Green-Schwarz action and loop calculations in superstrings

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Arguments are presented which show that the Green-Schwarz approach with explicit ten-dimensional supersymmetry and two-dimensional reparametrization invariance and without spin $\frac{1}{2}$ or $\frac{3}{2}$ fields on the world sheet may be useful to calculate loop corrections in superstring theory. We show how the Polyakov anomaly is canceled and we reproduce the result for the one-loop contribution to the four-particle amplitude.

1. INTRODUCTION

Great efforts were recently undertaken to develop a technique for multi-loop calculations in superstring theory. Most of the attempts are based on a two-dimensional super-generalization of the Polyakov integral. Here the multi-loop calculation is broken down into the following stages: a super-Riemannian surface of genus p is constructed with a given spin structure e; a space of super-moduli is constructed with a Mumford measure on it; integration over the odd moduli and summation over the spin structures e are carried out and the contributions of the left and right sectors are combined. The space-time supersymmetry (and along with it, finiteness of the amplitudes and vanishing of the statistical sums) is re-established only after summation over e.

At this time it is already clear that this program is not as simple as it seems. All its "constructive" parts have to a larger or smaller degree been realized. Explicit constructions of super-Riemannian surfaces and of Mumford's super-measure exist.[1] It is easy to perform at least the "naive"[2)] integration over the odd moduli[2] and obtain the obvious "naive" measures on the space of even moduli, which were proposed in Ref. 3 and explicitly constructed in Ref. 4.

Unfortunately the answer depends on the concrete choice of the odd moduli,[5,6] and the correct choice, which gives rise to the superstring while preserving the simple formulas of Ref. 4, is as yet unknown (problems appear already for $p = 2$).

The method of summation over the spin structures is also not given *a priori*. Moreover, it turns out to be not so simple to come up with it, if it is desired that space-time supersymmetry should appear with modular invariance simultaneously preserved (see Ref. 6 for a more detailed discussion of these difficulties and ways for overcoming them). In any event the currently available formalism is extraordinarily involved. This fact is particularly unsatisfactory because all answers for physical amplitudes in superstring theory are very simple. First of all, all statistical sums and 1-, 2-, and 3-point functions are equal to zero in all orders of perturbation theory (this was shown starting from general considerations in Ref. 7 and explicitly demonstrated in the two-loop case in Ref. 6). The expressions for the nonzero amplitudes also seem uncomplicated.

The scattering amplitude for four vector particles in the one-loop approximation is given accurate to within a kinematic factor by

$$\int \frac{d^2\tau}{(\operatorname{Im}\tau)^6} \int d^2z_1 \ldots \int d^2z_4 \langle \exp\{ip_1X(z_1)\}\ldots\exp\{ip_4X(z_4)\}\rangle. \tag{1}$$

The standard derivation of this exceptionally simple formula is quite laborious (we leave aside the operator method in its usual form; it demands even greater efforts and in practice does not generalize to higher loops). Starting from the super-generalization of the Polyakov integral and noting that only terms of the $\psi\tilde{\psi}$ type in the vertex operators[3)]

$$\int d^2z_\alpha |\psi\tilde{\psi}(z_\alpha)|^2 \exp\{ip_\alpha X(z_\alpha)\}$$

contribute to the answer (due to the GSO projection), it is easy to write down the following starting formula:

$$\int \frac{d^2\tau}{(\operatorname{Im}\tau)^6} \prod_{\alpha=1}^{4} d^2z_\alpha \langle \exp\{ip_1X(z_1)\}\ldots\exp\{ip_4X(z_4)\}\rangle \cdot \left|\frac{\det\bar\partial_2}{(\det\bar\partial_0)^5}\right|^2 \left|\sum_e \varepsilon_e \frac{(\det_e\bar\partial_{1/2})^5}{\det_e\bar\partial_{3/2}} \langle\psi\tilde{\psi}(z_1)\ldots\psi\tilde{\psi}(z_4)\rangle\right|^2. \tag{2}$$

The two factors following the correlator turn ultimately into unity, but this can be seen only upon making use of the explicit expressions for all the objects entering (2):

$$(\det\bar\partial_0)^{3/2} = (\det\bar\partial_0)^{1/2}\det\bar\partial_2 = \theta_*'(0),$$
$$(\det\bar\partial_0)^{1/2}\det_e\bar\partial_{1/2} = (\det\bar\partial_0)^{1/2}\det_e\bar\partial_{3/2} = \theta_e(0),$$
$$\langle\psi(x)\tilde{\psi}(y)\rangle_e = \frac{\theta_e(x-y)\theta_*'(0)}{\theta_e(0)\theta_*(x-y)},$$
$$\sum_e \varepsilon_e\theta_e^4(0) \equiv 0,$$
$$\sum_e \varepsilon_e\theta_e^2(z_1-z_2)\theta_e^2(z_3-z_4) = \frac{1}{2}\theta_*^2(z_1-z_2)\theta_*^2(z_3-z_4),$$
$$\sum_e \varepsilon_e\theta_e(z_1-z_2)\theta_e(z_2-z_3)\theta_e(z_3-z_4)\theta_e(z_4-z_1) = \frac{1}{2}\theta_*(z_1-z_2)\theta_*(z_2-z_3)\theta_*(z_3-z_4)\theta_*(z_4-z_1).$$

(All these well-known expressions are given here in a form which readily generalizes to the case of higher genus; see, e.g., Ref. 8 with respect to formulas for the determinants and Ref. 9 with respect to the Riemann identities.)

This calculation becomes even more complicated in the case of four-point amplitudes involving fermions [the answer is practically indistinguishable from (1), however Ref. 10, devoted to its calculation, contains over 40 pages and utilizes a nontrivial generalization of the Riemann identities].

Unfortunately the answers for the nonzero amplitudes,

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starting already with two loops, remain as yet unknown. There is, however, no doubt that they will look much simpler than all the intermediate formulas encountered in the conventional method for their derivation (the substantial simplifications that occur in the last stages of the calculation, when summation over the spin structures is carried out, were shown in Ref. 6). It is likely that everyone who has ever tried to calculate a supersymmetric amplitude has experienced the feeling, that many unnecessary steps are taken in the intermediate stages, giving rise to unwarranted complications.

It is less obvious, but in our opinion very likely, that these substantial complications should be blamed on the desire to introduce half-integer differentials—fields with half-integer two-dimensional spin. The impression is created that in the answers there is no memory of such fields (and not only that the dependence on the spin structure e disappears)—the final formulas contain, apparently, only Prim bidifferentials and other objects, naturally arising in a theory of integer-spin fields on Riemann surfaces. It is hard to rid oneself of the feeling that a formulation of superstring theory ought to exist, in which spinors on the world sheet and super-ghosts are absent.

Of course everybody knows that such a formalism exists—the Green-Schwarz action,[11] containing only scalars on the world sheet and usual (with integer spin) ghosts.

It was our aim in this Introduction to clarify why those who have ever used any standard Neveu-Schwarz-Ramond approach might wish to develop an alternative formalism.

Until now the only optimistic opinion on the functional integral in the Green-Schwarz formalism was expressed by Carlip.[12 4)] Due to difficulties with the local fermion symmetry he made use of some rather special devices in place of the normal procedure of gauge fixing. Below we shall depend on a method developed in Ref. 14 for fixing the gauge in this theory and will attempt to show that it is not as complicated as it is conventionally assumed. We shall also demonstrate impressive parallels between the Green-Schwarz and Neveu-Schwarz-Ramond approaches.

2. FUNCTIONAL INTEGRAL FOR THE GREEN-SCHWARZ SUPERSTRING

The first objection to the use of the Green-Schwarz action consists in the observation that this action is not quadratic in the fields and all the advantages of the Polyakov approach to string theory as a theory of free fields on Riemann surfaces appear to be lost. However this objection seems not very convincing.

The classical Green-Schwarz action for the heterotic string has the form

$$S=\int d^2z\{-{}^1/_2 g^{1/2}g^{\alpha\beta}\Pi_\alpha{}^\mu\Pi_{\beta\mu}+\varepsilon^{\alpha\beta}\partial_\alpha X^\mu\bar\theta\gamma^\mu\partial_\beta\theta+\mathscr{L}'\},\qquad(3)$$

where $\mathscr{L}'$ describes the gauge degrees of freedom of the heterotic string ("left sector")

$$\Pi_\alpha{}^\mu\equiv\partial_\alpha X^\mu-i\bar\theta\gamma^\mu\partial_\alpha\theta,$$

θ is an anticommuting ten-dimensional 16-component Majorana-Weyl spinor, which is a scalar on the world sheet. In the Lagrangian quantization of this theory problems arise in the general case, connected with an infinite sequence of ghosts with ghosts and with algebras that fail to close off shell. However, the analysis carried out in Ref. 14 has shown that it is possible to fix a special gauge, e.g.,

$$\gamma^+\theta=\sum_{a=1}^{2p-2}\delta^2(z-Q_a),\qquad g_{\alpha\beta}=\rho g_{\alpha\beta}^{(m)},$$

where $g_{\alpha\beta}^{(m)}$ is some background metric, depending on a finite number of moduli $m=(m_1,...,m_{3p-3})$. In conformal coordinates $\bar z$, z, in which

$$g_{zz}^{(m)}=g_{\bar z\bar z}^{(m)}=0,\quad g_{z\bar z}^{(m)}={}^1/_2,$$

the functional integral in fixed gauge turns out to be equal to

$$\sum_\chi e^{-4\pi\lambda\chi}\int dm\int DX^\mu D\theta\,|Db\,Dc|^2|\operatorname{Det}u_{\bar z}|^{-4}$$
$$\cdot\exp\Big\{-\int d^2z(\partial_z X^\mu\partial_{\bar z}X^\mu+\bar\theta\gamma^-\partial_{\bar z}X^+\partial_z\theta+\mathscr{L}'+b\bar\partial c$$
$$+\bar b\partial\bar c+O(Q_a)\Big\},\qquad(4)$$

where χ is the Euler characteristic and $O(Q_a)$ are inessential corrections. Here b, c, $\bar b$, $\bar c$ are the usual reparametrization ghosts. We have also introduced the notation $u_{\bar z}=\partial_{\bar z}X^+$. In (4) $(\operatorname{Det}u_{\bar z})-4$ enters the local integration measure and, at least formally, ensures that the theory does not depend on how the local fermion symmetry is fixed.[5)] It will be seen below that this symmetry, as well as conformal symmetry, is in principle anomalous. However the anomalies are easily removed.

Even in the action (4) it is seen that the cubic interaction will cause no problem, since only propagators of the type $\langle X^+X^-\rangle$ exist, and the expectation value of any number of X^+ fields without X^- vanishes (we recall that the Green-Schwarz action makes sense only in the ten-dimensional Minkowski space). We transform (4) to a simpler and more useful form by making use of the fact that the $SO(8)$-spinor $\theta(\gamma^+\theta=0)$ can be represented as two $SU(4)$-spinors, $\theta^k\equiv\eta^k$ and θ_k, $k=1,...,4$. Then

$$\bar\theta\gamma^-u_{\bar z}\partial_z\theta\Rightarrow 2\theta^k u_{\bar z}\partial_z\theta_k-\theta^k\theta_k\partial_z u_{\bar z}.$$

The second term $\theta^k\theta_k\partial\bar\partial X^+$ can be removed by redefining the field X^- (corresponding to going over to a chiral supersymmetric field basis). Now our functional integral in the conformal gauge looks as follows (we omit for the moment the integral over metrics on the world sheet):

$$\int DX^\mu D\eta_{\bar z}{}^k D\theta\,|DbDc|^2\exp\Big\{-\int d^2z(\partial X^\mu\bar\partial X^\mu+\eta_{\bar z}{}^k\partial_z\theta_k$$
$$+\mathscr{L}'+\mathscr{L}_{bc})\Big\},\qquad(5)$$

$\eta_{\bar z}{}^k\equiv u_{\bar z}\eta^k$.

The net result of these considerations may be formulated as follows. One may start from an action, invariant with respect to two-dimensional general-coordinate transformations, of the type

$$\begin{gathered}-{}^1/_2\int g^{1/2}(g^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\mu+P_-{}^{\alpha\beta}\theta_\alpha{}^k\partial_\beta\theta_k+\mathscr{L}')\,d^2z,\\ P_-{}^{\alpha\beta}=g^{\alpha\beta}-i\varepsilon^{\alpha\beta}/g^{1/2}=e_{\bar z}{}^\alpha e_z{}^\beta\end{gathered}\qquad(6)$$

and then fix reparametrization invariance in the usual way, by choosing the conformal gauge and introducing the ghosts b and c. It is of great relevance that the four space-time spinors θ_k are scalars (0-differentials) from the point of view of the world sheet, and the other four spinors $\eta_{\bar z}{}^k$ are anti-self-dual vectors (1-differentials); herein lies the key distinction from the light-cone formalism with eight spinors that are 1/2-differentials on the world sheet.[11]

The ten-dimensional supersymmetry is realized as follows:

$$\delta\theta=\varepsilon-\Pi^\mu\gamma_\mu\gamma^+\varepsilon(u_{\bar z})^{-1},$$
$$\delta X^\mu=i\varepsilon\gamma^\mu\theta-i\bar\theta\gamma^\mu\Pi^\lambda\gamma_\lambda\gamma^+\varepsilon(u_{\bar z})^{-1},$$
$$\delta g_{\alpha\beta}=0, \qquad (7)$$

or, in more detail,

$$\delta\theta_{\dot a}=0,\quad \delta\theta_a=\varepsilon_a+P_i\gamma^i_{a\dot a}\varepsilon_{\dot a},$$
$$\delta X^+=0,\quad \delta X^i=\varepsilon_{\dot a}\gamma^i_{\dot a a}\theta_a,\quad \delta X^-=\varepsilon_a\theta_a+P_i\delta X^i,$$
$$P_i=\partial_{\bar z}X^i(u_{\bar z})^{-1},\ \gamma^-\gamma^+\gamma_\mu\Pi_{\bar z}{}^\mu\gamma^+\gamma^-=u_{\bar z}\gamma^-,\ \Pi_{\bar z}{}^\mu=e_{\bar z}{}^\alpha\Pi_\alpha{}^\mu.$$

Passage to the transformation rules in terms of the variables θ_k, $\eta_{\bar z}{}^k$, $\tilde X^-=X^--\theta^k\theta_k$ presents no problems.

3. CANCELLATION OF THE CONFORMAL ANOMALY AND THE ANOMALY IN THE GAUGE FERMION SYMMETRY

It could turn out that in the action (6) there is a problem with the Polyakov anomaly in the Weyl symmetry of the action. Indeed, if the conventional real scalar X contributes 1/2 to the coefficient in front of the Liouville action $(48\pi)^{-1}\int R\Delta^{-1}Rd^2z$, then the reparametrization ghosts contribute -13. In the standard formalism one has in addition ten spin 1/2 fermions that contribute $(-d/2)(-1/2)=+5/2$ and reparametrization "superghosts" with spin 3/2 whose contribution equals $+11/2$. (The general formula for the contribution of a complex spin j field is: $C_j=6j^2-6j+1$.) The complete coefficient equals (for $d=10$)

$$\frac{d}{2}(+1)-13+\frac{d}{2}\left(-\frac{1}{2}\right)+\frac{11}{2}=\frac{3d}{4}-\frac{15}{2}=0.$$

In our case the spinors on the world sheet and the superghosts are absent and we have instead the four pairs of fields θ, $\eta_{\bar z}$.

If these fields were conventional 0- and 1-differentials, then their contribution to the anomaly would coincide with the contribution of the X-fields (but in place of $+5=+d/2$ one would obtain -4; the minus sign has to do with the opposite statistics). Then the coefficient in front of the anomalous action would be equal to $d/2-13-4=-12\neq0$. Naturally, this discussion is in error.

It turns out that our change of variables, transforming (4) into (5), presupposes that the regulator fields $\eta_{\bar z}$ do not have the conventional norm

$$\|\eta_{\bar z}\|^2=\int\eta_{\bar z}\bar\eta_{\bar z}\,d^2z,$$

characteristic of a 1 — differential. Instead their norm is induced from the norm of the 0-differentials θ:

$$\|\theta\|^2=\int d^2z\,g^{1/2}\theta\bar\theta\to\|\eta_{\bar z}\|^2=\int d^2z\frac{g^{1/2}}{|u_{\bar z}|^2}\eta_{\bar z}\bar\eta_{\bar z}.$$

Therefore the regulator integral has the form

$$\int|D\eta_{\bar z}|^2|D\theta|^2\exp\Big|-\int\eta_{\bar z}\partial\theta+\bar\eta_{\bar z}\bar\partial\bar\theta+M^2\frac{g^{1/2}}{|u_{\bar z}|^2}\eta_{\bar z}\bar\eta_{\bar z}.$$

Integration over the fields $\eta_{\bar z}$ yields

$$\int|D\theta|^2\exp\Big(-\int g^{1/2}\bar\theta\Delta\theta/M^2\Big),$$

with the Laplace operator equal to

$$\Delta=\frac{1}{g^{1/2}}\partial\frac{|u|^2}{g^{1/2}}\bar\partial. \qquad (8)$$

It is easily verified that the anomaly for the generalized Laplace operator

$$\Delta_{f,g}=f(z)\partial h(z)\bar\partial$$

(this is the Laplace operator for $\bar\partial$ that acts from a space with norm $\|c\|^2=\int f^{-1}|c|^2$ to a space with norm $\|b\|^2=\int h^{-1}|b|^2$) is described by the Liouville action

$$\exp\Big\{-\frac{1}{48\pi}\int\Big[\frac{|\partial f|^2}{f^2}-4\frac{\partial f\bar\partial h}{fh}+\frac{|\partial h|^2}{h^2}\Big]\Big\}. \qquad (9)$$

For the usual operator $\bar\partial_j$ one has in the conformal gauge $f=\rho^{j-1}$, $h=\rho^{-j}$ and (9) leads to the well-known answer:

$$\exp\Big(-\frac{C_j}{48\pi}\int|\partial\ln\rho|^2\Big),$$

$$C_j=(-j)^2-4(-j)(1-j)+(1-j)^2=6j^2-6j+1.$$

In our case (8) one has $f=\rho^{-1}$, $h=|u|^2\rho^{-1}$ and (9) gives

$$\exp\Big\{-\frac{1}{48\pi}\int[-2|\partial\ln\rho|^2-(2\ln\rho+\ln|u|^2)\partial\bar\partial\ln|u|^2]\Big\}. \qquad (10)$$

The first term enters with coefficient -2 (compare this with $c=+1$ for conventional scalars). This is precisely what is needed to cancel the Polyakov anomaly in (5):[6]

$$d/2(+1)-13-4(-2)=0 \text{ for } d=10.$$

The remaining two terms in (10) indicate the existence of additional conformal anomaly and anomaly in the gauge fermion symmetry. We recall that dependence on ρ violates conformal invariance, and dependence on $u_{\bar z}$ is in fact dependence on the method of fixing local fermion symmetry. The gauge condition may be written in terms of two null vectors[14]

$$n^\mu,\ m^\mu,\ n^2=m^2=mn-{}^1/_2=0,\ \gamma_\mu n^\mu\theta=\gamma_\mu n^\mu C_z=0.$$

Here C_z is a first generation ghost and

$$u_{\bar z}=\gamma_\mu m^\mu\gamma_\nu n^\nu\gamma_\lambda\Pi_{\bar z}{}^\lambda\gamma_\delta n^\delta\gamma_\eta m^\eta.$$

However these new anomalous terms in (10), dependent on $u_{\bar z}$, are proportional to $\bar\partial u=\partial\bar u=\partial\bar\partial X^+$ and can be eliminated by an appropriate shift of the field X^- in the action (5).

This calculation shows how the Polyakov anomaly cancellation takes place in the Green-Schwarz formalism.

Actually, it can be shown that the situation is no different in more general gauges, for example under the conditions $\gamma^+\partial_z\theta=0$ discussed in Ref. 14, which do not violate the linear realization of space-time supersymmetry. In that case the quantum action equals

$$\mathscr{L}_{cl}+\bar\pi^z\gamma^+\partial_z\theta+\bar C^z\gamma^-\partial_zC+b\bar\partial c+\bar b\partial\bar c. \qquad (11)$$

Here $\mathscr{L}_{cl}$ is the classical Green-Schwarz action (3) for the heterotic string, containing interaction terms of third and fourth degree; $\hat{C}_\alpha{}^z C_\alpha$, $\pi_\alpha{}^z$ are the Fadeev-Popov and Nielsen-Kallosh ghosts for local fermion symmetry. The local integration measure now equals $(\text{Det } u_{\bar z})^{-4}$, where $u_{\bar z} = \partial_{\bar z} X^+ - i\theta\gamma^+ \partial_{\bar z}\theta$. The corresponding functional integral is manifestly invariant under global supersymmetry transformations

$$\delta\theta=\varepsilon,\ \delta X^\mu = i\bar\varepsilon\gamma^\mu\theta.$$

It turns out that in the action (11) a change in variables is possible resulting in significant simplifications: the new action equals

$$\partial X^\mu \bar\partial X^\mu + \eta_{\bar z}^k \partial_z \theta_k + \pi_{\bar z}^{\dot a}\partial_z\theta_{\dot a} + \hat{C}_{\bar z}^a \partial_z C_a + b\bar\partial c + \bar b \partial \bar c. \qquad (12)$$

The global symmetry is now realized as follows:

$$\delta\theta_k = \varepsilon_k,\quad \delta\theta_{\dot a} = \varepsilon_{\dot a},\quad \delta\eta_{\bar z}{}^k = u_{\bar z}\varepsilon^k,$$
$$\delta X^- = i\varepsilon^k\theta_k,\quad \delta X^+ = i\varepsilon^{\dot a}\theta_{\dot a},\quad \delta X^i = i\varepsilon_a \gamma^i_{a\dot a}\theta_{\dot a}$$
$$\delta\pi_{\bar z}^{\dot a} = \varepsilon^{\dot a}\partial_{\bar z}X^- + \varepsilon^k(\partial_{\bar z}\theta_{\dot a})\theta_k + \varepsilon_a\gamma^i_{a\dot a}\partial X^i.$$

The structure of the action (12) shows that in this gauge the Polyakov anomaly is absent, as before. The contributions to the anomaly of the fields X, $\eta_{\bar z}{}^k$, θ_k and of the reparametrization ghosts cancel for reasons explained above. The contribution to the anomaly of the Fadeev-Popov ghosts $\hat{C}_{\bar z}{}^a$, C_a are canceled by the contributions of the Nielsen-Kallosh ghosts $\pi_{\bar z}{}^{\dot a}$ and the field $\gamma^+\theta$, denoted by $\theta_{\dot a}$ in (12). This can be seen from the fact that $\hat{C}_{\bar z}{}^a$ and $\pi_{\bar z}{}^{\dot a}$ are 1-differentials, C_a and $\theta_{\dot a}$ are 0-differentials (with conventional norms), and the statistics of $\hat{C}_{\bar z}{}^a$, C_a and $\pi_{\bar z}{}^{\dot a}$, $\theta_{\dot a}$ are different.

We also note that if in the general case (9) the Weyl and analytic anomalies are related as before, this could mean that the measure on the space of conventional moduli, corresponding to the superstring, is expressed in terms of holomorphic sections, and therefore the nonholomorphic structures that arise in the standard approach should cancel upon summation over the spin structures even in the case of the amplitude. Such a proposition was made in Refs. 15 and 5.

4. ZERO MODES AND ONE-LOOP CALCULATIONS

The action (5) is quadratic and therefore the functional integral reduces to a finitely-multiple integral over the moduli space of ratios of determinants of the corresponding differential operators, multiplied by the correlator of vertex operators. The contribution of nonzero modes to the ratio of determinants in one loop equals

$$\frac{d^2\tau}{(\text{Im}\,\tau)^6}\frac{|\det{}'\,\partial_2|^2 \bar F}{(\det{}'\,\partial_0)^5}(\det{}'\,\partial_0)^4. \qquad (13)$$

We shall not explain in detail the well-known reasons for the appearance of the various contributions in this formula:

$$\frac{d^2\tau}{(\text{Im}\,\tau)}|\det{}'\,\partial_2|^2$$

is connected with the determinant of the Fadeev-Popov reparametrization ghosts (with the conformal Killing vectors accurately taken into account);

$$\bar F/(\text{Im}\,\tau)^5(\det{}'\partial_0)^5$$

is the contribution of the right scalars and the left sector of the heterotic string. The last factor $(\det{}'\,\partial_0)^4$ is connected with the nonzero modes of the fields $\eta_{\bar z}{}^k$ and θ_k. For type 2 superstrings the contributions of the left and right sectors coincide and the formula takes on the form

$$\frac{d^2\tau}{(\text{Im}\,\tau)^6}\left|\frac{\det{}'\,\partial_2}{(\det{}'\,\partial_0)^5}(\det{}'\,\partial_0)^4\right|^2. \qquad (14)$$

In one loop all the determinants coincide (we note that for the nonsingular metrics admissible in that case of the type $g = dzd\bar z$ gravitational anomalies are absent in the determinants and correlators), and the required ratio of determinants turns into unity.

For this reason the key role in the evaluation of the scattering amplitude for a small number of particles in the Green-Schwarz formalism is played by the zero modes of the fields θ. These zero modes are constants on the world sheet. We note that the fields $\eta_{\bar z}{}^k$ which we introduced previously are not entirely arbitrary 1-differentials: they should be expressible in the form $\eta^k u_{\bar z}$ with $u_{\bar z} = \partial_{\bar z}X^+$, and therefore they cannot, for example, be equal to (anti) holomorphic 1-differentials, which at first sight correspond to zero modes of the action $\int\eta_{\bar z}{}^k\,\partial_z\theta_k$. The genuine zero modes of the fields $\eta_{\bar z}{}^k$ have the form $\eta_{0\bar z} = \eta_0{}^k u_{\bar z}$, where $\eta_0{}^k$ are constants on the world sheet. The action (5) is unchanged under the shift $\eta_{\bar z}{}^k \to \eta_{\bar z}{}^k + \eta_{0\bar z}{}^k$ provided the field X^- is simultaneously shifted by $\eta_0{}^k\theta_k$. If in the action (5) one performs only the shift of the field X^- by $\eta_{0k}\theta_k$ then the zero mode of $\eta_{\bar z}{}^k$ becomes manifest: for the action

$$\mathscr{L} = \partial X^\mu \bar\partial X^\mu - \eta_0{}^k\partial_z\theta_k\partial_{\bar z}X^+ + \eta_{\bar z}{}^k\partial_z\theta_k$$

the equation of motion has the form

$$\partial(\eta_{\bar z}{}^k - \eta_0{}^k u_{\bar z}). \qquad (15)$$

The field θ_k also has a constant on the world sheet zero mode θ_{0k}.

The appearance of these zero modes for the Grassmann fields θ explains the vanishing of the 0-, 1-, 2- and 3-point functions in the theory of superstrings; see also Ref. 12. In the Neveu-Schwarz-Ramond formalism fermion zero modes for certain even spin structures are absent, and this simple explanation has to be replaced by an involved analysis of the consequences of GSO projection, which rests on at least a nontrivial generalization of the Riemann identity and the Riemann theorem on zeros. In actuality this analysis has been carried out so far only for $p = 2$.

Even the four-point functions are nonvanishing. To calculate them we make use, following Ref. 11, of the vertex in the 10-dimensional supersymmetric form, but write it also in a 2-dimensional covariant form:

$$\int d^2z\, g^{1/2} g^{z\bar z}\rho_{\mu\nu}^{B,F}\,\eta_{B,F}^{\nu\lambda}\Pi_{\bar z\lambda}\Pi_{z\nu}e^{ipX(z)}. \qquad (16)$$

Here $\eta^{\mu\nu}$ are functions of $\bar\theta_\gamma{}^{\mu\nu\lambda}\theta p_\lambda$, and $\rho_{\mu\nu}$ are the second rank polarization tensors for the supergravity multiplet:

$$\rho_{\mu\nu}{}^B = \tau_\mu\bar\tau_\nu,\ \rho_{\mu\nu}{}^F = \bar\theta\gamma_\mu u\bar\tau_\nu.$$

As usual, in order to ensure the absence of the conformal anomaly that arises, generally speaking, when such a vertex is introduced, it is necessary that:

a) the condition of masslessness be satisfied, $p^2 = 0$,

b) the vertex should be constructed from the (1,1)-differential $\Pi_z \Pi_{\bar{z}}$.

For specific calculations we follow the approach of Ref. 11, connected with the transition to the limit $p^+ \to 0$. Then the right side of the vertex (16) turns into

$$V_B = \zeta_i(\Pi_{\bar{z}}^i - p^j\bar{\theta}\gamma^{ij}\theta u_{\bar{z}})$$

in the case of bosons, and into

$$V_F = u^a\theta_a u_{\bar{z}} + u^{\dot{a}}\gamma^i_{a\dot{a}}\theta^a(\Pi_{\bar{z}}^i - p^j\bar{\theta}\gamma^{ij}\theta u_{\bar{z}})$$

in the case of fermions.

It is seen from these formulas that four-boson and four-fermion amplitudes and amplitudes with two bosons and two fermions contain products $\eta_{\bar{z}}^{k_1}\ldots\eta_{\bar{z}}^{k_4}\theta_{l_1}\ldots\theta_{l_4}$, with fermion vertices taken in the $u_a u_{\dot{a}}$ combinations. Then all eight fields $\eta_{\bar{z}}^k$ and θ_k in the functional integral "eat" all fermion zero modes in the action.

It may seem that when $\eta_{\bar{z}}^k$ and θ_k are replaced in vertex operators of this kind by constant zero modes, they will yield in the integrand in (1) the superfluous product

$$\prod_{\alpha=1}^{4} u_{\bar{z}}(z_\alpha).$$

Of course, this is not so: the answer is precisely equal to (1). The reason is that the factors $u_{\bar{z}}$ arise from two sources: they enter the measure

$$D\theta^k D\theta_k/(\mathrm{Det}\, u_{\bar{z}})^4,$$

ensuring invariance with respect to gauge fermion symmetry, and they also arise upon integration over the fields θ with the action $\int\theta^k u_{\bar{z}}\partial_z\theta_k$. These contributions compensate each other. But if one introduces under the sign of functional integration $\theta^k u_{\bar{z}}\theta_k(z_\alpha)$, then the factor $u_{\bar{z}}(z_\alpha)$ in Det $\{u_{\bar{z}}\partial\}$ is absent—instead precisely such a factor is supplied from the vertex operator. (In other words, given the integral over two Grassmann variables ψ and χ,

$$\int d\psi\, d\chi \exp\{\psi u\chi\} = u,$$

then

$$\int d\psi\, d\chi\, \psi u\chi \exp\{\psi u\chi\}$$

also equals u, and not u^2.) In this manner (5) reproduces without particular difficulties the four-particle amplitude (1). We shall also give expressions for the kinematic factors, which arise in this calculation from the right degrees of freedom under standard assumptions on the kinematics of the external states.[11]

$$K = t_{i_1j_1\ldots i_4j_4}A^{i_1\ldots i_4j_1\ldots j_4},$$

where

$$A^{i_1\ldots j_4} = \tau_1^{i_1}\ldots\tau_4^{i_4}p_1^{j_1}\ldots p_4^{j_4}$$

in the case of the four-boson amplitude;

$$A^{i_1\ldots j_4} = \tau_1^{i_1}\tau_2^{i_2}p_1^{j_1}p_2^{j_2}\bar{u}_3\gamma^{i_3}\gamma^{i_4j_4}u_4p^{j_3}$$

for the scattering amplitude of two bosons and two fermions and

$$A^{i_1\ldots j_4} = \bar{u}_1\gamma^{i_1}\gamma^{i_2j_2}u_2p_1^{j_1}\bar{u}_3\gamma^{i_3}\gamma^{i_4j_4}u_4p_3^{j_3}$$

for the four-fermion amplitude. The tensor $t_{i_1\ldots j_4}$ is defined in Ref. 11 and equals

$$t^{ijklmnpq} \equiv \mathrm{tr}(R_0^{ij}R_0^{kl}R_0^{mn}R_0^{pq}) = -{}^1/_2\varepsilon^{ijklmnpq} + \ldots$$

Of course, a number of problems must still be solved in order to make use of the Green-Schwarz formalism in multiloop calculations. One of them has to do with the fact that one cannot pass to the gauge $\gamma^+\theta = 0$, which we used in the one-loop calculation, because the gauge transformation $\delta\theta = u_{\bar{z}}k_z$ does not change θ at those points on the world sheet where $u_{\bar{z}} = \partial_{\bar{z}}X^+$ vanishes. Such points exist even for the field X^+ in the general position, since for topological reasons the field of the 1-differential $\bar{\partial}X^+$ has $2p-2$ zeros on a Riemann surface of genus p. One's attention is called to the intriguing coincidence with the $2p-2$ number of odd moduli of a super-Riemann surface of genus p, which play a key role in the Neveu-Schwarz-Ramond approach. There may also occur "accidental" zeros of $\bar{\partial}X^+$, not required for topological reasons. However the corresponding fields X^+, evidently, form a set of measure zero. To be truthful one should mention the problem (present even in the case of the torus $p=1$) due to the fact that the exact differential $\bar{\partial}X^+$ must have zeros on a closed surface because of being exact. It is possible that these zeros are not terrible, since one can use a different gauge,[7] $\gamma^+\theta = \xi = \mathrm{const} \neq 0$, and every field on the torus is expressible as $\xi + (\partial_{\bar{z}}X^+)k_z$, for some k_z and some constant ξ with given $\partial_{\bar{z}}X^+$. It turns out that in this new gauge the action coincides with (5) and (6) after an appropriate shift of the variable X^i.

An important step would be also the use of the Green-Schwarz action in a manifestly supersymmetric gauge: Ref. 14 and equations (11) and (12). The zero modes for this action have the form

$$\theta_k = \theta_{0k} = \mathrm{const}, \quad \eta_{\bar{z}}^k = \eta_0^k u_{\bar{z}},$$

where now

$$u_{\bar{z}} = \partial_{\bar{z}}X^+ - i\bar{\theta}\gamma^+\partial_{\bar{z}}\theta.$$

In the proof of this assertion one needs not only the shift $X^- \to X^- - \eta_0^k\theta_k$, but also

$$\pi_{\bar{z}}^{\dot{a}} \to \pi_{\bar{z}}^{\dot{a}} + \eta_0^k(\theta_k\partial_{\bar{z}}\theta^{\dot{a}} + \partial_{\bar{z}}(\theta_k\theta^{\dot{a}})).$$

It could turn out that for higher genera the manifestly supersymmetric gauge (11) and (12) with linearly realized supersymmetry transformations will be more adequate than $\gamma^+\theta = 0$, since $u_{\bar{z}} = \partial_{\bar{z}}X^+ - i\bar{\theta}\gamma^+\partial_{\bar{z}}\theta$ is already not an exact form. It is therefore important that in the appropriate variables [see (12)] this theory is sufficiently simple and one can work with it.

5. DISCUSSION OF HIGHER LOOPS

We give here certain preliminary remarks with respect to multi-loop contributions to the four-point functions in the Green-Schwarz formalism. (We note that the answers in the standard formalism for $p \geqslant 2$ are as yet unknown. We shall attempt to show that something reasonable can be said on the subject even at a very naive level of understanding of the

Green-Schwarz formalism.)

The contribution of the usual ghosts and scalar fields X to the measure in the space of moduli equals

$$\frac{1}{(\det \mathrm{Im}\, T)^5}\left|\frac{\Lambda_2 \Pi\, dy}{\Lambda_0^5}\det{}^2\bar\partial_0\right|^2.$$

For example, in hyperelliptic coordinates[8,16]

$$\Lambda_0=(\det'\bar\partial_0)^{3/2}=(\det\sigma)^{3/2}\Pi(a)^{1/4}[s(R)/\Pi(R)]^{1/2},$$

$$\Lambda_2\Pi dy=(\Pi da/d\Omega)(\det\sigma)^{1/2}\Pi(a)^{-3/4}[s(R)/\Pi(R)]^{9/2},$$

$$\Lambda_2\equiv(\det\bar\partial_0)^{1/2}\det\bar\partial_2.$$

Here the following notation is used: the hyperelliptic curve is given by the equation

$$s^2(z)=\prod_{i=1}^{2p+2}(z-a_i),$$

$\{a_i\}$ are "branch points";

$$\Pi(a)\equiv\prod_{i<j}(a_i-a_j),$$

$\Pi da/d\Omega$ is the invariant measure. In hyperelliptic coordinates one has a natural choice of holomorphic 1-differentials:

$$v_i=\frac{z^{i-1}dz}{s(z)},\quad i=1,\ldots,p.$$

These differentials are not, however, the canonical ω_i, that satisfy the conditions

$$\oint_{A_k}\omega_j=\delta_{jk}.$$

Instead, one has $v_i=\sigma_{ij}\omega_j$.

Due to the gravitational anomaly the individual determinants depend on the choice of the metric. There is a preferred choice of metric: $|\nu_*(z)|^4$,

$$\nu_*^2(z)=\prod_{k=1}^{p-1}(z-R_k)\,dz/s(z).$$

Here $\{R_k\}$ are some $p-1$ branch points. Such sets of points $\{R_k\}$ naturally correspond to nonsingular odd theta-characteristics*. Finally,

$$s(R)\equiv\prod_{k=1}^{p-1}\Big(\prod_{a_i\neq R_k}(R_k-a_i)\Big)^{1/2},$$

$$\Pi(R)\equiv\prod_{k<l}(R_k-R_l).$$

From the most naive point of view the contribution of the θ fields equals simply $(\det\bar\partial_0)^4$, and the vertex operators $\theta^k u_{\bar z}\theta_k(z_\alpha)$ give rise to expressions independent of z_α. Therefore the most naive answer is:

$$\int\frac{1}{(\det\mathrm{Im}\,T)^5}\left|\frac{\Lambda_2\Pi\,dy}{\Lambda_0}\right|^2 d^2z_1\ldots d^2z_4\langle\exp(ip_1X(z_1))\ldots\exp(ip_4X(z_4))\rangle.$$

In the case of one loop this is the correct formula (except that Im τ should appear to the sixth power). However for $p\geqslant2$ this is already not so. This is clear if for no other reason than because Λ_2/Λ_0 is not an anomaly-free combination of determinants. In particular, the gravitational anomaly requires that Λ_2/Λ_0 be an 8-differential at each point R_k, where the metric $|\nu_*|^4$ has a double zero. Indeed, the Liouville action

$$\exp\Big[-\frac{C}{48\pi}\int|\partial\ln\rho|^2\Big],$$

to which the ratio $\det\Delta/|\det\bar\partial|^2$ is proportional, acquires under the infinitesimal coordinate transformation $z\to f(z)$ the factor

$$\exp\Big(-\frac{C}{48\pi}\int\ln|f'|^2\partial\bar\partial\ln\rho\Big)$$

(since $\rho\to\rho/|f'|^2$). For $\rho\sim|\nu_*|^4$

$$\partial\bar\partial\ln|\nu_*|^4=16\pi\sum_k\delta^2(z-R_k),$$

and this factor equals

$$\Big[\prod_k|f'(R_k)|^2\Big]^{-\frac{2}{3}C}$$

Since the determinant of the Laplace operator is invariant under coordinate transformations, $\det\bar\partial$ should transform with the factor

$$\Big[\prod_k f'(R_k)\Big]^{+\frac{2}{3}C},$$

i.e., behave like a $-(2/3)$ C-differential at each point R_k. Since $-(2/3)(C_2-C_0)=-(2/3)(13-1)=-8$, this "gravitational anomaly"[17] means that Λ_2/Λ_0 is a -8-differential at all R_k. In more detail, we have on the hyperelliptic surface

$$\frac{1}{(\det\mathrm{Im}\,T)^5}\left|\frac{\Lambda_2\Pi\,dy}{\Lambda_0}\right|^2=\frac{1}{G^5}\left|\frac{\Pi\,da/d\Omega}{\Pi(a)}\right|^2\left|\det\sigma\frac{s(R)}{\Pi(R)}\right|^8.$$

Here

$$G=|\det\sigma|^2\det\mathrm{Im}\,T=\int\left|\frac{z_1-z_2}{s(z_1)s(z_2)}dz_1\,dz_2\right|^2.$$

From our discussion of anomalies in Sec. 3 we know that they cancel out, and the contribution of the θ fields cannot be simply $(\det\bar\partial_0)^4$. The anomaly connected with this contribution should differ by the factor[8] -2 ($C=-2$ instead of 1). Consequently, a more accurate analysis is needed. We shall present here a preliminary and naive attempt at such an analysis.

The true contribution of the θ fields (which replaces the complicated structure

$$\sum_e\frac{\Lambda_{1/2}^5}{\Lambda_{3/2}}\Big\langle\prod_{\alpha=1}^{4}\psi\tilde\psi(z_\alpha)\prod_{\mu=1}^{2p-2}S(Q_\mu)\Big\rangle,\qquad S=\psi^\mu\partial X^\mu+\ldots$$

in the standard approach with spinors on the world sheet) equals

$$\prod_{\alpha=1}^{4}\left\{\frac{(\det\bar\partial_0)^{1/2}}{\operatorname{Det}u}\int D\eta\, D\theta \exp\left(\int \eta u\bar\partial\theta\right)\eta u\theta(z_\alpha)\right\}. \tag{17}$$

We omit the uninteresting factor $\langle \exp(ip_1X(z_1))\ldots \exp(ip_4X(z_4))\rangle$, due to X-fields correlators, as well as kinematic factors and integration over

$$\prod_{\alpha=1}^{4} d^2z_\alpha.$$

Let us perform an identity transformation on (17). First of all

$$\exp\left\{\int \eta u\bar\partial\theta\right\}$$

$$\equiv \int Db\, d\bar b \exp\left(\int b\bar\partial\theta\right)\exp\left(\int \eta u\bar b\right)\exp\left(\int b\bar b\right),$$

where $b, \bar b$ are Grassmann fields, which are (1,0)- and (0,1)-differentials. We expand now

$$\exp\left(\int b\bar b\right)=\sum_n\left(\int b\bar b\right)^n.$$

Then

$$\frac{(\det'\bar\partial_0)^{1/2}}{\operatorname{Det}u}\int D\theta\, D\eta(\eta u\theta(z)\exp\int \eta u\bar\partial\theta$$

$$\to \int_{d^2x_1}\ldots\int_{d^2x_p}[\det'\bar\partial_0]^{1/2}\int Db\, D\theta\,\theta(z)b(x_1)\ldots b(x_p)\exp\int b\bar\partial\theta\Big]$$

$$\cdot\left[\frac{1}{\operatorname{Det}u}\int D\bar b\, D\eta\,\eta u(z)\bar b(x_1)\ldots\bar b(x_p)\exp\int\eta u\bar b\right]. \tag{18}$$

The first factor in this expression is very close to Λ_0. We recall that according to Ref. 8

$$\Lambda_0\equiv(\det'\bar\partial_0)^{1/2}$$

$$=(\det'\bar\partial_0)^{1/2}\frac{\int Db\, D\theta\,\theta(z)b(x_1)\ldots b(x_p)\exp\int b\bar\partial\theta}{\det_{(ij)}\omega_i(x_j)}.$$

Therefore (17) equals

$$\Lambda_0^4\prod_{\alpha=1}^{4}\left\{\int_{d^2x_1}\ldots\int_{d^2x_p}\det_{(ij)}\omega_i(x_j)\right.$$

$$\left.\cdot\left[\frac{1}{\operatorname{Det}u}\int D\bar b\, D\eta\, u(z_\alpha)\bar b(x_1)\ldots b(x_p)\exp\int\eta u b\right]\right\}. \tag{19}$$

In the case of genus $p=1$

$$\frac{1}{\operatorname{Det}u}\int D\bar b\, D\eta\,\eta u(z)\bar b(x)\exp\left(\int\eta u b\right)=\delta^2(z-x)\,dz\,d\bar x.$$

and the square bracket in (19) equals

$$\omega(z_\alpha)=\frac{1}{\sigma}\frac{dz_\alpha}{s(z_\alpha)}.$$

Only one (pth) power of the integral $\int b\bar b$ contributes to (18) due to the absence of the propagator bb. For this reason for each θ field from the vertex operator we need precisely p operators b (there is one zero mode θ and p zero modes of the b fields).

For higher genera the integral

$$\frac{1}{\operatorname{Det}u}\int D\bar b\, D\eta\,\eta u(z)\bar b(x_1)\ldots\bar b(x_p)\exp\left(\int\eta u\bar b\right) \tag{20}$$

should be replaced by something, that behaves like a $+2$-differential at all points R_k. Clearly, in its present form (20) vanishes for $p>1$; $p-1$ fields η are still missing. To reproduce the gravitational anomaly one can, for example, introduce into the integrand the product

$$\prod_{k=1}^{p-1}\partial(u\eta)(R_k).$$

Such a prescription leads to the answer

$$\Lambda_0^4\prod_{\alpha=1}^{4}\det[\omega_i(z_\alpha)\omega_i'(R_1)\ldots\omega_i'(R_{p-1})]=\frac{\Lambda_0^4}{(\det\sigma)^4}$$

$$\cdot\prod_{\alpha=1}^{4}\det[v_i(z_\alpha)v_i'(R_1)\ldots v_i'(R_{p-1})]$$

for (18) and

$$\frac{1}{G^5}\left|\prod_{\alpha=1}^{4}\int d^2z_\alpha\right|\frac{\Pi\, da\, d\Omega}{\Pi(a)}\prod_{\alpha=1}^{4}\frac{s(R)}{\Pi(R)}\det[v_i(z_\alpha)v_i'(R_1)\ldots v_i'(R_{p-1})]$$

for the four-point function. As was explained in Ref. 6, this expression should still be summed over all possible choices of odd characteristics (i.e., sets of points $\{R_k\}$).

An alternative formula for $p=2$ could be, for example,

$$\sum_R s^4(R)[\det v_i(z_1)v_i(R)][\det v_i(z_2)v_i(R)][\det v_i(z_3)v_i''(R)]$$

$$\times[\det v_i(z_4)v_i''(R)] + \text{permutations}\quad z_1,\ldots,z_4$$

$$\to\sum_{i\neq j}\frac{(z_1-a_i)(z_2-a_i)(z_3-a_j)(z_4-a_j)}{(a_i-a_j)^2}+\text{permutations}$$

with derivatives of even order at the point R_* (derivatives of odd order give rise to a vanishing answer in the hyperelliptic case).

These expressions are free of anomalies and posseses the critical property of modular invariance (in hyperelliptic coordinates its role is played by projective invariance). The more general expression in the case of $p=2$, possessing this property, contains the structure

$$\frac{(z_1-a)(z_2-b)(z_3-c)(z_4-d)}{(a-b)(c-d)}+\text{permutations } z_1\ldots z_4.$$

We emphasize once more that in the case of multi-loop calculations a substantially more detailed analysis is needed. It should include a solution of the difficulties mentioned at the end of Sec. 4. Evidently it is also not possible to manage without an understanding of the role of total derivatives on the space of moduli.[5,18,6]

Our aim here has been to show that the Green-Schwarz approach may turn out to be less complicated than is usually assumed, and in that case it will be significantly more explicit and useful than the standard Neveu-Schwarz-Ramond formalism.

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[2)] We have in mind that the topologically nontrivial relation between the spaces of moduli and supermoduli remains uninvestigated. With it can be connected, in principle, nontrivial "boundary terms" in the measures on the space of moduli. It seems that this is the only possibility for a solution to the problem connected with the choice of odd moduli (see below).

[3)] Lorentz indices are omitted: the exact formula contains the structure $\varepsilon_{\mu\nu}p_\lambda p_\rho \psi^\mu \bar{\psi}^\lambda \bar{\psi}^\nu \bar{\psi}^\rho$.

[4)] One-loop calculations of the Green-Schwarz functional integral in the lighe-cone gauge were carried out in Ref. 13.

[5)] The origin of this factor (Det $u_{\bar{z}}$)$^{-4}$ may be connected with the nonpropagating Fadeev-Popov ghosts, corresponding to the fermion gauge symmetry $g^{1/2}g^{z\bar{z}}C\gamma^-\gamma^+\gamma^\mu\Pi_{\bar{z}\mu}\gamma^+\gamma - C_2 \to (\text{Det } u_{\bar{z}})^{-3}$, as well as with second class constraints in canonical quantization, which give rise to an additional (Det $u_{\bar{z}}$)$^{+4}$. In any case, if quantization is carried out in configuration space, as was done in Ref. 14, then the local measure can be reconstructed starting from the requirement of gauge invariance.

[6)] The first term in (10) was calculated (in original variables) by Carlip,[12] who was the first to verify the cancelation of the conformal anomaly in the Green-Schwarz formalism. The second term in (10) was found by Iengo and Toppan (private communication).

[7)] For higher genera gauges of the form

$$\gamma^+\theta(z) = \sum_{\alpha=1}^{2p-2} \zeta_\alpha \delta^2(z - Q_\alpha)$$

may turn out to be useful.

[8)] We note once more that the gravitational anomaly is localized at the points R_k, which are absent in the case of $p = 1$, which is why in that case the naive answer is the correct one; see below.

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