Effective ergospheres of magnetized black holes and the Kerr-Newman-Ernst solution

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The effective ergospheres of neutral and charged particles near a rotating black hole in a uniform external magnetic field are described. The magnetic field changes the chemical potential of the horizon and the entropy of the black hole. Magnetic entrainment of the frame of reference around a charged hole occurs in a magnetic field. A Harrison transformation will in general generate conical singularities. A modified Kerr-Newman-Ernst solution is constructed to describe a rotating charged black hole in an external magnetic field. The effect of the magnetic field on the metric is taken into account in this solution.

1. INTRODUCTION

Models of magnetized black holes in which a large-scale and fairly strong magnetic field is assumed to exist around a rotating black hole have recently been discussed actively in the literature. One such model, which was proposed in Refs. 1–3 in an effort to interpret the rapid release of energy in the cores of galaxies and quasars, was subsequently refined in Refs. 4 and 5. The model of a magnetized black hole opens up the possibility of an electrodynamic mechanism for extracting energy from black holes. In principle, this mechanism would be more efficient than the conventional mechanisms.[6]

Although several calculations, both analytic and numerical, have been carried out on the interaction of a magnetized plasma with a black hole, further analysis is still required to draw a general energy picture of the physics of magnetized black holes. This further analysis is one of the purposes of the present study, in which we offer a description of the energetics of black holes in a magnetic field in terms of effective ergospheres. We consider the ergospheres of charged particles around a rotating and slightly charged black hole in an asymptotically uniform test magnetic field (Sec. 2) and also the ergosphere of neutral particles which arises from the "magnetic" entrainment of the frame of reference around a nonrotating charged hole. We derive a theory of the superradiation and quantum evaporation of a black hole in an external magnetic field (Secs. 3 and 4).

A theory of magnetized black holes in which the organic relationship between electrodynamics and gravitation leads to new gravimagnetic effects is not only of direct astrophysical interest. In addition, this theory has recently attracted interest in connection with the possibility of deriving exact solutions of the Einstein-Maxwell equations which describe these effects.[7–11] Although the influence of a magnetic field on the space-time metric would seem to be weak under realistic astrophysical conditions, deriving such solutions is worthwhile not simply as a matter of principle but also to reach a better understanding of the electrodynamics of "ordinary" black holes.[10–12] The questions involved in the interpretation and analysis of magnetized solutions are not trivial, and there are points which remain unclear, despite the extensive literature on the subject (e.g., Refs. 13–16). For a Schwarzschild black hole in a magnetic universe,[7,17] the parameters in the solution have a completely definite physical meaning. When we make the transition to rotating and charged black holes, however, the situation becomes more complicated, since conical singularities arise near the polar axis in the solution derived, as suggested by Ernst and Wild,[8] from a "seed" Kerr-Newman metric through a Harrison transformation. The existence of conical singularities in the Kerr-Newman-Ernst solution was pointed out by Hiscock,[14] although the equations in his paper are incorrect. A more careful analysis (Sec. 5) shows that the appearance of conical singularities is a general property of the Harrison transformation,[18] which is used to derive magnetized metrics. This transformation should be supplemented with a rule for determining the limits on the range of the azimuthal coordinate in the resulting solutions.

Solutions with conical singularities have recently been discussed in connection with the theory of cosmic strings.[19] It can be shown (Refs. 20 and 21, for example) that the presence of conical points along some axis implies the existence of a δ-function singularity of the energy-momentum tensor. An energy-momentum tensor of this sort describes an infinitely thin rectilinear string with a longitudinal tension which is equal to minus the energy density in a system of units with $G = \hbar = c = 1$. The Kerr-Newman-Ernst solution in its original form is thus actually not a solution of the system of electrovacuum equations but instead a solution of the Einstein-Maxwell system of equations with a singular source. Eliminating the singularity by redefining the azimuthal coordinate leads to a new metric, which is a solution of the electrovacuum equations everywhere. This solution is derived and analyzed in Sec. 6. It leads to some relations for the physical parameters of a rotating charged black hole in a magnetic universe which are different from those derived[22] through the "naive" use of the Harrison transformation.

A preliminary analysis in terms of test fields (Secs. 2–4) promotes a better understanding of the exact magnetized solutions. In particular, it turns out that a Harrison transformation provides an elegant way to describe the magnetic entrainment of the frame of reference. We will be using a system of units with $c = G = \hbar = 1$ and a metric signature $(+ - - -)$.

2. EFFECTIVE ERGOSPHERE OF CHARGED PARTICLES AROUND A BLACK HOLE IN AN EXTERNAL MAGNETIC FIELD

Solutions of Maxwell's equations superposed on Schwarzschild and Kerr geometries have been studied in many places (Gal'tsov's book[11] describes the solutions and the methods by which they were derived and cites the original papers). In addition, a fairly detailed study has been made of the orbits traced out by charged particles in the resultant gravitational and electromagnetic fields. At this point we would like to mention one aspect of the dynamics of particles near magnetized holes which has not previously been discussed in the literature, although it is important for reaching an understanding of the mechanisms by which energy is extracted from black holes. We are talking about the effective ergosphere of charged particles in a magnetic field around a black hole, i.e., the region in which the particles have a negative total energy with respect to an observer at infinity. We know that the existence of an ergosphere makes possible extraction of energy from a black hole through the Penrose process and its wave analog, superradiation. A magnetic field deforms the effective ergosphere of the charged particles and may cause substantial spreading of these particles. In part, this effect of the magnetic field on the ergosphere stems from electrostatic induction. As was first pointed out by Wald,[23] an asymptotically uniform magnetic field in the Kerr metric has the property that it generates an electric field. The physical reason for the appearance of such a field lies in Faraday's law: a "rotation" of a Kerr metric creates an induced electric field, just as a field would be induced by rotating a loop in a magnetic field. The geometry of this field configuration is such that an induced potential difference arises between the event horizon and infinity. As a result of this potential difference, the black hole should preferentially acquire charges of the corresponding sign from the plasma surrounding it.[23,24] The hole may thereby acquire an electric charge $2JB$, where J is the angular momentum of the black hole, and B is the magnetic field. The same effect should result from electrodynamic pair production near a black hole, if the induced electric field which arises exceeds the Schwinger value $E_{cr} = \mu^2/e \approx 4.4\cdot 10^{13}$ G near the horizon.[25] These effects, like the change in the threshold for the superradiation of a rotating black hole in a magnetic field which was pointed out in Ref. 26 (see also Ref. 11), may be interpreted as a result of the deformation of the effective ergosphere of the charged particles near a black hole by an external magnetic field. In addition to this induction effect, the magnetic field will act directly on the effective ergosphere.

To describe the effective ergosphere of the charged particles near a rotating, weakly charged ($Q \ll M$) black hole, we consider the Hamilton-Jacobi equation

$$g^{\mu\nu}\left(\frac{\partial S}{\partial x^\mu}+eA_\mu\right)\left(\frac{\partial S}{\partial x^\nu}+eA_\nu\right)=m^2 \tag{2.1}$$

for a charge e (with mass m) which is moving in an electromagnetic field described by the 4-potential A_μ in a Kerr metric:

$$ds^2=\left(1-\frac{2Mr}{\Sigma}\right)dt^2+\frac{4Mar}{\Sigma}\sin^2\theta\,dt\,d\varphi - \frac{A\sin^2\theta}{\Sigma}d\varphi^2-\Sigma(\Delta^{-1}dr^2+d\theta^2), \tag{2.2}$$

where $A=(r^2+a^2)^2-\Delta a^2\sin^2\theta$, M is the mass, $a=J/M$ is the rotation parameter of the black hole, $\Delta=r^2+a^2-2Mr$, and $\Sigma=r^2+a^2\cos^2\theta$. There is an elegant way to introduce the 4-potential A_μ in a Kerr metric,[23] based on the following circumstance: In a vacuum gravitational field ($R_{\mu\nu}=0$) the equations for the Killing vectors $\mathcal{K}^\mu$, which characterize the symmetry of this field,

$$\mathcal{K}_{\mu;\nu}+\mathcal{K}_{\nu;\mu}=0 \tag{2.3}$$

(the semicolon means a covariant derivative), can be reduced to the following equation after repeated differentation and commutation of the derivatives in which we make use of Einstein's equations $R_{\mu\nu}=0$:

$$\mathcal{K}^{\mu;\nu}{}_{;\nu}=0. \tag{2.4}$$

This equation is the same as the equation for the 4-potential of an electromagnetic field in the Lorentz gauge, $A^\nu{}_{;\nu}=0$:

$$A^{\mu;\nu}{}_{;\nu}=0. \tag{2.5}$$

Consequently, each Killing vector field is associated with a corresponding electromagnetic field which has a definite internal relationship with the geometry of the space-time. As the 4-potential we adopt a linear combination $\mathcal{K}^\mu_{(t)}$ and $\mathcal{K}^\mu_{(\varphi)}$ of temporal and axial Killing vectors of the Kerr metric

$$A^\mu=\alpha\mathcal{K}^\mu_{(t)}+\beta\mathcal{K}^\mu_{(\varphi)}. \tag{2.6}$$

To assign a physical meaning to the parameters α and β, we consider the electromagnetic field tensor

$$\hat{F}=F_{\mu\nu}\,dx^\mu\wedge dx^\nu=\frac{2M}{\Sigma}\left(1-\frac{2r^2}{\Sigma}\right)(\alpha-\beta a\sin^2\theta)\cdot(d\hat{t}\wedge d\hat{r}+a\sin^2\theta\,d\hat{r}\wedge d\varphi) -\beta(2r\sin^2\theta\,d\hat{r}\wedge d\hat{\varphi}+A\Sigma^{-1}\sin 2\theta\,d\hat{\theta}\wedge d\hat{\varphi}) +2Mar\Sigma^{-2}\sin^2\theta[(\alpha a-\beta(r^2+a^2))d\hat{t}\wedge d\hat{\theta}\cdot +(r^2+a^2)(\alpha-\beta a\sin^2\theta)d\hat{\theta}\wedge d\hat{\varphi}] \tag{2.7}$$

in the asymptotic region $r \gg M$:

$$\hat{F}=-2\beta r\sin\theta(\sin\theta d\hat{r}\wedge d\hat{\varphi}+r\cos\theta d\hat{\theta}\wedge d\hat{\varphi}). \tag{2.8}$$

It is not difficult to see that we have $\beta = B/2$, where B is the uniform magnetic field, which is oriented along the polar axis (the z axis). At the same time, if we examine the Komar surface integrals[27] for the mass and angular momentum,

$$8\pi M=\oint \mathcal{K}^{\mu;\nu}_{(t)}\,d^2\Sigma_{\mu\nu},\quad 16\pi J=-\oint \mathcal{K}^{\mu;\nu}_{(\varphi)}\,d^2\Sigma_{\mu\nu}, \tag{2.9}$$

along with the corresponding expression for the electric charge

$$4\pi Q=\oint F^{\mu\nu}\,d^2\Sigma_{\mu\nu}, \tag{2.10}$$

we can show that we have

$$\alpha M-2\beta J=-{}^1/_2 Q,\quad \alpha=aB-Q/2M. \tag{2.11}$$

The 4-potential

$$A^\mu=\frac{1}{2}B\mathcal{K}^\mu_{(\varphi)}-\frac{Q-2aMB}{2M}\mathcal{K}^\mu_{(t)} \tag{2.12}$$

thus generates in a Kerr space-time a superposition of the Coulomb field of the charge Q which is at the singularity and an asymptotically uniform magnetic field which is directed along the polar axis.

It is not difficult to see that the physical electrostatic potential of the horizon, which is given by

$$\Phi_H = A_\mu(\mathscr{K}^\mu_{(t)} + \Omega_H \mathscr{K}^\mu_{(\varphi)}), \tag{2.13}$$

according to Carter[28] [$\Omega_H = a/2Mr_+$, $r_+ = M + (M^2 - a^2)^{1/2}$ is the angular rotation velocity of the horizon], is a quantity which is constant at the horizon and equal to zero. At infinity (where we have $\Omega_H = 0$), however, this potential is nonzero, having the value

$$\Phi_\infty = -(Q - 2aMB)/2M. \tag{2.14}$$

The potential difference between the event horizon and an infinitely remote point,

$$\Delta\Phi = \Phi_H - \Phi_\infty = (Q - 2aMB)/2M, \tag{2.15}$$

turns out to be the same as that which would be produced by an electric charge

$$\tilde{Q} = Q - 2aMB. \tag{2.16}$$

For describing the effective charged-particle ergosphere, however, we find potential (2.12) inconvenient, because it does not vanish at infinity. To make the tranformation to a gauge with $\Phi_\infty = 0$, it is sufficient to carry out a transformation of the 1-form $\tilde{A} = A_\mu dx^\mu$:

$$\hat{\tilde{A}} = \hat{A} + (\tilde{Q}/2M)\, d\hat{t}. \tag{2.17}$$

The potential of the horizon becomes finite [equal to the right side of (2.15)]. The quadratic quantity $\tilde{A}_\mu \tilde{A}^\mu$, however, diverges at the event horizon. This divergence does not involve a divergence of physical quantities, since the potential is not observable. The nonvanishing contravariant components of the 4-potential $\tilde{A}^\mu$ are

$$\tilde{A}^0 = \tilde{Q}r(r^2+a^2)/\Delta\Sigma, \quad \tilde{A}^\varphi = B/2 + \tilde{Q}ra/\Delta\Sigma. \tag{2.18}$$

It can be seen from these expressions that near the event horizon ($\Delta \to 0$) the magnetic field should be manifested primarily through the effective charge (2.16), while the additional contribution (the first term in $\tilde{A}^\varphi$) will become relatively small.

Substituting the potential $\tilde{A}^\mu$ into Eq. (2.1), and writing the action in the form

$$S = -Et + L\varphi + f(r, \theta), \tag{2.19}$$

we find the following equation for $f(r, \theta)$:

$$\Delta\left(\frac{\partial f}{\partial r}\right)^2 + \left(\frac{\partial f}{\partial \theta}\right)^2 - \frac{A}{\Delta}E^2 - \frac{\Sigma - 2Mr}{\Delta \sin^2\theta}L^2 + \frac{4Mar}{\Delta}EL + 2e\left[r(r^2+a^2)\frac{\tilde{Q}E}{\Delta} - L\left(\frac{ra}{\Delta}\tilde{Q} + \frac{B}{2}\Sigma\right)\right] + e^2\left(\frac{B^2\sin^2\theta}{4}A + \tilde{Q}Bar\sin^2\theta - \frac{\tilde{Q}^2r^2}{\Delta}\right) + m^2\Sigma = 0. \tag{2.20}$$

The effective charged-particle ergosphere is the region of those values of (r, θ) for which real solutions of Eq. (2.20) with $E < 0$ (and with all possible values of the angular momentum L) exist. It is not possible to describe the ergosphere boundary analytically in the general case, but one can find an explicit expression for the boundary of the intersection of this surface with the $\theta = \pi/2$ plane. It is clear from symmetry considerations that in this case we have $\partial f/\partial\theta = 0$ and $f \equiv f(r)$. From Eq. (2.20) with $E = 0$ we find the following expression for $f(r)$:

$$f(r) = \int \frac{dr}{\Delta^{1/2}}\left[\frac{2Mr - r^2}{\Delta}L^2 - m^2r^2 + 2eqL - e^2\left(\frac{bB^2}{4} + \tilde{Q}Bra - \frac{\tilde{Q}^2r^2}{\Delta}\right)\right]^{1/2},$$
$$q = ra\tilde{Q}/\Delta + Br^2/2, \quad b = (r^2+a^2)^2 - \Delta a^2. \tag{2.21}$$

The motion may occur in that region of r values in which the expression in brackets is positive. Clearly, in the case $E < 0$ a particle cannot go off to infinity, so this region is bounded by some r_0. The value of r_0 at a fixed $E < 0$ depends in turn on the projection of the orbital angular momentum L onto the polar axis. Equating expression (2.21) to zero, we find the corresponding values of the projection of the orbital angular momentum:

$$L = \frac{\Delta}{2Mr - r^2}\left\{-eq \pm \left[e^2q^2 + \frac{2Mr - r^2}{\Delta}\left(m^2r^2 + e^2\left(\frac{bB^2}{4} + \tilde{Q}Bar - \frac{\tilde{Q}^2r^2}{\Delta}\right)\right)\right]^{1/2}\right\}. \tag{2.22}$$

In turn, the expression in brackets on the right side of (2.22) must be nonnegative. From the condition that this expression vanish we find the equation

$$e^2q^2 + \frac{2Mr - r^2}{\Delta}\left(m^2r^2 + e^2\left(\frac{bB^2}{4} + \tilde{Q}Bar - \frac{\tilde{Q}^2r^2}{\Delta}\right)\right) = 0. \tag{2.23}$$

The larger of the real roots of this equation determines the line at which the ergosphere intersects the equatorial plane. Let us consider several particular cases: a) the ergosphere of a charged, nonrotating black hole in the absence of a magnetic field ($a = 0$, $B = 0$), in which case we find from (2.23)

$$r_0 = M + (M^2 + e^2Q^2/m^2)^{1/2}; \tag{2.24}$$

b) a nonrotating charged black hole in a magnetic field ($a = 0$, $\tilde{Q} \neq 0$, $B \neq 0$), in which case the result is the same as that in (2.24), i.e., the magnetic field does not affect the boundary of the ergosphere; c) a rotating black hole in a magnetic field for the particular value $Q = 2aMB$ of the electric charge, in which case we have

$$r_0 = M + \left(M^2 + \frac{e^2}{m^2}a^2B^2\right)^{1/2}; \tag{2.25}$$

and d) the general case $a \neq 0$, $\tilde{Q} \neq 0$, $B \neq 0$ in which we find

$$r_0 = M + [M^2 + (e^2/m^2)(Q - aMB)^2]^{1/2}. \tag{2.26}$$

Analogously, we can find the ergosphere boundary for $\theta = 0, \pi$. In this case we should also set $\partial f/\partial\theta = 0$ and $E = 0$, so we find

$$e^2\tilde{Q}^2r^2 - \Delta m^2(r^2+a^2) = 0. \tag{2.27}$$

Under the condition $a \ll M$, the solution is

$$r_0 = M + (M^2 + (e^2/m^2)\tilde{Q})^{1/2}. \tag{2.28}$$

We thus see that in the limit $\tilde{Q} \to 0$ the boundary of the ergosphere merges with the horizon surface.

3. CHEMICAL POTENTIAL OF THE HORIZON AND SUPERRADIATION IN A MAGNETIC FIELD

A deformation of the effective charged-particle ergosphere near a rotating charged black hole in a magnetic field will also be manifested through wave (or quantum) effects: superradiation[29,30] and quantum evaporation.[31] We begin with a more detailed look at the superradiation in a magnetic field.

We write the Klein-Gordon equation for a massive charged scalar particle in an asymptotically uniform magnetic field in the Kerr metric:

$$(\mathscr{D}^\mu \mathscr{D}_\mu + \mu^2)\psi = 0, \quad \mathscr{D}_\mu \psi = \psi_{;\mu} + ieA_\mu \psi. \tag{3.1}$$

Substituting the 4-potential (2.18) into this equation, and using the metric (2.2), we find

$$\frac{\partial}{\partial r}\left(\Delta \frac{\partial \psi}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \psi}{\partial \theta}\right) - \frac{A}{\Delta}\frac{\partial^2 \psi}{\partial t^2} - \frac{4Mra}{\Delta}\frac{\partial^2 \psi}{\partial t\, \partial \varphi}$$
$$+ \left(\frac{1}{\sin^2\theta} - \frac{a^2}{\Delta}\right)\frac{\partial^2 \psi}{\partial \varphi^2} - 2ie\Sigma\left[\frac{r(r^2+a^2)}{\Delta}\tilde{Q}\frac{\partial \psi}{\partial t} + \left(\frac{B}{2} + \frac{ar}{\Delta\Sigma}\tilde{Q}\right)\frac{\partial \psi}{\partial \varphi}\right]$$
$$- e^2\left(\frac{B^2 \sin^2\theta}{4}A + \tilde{Q}Bar\sin^2\theta - \frac{\tilde{Q}^2 r^2}{\Delta}\right)\psi - \mu^2 \Sigma \psi = 0. \tag{3.2}$$

It is not possible to completely separate variables in this equation, but if the magnetic field is sufficiently weak we can use the following arguments. Clearly, the representation of a uniform magnetic field which stretches out to infinity is nonphysical, and in any realistic case such a field would have to merge, at some distance $\bar{r}$ from the hole, with a magnetic field which decays at infinity. For $\bar{r} \gg r_+$, this region may be thought of as the asymptotic region, and we can calculate the flux of particles which are produced as a result of spontaneous superradiation specifically in this region. The behavior of the magnetic field for $r > \bar{r}$ thus becomes unimportant for the given problem. If the field satisfies the condition $eB\bar{r}^2 \ll 1$, we can ignore the terms in Eq. (3.2) which are quadratic in B, except the term $\tilde{Q}^2 r^2/\Delta$, which increases rapidly toward the horizon. In this approximation, Eq. (3.2) allows a complete separation of variables in the following way:

$$\psi = \sum_{lm} u_{lm}(r)\,(r^2+a^2)^{-1/2} S_{lm}(\theta)\, e^{-i\omega t + im\varphi}, \tag{3.3}$$

where $S_{lm}(\theta)$ are spheroidal functions, and the radial functions $u(r)$ satisfy

$$\frac{d^2 u}{dr^{*2}} - V_{eff} u = 0, \qquad dr^* = \frac{r^2+a^2}{\Delta} dr, \tag{3.4}$$

where the effective potential $V_{\rm eff}$ takes the following form in the limits $r^* \to \pm\infty$:

$$V_{eff} = \begin{cases} k^2 = \omega^2 - \mu^2 + eBm, & r^* \to \infty \\ p^2 = \left[\omega - m\Omega_H - \dfrac{e}{2M}(Q - 2aMB)\right]^2, & r^* \to -\infty \end{cases} \tag{3.5}$$

Far from the black hole, the potential acquires an increment eBm, which corresponds to a Zeeman shift of the energy of a charged particle in a magnetic field. In accordance with (3.5), we select the following boundary conditions for the two linearly independent solutions of Eq. (3.4):

$$\overleftarrow{u} \approx (2\pi k)^{-1/2}(e^{-ikr^*} + \overleftarrow{A} e^{ikr^*}), \quad \overrightarrow{u} \approx (2\pi p)^{-1/2} \overrightarrow{B} e^{ikr^*}, \quad r^* \to \infty,$$
$$\overleftarrow{u} \approx (2\pi k)^{-1/2} \overleftarrow{B} e^{-ipr^*}, \quad \overrightarrow{u} \approx (2\pi p)^{-1/2}(e^{ipr^*} + \overrightarrow{A} e^{-ipr^*}), \quad r^* \to -\infty. \tag{3.6}$$

These solutions have been normalized in such a way that the corresponding "incident" waves (the terms which do not contain the constants A and B) are orthonormal:

$$(\overleftarrow{\psi}_{\omega' l' m'}, \overleftarrow{\psi}_{\omega l m}) = \frac{\omega}{|\omega|}\delta(\omega - \omega')\delta_{ll'}\delta_{mm'},$$
$$(\overrightarrow{\psi}_{\omega' l' m'}, \overrightarrow{\psi}_{\omega l m}) = \frac{p}{|p|}\delta(\omega - \omega')\delta_{ll'}\delta_{mm'}. \tag{3.7}$$

In the scalar-product sense, we have

$$(\psi, \phi) = \frac{i}{2}\int g^{0\mu}(-g)^{1/2}(\psi^* \mathscr{D}_\mu \phi - (\mathscr{D}_\mu \psi)^* \phi)\, d^3x. \tag{3.8}$$

Through a standard calculation[32] of the vacuum expectation value of the components T_{r0} and $T_{r\varphi}$ of the operator which represents the energy-momentum tensor, we find

$$\frac{d}{dt}\binom{E}{J} = \frac{1}{2\pi}\sum_{lm}\int_0^{\omega_{max}} d\omega \binom{\omega}{m}(1 - |\overleftarrow{A}_{lm}^{\omega}|^2), \tag{3.9}$$

where

$$\omega_{max} = m\Omega_H + e(Q - 2aMB)/2M \tag{3.10}$$

is the threshold frequency at which the superradiation ceases. We thus see that the external magnetic field shifts the superradiation threshold (the chemical potential of the horizon) by an amount corresponding to the additional electric charge of the hole ($-2aMB$). If the relation

$$\frac{e}{2Mr_+}(Q - 2aMB) \gtrsim \frac{\mu^2}{e} \tag{3.11}$$

holds, intense pair production will begin as a result of the electrodynamic instability of the vacuum,[25] and the black hole will acquire a charge $Q = 2aMB$.

We can use the same approximation to carry out calculations on the quantum evaporation of a black hole in an external magnetic field. As Unruh has shown,[33] the problem reduces to one of constructing the complete system of modes and carrying out a second quantization on an expanded Kruskal manifold. As the positive-frequency modes we should select those which exhibit this property with respect to a timelike Killing vector on the Cauchy surface for the given mode. If there is an external field, the quantization procedure remains the same in principle, but there are changes in the asymptotic conditions for the modes. Furthermore, a complete separation of variables is no longer possible. Repeating the discussion in Ref. 33 regarding the relationship between the basis functions specified on an expanded Kruskal manifold with the modes $\overleftarrow{u}$ and $\overrightarrow{u}$, we find the following expressions for the loss of mass and angular momentum of the black hole in this approximation in terms of the strength of the magnetic field:

$$\frac{d}{dt}\binom{E}{J}=\frac{1}{2\pi}\int_{\mu}^{\infty}d\omega\sum_{lm}|\overleftarrow{B}_{\omega m}^{l}|^{2}(e^{2\pi\rho/\varkappa}-1)^{-1}\binom{\omega}{m}, \quad (3.12)$$

where $\varkappa$ is the surface gravitation.

4. "MAGNETIC" ENTRAINMENT OF THE FRAME OF REFERENCE NEAR A NONROTATING CHARGED BLACK HOLE

We turn now to another effect which is associated with the imposition of a magnetic field on a black hole. This is an effect which we pointed out previously[26] and which is important for reaching a correct interpretation of the exact solutions of Ernst *et al.*[7,8] The now familiar analogy between the gravitational field[34] of rotating masses and a magnetic field suggests that there may be a gravitational analog of the electromagnetic effects which we discussed above. It turns out that this is the case. The gravitational field produced by superimposing the Coulomb electric field of a charge and a uniform magnetic field contains a rotational component $g_{0\varphi}$ if the original background field has $g_{0\varphi}=0$. In other words, an ergosphere for neutral particles should arise around a charged nonrotating black hole in an external magnetic field.

We seek a quantitative description of the effect. As the background metric we adopt a Schwarzschild metric, and we assume that the black hole has a small electric charge $Q\ll M$; the electromagnetic field of this charge can be thought of as a test field. We assume that the external magnetic field is a test field; i.e., we choose the overall 4-potential in the form (2.12) with $a=0$. The corresponding electromagnetic field tensor will have the form of (2.7), where the values of the parameters α and β from (2.11) (with $a=0$) must be taken into account. We write the energy-momentum tensor in the form

$$T^{\mu\nu}=T^{\mu\nu}_{(Q)}+T^{\mu\nu}_{(B)}+T^{\mu\nu}_{(QB)}, \quad (4.1)$$

where $T^{\mu\nu}_{(Q)}$ is the energy-momentum tensor of the Coulomb field of the charge (which is proportional to Q^2), $T^{\mu\nu}_{(B)}$ is the energy-momentum tensor of the magnetic field (which is proportional to B^2), and the third term, which is the term in which we will be interested below, represents the interference contribution of the Coulomb field and of the uniform magnetic field (it is proportional to the product QB). Here is the explicit expression for the one nonvanishing component, $T^{0\varphi}_{(QB)}$:

$$T^{0\varphi}_{(QB)}=\frac{1}{4\pi}\frac{QB}{r^3}. \quad (4.2)$$

The tensor $T^{\mu\nu}_{(QB)}$ is covariantly conserved $T^{\mu\nu}_{(QB);\nu}=0$. Those corrections $h^{\nu\mu}$ to the space-time metric in which we are interested satisfy the following equation in the linearized theory of gravitation in the gauge $h^{\mu\nu}_{;\nu}=0$ and under the condition $h=h^{i}_{i}=0$:

$$h^{\mu\nu}{}_{;\lambda}{}^{;\lambda}+2R^{\mu\lambda\nu\tau}h_{\lambda\tau}=16\pi T^{\mu\nu}, \quad (4.3)$$

where $R^{\nu\lambda\nu\tau}$ is the curvature tensor, and the covariant derivatives are calculated from the Schwarzschild background metric. Since Eq. (4.3) is linear, the gravitational fields generated by the Coulomb field, the magnetic field, and their interference (4.2) can be analyzed independently. Let us consider the contribution of the interference term, (4.2).

Writing Eq. (4.3) in component form, we see that its physical solution with energy-momentum tensor (4.2) is described by the one nonvanishing component $h^{0\varphi}$, which depends on the single variable r and for which we find the following equation:

$$\Delta\frac{d^2}{dr^2}h^{0\varphi}+4(r-M)\frac{d}{dr}h^{0\varphi}+\frac{4M}{r}h^{0\varphi}=\frac{4BQ}{r}. \quad (4.4)$$

Introducing the new function $\tilde{\varphi}=\Delta h^{0\varphi}$, we can put Eq. (4.4) in the form

$$\frac{d^2}{dr^2}\tilde{\varphi}-\frac{2}{r^2}\tilde{\varphi}=\frac{4BQ}{r}. \quad (4.5)$$

A general solution of this equation is

$$\tilde{\varphi}(r)=C_1r^2-C_2/r-2BQr. \quad (4.6)$$

The gravitational field in which we are interested should vanish in the case $B=0$, so we set $C_1=C_2=0$. As a result we reach the conclusion that the correction to the Schwarzschild metric for a weakly charged black hole in a uniform external magnetic field is

$$h^{0\varphi}=-2BQr/\Delta. \quad (4.7)$$

Omitting the indices, we have

$$h_{0\varphi}=2BQr\sin^2\theta. \quad (4.8)$$

Comparing this expression with the component $g_{0\varphi}=2Ma\times\sin^2\theta/r$ of the metric of a Kerr field with is linearized in the rotation parameter a, we can easily find that angular velocity of the magnetic entrainment of the frame of reference which stems from the correction (4.8) in the immediate vicinity of the event horizon, $\Omega_B=QB/M$. An important distinction between the correction (4.8) and the corresponding Kerr component of the metric is that the latter asymptotically vanishes in the limit $r\to\infty$, while the correction (4.8) increases. The reason for this behavior is that we have assumed that the uniform magnetic field stretches out to infinity. However, the fact that a "rotational" component of the metric, $g_{0\varphi}$, arises is not a consequence of the representation of a uniform magnetic field stretching out to infinity. To see this, let us assume that the magnetic field exists only in a finite region. We then see that the arguments above remain valid in this region, and the solution which we have derived, (4.8), must then be joined with the external, asymptotically flat solution. It is not difficult to see that the solution which we have found [in the form in (4.8)] remains meaningful in the absence of a black hole; i.e., this solution describes the gravitational field which is generated by the superposition of the electrostatic field of a charged sphere and a uniform magnetic field in Minkowski space.

5. HARRISON TRANSFORMATION AND COSMIC STRINGS

Another way to take account of the effect of an external magnetic field on the geometry of space-time was pointed out by Ernst,[7] who constructed an exact solution of the Einstein-Maxwell equations describing a Schwarzschild black hole in a strong external magnetic field. Although that solution is not asymptotically flat, it is quite simple to interpret physically, because in the case $B\ll B_M$ (where $B_M=1/M\approx2.4\cdot10^{19}M_{\odot}/M$ G; Ref. 11) there exists an intermediate asymptotic region $r_+\ll r\ll B^{-1}$ in which the space-time is approximately of a Schwarzschild nature, and the magnetic field is the same as the test field described in Sec. 2 in the case of charged and rotating black holes, however, it is a less trivial matter to find a physical interpretation of such solutions,[7–10] since the parameter values of the seed solutions (M, a, and Q_0) are not the same as the corresponding parameter values of the magnetized solutions. These solutions and their physical interpretation were recently discussed by Dokuchaev,[22] who attempted to derive relations between the physical parameters of the seed solution and of the Harrison-transformed solution. However, Dokuchaev did not incorporate an important general property of Harrison transformations for axisymmetric steady-state fields of the electrovacuum. This general property requires that the resultant fields be interpreted in a manner different from that of Ref. 22. Specifically, it is difficult to believe that solutions generated through a Harrison transformation from Reissner-Nordström and Kerr-Newman metrics have singularities at the polar axis. The singularities are of the nature of conical points, and their presence indicates that the corresponding solution in fact does not satisfy the system of electrovacuum equations throughout space.

Here we would like to show that the generation of conical singularities by Harrison transformations is a general property. We would also like to discuss the physical meaning of such solutions in light of some recent results in the theory of cosmic strings. Let us review the content of the method for deriving exact magnetized solutions.

We consider an interval of a steady-state axisymmetric space-time in cylindrical coordinates:

$$ds^2=-f(d\varphi-\omega dt)^2-f^{-1}[e^{2\gamma}(d\rho^2+dz^2)-\rho^2dt^2], \qquad (5.1)$$

where f, ω, and γ are real functions. We introduce the complex electromagnetic potential $\Phi = A_\varphi + iB_\varphi$ and the complex gravitational potential $\mathscr{E} = f - i\psi$ (B_φ is a component of the magnetostatic potential), which generate a dual electromagnetic field tensor $\tilde{F}_{\mu\nu} = 2B_{[\nu,\mu]}$. The existence of this tensor, like that of the potential ψ, follows immediately from the corresponding Einsein-Maxwell equations. As a result, the system of Einstein-Maxwell equations reduces to a system of two nonlinear equations for the potentials $\mathscr{E}$ and Φ (Ref. 35):

$$\begin{aligned} f\Delta\Phi&=\partial_a\Phi(\partial^a\mathscr{E}-2\Phi^*\partial^a\Phi),\\ f\Delta\mathscr{E}&=\partial_a\mathscr{E}(\partial^a\mathscr{E}-2\Phi^*\partial^a\Phi), \end{aligned} \qquad (5.2)$$

where

$$f=\mathrm{Re}\,\mathscr{E}-|\Phi|^2,\quad \Delta=\frac{\partial^2}{\partial\rho^2}+\frac{\partial^2}{\partial z^2}+\rho^{-1}\frac{\partial}{\partial\rho},$$

$$\partial_a=\frac{\partial}{\partial x^a},\quad a=1,2\equiv\rho,z.$$

The indices are raised and lowered by means of the two-dimensional tensor $g_{ab} = g^{ab} = \mathrm{diag}\ (1,1)$. The potentials B_φ and ψ satisfy the equations

$$\nabla A_0+\omega\nabla A_\varphi=-i\rho f^{-1}\nabla B_\varphi, \qquad (5.3)$$

$$\nabla\psi-2i\Phi^*\nabla\Phi=i\rho^{-2}f\nabla\omega, \qquad (5.4)$$

where the operator ∇ is given by $\nabla = \partial/\partial\rho + i\partial/\partial z$. System (5.2) is invariant under the group $SU(2,1)$ of transformations of complex potentials.[36] Applying one of the transformations of this group[18] to some seed solution of the Eisntein-Maxwell equations, we can construct a new solution, which corresponds physically to the introduciton of a uniform magnetic field:

$$\mathscr{E}'=\Lambda^{-1}\mathscr{E},\quad \Phi'=\Lambda^{-1}(\Phi-{}^1/_2B\mathscr{E}),\quad \Lambda=1-B\Phi+{}^1/_4B^2\mathscr{E}. \qquad (5.5)$$

Here $f\to f'$ and $\omega\to\omega'$ where

$$f'=\mathrm{Re}\,\mathscr{E}'-|\Phi'|^2=|\Lambda|^{-2}f, \qquad (5.6)$$

$$\nabla\omega'=|\Lambda|^2\nabla\omega-\rho f^{-1}(\Lambda^*\nabla\Lambda-\Lambda\nabla\Lambda^*), \qquad (5.7)$$

while the other quantities in (5.1) remain unchanged.

Let us examine the behavior of seed solution (5.1) and of the transformed solution (5.5)–(5.7) near the polar axis ($\rho = 0$). The condition that there be no conical singularities in the seed solution is the requirement

$$\lim_{\rho\to 0}(\rho e^\gamma f^{-1})=1. \qquad (5.8)$$

The function f is subjected to an extension by a factor of $|\Lambda|^2$ in the course of the Harrison transformation (5.6). Consequently, if we are to avoid the appearance of conical singularities we must require

$$\lim_{\rho\to 0}|\Lambda|^2=1. \qquad (5.9)$$

In general, however, this condition does not hold. In particular, it does not hold for magnetized Kerr-Newman-Ernst solutions. We thus conclude that in general Harrison transformations lead to solutions which do have conical singularities. Here there is no contradiction with the theorem[36] regarding the symmetry of system (5.2) under the group $SU(2,1)$, since the theorem guarantees that the transformed solutions will satisfy the system only in a local sense. In particular, no limits are imposed on the ranges of the coordinates in the transformed solution.

Space-times with conical singularities along some spacelike axis have recently attracted interest in connection with the hypothesis that cosmic strings—topological defects of the vacuum of quantum field theory—may exist in certain Grand Unification models.[37] Such strings might have formed as result of phase transitions in the early universe[19,38] and might subsequently have served as sites of the condensation of matter into structures of various scales. As Vilenkin[39] has mentioned, the gravitational field of an infinitely thin string having an energy-momentum tensor

$$T_\mu{}^\nu=\varepsilon(1,0,0,1)\frac{1}{2\pi\rho}\delta(\rho) \qquad (5.10)$$

(ε is the energy per unit length along the string; the tension along the z axis is negative and equal to ε in absolute value) corresponds to a locally plane space which has conical singularities along the z axis:

$$ds^2=dt^2-dz^2-d\rho^2-\rho^2(1-4\varepsilon)^2d\varphi^2. \qquad (5.11)$$

6. PHYSICAL INTERPRETATION OF THE KERR-NEWMAN-ERNST SOLUTION

Substituting Φ and $\mathscr{E}$ for the Kerr-Newman solution into the Harrison-transformation equations, (5.5),

$$\Phi = Q_0\,[i\cos\theta - a\sin^2\theta\,(r+ia\cos\theta)^{-1}],$$
$$\mathscr{E} = (r^2+a^2)\sin^2\theta + Q^2\cos^2\theta - 2iaM(3-\cos^2\theta)$$
$$\cdot\cos\theta + 2a\sin^2\theta\,(r+ia\cos\theta)^{-1}(Ma\sin^2\theta + iQ_0^2\cos\theta), \qquad (6.1)$$

we find the function $\Lambda(r, \theta)$ and the transformed solution

$$ds^2 = \left(\frac{\Delta}{A}dt^2 - \frac{dr^2}{\Delta} - d\theta^2\right)\Sigma|\Lambda|^2 - \frac{A\sin^2\theta}{\Sigma|\Lambda|^2}(d\varphi - \omega'\,dt)^2, \qquad (6.2)$$

where $\omega'(r, \theta)$ obeys Eq. (5.7). It is not difficult to see that the quantity Λ is complex; its values at $\theta = 0$ and π are different from unity and are complex conjugates of each other:

$$\Lambda_0 = \Lambda(\theta=0) = \Lambda^*(\theta=\pi) = 1 + B^2Q_0^2/4 - iB(Q_0 + aMB). \qquad (6.3)$$

We now find that the energy density of the string which corresponds to a conical singularity of solution (6.2) is

$$\varepsilon = {}^1\!/_4(|\Lambda_0|^2 - 1)\,|\Lambda_0|^{-2}. \qquad (6.4)$$

A nonvanishing value of ε indicates the existence of a conical singularity, so the metric (6.2) no longer satisfies the Einstein-Maxwell equations throughout space (including the polar axis). To eliminate the conical similarity, i.e., to find a solution which satisfies the system of electrovacuum equations everywhere, we should change the limits on the azimuthal coordinate, $0 \leqslant \varphi \leqslant 2\pi|\Lambda_0|^2$. Alternatively and equivalently, we can introduce a new azimuthal coordinate: $\varphi \to \varphi\,|\Lambda_0|^2$. As a result, we find the "corrected" solution

$$ds^2 = \left(\frac{\Delta}{A}dt^2 - \frac{dr^2}{\Delta} - d\theta^2\right)\Sigma|\Lambda|^2$$
$$- \frac{A\sin^2\theta}{\Sigma|\Lambda|^2}(d\varphi|\Lambda_0|^2 - \omega'\,dt)^2, \qquad (6.5)$$

where Λ_0 is given by (6.3) (the angle φ again has the range $0 < \varphi\ 2\pi$). The quantity $\omega'(r, \theta)$ is given by a simple expression in the approximation linear in the magnetic field B (Ref. 8):

$$\omega' = A^{-1}[(2Mr - Q_0^2)a - 2BQ_0r(r^2+a^2)], \qquad (6.6)$$

where the constant of integration has been set equal to zero. In the case $a = 0$, this expression leads to Eq. (4.8), which was derived above. In the more general case in which the angular momentum of the hole is not zero, expression (6.6) describes magnetic entrainment of the frame of reference which is caused by the gravitational effect of the magnetic field on the metric of the space-time of a charged black hole.

How does a magnetic field influence the event horizon of a black hole? From (6.5) we see that a two-dimensional cross section of the horizon is again a sphere of radius $r_+ = M + (M^2 - a^2 - Q_0^2)^{1/2}$, but now we have a different expression for the surface area of the horizon:

$$S = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta\,|g_{22}g_{33}|^{1/2} = 4\pi|\Lambda_0|^2(r_+^2 + a^2). \qquad (6.7)$$

We see that a strong magnetic field increases the surface area of the event horizon; for a seed charge $Q_0 \neq 0$ of the Kerr-Newman solution, this change is proportional to the square of the magnetic field, while in the case $Q_0 = 0$ it is proportional to the quantity $(a/M)^2(B/B_M)^4$. What is the physical meaning of the parameter Q_0 in solution (6.5)? To answer this question, we calculate the total electric-field flux through a closed surface around the hole. Making use of the properties of Ernst potential,[10,11] we find the following expression for the physical value of the electric charge:

$$Q = -{}^1\!/_2|\Lambda_0|^2\,\mathrm{Im}(\Phi'(\pi) - \Phi'(0)) = Q_0 + 2aMB - Q_0^3B^2/4. \qquad (6.8)$$

In precisely the same way, we can show that the magnetic charge remains equal to zero.

If we evaluate the Komar surface integrals[27] (2.9) which determine the mass M and the angular momentum J in the case of an asymptotically planar space-time, we find that they depend on the integration surface. They generally diverge as this surface is removed to infinity. This result is not surprising, since the angular momentum and mass of an external field which is nonzero throughout space would be infinite. In the approximation linear in B, an integration over a spherical surface of radius $r_0 \gg M$ leads to the result

$$M_\infty = M - {}^2\!/_3 aBQ_0, \quad J_\infty = aM - {}^1\!/_3 Q_0Br_0^2; \qquad (6.9)$$

i.e., the angular momentum of the field diverges even in the linear approximation in B (if $Q_0 \neq 0$).

An evaluation of the Komar integrals over the surface of the event horizon under the condition $a^2 \ll M^2$ yields

$$M_H = M - \frac{Q^2}{r_+}\left(1 + \frac{1}{3}\frac{a^2}{r_+^2}\right) + \frac{2}{3}aQ_0^3Br_+^{-2} - \frac{8}{3}aQ_0B, \qquad (6.10)$$

$$J_H = aM - {}^2\!/_3 aQ_0^2r_+^{-1} - {}^1\!/_3 Q_0Br_+^2. \qquad (6.11)$$

It can be seen from these expressions that the seed values of M and aM are not the same as the quantities in (6.9)–(6.11). Consequently, when there is an external magnetic field the parameters Q_0, M, and a can no longer be interpreted as the electric charge, mass, and specific angular momentum of the black hole.

Expressions (6.7) and (6.8) are not the same as the corresponding expressions of Ref. 22, where the nonphysical solution (6.2), which is singular at the polar axis, was used in place of (6.5).

7. CONCLUSION

The theory of magnetized black holes predicts effects which stem from the simultaneous existence of gravitational and electromagnetic fields and the influence of these fields on each other. These effects are not the simple sum of the effects of electrodynamics and gravitation; they are instead manifestations of a synthesis of the electrodynamic and gravitational effects and of an internal relationship. They lead to new electrodynamic mechanisms for an extraction of energy from black holes, which may be pertinent to real astrophysical situations. In this paper we have attempted to draw an overall picture of the change in the energetics of a black hole in the presence of an external magnetic field, including both the electromagnetic effect on charged particles and the gravitational effect on the metric due to the external field. The latter effect is of course extremely small in most astrophysical systems, but it is definitely of fundamental interest since it allows a deeper understanding of the nature of the gravitational interaction. We have also attempted to refine the existing interpretation of the interesting and physically rich Kerr-Newman-Ernst solutions, by pointing out the need to incorporate the conical singularities which arise during Harrison transformations. If the resulting solution is to satisfy the system of electrovacuum equations everywhere, this transformation must be supplemented with an appropriate change in the range of the azimuthal coordinate: $0\leqslant\varphi\leqslant 2\pi|\Lambda_0|^2$. If this circumstance is ignored, one will draw incorrect conclusions regarding the physical meaning of the parameters of magnetized black holes.

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