in a three-dimensional ferromagnet; the dispersion law of the latter mode changes substantially when the interaction of this mode with the spin waves is taken into account.

5. ALLOWANCE FOR ANISOTROPY

We now consider an antiferromagnet with anisotropy of the "easy plane" type. In this case we must add to the Hamiltonian (1) a term describing the anisotropy

$$H_{an} = {}^1/_2\chi b m^2 (\mathbf{n}\mathbf{v})^2. \tag{28}$$

The vector $\mathbf{v}$ in (28) indicates the direction of the anisotropy axis. We shall assume that the anisotropy is sufficiently weak for the condition $m \ll \Lambda$ to be fulfilled. In this case there is a broad region of wave-vector values in which leading fluctuation corrections containing $\ln\Lambda$ arise. If, however, m is comparable to Λ, the fluctuation contributions are certainly smaller.

In the presence of weak anisotropy m determines only the characteristic scale at which the renormalizations stop, and therefore the correction to the spin-wave velocity is determined as before by the formula (21), in which, however, it is necessary to replace the lower limit of the logarithm by m in the case $k < m$. The calculation of the corrections to the imaginary part of the spectrum that do not contain $\ln\Lambda$ requires a more detailed analysis. It is carried out most simply using a parametrization of the order parameter by means of the spherical angles θ and φ in a fixed coordinate system, assuming that in the equilibrium state the vector $\mathbf{n}$ lies in the direction of the x axis while the anisotropy axis coincides with the z axis. Then for small fluctuations about the uniform equilibrium state the variables φ and $\psi = \cos\theta$ can be regarded as unbounded fields, with zero average values. To avoid confusion we emphasize that we are concerned with a locally ordered state arising in a region with size of the order of m^{-1}. In this state the local moments combine into a single "block" spin, and the field φ can be assigned a definite value. This variable describes Goldstone excitations with dispersion law $\omega_\varphi = b^{1/2}k$, while ψ corresponds to a gap mode with spectrum $\omega_\psi = b^{1/2}(k^2+m^2)^{1/2}$. This representation is convenient because the anisotropy is taken into account exactly and, in the diagram technique, there are no vertices associated with the anisotropy. As a consequence of this, each individual diagram making a contribution to Σ_{-+} or Π_{-+} is infrared-finite.

Analysis of the dissipative corrections shows that the imaginary part of the spectrum of the Goldstone mode can be represented in the form $\delta\omega_\varphi = -ik\gamma_\varphi(k/m)$. The function $\gamma_\varphi(k/m)$ takes values of order $g^2\ln(1/g^2)$ in the region $k \gtrsim m$. For $k \ll m$ the function $\gamma_\varphi(k/m)$ becomes linear in k, corresponding to quadratic (in k) damping of the gapless mode in the given region. The imaginary part $\delta\omega_\psi = -im\gamma_\psi(k/m)$ of the gap mode does not vanish as $k \to 0$. The asymptotic forms of the function $\gamma_\psi(k/m)$ are

$$\gamma_\psi\left(\frac{k}{m}\right) = \begin{cases} \gamma_1 k/m, & k \gg m, \\ \gamma_2, & k \ll m. \end{cases} \tag{29}$$

The values of the coefficients $\gamma_{1,2}$ are also of order $g^2\ln(1/g^2)$.

For the self-consistent calculations, analogous to those performed in Sec. 3, we require the correlators $\langle\varphi\varphi\rangle = -\mathscr{D}_\varphi$ and $\langle\psi\psi\rangle = -\mathscr{D}_\psi$ in the coordinate representation. As in the isotropic case, the corrections to the spectrum that do not contain $\ln\Lambda$ diverge on the light cone when the damping is neglected. A nonzero mass m does not remove the kinematic singularity. The reason for this is that the corresponding corrections, which are the result of differentiations of the functions $\mathscr{D}_{\varphi,\psi}(\mathbf{r}, t)$, depend on the "interval" $(r^2 - bt^2)^{1/2}$ (see formulas (23) and (30)). This dependence leads to singular (in the limit $\gamma_{\varphi,\psi} \to 0$) integrands in the integration over t when $t \to rb^{-1/2}$, while the important region of integration over r is the region $r \sim m^{-1}$.

It is not possible, of course, to calculate the coordinate representations for the functions $\mathscr{D}_\varphi$ and $\mathscr{D}_\psi$ in the case of arbitrary functions γ_φ and γ_ψ. However, if $\ln(1/g^2)$ can be regarded as a large number, then (since the ranges of variation of the functions γ_φ and γ_ψ are limited), it is sufficient, to the given accuracy, to confine ourselves to an approximate form of the spectrum:

$$\omega_\varphi = (b^{1/2} - i\bar\gamma)k, \quad \omega_\psi = (b^{1/2} - i\bar\gamma)(k^2+m^2)^{1/2},$$

where $\bar\gamma$ is a quantity of the same order as the "average" values of the functions $\gamma_\varphi(k/m)$ (for $k \gtrsim m$) and $m(k^2+m^2)^{-1/2}\gamma_\psi(k/m)$. That this procedure is possible is due to the fact that, in leading order, the regularized (with allowance for the damping) expressions for the corrections contain, as in the isotropic case, a factor $\ln(b^{1/2}/\overline{\gamma})$, which we assume to be large (otherwise, the answer is known only to within a coefficient of order unity). In the calculation in the coordinate representation for $k \gg m$, the important region of integration is the region $r \sim k^{-1}$, and, therefore, the result does not contain any substantial dependence on m. In the given case, the above approximate form of the spectrum is certainly justified.

For $k \lesssim m$ the main contribution comes from the region $r \sim m^{-1}$, in which the functions γ_φ and γ_ψ take values that do not differ in order of magnitude from the isotropic values. Therefore, in the calculation of the corrections the behavior of the functions $\mathscr{D}_{\varphi,\psi}(\mathbf{r}, t)$ for $r \gg m^{-1}$ is unimportant, and the approximate form of the spectrum is also admissible. For $\mathscr{D}_\varphi(\mathbf{r}, t)$ we obtain an expression coinciding with (23), while $\mathscr{D}_\psi(\mathbf{r}, t)$ is given by the formula

$$\mathscr{D}_\psi(\mathbf{r},t)=\frac{g}{2\pi}\left[-K_0(mr)+\theta(b^{1/2}|t|-r)\int_0^M \frac{dz\cos mz}{(z^2+r^2)^{1/2}}\right], \tag{30}$$

where $M^4 = (r^2 - bt^2)^2 + 4(\gamma bt^2)^2$, and $K_0(x)$ is a modified Bessel function of the second kind.

Calculating diagrams of the type depicted in Fig. 5, we find that the correction to the imaginary part of the Goldstone mode is determined by the equation

$$\gamma_\psi\left(\frac{k}{m}\right)=\frac{g^2}{4\pi^2}\ln\left(\frac{b^{1/2}}{\overline{\gamma}}\right)\int_0^\infty K_1(u)\left[J_0\left(\frac{ku}{m}\right)\sin\frac{ku}{m}+J_1\left(\frac{ku}{m}\right)\cos\frac{ku}{m}\right]du, \tag{31}$$

where $J_i(x)$ are Bessel functions. For $k \gg m$ the function γ_φ takes a value coinciding with (25). In the limit $k \ll m$ we obtain from (31)

$$\gamma_\varphi\left(\frac{k}{m}\right)=\frac{g^2}{4\pi}b^{1/2}\frac{k}{m}\ln\frac{1}{g^2}. \tag{32}$$

The damping of the gapless mode becomes analytic in k in the long-wavelength region. Consequently, for configurations of the field $\mathbf{n}$ with characteristic length scale $r \gg m^{-1}$ one can obtain an effective equation of motion in which local dissipative terms of fluctuational origin appear. These terms are the expansion of the complete (nonlocal) dissipative term in the parameter k^2/m^2.

An analogous calculation of the damping of the gap mode gives

$$\gamma_\psi\left(\frac{k}{m}\right)=\frac{1}{8\pi}g^2b^{1/2}\left(1+\frac{k^2}{m^2}\right)^{1/2}\ln\frac{1}{g^2}. \tag{33}$$

In the leading approximation the constants γ_1 and γ_2 from (29) coincide (the discarded terms are of order g^2). In the isotropic limit γ_ψ coincides with (25), while in the limit $k \ll m$ the expression (33) gives the damping of the uniform oscillation of $\mathbf{n}$ corresponding to the gap mode with $\omega = b^{1/2}m$.

It should be noted that the statement made above about the damping of the gap mode does not apply to uniform precession of $\mathbf{s}$, which, in the approximation of the Hamiltonian (1), (28), is nondissipative, since in this approximation the component $\mathbf{s}\cdot\mathbf{v}$ of the density of the moment is conserved.

When (32) and (33) are taken into account the effective equation of motion for $\mathbf{n}$ in the region of scales large in comparison with m^{-1} takes the form

$$\left[\mathbf{n}\frac{\partial^2\mathbf{n}}{\partial t^2}\right]-\left(b+\frac{4\gamma}{m}\mathscr{P}\frac{\partial}{\partial t}\right)[\mathbf{n}\nabla^2\mathbf{n}]+\left(bm^2+2\gamma m\mathscr{P}\frac{\partial}{\partial t}\right)([\mathbf{nv}](\mathbf{nv}))=0,$$
$$\mathscr{P}_{ab}=\delta_{ab}-n_an_b, \tag{34}$$

where $\gamma = (1/8\pi)g^2\ln(1/g^2)$. The equation (34) can be used to study the dynamics of the collective excitations, e.g., vortices (vortex pairs). We note that in the present case of an easy-plane antiferromagnet with weak anisotropy at a temperature below the Kosterlitz-Thouless transition temperature T_c (Ref. 10), to which corresponds $g_c = 2\pi/\ln(\Lambda/m)$, the expansion of the corrections to b is performed, in essence, in the parameter $(T/T_c)^2$. As T_c is approached it is necessary to take higher corrections into account, and the expression (21) (with k replaced by m) becomes inapplicable.

We note also that, as can be shown, the diffusion mode in the region $k < m$ becomes uniform relaxation with dispersion law $\omega_{\text{dif}} = -i\eta' m^2$. In the long-wavelength region it is not important to take this mode into account.

In calculating the fluctuational damping we assumed above that the fluctuational contributions are the main contributions, and did not take into account the bare dissipative term quadratic in k. For the bare value of η we can use the estimate $\eta \sim b^{1/2}\Lambda^{-1}$. The results of the preceding calculations should apply to the region $k < \gamma/\eta \sim \gamma\Lambda/b^{1/2}$, in which the fluctuational terms are the leading terms. At larger values of k the corrections are small in comparison with the bare (nonhydrodynamic) damping.

6. FLUCTUATIONAL CORRECTIONS TO THE SOUND-WAVE SPECTRUM

In this section we shall consider the interaction between the spin waves and sound waves. The anisotropy of the elastic constants of materials that are layer magnets is, as a rule, substantially smaller than the anisotropy of the exchange integrals, and therefore the lattice vibrations take the form of ordinary three-dimensional sound. The fluctuating field describing the sound vibrations is the momentum density $\mathbf{j}$, related to the displacement vector $\mathbf{u}$ by $\mathbf{j} = \rho\partial\mathbf{u}/\partial t$. By virtue of the continuity equation

$$\partial\rho/\partial t+\nabla_kj_k=0$$

the density variation $\delta\rho$ can be expressed in terms of $\mathbf{j}$.

The nondissipative part of the hydrodynamic action describing the fluctuations of j_k contains the auxiliary fields ξ_k and has the form

$$\mathscr{L}_s=\xi_k(\partial j_k/\partial t+\nabla_lT_{kl}), \tag{35}$$

where T_{kl} is the stress tensor. Its elastic part can be written in the form

$$T_{kl}^{(e)}=\rho^{-1}(j_kj_l+\mathbf{j}^2\delta_{kl})-E\delta_{kl}, \tag{36}$$

where E is the internal-energy density, and the part describing the interaction with the spin degrees of freedom is

$$T_{\mu\nu}^{(sw)}=\nabla_\mu \mathbf{n}\frac{\delta H}{\delta \nabla_\nu \mathbf{n}}+\delta_{\mu\nu}\left(\rho\frac{\delta H}{\delta\rho}-H+\mathbf{s}\frac{\delta H}{\delta \mathbf{s}}\right). \qquad (37)$$

The spin Hamiltonian H is given by Eq. (1). Here and below, the spatial indices take two values in the plane of the magnetic layers, and the expression (37) is a two-dimensional density.

Additional interaction terms arise from taking terms $-\rho^{-1}\nabla_\mu(j_\mu \mathbf{n})$ and $-\rho^{-1}\nabla_\mu(j_\mu \mathbf{s})$ into account in Eqs. (3a) and (3b), respectively, and also from taking account of the dependence of the coefficients b and χ on the variable ρ. We do not take into account the effect of the dissipative terms from Eq. (4). The diffusion part of the spin action is also unimportant.

Introducing the constants

$$\beta_1=\frac{\partial \ln\chi}{\partial \ln\rho},\quad \beta_2=\frac{\partial \ln(b\chi)}{\partial \ln\rho},$$

we write the total interaction Lagrangian in the form

$$\mathscr{L}_{int}=-\chi\nabla_\mu\xi_\nu\left\{b\nabla_\mu\mathbf{n}\nabla_\nu\mathbf{n}+{}^1/_2\delta_{\mu\nu}\left[(\beta_1+1)\left(\frac{\partial\mathbf{n}}{\partial t}\right)^2 -b(\beta_2-1)(\nabla\mathbf{n})^2\right]\right\}$$
$$-\frac{1}{\rho}j_\mu\left(\nabla_\mu\mathbf{p}\left[\mathbf{n}\frac{\partial\mathbf{n}}{\partial t}\right]+\frac{\partial\mathbf{p}}{\partial t}[\mathbf{n}\nabla_\mu\mathbf{n}]\right)$$
$$+\frac{b}{\rho}(\beta_1-\beta_2)\nabla_\nu\mathbf{p}[\mathbf{n}\nabla_\nu\mathbf{n}]\left(\frac{\partial}{\partial t}\right)^{-1}[\nabla_\mu j_\mu]. \qquad (38)$$

The expansion (38) gives, in lowest order, the three-point interaction vertices, and therefore the fluctuational corrections to (35), with general form

$$\delta\mathscr{L}_s=i\xi_\mu\Sigma_{\mu\nu}j_\nu+{}^1/_2 i\xi_\mu\Pi_{\mu\nu}\xi_\nu, \qquad (39)$$

come from diagrams of the type depicted in Fig. 2. The internal lines correspond to the spin correlators (12) and (13), and the external points are ξ_μ and j_μ. For the self-energy parts introduced in (39) the relation

$$\operatorname{Im}\Sigma_{\mu\nu}=-\Pi_{\mu\nu}/2T\rho \qquad (40)$$

is fulfilled. Calculation of $\Pi_{\mu\nu}$, which gives the damping of the sound waves on account of interaction with a magnetic layer, leads to the expression

$$\Pi_{\mu\nu}=\frac{T^2}{8\pi^2}\left[k_\mu k_\nu(1+\beta_1+\beta_2)^2+\frac{1}{2}\delta_{\mu\nu}k^2\right]\int\frac{d^2q}{\Delta(q)}, \qquad (41)$$

where $\Delta(q)$ includes both the bare and the fluctuational part of the spin-wave damping. To take account of the effect of all the layers it is necessary to sum over them. As in Ref. 3, we replace this summation by integration over the coordinate z in the direction orthogonal to the layers, with allowance for the interlayer spacing a. The sound-wave spectrum is determined from the equation

$$\det\left[(c^2k^2-\omega^2)\delta_{mn}-i\omega\Pi_{mn}/2a\rho T\right]=0, \qquad (42)$$

where c is the velocity of sound in the isotropic crystal (for simplicity we make no distinction between the longitudinal mode and the transverse modes). For $m, n = 1, 2$ the operator Π_{mn} is given by formula (41), and in the remaining cases this term vanishes. An analogous contribution to the damping of sound has been obtained in the framework of the model of Ref. 3. The corrections that follow from (41) and (42) give sound damping $\propto k^2$, in the same way as do the usual terms due to viscosity and thermal conduction for dielectrics.[11] At the same time, this damping is anisotropic (it depends only on the components of $\mathbf{k}$ in the plane of the magnetic layers), and, in addition, it can appear with a large coefficient. To explain the latter circumstance we consider the integral $\int d^2q/\Delta(q)$. When only the bare damping of the spin waves is taken into account this expression contains $\eta^{-1}\ln(\Lambda/k)$, in analogy with the results of Ref. 3. However, the part of $\Delta(q)$ linear in q, which has a fluctuational origin, makes the integral convergent at small q.

For high-frequency sound with $k>\gamma/\eta$ the dependence $\eta^{-1}\ln(\Lambda/k)$ is preserved, but for small $k<\gamma/\eta$ the resulting logarithmic factor $\eta^{-1}\ln(b^{1/2}/\gamma)$ does not contain k and the correction to the sound-damping coefficient remains regular as $k\to 0$.

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