

Theory of inelastic multiquantum resonant tunneling of electrons

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Coherent inelastic resonance tunneling of electrons through a structure with a quantum well is examined taking into account the essentially nonadiabatic processes of oscillator interaction. An expression is derived for the multiquantum tunneling amplitude in a resonance approximation. An inelastic resonant tunneling molecular spectroscopy theory is developed for application that describes the I-V characteristics of the resonance structure in the presence of molecular impurities with a preferred mode of frequency ω . Cases are examined where the electron/vibration interaction strength is fairly high and multiquantum processes become important. The generation of inelastic tunneling channels with increasing applied voltage, as in regular inelastic tunneling molecular spectroscopy, produces an irregular increase in the differential conductivity. These jumps alternate with irregular decays associated with the saturation of inelastic resonance channels. The derived amplitude describes resonant tunneling of electrons in an oscillating electric field in the limit of large quantum numbers. The magnitude of the multiquantum photovoltaic effect for the case where the structure is irradiated by infrared radiation is found as an application of this result. The photovoltaic current is proportional to the radiation intensity for a weak oscillating field (in one-quantum excitation), while it is proportional to the root of the intensity for a strong field (in the multiquantum limit).

The significant number of recent studies devoted to the physics of resonant tunneling of electrons can be attributed to both expanding interest in investigating the effect of resonance phenomena on the tunnel conductivity of disordered systems¹⁻⁵ and the prospects for using structures with resonant tunnel conductivity (S-D-S-D-S, M-D-M-D-M, etc.) in microelectronics⁶⁻¹⁰ and quantum electronics.¹¹ Modern technology makes it possible to fabricate structures containing barriers and quantum wells with a predetermined type of potential.⁷ The fabrication of ultrathin and thick barriers¹² makes it possible to reduce the influence of spatial inhomogeneity on the conductivity of such structures and makes it possible to investigate nonadiabatic processes in electron tunneling.

If the electron lifetime in the coupled state is long enough, the time-independent formulation of the problem is no longer valid and it is necessary to take into account electron interaction with local oscillations, phonons and electrons.²⁻⁴ In order to establish the conductivity mechanism it is necessary above all to compare the characteristic coherence jump time τ_c of the electrons to their lifetime in the well, τ . For $\tau_c \gg \tau$ the series of internal reflections in the well produces constructive interference and the resonant transparency for barriers with the same permeability is of order unity. In the opposite case, $\tau_c \ll \tau$, inelastic interaction with the electron can be assumed to be random and resonant smearing occurs, as described in studies Ref. 4, 13.

A theory of resonant tunneling through a nonstationary δ -potential has been formulated in Ref. 14, 15 in order to carry out a qualitative investigation of the influence of an oscillatory field as well as vibrations of the medium ignoring noncoherent processes. The case of tunneling through a double-humped quasiclassical potential vibrating at a whole is investigated in Ref. 13. The numerical analysis algorithm for calculating resonant tunneling in an oscillatory field and a theory of exchange effects are proposed in Ref. 16. A nu-

merical investigation of resonant tunneling in an oscillatory field in the one-quantum approximation is carried out in Ref. 11.

Reference 3 has formulated a theory of inelastic multiquantum resonant tunneling of electrons through a center that takes into account its vibrational degrees of freedom. It was assumed in the adiabatic approximation that the frequency of the vibrations of the center ω is much less than the frequency of electron transitions in the well. Electron interaction with the vibrations in this problem is localized in the immediate vicinity of the center.

The purpose of the present study is to investigate coherent nondispersive inelastic resonant tunneling through structures with a quantum well, taking into account the nonadiabatic processes of interaction with vibrations of the medium both in the well and in the subbarrier region. The quantity ωT ($\omega \leq 0.1$ eV, ω is the frequency of the oscillations, T is the transit time) in the subbarrier region and in the well is generally of order of unity in the experiments of Ref. 6-9. The typical coherence jump time is $\tau_c \sim 10^{-12}$ sec, while the lifetime τ in the well for the barriers examined in this study is of order 10^{-13} sec, and the coherence condition can be satisfied.

References 2, 17 have identified the importance of the many-particle stationary and nonstationary effects associated with electron-electron interaction that can be attributed to distortions in the potential due to oscillations of the charge in the well. We will assume that the period $2\pi/\omega$ of the oscillations examined below is significantly less than the lifetime in the well, which characterizes the charge accumulation time.

It should be noted that the cleanest experiments for studying resonant tunneling of electrons can be carried out by using a scanning electron microscope focused on the surface of an M-D-M structure. It is possible in this case to investigate individual local oscillations. The role of resonant

amplification of inelastic processes in this case was identified in Ref. 18 (see also Ref. 19).

Below it is assumed for simplicity that the only significant interaction occurs with a single oscillation of frequency ω . The first section of the study presents a formulation of the problem and provides an expression for the amplitude of inelastic resonant tunneling through a two-barrier structure. Section 4 presents a method of calculating the amplitude that differs from the technique used to solve similar problems based on the Green's function technique (see Ref. 3 and its references) that is also more convenient for our purposes.

The second section examines an application of the theory to investigate the I-V characteristics of M-D-M-D-M structures with molecular impurities having a preferred mode of frequency ω . Cases where the electrons interact strongly with the oscillations and where multi-quantum processes become significant (an exponential subbarrier increase in interaction intensity, and amplification as the frequency ω approaches the resonant frequency and the adiabatic limit in the well) are investigated. The development of inelastic resonant tunneling channels for $U = n\omega$, $n = 1, 2, \dots$, that develop with increasing applied voltage U , as in regular tunneling molecular spectroscopy,²⁰ generates a discontinuous growth of the differential conductivity. This irregular growth in conductivity also exists when the bottom of the quasienergy band in the quantum well $E_0^{(x)} + n\omega$ intersects the Fermi level E_F (Fig. 1, a). This intersection indicates the opening of the resonant tunneling channel through this band. These jumps alternate with discontinuous decays corresponding to the saturation of the inelastic resonant current channels. Saturation occurs after the bottom of the qua-

sienergy band $E_0^{(x)} + n\omega$ rises above the level $E_F - U$, and no expansion of the resonant tunneling channel through this band occurs with further increase in voltage.

An expression is derived in the third section as a limiting case of the formulae in Sec. 1 for the amplitude of the inelastic resonant tunneling transitions through the two-barrier structure in an oscillating electric field. The features of the differential conductivity as a function of the applied voltage in this case are analogous to those examined in the second section. Primary attention is therefore devoted to an investigation of the dependence of the current on the alternating field magnitude. The extent of the multi-quantum photovoltaic effect is found for the case where the structure is irradiated by infrared radiation. For a weak oscillating field (in one-quantum excitation conditions) the photovoltaic current is proportional to the radiation intensity, while for a weak field (in the multi-quantum limit) it is proportional to the square root of the intensity.

1. FORMULATION OF THE PROBLEM; AMPLITUDE OF INELASTIC RESONANT TUNNELING TRANSITIONS

We will describe the structure under examination (Fig. 1, a) in the free-electron approximation, assuming for simplicity that the effective mass of the electrons is identical in all regions. Suppose that the electrons interact significantly only with a single vibrational mode of frequency ω . We will assume that the structure is a one-dimensional structure and the mode interaction depends adiabatically on the transverse electron coordinates. The latter assumption is generally an idealization for the interaction with localized molecular vibrations examined in Sec. 2 [see Ref. 20 and the commentary following formula (2.6)]. This assumption holds for the interaction with an oscillatory electric field examined in Sec. 3. In these conditions the Schrödinger equation describing electron interaction with a harmonic oscillation, after separation of the transverse coordinates, takes the form (using a system of units in which the electron and oscillator masses as well as Planck's constant are equal to unity)

$$\frac{1}{2} \left(\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial u^2} \right) + \left[\epsilon - V(x) - W(x)u - \frac{\omega^2 u^2}{2} \right] \Psi = 0, \quad (1)$$

where x is the longitudinal coordinate of the electron, u is the oscillator coordinate, $W(x)u$ is their interaction potential, and the double-humped potential $V(x)$ is shown in Fig. 1, a. It is assumed that $V(x)$ (with the exception of the jumps at the points x_j) and $W(x)$ are slowly varying quasiclassical functions and, moreover, that the oscillator energy and the interaction energy with the oscillator are small compared to the electron energy.²¹ The latter conditions hold for M-D-M-D-M type structures.¹⁰ For S-D-S-D-S structures the kinetic electron energy in region 1 is normally of order ω , although it can also be made much greater than ω (see, for example, Ref. 8).

We will consider the problem of inelastic electron scattering by a double-humped potential $V(x)$. In the range of large x , where $W(x)$ is assumed to be constant, $W(x) \rightarrow W_{s1}$ for $x \rightarrow \pm \infty$, the particular solutions of Eq. (1.1) take the form

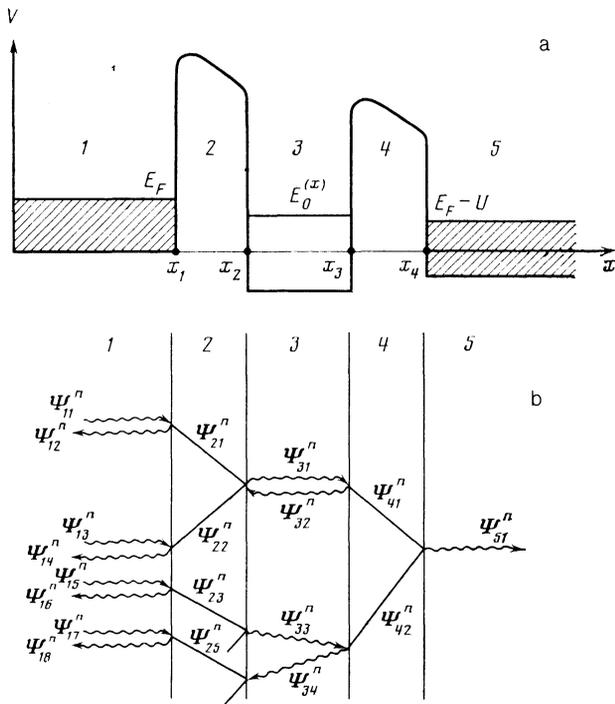


FIG. 1. Energy diagram of the analyzed structure (a) and the scheme for the continuation of the solution Ψ_{s1}^n from region 5 to region 1 (b).

$$\Phi_{jn}^{\pm} = \left[2^n n! \left(\frac{\pi}{\omega} \right)^{1/2} p \right]^{-1/2} H_n \left(\omega^{1/2} \left(u + \frac{W_j}{\omega^2} \right) \right) \cdot \exp \left\{ \pm i \int_{x_{j-1}}^x \left[p - \left(n + \frac{1}{2} \right) \frac{\omega}{p} + \frac{W_j^2}{2\omega^2 p} \right] dx - \frac{\omega}{2} \left(u + \frac{W_j}{\omega^2} \right)^2 \right\}, \quad p(x) = \{2[\varepsilon - V(x)]\}^{1/2},$$

$$x_0 = x_1, \quad j=1, 5, \quad (1.2)$$

where $H_n(x)$ is the Hermitian polynomial, the number j corresponds to the number of the region in Fig. 1 and, consistent with these assumptions, $p^2 \gg (n + 1/2)\omega$. For the case of interaction with local vibrations we have $W_1 = W_5 = 0$. In analyzing the problem of tunneling in an oscillatory electric field we assume $W_1 = 0$, while W_5 , generally, is nonzero.

Let the oscillator be in state m before scattering. We will assume an incident wave Φ_{1m}^+ normalized to a unit flux at $-\infty$, and we will look for the reflected wave Φ_r^m and the transmitted wave Φ_t^m as

$$\Phi_r^m = \sum_{k=0}^{\infty} R_{mk} \Phi_{1k}^-, \quad \Phi_t^m = \sum_{n=0}^{\infty} S_{mn} \Phi_{5n}^+. \quad (1.3)$$

In order to find the inelastic multiquantum transmission amplitude S_{mn} and reflection amplitudes R_{mk} (elements of the S -matrix), it is necessary to continue the wave function Φ_t^m from region 5 to region 1 and to join with the wave Φ_{1m}^+ at $-\infty$. The calculation method is given in Sec. 4. Here we will provide only the final results obtained from expression (4.14) in the Breit-Wigner approximation. In the resonance approximation, i.e., when the energy ε is much closer to the poles of the S -matrix (quasistationary state) \mathcal{E}_q than the distance between the poles $\Delta\mathcal{E}_q$, and when the width of the quasistationary state satisfies $\Gamma_q = -2\text{Im}\mathcal{E}_q \ll \Delta\mathcal{E}_q$ (specifically when $\Gamma_q \ll \omega$ holds) it is possible to represent the expression for the resonant tunneling amplitude as

$$S_{mn}^q(\varepsilon) = \frac{\gamma_{mq}^{(2)} \gamma_{nq}^{(4)}}{(\varepsilon - \varepsilon_q) + i/2\Gamma_q}, \quad \varepsilon_q = \text{Re}\mathcal{E}_q, \quad (1.4)$$

where $\gamma_{mq}^{(2)}$, $\gamma_{nq}^{(4)}$ are the partial decay amplitudes of the quasilevel through the right and left barriers, respectively:

$$\gamma_{mq}^{(2)} = \left(\frac{q!}{m!} \Gamma_2^{(0)} \right)^{1/2} [(2\omega)^{1/2} B_2^+]^{m-q} L_q^{m-q} (2\omega B_2^+ + B_2^-) e^{n_2}, \quad (1.5)$$

$$\gamma_{nq}^{(4)} = \left(\frac{q!}{n!} \Gamma_4^{(0)} \right)^{1/2} [(2\omega)^{1/2} B_4^-]^{n-q} L_q^{n-q} (2\omega B_4^- + B_4^+) e^{n_4}. \quad (1.6)$$

In Eqs. (1.5), (1.6) $L_m^n(x)$ are Laguerre polynomials. The width of the quasistationary state

$$\Gamma_q = \Gamma_{2q} + \Gamma_{4q}, \quad (1.7)$$

where

$$\Gamma_{2q} = \Gamma_2^{(0)} L_q(-\alpha_2) e^{\beta_2} \quad (1.8)$$

is the left-side decay width of the quasilevel and

$$\Gamma_{4q} = \Gamma_4^{(0)} L_q(-\alpha_4) e^{\beta_4} \quad (1.9)$$

is the right-side decay width of the quasilevel. Here $\Gamma_2^{(0)}$, $\Gamma_4^{(0)}$ are the left and right decay widths of the quasilevel in the stationary case for $W \equiv 0$.²² B_j^{\pm} are linear quantities, while α_j , β_j and n_j are quadratic interaction functionals of $W(x)$. Explicit expressions for these are given in the Appendix. Using familiar expansion formulae for the Laguerre polynomials²³ it is possible to represent the widths Γ_{2q} and Γ_{4q} as a sum of partial widths:

$$\Gamma_{4q} = \sum_{k=0}^{\infty} |\gamma_{kq}^4|^2. \quad (1.10)$$

Interaction with the vibrations causes splitting of the quasilevel in the well into a number of quasilevels separated by ω . The quantities β_2 and β_4 determining the change in the width of the quasilevels due to interaction for $q = 0$ can be both positive and negative. For $q > 0$ the Laguerre polynomials also cause smearing of the width of the quasilevels [$L_q(-\alpha_j) > 1$, since $\alpha_j > 0$].

We introduce the transit time for region j :

$$T_j = \int_{x_{j-1}}^{x_j} \frac{dx}{|p|}. \quad (1.11)$$

An analysis of the derived expression for the inelastic resonant tunneling amplitude reveals that if the interaction quantities in the well and in the subbarrier region are of the same order, for $\omega T_2 \ll 1$ and $\omega T_4 \ll 1$ the contribution from interaction in the subbarrier region is relatively small. In this case, which is a natural one for applications, it is possible to obtain an expression for the S -matrix with a random ratio between Γ_q and ω , and not only for $\Gamma_q \ll \omega$ (the wave functions $\Psi_{1,2,3,4}^k$ in (4.5) with $s = 1-4$ have a linear dependence). Therefore the expression for the resonant tunneling amplitude takes the form

$$S_{mn} = \sum_{q=0}^{\infty} \frac{\gamma_{mq}^{(2)} \gamma_{nq}^{(4)}}{(\varepsilon - \varepsilon_q) + i\Gamma_q/2}. \quad (1.12)$$

With adiabatic electron motion in the well ($\omega T_3 \ll 1$) this expression is in agreement with Eq. (33) of Ref. 3.

2. INELASTIC TUNNELING RESONANT SPECTROSCOPY

In this section we will consider the application of the above results to an investigation of the I-V characteristics of M-D-M-D-M structures with molecular impurities having a distinct mode of frequency ω . These experiments are appropriately labeled inelastic tunnel resonant spectroscopy, which has a number of features that distinguish it from regular inelastic tunnel spectroscopy.²⁰ Before proceeding to an investigation of the I-V characteristics, we will consider the case where interaction with the molecular vibrations is localized in one of the regions indicated in Fig. 1 and we will analyze the conditions that facilitate an enhancement of the multiquantum transitions. For simplicity the parameters of the multiquantum transition amplitude are given explicitly below.

a. Let the interaction be localized in the vicinity of the well. In this case the expressions for the parameters of the

resonance amplitude are significantly simplified. In formulae (1.4)–(1.9) we now set

$$\begin{aligned} B_2^\pm &= A_2 \exp(\mp \omega T_2), & B_4^\pm &= A_4 \exp(\pm \omega T_4), \\ \alpha_j &= 8\omega A_j^2 \operatorname{sh}(\omega T_j), & \beta_j &= 2\omega A_j^2 [\exp(-2\omega T_j) - 1], \\ \eta_j &= -\omega A_j^2, \end{aligned} \quad (2.1)$$

where

$$A_{2,4} = -(2\omega \sin \omega T_{2,4})^{-1} \int_{x_1}^{x_2} \cos\left(\omega \int_{x_1}^x \frac{dx}{p}\right) W \frac{dx}{p}, \quad (2.2)$$

and when the adiabatic condition is satisfied in the well ($\omega T_3 \ll 1$)

$$A_2 = A_4 = -\frac{\langle W \rangle}{2\omega^2}, \quad \langle W \rangle = \frac{1}{T_3} \int_{x_1}^{x_2} W(x) \frac{dx}{p}, \quad (2.3)$$

where $\langle W \rangle$ is the quasiclassical average of the interaction in the well.

In the case of adiabatic motion in the subbarrier region for $\omega T_2, \omega T_4 \ll 1$ we have from (2.1)

$$B_2^\pm = A_j, \quad \eta_j = -\omega A_j^2, \quad \alpha_j, \beta_j = O((\omega T_j)^2). \quad (2.4)$$

It is clear from (2.2) that the interaction strength can be increased significantly by two possible features of the structure. The first is associated with the exponential gain attributable to the nonadiabaticity of motion in the subbarrier region which has been investigated previously.^{21,24,25} For single barriers the adiabaticity parameter ωT_j will in reality reach a value of approximately 10 (the limit is related to the possibility for an experimental observation of very low current²⁵). However when ωT_j is large and, consequently, the penetrability of the corresponding barrier is extraordinarily low it is necessary for the other barrier to be significantly more penetrable in order to retain constructive interference superposed on the inelastic processes.⁴

The one additional possibility for increasing interaction strength in quasiclassical conditions is achieved near classical resonance, corresponding to the zeros of the resonant denominator $\sin \omega T_3$ in (2.2). In other words this occurs when the separation between the levels in the well is close to ω or when the adiabatic condition $\omega T_3 \rightarrow 0$ is satisfied. In direct proximity to resonance, where n_j and $2\omega B_j^+ B_j^-$ from (1.5), (1.6) become greater than or of the order of the action in the well, the method used here becomes useless. However multiquantum processes become significant as early as $n_j, 2\omega B_j^+ B_j^- \sim 1$.

b. Suppose that the interaction is concentrated in the subbarrier region 4. Then we have

$$\begin{aligned} \alpha_2 = \beta_2 = \eta_2 = 0, & & B_4^\pm &= G_4^\mp(x_3), & \alpha_4 &= 2\omega [G_4^+(x_3) - G_4^-(x_3)]^2, \\ \eta_4 &= -2\omega \int_{x_1}^{x_2} G_3^+(x) \frac{d}{dx} G_3^-(x) dx, & \beta_4 &= 2[G_4^+(x_3)]^2 + 2\eta_4. \end{aligned} \quad (2.5)$$

The following convention is used in (2.5)

$$G_j^\pm = \pm \frac{1}{2\omega} \int_{x_j}^x \exp\left(\pm \omega \int_{x_j}^x \frac{dx}{|p|}\right) W \frac{dx}{|p|}.$$

It is also simple to obtain similar expressions for the case of interaction localization in region 2. Since we have $\beta_4 > 0$ in (2.5), interaction with vibrations in the subbarrier region always causes smearing of the quasilevel width. In these cases there is no resonant enhancement of interaction strength, and the strength can be increased only by the exponential amplification.

We will use our results to analyze the resonant tunnel conductivity in the case where the interaction is strong and multiquantum processes become important. We consider the low temperature case. We take into account that electrons tunneling from region 1 are actually three-dimensional. Therefore in addition to the initial longitudinal electron momentum $p_x = (2E - \omega)^{1/2}$ we introduce the momenta p_y and p_z , since the total initial electron energy is $E = (1/2)(p_x^2 + p_y^2 + p_z^2)$. In the one-particle approximation by analogy to calculations from Ref. 20, 26 the tunneling resonant current can then be calculated from

$$\begin{aligned} I(U) &= \frac{2}{(2\pi)^3} \int dp_x dp_y dp_z p_x \\ &\cdot \sum_{n,q} \theta(E_F - E) \theta(E - \omega n - E_F + U) |S_{0n}^q(\varepsilon)|^2, \end{aligned} \quad (2.6)$$

where $\theta(x)$ is a step function, U is the applied voltage, and E_F is the Fermi energy of the electrons in region 1 (Fig. 1,a).¹⁾ Expression (2.6) is generally valid only for barriers of variable penetrability. Inelastic exchange effects become important as the barriers assume identical penetrability (see Sec. 3 for a detailed discussion). It is necessary to average the current in (2.6) over the transverse electron coordinates. However, as we shall see from the I–V characteristics given below, this averaging will not result in a qualitative change in their form. Therefore, bearing in mind the model nature of our assumption of the adiabaticity of the transverse coordinates and the fact that interaction $W(x)$ generally is unknown, we will not carry out averaging, and will assume that the parameters entering into the formulae for the resonance amplitudes in (1.4)–(1.9) are model parameters.

Figure 2 illustrates the geometric picture of the successive opening and saturation of inelastic tunneling channels with increasing U that, consistent with (2.6), we will label by two indices (q, n). A significant feature of this process is the fact that the three-dimensional electrons experience resonant tunneling through the one-dimensional level $E_0^{(x)}$ in the well; this level is at the same time an energy band with $E_0^{(x)}$ as the bottom for the three-dimensional problem. The bold circle in the momentum space (p_x, p_y, p_z) represents the Fermi sphere of radius $p_F = (2E_F)^{1/2}$. The figure also shows the cross sections $\sigma^{(q)}(U)$ of this sphere by the planes $p_x = p_x^{(q)}(U) = \{2[E_0^{(x)}(U) + q\omega]\}^{1/2}$ on which the one-dimensional quasienergies of the well electrons lie. The number q varies from zero to the integer-valued quantity m of $[E_F - E_0^{(x)}(U)]/\omega$, since there are no electrons with an energy component on the p_x axis greater than E_F at zero temperature. The light circles represent the family of spheres of radii $p^{(n)}(U) = [2(E_F - U + n\omega)]^{1/2}$, $n \geq 0$. The geometric location of the points corresponding to electrons experiencing resonant tunneling through the channel (q, n) appears as a small [thickness of the order of $\Gamma_q/p^{(n)}$] neighborhood of the ring segmented by the sphere $p^{(n)}(U)$ from

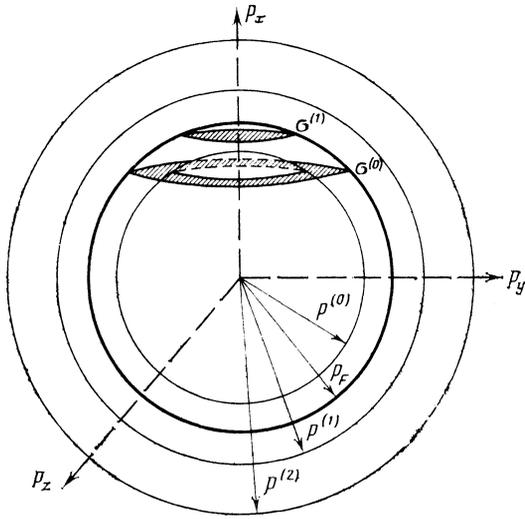


FIG. 2. Geometric pattern of the successive opening and saturation of inelastic resonant tunneling channels with increasing U . The situation where $\omega < E_F - E_0^{(x)}(U) < 2\omega$, $U < \omega$ is shown. Channels (0, 0), (1, 0) are open. The (1, 0) channel is saturated.

the section $\sigma^{(q)}(U)$. The intersection of the sphere $p^{(n)}(U)$ with sphere p_F represents the opening of $m+1$ resonant channels with numbers (q, n) , $0 \leq q \leq m$. The addition to the current resulting from these channels appears for $p^{(n)}(U) > p_F$, equivalent to the condition $U > n\omega$ and proportional to the area of each of the m rings, equal to $\pi[p_F^2 - (p^{(n)}(U))^2] = \pi(U - n\omega)$ (this area is independent of q). Therefore the current $I(U)$ at the point $U = n\omega$ has a discontinuity, and a positive jump appears in the plot of the differential conductivity $G(U)$. The sharp growth of conductivity in the interval $\Delta U \sim \Gamma_q \ll \omega$ also appears with the generation (with increasing U) of the cross section $\sigma^{(q)}(U)$ at the upper point of the sphere p_F that occurs for $p_F = p_x^{(q)}(U)$, equivalent to $E_F = E_0^{(x)}(U) + q\omega$. Indeed, when $E_F > E_0^{(x)}(U) + q\omega$ an addition to the current appears that is proportional to the area of the cross section $\sigma^{(q)}(U)$, equal to $\pi[p_F^2 - (p_x^{(q)}(U))^2] = \pi[E_F - E_0^{(x)}(U) + q\omega]$. Finally when the sphere $p^{(n)}(U)$ makes contact with the cross section $\sigma^{(q)}(U)$ [when $p^{(n)}(U) = p_x^{(q)}(U)$ equivalent to $E_F = E_0^{(x)}(U) + U + (q-n)\omega$], the channel (q, n) is saturated. Proceeding analogously we find that this corresponds in the conductivity plot to a reduction in its value in the interval $\Delta U \sim \Gamma_q$.

The conductivity plot for this structure will therefore consist of flat sections separated by positive or negative jumps. The jumps can be attributed to the topological features of the configuration of the spheres $p_F, p^{(n)}(U)$ and the cross sections $\sigma^{(q)}(U)$, which have a rather general nature, unrelated to the interaction model selected in the present study or to the assumptions made in solving the Schrödinger equation.

We will consider the case of small $U \sim \omega$ assuming that $\gamma_{nq}^{(j)}$ and Γ_q remain constant in this interval, while $E_0^{(x)}(U) = E_0^{(x)}(0) - \beta U$. These conditions are equivalent to assuming adiabatic electron motion. Using the expression for the matrix elements (1.4) we obtain from (2.6) for the differential conductivity

$$G(U) = \frac{\partial I}{\partial U} = \frac{1}{\pi} \sum_{q,n} \theta(U - n\omega) \frac{|\gamma_{0q}^{(2)} \gamma_{qn}^{(4)}|^2}{\Gamma_q} \left[\frac{\pi}{2} + \beta \operatorname{arctg} \frac{2[E_F - E_0^{(x)}(U) - q\omega]}{\Gamma_q} + (1-\beta) \operatorname{arctg} \frac{2[E_F - U - E_0^{(x)}(U) + (n-q)\omega]}{\Gamma_q} \right]. \quad (2.7)$$

Here summation is carried out over all resonant tunneling open channels identified above.

In investigating the I-V characteristics it is possible to change the potential difference between the well (base), region 1 (the emitter) and region 5 (the collector). The opening of elastic and inelastic resonant tunneling channels will therefore depend on the relative position of the level in the well, the Fermi level of the emitter and the Fermi level of the collector. If the interaction is localized under the first (second) barrier, either the change in the potential difference between the emitter and collector will be nontrivial or the change between the base and the emitter (collector) will be. Figure 3 provides conductivity graphs produced by numerical analysis using Eq. (2.7) for the case where the difference in potentials between the base and collector changes. It is assumed that the level in the well $E_0^{(x)}$ is fixed with respect to the Fermi level of the emitter E_F ($\beta = 0$) where $E_0^{(x)} < E_F$ holds (the cross sections $\sigma^{(q)}$ shown in Fig. 2 are fixed). Positive jumps for $U = k\omega$ corresponding to opening of inelastic resonance channels that alternate with negative jumps for $U = E_F - E_0^{(x)} - k\omega$ corresponding to channel absorption can be observed in the conductivity plots. When the interaction is weak the maximum positive jump corresponds to the opening of the first inelastic channel for $U = \omega$, while the maximum negative jump corresponds to

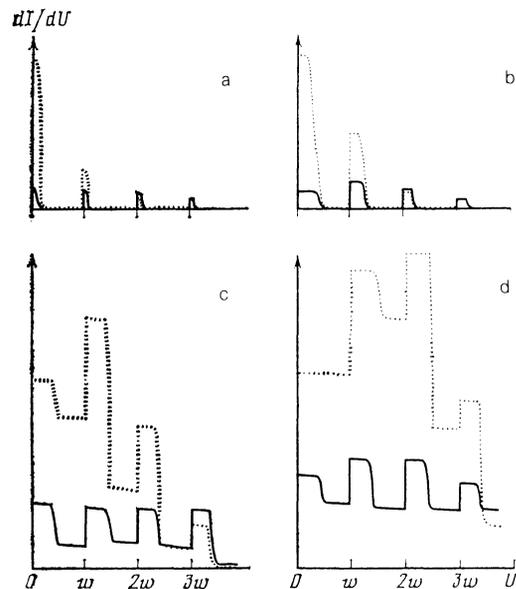


FIG. 3. The differential conductivity plotted as a function of applied voltage U for $\Gamma/\omega = 2.5 \cdot 10^{-3}$. The points represent the relations for electron-vibrational interaction parameters $2\omega B_2; B_2 = 2\omega B_4; B_4 = 0.5$, while the traces represent the relations for $2\omega B_j; B_j = 1.5$: a— $E_F = E_0^{(x)}$; b— $E_F - E_0^{(x)} = 0.3\omega$; c— $E_F - E_0^{(x)} = 1.3\omega$; d— $E_F - E_0^{(x)} = 2.3\omega$.

opening of an elastic channel for $U = E_F - E_0^{(x)}$. Characteristic peaks are clear in the conductivity plot (Fig. 3,a) for similar values of $E_F - E_0^{(x)}$ and ω . Physically these peaks clearly differ from those that arise from an investigation of resonant tunneling through a three-dimensional level.³

3. RESONANT TUNNELING OF ELECTRONS IN AN OSCILLATORY ELECTRIC FIELD; EXCHANGE EFFECTS; THE PHOTOVOLTAIC EFFECT

The problem of resonant tunneling in an oscillatory electric field can be considered as a limiting case of the problem solved above which corresponds to large quantum numbers of the oscillator. For large m and fixed q it is possible to use the asymptotic forms containing Bessel functions for the Laguerre polynomials^{2,3}:

$$L_m^q(x) \approx \frac{(m+q)!}{m!} (mx)^{-q/2} e^{x/2} J_q(2(mx)^{1/2}). \quad (3.1)$$

In this case the oscillator is quasiclassical and its action on the electron is reduced to a variable correction to the potential:

$$\tilde{W}(x) \cos \omega t = (2m/\omega)^{1/2} W(x) \cos \omega t. \quad (3.2)$$

Let $\tilde{W}(x) \rightarrow \tilde{W}_1 = 0$ for $x \rightarrow -\infty$, $W(x) \rightarrow \tilde{W}_5$ for $x \rightarrow +\infty$, where in the general case we have $\tilde{W}_5 \neq 0$.

Passing to the limit in (1.4)–(1.9) subject to (3.1), (3.2) we obtain for the resonant tunneling amplitude of an electron absorbing d - q quanta having an initial energy near the quasilevel $E_q^{(x)}$

$$S_{qd}(E^{(x)}) = \frac{\gamma_q^{(2)} \gamma_d^{(4)}}{(E^{(x)} - E_q^{(x)}) + i\Gamma/2}, \quad \Gamma = \Gamma_2 + \Gamma_4, \quad (3.3)$$

where $E_q^{(x)} = E_0^{(x)} + q\omega$, while $E_0^{(x)}$ is the level in the stationary well. For the other parameters of Eq. (3.3) we have

$$\begin{aligned} \Gamma_2 &= \Gamma_2^{(0)} I_0((2\omega\tilde{\alpha}_2)^{1/2}), \quad \Gamma_4 = \Gamma_4^{(0)} I_0((2\omega\tilde{\alpha}_4)^{1/2}), \\ \gamma_q^{(2)} &= (-1)^q (\Gamma_2^{(0)})^{1/2} \mu_2^{q/2} J_q(\chi_2), \\ \gamma_d^{(4)} &= (-1)^d (\Gamma_4^{(0)})^{1/2} \mu_4^{d/2} J_d(\chi_4), \\ \mu_2 &= \tilde{B}_2^- / \tilde{B}_2^+, \quad \mu_4 = \tilde{B}_4^+ / \tilde{B}_4^-, \quad \chi_j = 2\omega (\tilde{B}_j^+ \tilde{B}_j^-)^{1/2}. \end{aligned} \quad (3.4)$$

Here $I_0(y)$ is a modified Bessel function, while the parameters \tilde{B}_j^\pm and $\tilde{\alpha}_j$ are calculated in the same manner as B_j^\pm and α_j , replacing $W(x)$ with $\tilde{W}(x)$.

It follows from (3.4) that in this limiting case the quasilevel width is independent of its number. An increase in the intensity of the oscillation will always result in smearing of the quasilevel width. In the adiabatic limit $\omega T_2, \omega T_4 \rightarrow 0$ we have $\mu_j \rightarrow 1$.

In considering the application of our results we will analyze the influence of an IR field of frequency ω significantly less than the characteristic energy parameters of the structure on the resonant tunneling process. In this case if $\tilde{W} \gtrsim \omega$, multiquantum transitions may be observed. Typically such a field penetrates into the metal a distance of order 10^2 – 10^3 Å. This depth can be smaller for ultrapure specimens. The possibility of identifying the inelastic processes of electromagnetic field interacting with electrons on top of other inelastic processes in a solid has been discussed, for example, in Ref. 27.

In our one-particle approximation the inelastic channels cause electrons with different initial energies to end up in identical final states. Therefore the need to take into account exchange effects caused by inelastic processes becomes an issue. A theory of electron tunneling in an oscillatory electric field taking into account exchange effects has been developed in Ref. 16. The algorithm proposed in Ref. 16 for finding the one-particle S -matrix (used to determine many-particle exchange effects) approximates the functions $V(x)$ and $\tilde{W}(x)$ by piecewise-constant functions. Therefore for actual relations its calculation requires rather laborious numerical techniques. Our approach makes it possible to determine the contribution of the exchange current analytically. We will not provide the cumbersome expressions for this value obtained by substitution of the amplitudes (3.4) into the general expressions of Ref. 16. We will simply point out that the ratio of the inelastic current to the exchange current is determined to within an order of magnitude by the ratio $\Gamma_2 \Gamma_4 / (\Gamma_2 + \Gamma_4)^2$. It is then clear that when the right and left barriers have similar penetrabilities, $\Gamma_2 \sim \Gamma_4$, the inelastic and exchange current are of the same order and the problem is essentially a many-particle problem. In the opposite case, $\Gamma_2 \gg \Gamma_4$ or $\Gamma_2 \ll \Gamma_4$, it is possible to neglect the exchange effects attributable to inelastic processes.

We will limit the analysis to barriers of different penetrabilities. In this case the resonant current through the structure with these assumptions can be determined analogous to Ref. 20, 26 by the formula

$$I(U) = \frac{2}{(2\pi)^3} \sum_{q,d=-\infty}^{\infty} \int dp_x dp_y dp_z p_x \{ f(E) - f[E+U+(q-d)\omega] \} \left| S_{qd} \left(\frac{p_x^2}{2} \right) \right|^2. \quad (3.5)$$

We will consider the case of zero temperature when $f(E) = \theta(E_F - E)$. Then for sharp resonances ($\Gamma \ll \omega$) the integral is evaluated in (3.5). As a result, taking into account the relation

$$\Gamma_j = \sum_{m=-\infty}^{\infty} |\gamma_m^{(j)}|^2 \quad (3.6)$$

we can write Eq. (3.5) in the form

$$I(U) = \frac{1}{\pi\Gamma} \left\{ \Gamma_4 \sum_{m=-\infty}^{m_0} (E_F - E_0^{(x)} - m\omega) |\gamma_m^{(2)}|^2 - \Gamma_2 \sum_{n=-\infty}^{n_0} (E - U - E_0^{(x)} - n\omega) |\gamma_n^{(4)}|^2 \right\}, \quad (3.7)$$

using the integral parts $m_0 = [(E_F - E_0^{(x)})/\omega]$ and $n_0 = [(E_F - U - E_0^{(x)})/\omega]$. Here unlike Eq. (2.7) for simplicity we have replaced functions of the form $\text{arctg}(\dots/\Gamma)$ with step functions ($\Gamma \ll \omega$). The photocurrent is determined by Eq. (3.7) if we set $U = 0$ in this formula. On the other hand assuming $I(U) = 0$, it is possible to determine the photo-EMF from (3.7). For example we assume that $m_0 = n_0 = 0$, i.e., that the level in the well $E_0^{(x)}$ lies above the Fermi level E_F , but not above ω and also that the voltage U is not substantial: $\Gamma < U < E_F - E_0^{(x)} < \omega$. Moreover, we let $\tilde{B}_j^+ = (\tilde{B}_j^-)^*$ (see below). Then the sums in (3.7) can be calculated analytically and we obtain

$$I(U) = \frac{\Gamma_2^{(0)} \Gamma_4^{(0)}}{2\pi(\Gamma_2^{(0)} + \Gamma_4^{(0)})} \{y(2\omega |B_2^+|) - y(2\omega |B_4^+|) + U[1 + J_0^2(2\omega |B_4^+|)]\},$$

$$y(z) = (E_F - E_0^{(x)}) J_0^2(z) + \omega z^2 [J_0(z) J_1'(z) + J_1^2(z)]. \quad (3.8)$$

For relatively weak radiation in one-quantum excitation conditions $2\omega |B_j^+| \ll 1$ it is possible to set

$$y(z) = (E_F - E_0^{(x)}) + (\omega + E_0^{(x)} - E_F) z^2/2$$

and the photocurrent is proportional to the radiation intensity. In the opposite multiquantum limit $2\omega |B_j^+| \gg 1$ we have $y(z) \approx 2\omega z/\pi$ and the photocurrent is proportional to the square root of the intensity. Similar results are also obtained for other final values of m_0 and n_0 . For $\omega/(E_F - E_0^{(x)}) \rightarrow 0$ the indices satisfy $m_0, n_0 \rightarrow \infty$ and from (3.7) we obtain

$$I(U) = \frac{1}{\pi\Gamma} (\Gamma_2 \Gamma_4^{(0)} \xi_4 + \Gamma_4 \Gamma_2^{(0)} \xi_2 + U \Gamma_2 \Gamma_4),$$

$$\xi_j = \omega^2 \frac{|B_j^+|^2 - |B_j^-|^2}{|B_j^+ - (B_j^-)^*|} I_1(2\omega |B_j^+ - (B_j^-)^*|). \quad (3.9)$$

Applying a specific voltage to the well and keeping fixed the positions of the Fermi levels of the electrons, we can change the position of $E_0^{(x)}$ with respect to E_F . We therefore have the capability to examine a variety of situations.

Assume that outside the structure in regions 1, 5 we have

$$\tilde{W}(x) = \tilde{W}(x_{1,4}) \exp(-\lambda_{1,5} |x - x_{1,4}|).$$

We assume that the adiabatic conditions are satisfied in the structure, $\omega T_j \ll 1$. The depth of penetration of the oscillatory field λ_j^{-1} can be sufficiently great that $\omega \gtrsim \lambda_j p_j^\infty, p_{1,5}^\infty = p(\pm \infty)$. Then we obtain

$$\tilde{B}_{2,4}^\pm = \frac{\tilde{W}(x_{1,4}) - \tilde{W}_{1,5}}{2\omega(\omega \pm i\lambda_{1,5} p_{1,5}^\infty)} - \frac{\langle \tilde{W} \rangle - \tilde{W}_{1,5}}{2\omega^2}, \quad \langle \tilde{W} \rangle = \frac{1}{T_3} \int_{x_2}^{x_4} \tilde{W} \frac{dx}{p}, \quad (3.10)$$

where $\langle \tilde{W} \rangle$ is the quasiclassical average of \tilde{W} in the well. In this case, as assumed in deriving (3.8), $\tilde{B}_j^+ = (\tilde{B}_j^-)^*$. Violation of this equality is due to deviation from adiabatic motion in the subbarrier region, which, consistent with (3.9), produces the photovoltaic effect at a low frequency ω .

It is possible to detect experimentally the photovoltaic effect by irradiating the structure with short IR pulses so that the temperature is not substantially increased.²⁷ In the general case it is necessary to take into account interaction with the phonons and the temperature rise.

It is clear that any contact asymmetry will produce a photovoltaic effect to some extent. However, for an asymmetric single barrier the photo-EMF in this approximation is equal to zero.

It is easy to determine from Eq. (3.7) that positive jumps with integer-valued $[E_F - E_0^{(x)}(U)]/\omega$ and negative jumps with integer-valued $[E_F - U - E_0^{(x)}(U)]/\omega$ will be observed in this case in a plot of the conductivity $G = \partial I / \partial U$ as a function of U . Only negative jumps will be

observed in the case analyzed in the preceding section where $E_0^{(x)}$ is fixed ($\beta = 0$) unlike the plots shown in Fig. 3.

4. METHOD OF DETERMINING THE S-MATRIX

An asymptotic solution of Eq. (1.1) in the j region that is valid under the conditions outlined in the first section of the present study was obtained in Ref. 21. We will label this solution [see Eq. (A1) in the Appendix] by Ψ_{jl}^n , where n is the order of the Hermitian polynomial in this solution, while l is the number of the solution in region j (see below and Fig. 1.). In order to find the inelastic resonant tunneling S-matrix we continue the transmitted wave Φ_{5l}^m [see (1.3)] from region 5 to region 1 and join it with the incident wave Φ_{1m}^+ . For this purpose we represent the transmitted wave Φ_{5l}^m in the form of an expansion:

$$\Phi_{5l}^m = \sum_{k=0}^{\infty} c_{mk} \Psi_{51}^k. \quad (4.1)$$

It is easily demonstrated that the elements of the S-matrix S_{mk} are related to the coefficients c_{mk} by the expression

$$S_{mn} = \sum_{k=0}^{\infty} c_{mk} r_{kn}, \quad (4.2)$$

where r_{kn} are the coefficients of the expansion of the wave Ψ_{51}^k in terms of the complete set of solutions $\{\Phi_{5n}^+\}$ expressed through Laguerre polynomials.

We continue each of the waves Ψ_{51}^k from region 5 to region 1. We require the wave function and its first derivative to be continuous in the quasiclassical approximation at the points x_j where the potential $V(x)$ has a jump (Fig. 1,a). In this approximation the behavior of the potential $V(x)$ close to the turning points x_j is not important for expressions (1.4)–(1.9) to remain valid. Only $\Gamma_j^{(0)}$ and ε_q will be dependent on it. The exciting wave Ψ_{51}^k after joining at the points x_4, x_3, x_2, x_1 subsequently splits into two and, consequently, in region 1 generally includes 16 linearly independent waves (8 incident waves and 8 reflected waves). However in the quasiclassical approximation four of these waves (the lowest waves in region 1, Fig. 1,b) are exponentially small and can be ignored. Proper selection of the free parameters of wave functions Ψ_{3l}^n in the well in accordance with Ref. 21 makes it possible to reduce the number of waves in region 1 to four incident and four reflected waves:

$$\Psi_{51}^k \rightarrow \sum_{s=1}^4 d_{2s-1}^k \Psi_{1,2s-1}^k + \sum_{s=1}^4 d_{2s}^k \Psi_{1,2s}^k. \quad (4.3)$$

We will drop the coefficients d_{ij}^k for simplicity. In the general case the waves Ψ_{1i}^k are linearly-independent. As a result the wave Φ_{5l}^m in region 1 becomes the sum of incident waves φ_i^m and reflected waves φ_r^m :

$$\varphi^m = \varphi_i^m + \varphi_r^m, \quad (4.4)$$

$$\varphi_i^m = \sum_{k=0}^{\infty} c_{mk} \left(\sum_{s=1}^4 d_{2s-1}^k \Psi_{1,2s-1}^k \right),$$

$$\varphi_r^m = \sum_{k=0}^{\infty} c_{mk} \left(\sum_{s=1}^4 d_{2s}^k \Psi_{1,2s}^k \right). \quad (4.5)$$

Using the completeness and orthogonality of the system of Hermite polynomials, we expand in region 1 each of the incident waves $\Psi_{1,2\infty-1}$ in (4.5) in terms of the set of waves $\{\Psi'_{11}\}$:

$$\Psi_{1,2\infty-1}^h = \sum_{l=0}^{\infty} b_{2s-1,l}^h \Psi_{11}^l, \quad b_{1,l}^h = \delta_{hl}. \quad (4.6)$$

Substituting (4.6) into (4.5) we obtain

$$\varphi_i^m = \sum_{l=0}^{\infty} \Psi_{11}^l \left(\sum_{h=0}^{\infty} c_{mh} \sum_{s=1}^4 d_{2s-1}^h b_{2s-1,l}^h \right). \quad (4.7)$$

On the other hand we can also expand the wave Φ_{1m}^+ in terms of the set $\{\Psi'_{11}\}$:

$$\Phi_{1m}^+ = \sum_{l=0}^{\infty} a_{ml} \Psi_{11}^l. \quad (4.8)$$

Equating (4.8) and (4.7) we obtain an infinite system of linear algebraic equations for the coefficients c_{mk} :

$$a_{mi} = \sum_{h=0}^{\infty} z_{hi} c_{mh}, \quad z_{hi} = \sum_{s=1}^4 d_{2s-1}^h b_{2s-1,i}^h. \quad (4.9)$$

Suppose that the resonances are sharp, i.e., assume the condition $\Gamma_q \ll \omega$ holds, where Γ_q is the quasilevel width in the well. In this case the poles of the S -matrix \mathcal{E}_{qn} corresponding to the quasi-levels, as is clear from (4.12), are determined by the zeros of the diagonal matrix elements z_{kk} of Eqs. (4.9). We then find that the quasilevel energy $\varepsilon_{qn} = \text{Re } \mathcal{E}_{qn}$ is determined by the relation

$$\varepsilon_{qn} = E_{0n}^{(n)} + \omega(q + 1/2) + \lambda(W), \quad (4.10)$$

where $n, q \geq 0$ are the integers, $E_{0n}^{(x)}$ are the one-dimensional electron levels in the well with zero interaction, and $\lambda(W)$ is the quadratic functional of the interaction determined by Eq. (A4) in the Appendix. We drop the index n , assuming it to be fixed.

Far from the poles of the S -matrix for energies ε such that $|\varepsilon - \varepsilon_q| \gg \Gamma_q$, the conditions $d_1^k \gg d_l^k, l = 2, 3, 4$ holds and the matrix of the system (4.9) becomes diagonal. As a result in the nonresonance case we have

$$c_{mk} = a_{mk} / d_1^k. \quad (4.11)$$

For energies satisfying the condition $|\varepsilon - \varepsilon_q| \ll \omega$ in the q^{th} equation in system (4.9) the primary contribution comes from one term with $k = q$. Therefore in the resonant approximation (zeroth order in Γ_q/ω) we obtain

$$c_{mq} = a_{mq} \left(\sum_{s=1}^4 d_{2s-1}^q b_{2s-1,q}^q \right)^{-1}, \quad c_{mq} \gg c_{mk}, \quad k \neq q. \quad (4.12)$$

In region 5 the solution takes the form

$$\varphi_i^m = c_{mq} \Psi_{51}^q. \quad (4.13)$$

In the nonresonant case Eq. (4.12) becomes (4.11). We obtain from (4.12), (4.13) and (4.2) for the resonant tunneling amplitude.

$$S_{mn}^q = a_{mq} r_{qn} \left(\sum_{s=1}^4 d_{2s-1}^q b_{2s-1,q}^q \right) \quad (4.14)$$

The resonant reflection amplitudes can be found analogously.

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APPENDIX

The asymptotic solution of Eq. (1.1) in region j takes the following form:

$$\begin{aligned} \Psi_{j1}^n = & D_{j1} p^{-n} H_n(\omega^{1/2} [u - f_{j1}(x)]) \exp \left\{ -i(-1)^l \int_{x_{j-1}}^x p dx - \frac{\omega u^2}{2} \right. \\ & + u \left[\omega f_{j1}(x) - i(-1)^l p \frac{df_{j1}}{dx} \right] + \frac{i}{2} \int_{x_{j-1}}^x \left[\omega f_{j1}(x) \right. \\ & \left. \left. - i(-1)^l p \frac{df_{j1}}{dx} \right]^2 \frac{dx}{p} \right\}, \quad (A1) \end{aligned}$$

where D_{j1} is a free parameter and $x_0 = x_1$. By p in the classically-forbidden range, we will understand $i|p|$. In the classically allowed region waves incident in the positive direction correspond to odd l in (A1), while waves incident in the negative direction correspond to even l . The function f_{j1} obeys the differential equation

$$p(pf_x)_x + \omega^2 f + W(x) = 0$$

and takes the form

$$\begin{aligned} f_{j1}(x) = & A_{j1}^+ \exp \left(i\omega \int_{x_{j-1}}^x \frac{dx}{p} \right) + A_{j1}^- \exp \left(-i\omega \int_{x_{j-1}}^x \frac{dx}{p} \right) \\ & + \frac{1}{2i\omega} \left[\exp \left(-i\omega \int_{x_{j-1}}^x \frac{dx}{p} \right) \int_{x_{j-1}}^x \exp \left(i\omega \int_{x'}^x \frac{dx'}{p} \right) W \frac{dx'}{p} \right. \\ & \left. - \exp \left(i\omega \int_{x_{j-1}}^x \frac{dx}{p} \right) \int_{x_{j-1}}^x \exp \left(-i\omega \int_{x'}^x \frac{dx'}{p} \right) W \frac{dx'}{p} \right], \quad (A2) \end{aligned}$$

where A_{j1}^{\pm} are free parameters.

We introduce the following conventions:

$$E_{jk}^{\pm} = \exp \left(\pm i\omega \int_{x_j}^{x_k} \frac{dx}{p} \right),$$

$$G_{jk}^{\pm} = \mp \frac{1}{2i\omega} \int_{x_j}^{x_k} \exp \left(\pm i\omega \int_{x_j}^x \frac{dx}{p} \right) W \frac{dx}{p},$$

$$G_{45}^{\pm} = \mp \frac{1}{2i\omega_{x_4}} \int_{x_4}^{\infty} \exp \left(\pm i\omega \int_{x_4}^x \frac{dx}{p} \right) [W(x) - W_5] \frac{dx}{p},$$

$$A_{2,4} = -(2\omega \sin \omega T_2)^{-1} \int_{x_2}^{x_4} \cos \left(\omega \int_{x_2}^x \frac{dx}{p} \right) W \frac{dx}{p},$$

$$\xi_{jk}^{\pm} = \frac{1}{2\omega} \int_{x_j}^{x_k} \int_{x_j}^{x_k} \exp\left(\pm i\omega \int_x^{x'} \frac{dx}{p}\right) \frac{W(x')W(x)}{p(x')p(x)} dx' dx,$$

$$\xi_{45}^{\pm} = \frac{1}{2\omega} \int_{x_4}^{\infty} \int_{x_4}^{\infty} \exp\left(\pm i\omega \int_x^{x'} \frac{dx}{p}\right) \cdot \frac{[W(x') - W_5][W(x) - W_5]}{p(x')p(x)} dx' dx,$$

$$C_2^{\pm} = A_2 E^{\mp} + G_{12}^{\mp}, \quad C_4^{\pm} = A_4 E_{34}^{\pm} - G_{34}^{\mp} E_{34}^{\pm},$$

where $x_5 = \infty$. then the parameters entering into the expression for the resonant tunneling amplitude will be determined by the relations

$$\begin{aligned} B_2^{\pm} &= C_2^{\pm} - G_{10}^{\mp}, & B_4^{\pm} &= C_4^{\pm} - G_{45}^{\mp} + W_5/2\omega^2, \\ \alpha_j &= 2\omega (C_j^+ - C_j^-)^2, \\ \beta_2 &= 2\omega (C_2^+)^2 - 4\omega C_2^+ G_{12}^+ - 2\omega A_2^2 - 2\xi_{12}^-, \\ \beta_4 &= 2\omega (C_4^-)^2 + 4\omega A_4 G_{34}^- - 2\omega A_4^2 + 2\xi_{34}^+, \end{aligned} \quad (A3)$$

$$\begin{aligned} \eta_2 &= 2\omega C_2^+ G_{10}^+ - 4\omega C_2^+ G_{12}^+ - 2\omega A_2^2 - 2\xi_{12}^- - \xi_{10}^+, \\ \eta_4 &= 2\omega \left(C_4 + \frac{W_5}{2\omega^2} \right) G_{45}^- + 4\omega A_4 G_{34}^- - 2\omega A_4^2 - \frac{W_5 C_4^-}{\omega} \\ &\quad - \xi_{45}^- + 2\xi_{34}^+ - \frac{W_5^2}{4\omega^3} + \frac{iW_5}{\omega^2} \int_{x_4}^{\infty} [W(x) - W_5] \frac{dx}{p}. \end{aligned}$$

The parameter entering into the quantization rule (4.10) takes the form

$$\lambda(W) = \text{Re} (A_2 G_{23}^- + \xi_{23}^-) / T_3. \quad (A4)$$

¹¹We are not summing over the levels in the well, but rather are assuming for simplicity that the quasilevels corresponding to one well make the principal contribution to the current.

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