Special features of the current-voltage characteristic of warm two-dimensional electrons

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It is shown that in view of the threshold for electrons scattering by optical phonons the current-voltage characteristic of a semiconductor is nonanalytic in the range of weak electric fields. In the case of two-dimensional electrons this nonanalyticity is manifested by the fact that the series expansion of the conductivity in terms of the field does not begin with the term E^2 but with $|E|$. Numerical estimates of this effect are given.

1. INTRODUCTION

In the case of weak heating of carriers (in what is known as the warm-electron range) the properties of a semiconductor are usually represented by a regular series expansion in powers of a weak electric field. For example, the current-voltage characteristic can be written in the form[1]

$$\mathbf{j}=\sigma_0\mathbf{E}(1+\beta E^2). \tag{1.1}$$

The first nonohmic correction to the conductivity is usually regarded as a quadratic function of the field, which is explained by the existence of a center of inversion in the **p** space of the kinetic (transport) equation. In special cases such as that of noncentrosymmetric crystals when carriers experience non-Born scattering by anisotropic scattering centers, the expansion of the conductivity includes also the linear terms.[2] However, the regular expansion in terms of the field E is not obtained in all cases and even when a semiconductor is isotropic and has a center of inversion there may be nonanalytic corrections preceding the standard correction βE^2. In fact, if the kinetic description of the electron system in a semiconductor is adopted, an expression of the type given by Eq. (1.1) can be obtained by expanding the kinetic equation

$$e\mathbf{E}\frac{\partial}{\partial \mathbf{p}}f(\mathbf{p})=S(f) \tag{1.2}$$

in terms of the quantity on the left-hand side which contains the field. Since the collisional term $S(f)$ is usually an integral operator, it follows from Eq. (1.2) that the expansion in terms of E is in fact in terms of the highest derivative of the equation. It is known that such expansions sometimes lead to nonanalyticity of the solution so that expansions of the type given by Eq. (1.1) are no longer valid.

The nonanalyticity of the solution usually occurs in the presence of any singularities in the coefficients of the equation, for example, in the case of threshold effects due to inelastic scattering of electrons by optical phonons.

We shall consider the situation by tackling the example of the current-voltage characteristic of warm two-dimensional electrons, the heating of which has been studied in the last decade both in silicon metal-oxide-semiconductor structures[3] and in heterojunctions made of III–V compounds.[4] It has been shown that because of the threshold for scattering of electrons by optical phonons the direct E expansion for the current of the type given by Eq. (1.1) diverges. After a suitable modification of the series we can obtain a nonanalytic correction to the conductivity which is of the following form:

$$\frac{\Delta\sigma}{\sigma_0}=-\frac{2^{1/2}a_0}{2}\left(\frac{\hbar\omega_0}{kT}\right)^{3/2}\exp\left(-\frac{\hbar\omega_0}{kT}\right)\frac{e^2(\Delta\tau)^2}{\mu_0 m^{3/2}(kT)^{1/2}}|E|, \tag{1.3}$$

where ω_0 is the limiting frequency of an optical phonon; T is the lattice temperature; μ_0 is the ohmic mobility; m is the mass of a carrier; $\Delta\tau=\tau^- - \tau^+$ is a discontinuity of the momentum relaxation time at an energy $\varepsilon=\hbar\omega_0$, which is due to the different efficiencies of the scattering of carriers in the passive ($\varepsilon<\hbar\omega_0$) and active ($\varepsilon>\hbar\omega_0$) regions of the momentum space; a_0 is a numerical coefficient of the order of unity.

The results obtained are mathematically analogous to the nonanalytic corrections to the magnetoresistance of two-dimensional electrons predicted in Ref. 5 and also due to singularities of the coefficients in the kinetic equation.

2. KINETIC EQUATION FOR TWO-DIMENSIONAL ELECTRONS

We shall consider a system of nondegenerate two-dimensional electrons described by the kinetic equation (1.2). Then, the collisional term of the equation is of the form

$$S(f)=\int d^2\mathbf{p}'\{W(\mathbf{p}',\mathbf{p})f(\mathbf{p}')-W(\mathbf{p},\mathbf{p}')f(\mathbf{p})\}. \tag{2.1}$$

The linearity of the operator does not exclude the possibility of allowance for the electron-electron scattering, because in the range of weak fields of interest to us the operators describing the *ee* scattering also become linear.

We shall consider only the isotropic case and represent the probability of the scattering by the following expansion:

$$W(\mathbf{p},\mathbf{p}')=\frac{1}{2\pi}W_0(\varepsilon,\varepsilon')+\frac{1}{\pi}\sum_{n=1}^{\infty}W_n(\varepsilon,\varepsilon')\cos(n\chi). \tag{2.2}$$

Here, χ is the angle between the vectors **p** and **p**′ and $\varepsilon=p^2/2m$ is the electron energy.

We shall also represent the distribution function $f(\mathbf{p})$ as a Fourier series expansion in terms of the angle φ between the vectors **p** and **E**:

$$f(\mathbf{p})=\frac{1}{2}f_0(\varepsilon)+\sum_{n=1}^{\infty}f_n(\varepsilon)\cos(n\varphi). \tag{2.3}$$

Substituting now Eq. (2.3) and also Eqs. (2.1) and (2.2)

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into the original kinetic equation (1.2), we obtain a system of equations for the components of the distribution function $f_n(\varepsilon)$. We shall write this system in the following dimensionless form:

$$\mathscr{E}\frac{d}{dx}(x^{1/2}f_1)=S_0(f_0), \tag{2.4}$$

$$\frac{1}{2}\mathscr{E}\left\{x^{n/2}\frac{d}{dx}(x^{(1-n)/2}f_{n-1})+x^{-n/2}\frac{d}{dx}(x^{(1+n)/2}f_{n+1})\right\}=S_n(f_n),\quad n\geqslant 1. \tag{2.5}$$

We have introduced here the dimensionless electron energy

$$x=\varepsilon/kT \tag{2.6}$$

and the dimensionless electric field

$$\mathscr{E}=E/E^*,\quad E^*=\frac{1}{e\tau^-}\left(\frac{mkT}{2}\right)^{1/2}, \tag{2.7}$$

where

$$\tau^-=\tau_0(\hbar\omega_0)=\left\{\int d^2\mathbf{p}'W(\mathbf{p},\mathbf{p}')\right\}^{-1}\Big|_{p=(2m\hbar\omega_0)^{1/2}} \tag{2.8}$$

is the scattering time at the boundary of the passive region. The symbols $S_n(f_n)$ denote the corresponding original terms of the equations normalized to the value of τ^-.

We shall postulate the following normalization of the distribution functions:

$$\int_0^\infty dx\, f_0(x)=1. \tag{2.9}$$

We shall avoid the necessary complications by limiting our treatment to the simplest case and including in the collisional terms of the kinetic equations only the deformation scattering by acoustic and optical phonons. The collisional term for the symmetric part of the distribution function contains a differential operator corresponding to the acoustic scattering and a difference part corresponding to the scattering by optical phonons. The collisional terms for the other components of the distribution function ($n\neq 0$) are identical and have the form of the relaxation time, i.e., the system of equations (2.5) can be represented as follows:

$$\frac{1}{2}\mathscr{E}\left\{x^{n/2}\frac{d}{dx}(x^{(1-n)/2}f_{n-1}) + x^{-n/2}\frac{d}{dx}(x^{(1+n)/2}f_{n+1})\right\} = -\frac{1}{\tau(x)}f_n,\quad n\geqslant 1. \tag{2.10}$$

The energy dependence of the normalized, to τ^-, relaxation time $\tau(x)=\tau_0(\varepsilon)/\tau^-$ is shown in Fig. 1, where $u=\hbar\omega_0/kT$ is the dimensionless energy of an optical phonon. A discontinuity δ of the relaxation time appears at the boundary of the passive region because of the threshold-like activation of spontaneous emission of optical phonons by electrons.

The expression for the dissipative current which we shall calculate is of the form

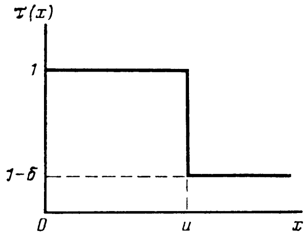

FIG. 1. Momentum relaxation time.

$$J=ne\left(\frac{2kT}{m}\right)^{1/2}j, \tag{2.11}$$

$$j=\int_0^\infty dx\, x^{1/2}f_1(x). \tag{2.12}$$

3. EXPANSION IN TERMS OF THE FIELD E

We can obtain the field expansion for the current by representing the components of the distribution function $f_n(x)$ as follows:

$$f_n(x)=\sum_{k=0}^\infty \mathscr{E}^k f_n^{(k)}(x), \tag{3.1}$$

and iterating the field terms of Eqs. (2.4) and (2.10). We shall show that the series obtained for the current

$$j=\sum_{k=0}^\infty \mathscr{E}^k j^{(k)} \tag{3.2}$$

diverges. We shall do this by calculating the first few orders of the series.

In the zeroth order the current $j^{(0)}$ is naturally zero because there is only a symmetric part of the distribution function, which is identical with the equilibrium function

$$f_0^{(0)}(x)=e^{-x}. \tag{3.3}$$

In the first order there is a component $f_1^{(1)}$ which we can find from Eq. (2.10),

$$f_1^{(1)}(x)=\left(-\frac{1}{2}\tau(x)x^{1/2}\frac{d}{dx}\right)e^{-x}, \tag{3.4}$$

and the corresponding contribution to the dissipative current

$$j^{(1)}=\int_0^\infty dx\, x^{1/2}\left(-\frac{1}{2}\tau(x)x^{1/2}\frac{d}{dx}\right)e^{-x}. \tag{3.5}$$

The subsequent course of calculation of the $\mathscr{E}$ expansion is obvious and we can easily show that the next contribution to the current appears in the third order and consists of two parts differing in respect of the component (zeroth or second) associated with the second-order iteration, i.e.,

$$j^{(3)}=j_0^{(3)}+j_2^{(3)}, \tag{3.6}$$

where

$$j_0^{(3)}=\int_0^\infty dx\,x^{1/2}\left(-\frac{1}{2}\tau(x)x^{1/2}\frac{d}{dx}\right)\left(\hat{S}_0^{-1}\frac{d}{dx}x^{1/2}\right)$$

$$\times\left(-\frac{1}{2}\tau(x)x^{1/2}\frac{d}{dx}\right)e^{-x}, \quad (3.7)$$

$$j_2^{(3)}=\int_0^\infty dx\,x^{1/2}\left(-\frac{1}{2}\tau(x)\frac{1}{x^{1/2}}\frac{d}{dx}x\right)\left(-\frac{1}{2}\tau(x)x\frac{d}{dx}\frac{1}{x^{1/2}}\right)$$

$$\times\left(-\frac{1}{2}\tau(x)x^{1/2}\frac{d}{dx}\right)e^{-x}. \quad (3.8)$$

In Eq. (3.7) we have the inverse operator $\hat{S}_0^{-1}$ the effect which can be regarded as the solution of Eq. (2.4). Although very frequently numerical methods have to be used to solve this equation, this causes no fundamental difficulties. It should be pointed out that if we retain only the first term in Eq. (3.6), the standard expression of Eq. (1.1) is obtained. This is usually so in the three-dimensional case. The justification for this is the smallness of the scattering inelasticity coefficient [which can be written symbolically in the form $S_0^{-1}\gg\tau(x)$].

Inclusion of the additional contribution represented by Eq. (3.8) changes the situation significantly. In fact, after single integration by parts, we readily see that the expression given by Eq. (3.8) acquires a term of the type

$$\int dx\left[\frac{d}{dx}\tau(x)\right]^2\sim\int dx[\delta(x-u)]^2, \quad (3.9)$$

which diverges in the vicinity of $x=u$. The divergence of integrals in the third order is evidence of the inconsistency of the expansion described by Eq. (1.1). It should be pointed out that these difficulties do not appear when the contribution of Eq. (3.7) is allowed for because of the smoothing out action of the operator S_0^{-1}, which includes the differential form of the operator representing the scattering by acoustic phonons. We can easily show that stronger divergences occur in higher orders of the series. Consequently, even if we limit the expansion of the current-voltage characteristic to the terms cubic in respect of the electric field, we have to carry out summation of these singular terms and thus modify further the series.

4. SUMMATION OF SINGULAR TERMS

The analysis given in the preceding section shows that the most singular contribution is obtained when all the differentiation operators act on nonanalytic functions $\tau(x)$. Therefore, in the calculation of the main nonanalytic contribution in all the orders of the series we have to retain only this dependence on x and replace the other x's with u. The summation of the most singular terms of the series obtained in this way is an easy task and it naturally yields a system of equations which are identical in form with the system (2.10). It is this contribution that is determined by the solution of the system of equations

$$\frac{1}{2}\mathscr{E}u^{1/2}\frac{d}{dx}(f_{n-1}^s+f_{n+1}^s)=-\frac{1}{\tau(x)}f_n^s-\delta_{n,2}\frac{1}{4}\mathscr{E}^2ue^{-u}\frac{d}{dx}\tau(x),$$

$$n\geq1; \quad (4.1)$$

$$f_0^s(x)=0,$$

whereas the corresponding contribution to the dissipative current is

$$j^s=u^{1/2}\int_0^\infty dx\,f_1^s(x). \quad (4.2)$$

The correctness of the above conclusion is easiest to demonstrate by expanding the solution of the system (4.1) as a $\mathscr{E}$ series and comparing it term-by-term with the most singular terms of the $\mathscr{E}$ series considered in Sec. 3 for the system comprising Eqs. (2.4) and (2.10). The symmetric part of the distribution function $f_0^s(x)$ is excluded as less singular because of the smoothing effect of the operator $\hat{S}_0^{-1}$

It is remarkable that the coefficients of the resultant system of equations (4.1) are in fact independent of x. This makes it possible to eliminate the field $\mathscr{E}$ from Eq. (4.1) by a suitable transformation of the variables and obtain directly the field dependence of the nonanalytic correction to the current. This transformation is of the form

$$x=u+\mathscr{E}u^{1/2}r\begin{cases}1-\delta, & r>0\\ 1 & r<0\end{cases}, \quad (4.3)$$

$$f_n^s={}^1/_2\mathscr{E}u^{1/2}e^{-u}\delta F_n. \quad (4.4)$$

Substituting these expressions into Eqs. (4.2) and (4.1), we obtain

$$j^s=-{}^1/_2u^{3/2}e^{-u}\delta A\mathscr{E}|\mathscr{E}|, \quad (4.5)$$

where

$$A=-\int_{-\infty}^0 dr\,F_1(r)+(\delta-1)\int_0^\infty dr\,F_1(r), \quad (4.6)$$

and the functions $F_n(r)$ satisfy the following system of equations:

$$\frac{1}{2}\frac{d}{dr}(F_{n-1}+F_{n+1})=-F_n+\frac{1}{2}\delta_{n,2}\delta(r), \quad n\geq1, \quad (4.7)$$

$$F_0(r)=0,$$

and the boundary conditions

$$F_n(\pm\infty)=0. \quad (4.8)$$

For convenience we have replaced the lower integration limit in Eqs. (4.6) and (4.8) by $-\infty$, which in the final analysis results only in an unimportant exponential $[\exp(-1/\mathscr{E})]$ error.

Equation (4.5) contains the modulus of the electric field $|\mathscr{E}|$ because the inhomogeneous term of Eq. (4.1) in fact contains the δ function, which after transformation of the variables changes from $\delta(\mathscr{E}r)$ to $\delta(r)/|\mathscr{E}|$. The occurrence of $|\mathscr{E}|$ in Eq. (4.5) guarantees the necessary symmetry properties of the current.

It should be noted that the appearance of the quadratic correction for the current given by Eq. (4.5) is a relatively common occurrence. It is simply due to the presence of a discontinuity of the scattering intensity at the boundary of the passive region and the resultant injection of electrons into a thin boundary layer in the momentum space adjoining directly this boundary. The specific model of an isotropic semiconductor with the deformation scattering of carriers by acoustic and optical phonons determines only the actual form of the system of equations (4.7), which governs the dimensionless factor A in Eq. (4.5).

In the case under discussion we can simplify somewhat Eq. (4.7). We note that since $\delta(r)$ is present only in the equation with $n=2$, all the even components of $F_{2n}(r)$ should be continuous at the point $r=0$. Then, integrating term-by-term the equation with $n=1$ from $-\infty$ to 0 and from 0 to $+\infty$, and applying the boundary conditions of Eq. (4.8), we obtain

$$A={}^1/{}_2\delta a_0, \tag{4.9}$$

where

$$a_0=F_2(0). \tag{4.10}$$

Substituting now the constant A in Eq. (4.5) and then in Eq. (2.11), and returning to the dimensional variables, we obtain the final expression for the nonanalytic correction to the conductivity given by Eq. (1.3)

The parameter a_0 can be estimated numerically from Eq. (4.10), retaining only a small number of functions in the system of equations (4.7). The value of this parameter is approximately $a_0\approx0.5$.

5. NUMERICAL ESTIMATES

Allowing for the nonanalytic correction obtained above, the electrical conductivity of two-dimensional electrons in weak heating fields can be represented as follows:

$$\Delta\sigma/\sigma=\alpha|E|+\beta E^2. \tag{5.1}$$

It follows from Eq. (1.3) that the nonanalytic correction coefficient α is small and may amount to no more than $0.2/E^*$, whereas β is of the order of $(E^*)^{-2}$. Therefore, we shall obtain some numerical estimates in order to estimate specifically the possibility of detecting experimentally this nonanalytic correction.

The coefficient α is given by Eq. (1.3). The coefficient β is calculated using Eq. (3.7) and the inverse operator $\hat{S}_0^{-1}$ is calculated by numerical solution of the relevant equation employing the two-part Monte Carlo method suggested earlier.[6] It is assumed that the thickness d of the layer in question is such that the separation between the levels in the resultant quantum well is of the order of $4\hbar\omega_0$ and if $u\gtrsim1$, we need to include only the main term. The constants and the operators of the electron-phonon interaction are derived from three-dimensional analogs following Ref. 7.

Figure 2 shows the results for a layer of thickness $d=50$ Å in a semiconductor with a parabolic isotropic energy band and parameters corresponding to the heavy-hole band of p-type Ge (Ref. 8). It seems to us that this material is best for the experimental detection of the linear correction to the electrical conductivity because of the strong interaction of holes with optical phonons. It is clear from Fig. 2 that in a relatively wide temperature range the value of β is at least two orders of magnitude less than α and it follows from Eq. (5.1) that the nonanalytic correction should be a few percent of the total conductivity in electric fields, when both corrections become comparable. Such corrections can be detected experimentally using apparatus currently available. It should be pointed out that β changes its sign at $T\sim200$ K, which improves the conditions for the experimental observation of the nonanalytic correction.

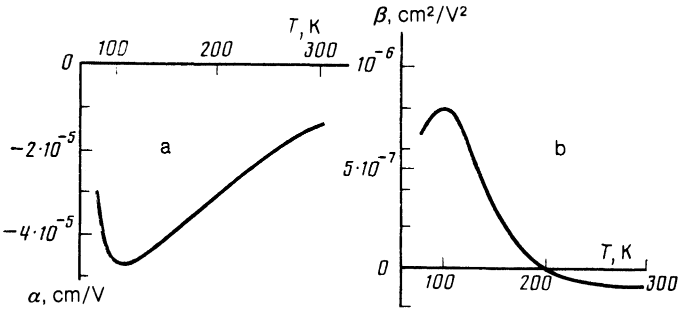

FIG. 2. Coefficients α and β for a covalent semiconductor.

Systems of two-dimensional electrons in heterostructures based on GaAs are at present attracting most attention. We obtained a qualitative estimate of the coefficients α and β for a layer of thickness $d=100$ Å in GaAs and in the case of asymmetric collisional operators we used the relaxation time approximation, borrowing the parameters from Ref. 9. This calculation showed that the temperature dependences of α and β are qualitatively similar to those shown in Fig. 2, although the absolute values are greater than for p-type Ge. For example, the value of α decreases monotonically from -2.6×10^{-4} cm/V at $T=80$ K to -3.8×10^{-5} cm/V at $T=300$ K, whereas β is approximately two orders of magnitude greater than the corresponding coefficient for p-type Ge. It is important to note that an inversion of the sign of β again occurs at $T\sim190$ K.

In addition to the inversion of the sign of the coefficient β the experimental detection of the nonanalytic correction to the conductivity should be facilitated by various inertial properties of the coefficients α and β. It is known that the dispersion of β should manifest itself at frequencies $\omega\propto\tau_\varepsilon^{-1}$, where τ_ε is the energy relaxation time, whereas α is determined by the momentum relaxation time and its dispersion should not appear right up to frequencies $\omega\propto(\tau^+)^{-1}$.

6. CONCLUSIONS

Numerical estimates obtained in the preceding section demonstrate that in the case of two-dimensional electrons in GaAs the nonanalytic correction to the conductivity can be detected at room and liquid nitrogen temperatures using the apparatus currently available.

It should also be pointed out that in reality there is always some smearing of the discontinuity of $\tau(x)$ at the boundary of the passive region. This smearing is due to quantum-mechanical corrections and also due to the diffusion corrections to the acoustic scattering operators which are ignored in the system (2.10). This smearing limits the validity of Eq. (5.1) on the weak field side, so that the validity of Eq. (5.1) is restricted to the range

$$E^*(\Delta\varepsilon/\hbar\omega_0)<E<E^*, \tag{6.1}$$

here $\Delta\varepsilon$ is the characteristic smearing of the discontinuity. Bearing in mind the broadening mechanisms mentioned above, this smearing can be estimated as follows:

$$\frac{\Delta\varepsilon}{\hbar\omega_0} \sim \max\left\{\frac{1}{\omega_0\tau_\varepsilon}, \frac{\tau^+}{\tau_\varepsilon}\right\}, \tag{6.2}$$

which in the cases under discussion here does not exceed 0.1.

We shall conclude by noting that the nonanalytic correction to the conductivity occurs also in principle in the three-dimensional case. However, since a singularity of the momentun relaxation time at the boundary of a passive region in the three-dimensional case is only of the square-root type, this nonanalyticity is weaker and is of the form $|E^{11/6}|$.

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