Phase transitions in crystals with low-symmetry point defects

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The critical behavior of crystals (in which the phase transition is described by an m-component order parameter with hypercubic symmetry) containing frozen point defects is studied by the ε-expansion method in the two-loop approximation. The random field of the point defects is described by symmetric matrices V_{ij}. It is shown that in the case of diagonal matrices V_{ij} that are not multiples of the unit matrix all the components of the fluctuating order-parameter field are decoupled asymptotically in the impurity critical region, and the effective Hamiltonian of the system decomposes into a sum of m Hamiltonians of impurity Ising models. In the case of matrices V_{ij} of general form a tendency of the fluctuating field to become isotropic is observed, and this makes it possible to conjecture the possibility of a phase transition to a state of the spin-glass type.

INTRODUCTION

The effect of randomly distributed "frozen" point defects [describable by the introduction of a scalar field—the "random temperature" of the phase transition (PT)] on the properties of a second-order PT in the fluctuation region has been studied in a number of papers,[1-4] in which it has been shown that at a certain temperature value $T = T_c^*$ in such systems there occurs an unsmeared PT describable by critical indices that do not depend on the impurity concentration. Their values differ from the values of the indices for the pure crystal, if its specific heat diverges; otherwise, the presence of defects does not change the critical behavior of the crystal.

To describe the structure of certain rare-earth amorphous magnets the authors of Ref. 5 proposed a model of a spin system with a random single-ion-anisotropy axis. In the case of an isotropic distribution of the directions of the random axis, with the aid of arguments analogous to those given in Ref. 6 for defects of the "random field" type it was established that for systems with a continuous symmetry group for spatial dimensions $d \leqslant 4$ the uniform low-temperature phase is unstable (irrespective of the defect concentration).[7-9] The similarity of the low-temperature state to the spin-glass phase has been pointed out in a number of papers (see, e.g., Refs. 7, 8, 10, and 11). In particular, recently, the authors of Ref. 12 proved the existence of a PT to an Ising spin glass state in a three-dimensional model of an amorphous magnet with strong random anisotropy and long-range interaction in the limit $m \to \infty$ (m is the number of spin components).

In the case of structural phase transitions to commensurate phases describable by many-component order parameters the symmetry group of the crystal is discrete, and the crystalline anisotropy, as a rule, is not small. The low-symmetry point defects present in the crystal (and violating the symmetry of the crystal class) can locally mix the coordinates of the normal soft modes and (or) lift their degeneracy. In particular, the mixing property is possessed in an obvious way by elastic defects creating internal shear stresses.

It is clear that such defects do not reduce, generally speaking, to models with a random anisotropy axis or a random PT temperature. In the present paper we investigate the question of the influence of randomly located low-symmetry point defects on the character of the PT in crystals in which the order parameter has hypercubic symmetry.

1. EFFECTIVE HAMILTONIAN AND RENORMALIZATION-GROUP EQUATIONS

We write the Hamiltonian of our model with an m-component order parameter $\varphi_i(\mathbf{x})$ in the form

$$H = \int d^d x [H_c(\mathbf{x}) + H_{imp}(\mathbf{x})],$$

$$H_c = \sum_{i=1}^{m} \frac{1}{2} [r_0 \varphi_i^2 + (\nabla \varphi_i)^2] + \sum_{i,k=1}^{m} [g_0 + (\lambda_0 - g_0)\delta_{ik}] \varphi_i^2 \varphi_k^2, \quad (1)$$

$$H_{imp} = \sum_{i,k,t} \tilde{V}_{ik}^t(\mathbf{x}) \varphi_i(\mathbf{x}) \varphi_k(\mathbf{x}).$$

where H_c is the Hamiltonian of the perfect crystal with order parameter possessing hypercubic symmetry and H_{imp} is the Hamiltonian of the interaction with the defects, written in the harmonic approximation. The symmetric random matrices $\tilde{V}_{ik}^t(\mathbf{x})$ describe the local change of the force constants in the presence of the defects. The index t indicates the type of the given point defect; matrices with different values of t can be assigned both to defects with essentially different structures and to identical defects situated at different but crystallographically equivalent positions. The role of these latter defects can be taken by noncentral ions, impurity centers in dumbbell configurations, bivacancies, pairs of the impurity-impurity or impurity-vacancy type, and other defect complexes having several equivalent energy minima. For such defects the matrices V_{ik}^t go over into each other under the action of a set of operators $\{G\}$ corresponding to elements of the symmetry class of the crystal. We shall assume that the presence of the defects does not break the hypercubic symmetry of the crystal on the average, i.e., the occupancies of defect states whose matrices are connected by the transformations $\{G\}$ are equal. We shall assume that the arrangement of the defects is random and uncorrelated, and that their total concentration $\Sigma_t c_t$ is substantially below the percolation threshold. In accordance with the assumptions made, the correlation properties of the random field

$$V_{ik}(\mathbf{x}) = \sum_t V_{ik}^t(\mathbf{x})$$

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can be written in the form

$$\langle V_{ik}(\mathbf{x})\rangle = V\delta_{ik},$$
$$\langle V_{ik}(\mathbf{x})V_{lm}(\mathbf{y})\rangle - \delta(\mathbf{x}-\mathbf{y})\langle V_{ik}(\mathbf{x})\rangle\langle V_{lm}(\mathbf{y})\rangle = \delta(\mathbf{x}-\mathbf{y})\,[2v_0\delta_{ik}\delta_{lm}+w_0(\delta_{il}\delta_{km}+\delta_{im}\delta_{kl}) + 2(u_0-v_0-w_0)\delta_{ik}\delta_{lm}\delta_{il}], \tag{2}$$

where $\langle\ldots\rangle$ denotes a configurational average. From (1) and (2), using the standard replica method[4] we find a translationally invariant effective Hamiltonian:

$$H_{eff} = \int d^dx \Big\{ \sum_{i=1}^{m} \Big[\frac{1}{2}(\tilde{r}_0\Phi_i^2 + |\nabla\Phi_i|^2) + \sum_{\alpha=1}^{n}\Big(\lambda_0\varphi_{i\alpha}^4 + g_0\varphi_{i\alpha}^2\sum_{k\neq i}\varphi_{k\alpha}^2\Big)\Big] - u_0\sum_{i=1}^{m}(\Phi_i^2)^2 - 2v_0\sum_{1\leqslant i<k\leqslant m}\Phi_i^2\Phi_k^2 - 2w_0 \times \sum_{1\leqslant i<k\leqslant m}\Big(\sum_{\alpha=1}^{n}\varphi_{i\alpha}\varphi_{k\alpha}\Big)^2\Big\}, \tag{3}$$

where

$$\tilde{r}_0 = r_0+2V, \quad \Phi_i = \{\varphi_{i1},\ldots,\varphi_{in}\}, \quad \Phi_i^2 = \sum_{\alpha=1}^{n}\varphi_{i\alpha}^2$$

and the region of the bare values of the impurity vertices is $u_0 \geqslant 0$, $u_0 - v_0 \geqslant 0$, $w_0 \geqslant 0$. In particular, for a vector order parameter (with $m = 2, 3$),

$$u_0 = \sum_t c_t(1-c_t)(V_{11}^t)^2, \quad v_0 = \sum_t c_t(1-c_t)V_{11}^tV_{22}^t,$$
$$w_0 = \sum_t c_t(V_{12}^t)^2.$$

By the ε-expansion method, for the effective Hamiltonian (3) in the one-loop approximation we obtain the following system of Gell-Mann–Low (GML) equations for the renormalized invariant charges λ, g, v, u, and w for $n = 0$:

$$\begin{aligned} d\lambda/dt &= \varepsilon\lambda/2 - 36\lambda^2 - 4(m-1)g^2 + 48\lambda u + 8(m-1)gw,\\ dg/dt &= g[\varepsilon/2 - 24\lambda - 4(m+2)g + 16(u+2v) + 8mw] + 24\lambda w,\\ du/dt &= u[\varepsilon/2 - 24\lambda + 32u] - 8(m-1)(g-w)v + 4(m-1)w^2,\\ dv/dt &= v[\varepsilon/2 - 24\lambda - 8(m-2)(g-w) + 16(u+v)] - 8(g-w)u + 4w^2,\\ dw/dt &= w[\varepsilon/2 - 16g + 16(u+2v) + 4mw]. \end{aligned} \tag{4}$$

In (4) $t = \ln(1/r)$, where $r^{-1/2}$ is the correlation length. We start the discussion of the properties of the solutions of the GML equations with an analysis of the special case $w_0 = 0$, to which correspond diagonal defect matrices $V_{ik}^t = \delta_{ik}V_{ii}^t$. In this case the system (4) is found to be isomorphic to the system of GML equations obtained in Ref. 13, in which 14 fixed points (FP) of this system were found and it was shown that none of them is stable. Correspondingly, in Ref. 13 it was concluded that the trajectories "run away to infinity," and it was conjectured that this could be interpreted as the smearing of the PT in such a defect system. Alternatively, some of the phase trajectories could leave the stability region of the Hamiltonian, leading to a first-order PT. However, as will be shown, the conclusion that the PT is smeared in such a model is incorrect. In fact, the system of GML equations with $w = 0$ in the one-loop approximation possesses nonobvious random degeneracy, and in the two-loop approximation has a stable FP. Thus, to analyze the critical behavior of the model (3) it is necessary to use the system of GML equations written in the two-loop approximation:

$$\begin{aligned} d\lambda/dt &= \beta_\lambda^{(1)} - \eta\lambda + 4^3\{27\lambda^3 - 72\lambda^2u + 42\lambda u^2 + (m-1)[3\lambda g^2 + 2g^3 - 2g^2(u+2v+3w) - 6\lambda g(v+w) + {}^3/_2\lambda w^2 + 6\lambda vw + gw(4u+8v+3w)]\},\\ dg/dt &= \beta_g^{(1)} - \eta g + 4^3\{9\lambda^2 g + 18\lambda g^2 + 5(m-1)g^3 - 9\lambda^2 w - 3\lambda g[6(u+2v)+10w] - 2g^2[8u+(6m-5)v] - (14m-15)g^2w + 3\lambda w[4(u+2v)+3w] + g[6u^2+24uv+16uw+2(9m-11)vw + 12v^2 + {}^1/_2(17m-19)w^2]\},\\ du/dt &= \beta_u^{(1)} - \eta u + 4^3\{9\lambda^2u - 36\lambda u^2 + 22u^3 + (m-1)[g^2(u+2v) - 6gv(u+v) - {}^1/_2(3\lambda+5g)w^2 - 2gw(u+2v) + {}^5/_2w^2(u+2v) + 6wv(u+v) + {}^3/_2w^3]\},\\ dv/dt &= \beta_v^{(1)} - \eta v + 4^3\{9\lambda^2v + g^2[2u+(3m-5)v] - 18\lambda v(u+v) - {}^3/_2\lambda w^2 - 2g(4uv+2uw+u^2) - 2(6m-11)gv^2 - 2(3m-5)gvw - (m+{}^1/_2)gw^2 + 6uv(u+2v) + 4v^3 + 2uw(u+4v+2w) + 2(6m-11)v^2w + {}^1/_2(3m+1)vw^2 + {}^3/_2(m-1)w^3\},\\ dw/dt &= \beta_w^{(1)} - \eta w + 4^3w\{6\lambda g + mg^2 - 6\lambda(u+2v+w) - 16gu - 2(3m+1)gv - (5m-6)gw + 6u(u+4v) + 12v^2 + 10uw + (12m-16)vw + {}^5/_2(m-1)w^2\},\\ \eta &= 32\{3\lambda^2 - 6\lambda u + 2u^2 + (m-1)[g^2 - 2g(v+w) + 2vw + {}^1/_2w^2]\}. \end{aligned} \tag{5}$$

In (5) $\beta_X^{(1)}$ are the GML functions of the one-loop approximation. We note that for $m = 2$, as for the case of a perfect tetragonal crystal,[14] there exists a covariant transformation $\tilde{\varphi}_{1\alpha,2\alpha} = (\varphi_{1\alpha} \pm \varphi_{2\alpha})/\sqrt{2}$ that preserves the form of the effective Hamiltonian (3) apart from replacement of the set of bare vertex values $\{X\} \to \{\tilde{X}\}$:

$$\tilde{\lambda}_0 = {}^1/_2(\lambda_0+g_0), \quad \tilde{g}_0 = {}^1/_2(3\lambda_0-g_0),$$
$$\tilde{u}_0 = {}^1/_2(u_0+v_0+w_0), \quad \tilde{v}_0 = {}^1/_2(u_0+v_0-w_0), \quad \tilde{w}_0 = u_0-v_0. \tag{6}$$

The invariance under the transformation (6) leads to the appearance of functional equations that should be satisfied by the exact GML functions β_X, and the values of the indices η and ν, of course, do not change:

$$\begin{aligned} &\beta_\lambda(\tilde{X}) = {}^1/_2[\beta_\lambda(X)+\beta_g(X)], \quad \beta_g(\tilde{X}) = {}^1/_2[3\beta_\lambda(X)-\beta_g(X)],\\ &\beta_u(\tilde{X}) = {}^1/_2[\beta_u(X)+\beta_v(X)+\beta_w(X)],\\ &\beta_v(\tilde{X}) = {}^1/_2[\beta_u(X)+\beta_v(X)-\beta_w(X)],\\ &\beta_w(\tilde{X}) = \beta_u(X)-\beta_v(X), \quad \eta(\tilde{X}) = \eta(X), \quad \nu(\tilde{X}) = \nu(X). \end{aligned} \tag{7}$$

2. CRITICAL BEHAVIOR OF A CRYSTAL WITH DEFECTS DESCRIBABLE BY DIAGONAL MATRICES

If the structure of the defects is such that the matrices V_{ik}^t are diagonal, the bare vertex satisfies $w_0 = 0$. The search for fixed points of the system (5) and analysis of their stability show that in this case, for any value of m, there is a unique physically accessible[1] stable FP with coordinates

$$\lambda^* = {}^1/_{12}(6\varepsilon/53)^{1/2} + O(\varepsilon), \quad u^* = {}^1/_{16}(6\varepsilon/53)^{1/2} + O(\varepsilon),$$
$$g^* = v^* = 0. \tag{8}$$

The degenerate FP with coordinates

$$\lambda^*=\tfrac{1}{12}(6\varepsilon/53)^{1/2}+O(\varepsilon), \quad g^*=0,$$
$$u^*=v^*=\tfrac{1}{16}(6\varepsilon/53)^{1/2}+O(\varepsilon), \tag{9}$$

which was also not found in Ref. 13, is unstable. The FP (8) corresponds to the situation in which the system reaches a Khmel'nitskiĭ fixed point[2] in each of m noninteracting Ising models with scalar impurities. The stability indices of the FP (8) in the (λ, u) plane are known from the solution of the problem of the PT in the impurity Ising model,[2] and the stability indices with respect to the renormalized invariant charges g and v are equal to

$$\lambda_g=\lambda_v=-(6\varepsilon/53)^{1/2}+O(\varepsilon). \tag{10}$$

There is a simple relation connecting these quantities with the specific-heat index in the impurity Ising model; this can be obtained without invoking the expansion of λ_g and λ_v in powers of $\varepsilon^{1/2}$. In fact, near the FP (8) the fields Φ_i fluctuate independently; consequently, the operators Φ_i^2 on the basis (8) have dimension $r^{(1-\alpha)/2\nu}$ (α and ν are the indices of the specific heat and correlation length in the impurity Ising model). It follows from (3) that $\lambda_g=\lambda_v$, and for the index λ_v we obtain

$$\lambda_v=d/2-(1-\alpha)/\nu=\alpha/2\nu<0. \tag{11}$$

Similar arguments were employed by the authors of Ref. 15 in an analysis of the critical behavior of alloys from materials with m_1- and m_2-component order parameters of the competing-anisotropies type. In Ref. 15 it was shown that the two order parameters become asymptotically independent as the system approaches the tetracritical point. In the particular case $m=2$ the stable FP (8) coincides with the FP obtained in Ref. 15 for the values $m_1=m_2=1$. We stress that our conclusion that the critical behavior of the model with $w_0=0$ is equivalent to the behavior of a set of independent impurity Ising models remains valid for all values of m. We note that the critical behavior of the model with $m=2$ and diagonal matrices V_{ij}^t is isomorphic to the behavior of the model with nondiagonal defect matrices of the form $V_{11}^t=V_{22}^t$, $V_{12}^t=V_{21}^t$, by virtue of the existence of the transformations (6).

The region of stability of the FP (8) for the case of small bare values of the impurity vertices coincides approximately, on the (λ, g) plane, with the basin of attraction of the stable FP of the pure crystal. This situation for $m=3$ is illustrated in Fig. 1.

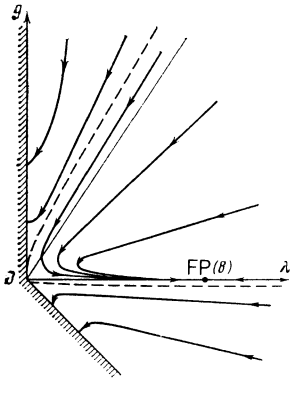

FIG. 1. Qualitative picture of trajectories of the GML equations (5) for $w_0=0$ in projection on to the (λ, g) plane for the case $m=3$ and fixed initial values $u_0, v_0 \ll \lambda_0, g_0$. The solid lines indicate the boundaries of the basin of attraction of the "pure" FP, while the dashed lines indicate the corresponding boundaries for the FP (8). The region of instability of the Hamiltonian (3) is indicated by the shading.

To conclude this section we note that recently, in Ref. 16, for the model of a tetragonal crystal ($m=2$) with scalar point defects, the degeneracy of the GML equations in the one-loop approximation was also pointed out. In the two-loop approximation the corresponding FP was discovered (in Ref. 16 it was labelled VIII). However, the expressions given in Ref. 16 for the GML functions contain a number of numerical errors, as is immediately clear from the fact that these expressions do not satisfy the exact relations (7) with $w=0$ and $u=v$. Correspondingly, the coordinates of the FP were also wrongly determined; in fact, they can be obtained from the coordinates of the FP (9) by the symmetry transformation (6). We shall not give the corrected values of the corresponding critical indices, since this FP is unstable.

3. CRITICAL BEHAVIOR OF CRYSTALS WITH DEFECTS DESCRIBABLE BY ARBITRARY MATRICES

We now consider the case of arbitrary low-symmetry defects describable by symmetric matrices V_{ij}^t of general form, when the vertex $w\neq 0$. It is not difficult to convince oneself that the FP (8), stable for $w=0$, becomes unstable for all values of m when a nonzero vertex w is included. Thus, to analyze the character of the solutions of the system of GML equations we need to find the fixed points with values $w^*\neq 0$ and to ascertain their stability. For this it is convenient to introduce new renormalization-group variables:

$$a=\lambda/w, \quad b=g/w, \quad c=(u-v)/w, \quad f=v/w.$$

Using the GML system (4), in the one-loop approximation we obtain the following system of equations:

$$\begin{aligned}
da/dt&=w[-36a^2+16ab-4(m-1)b^2+32ac+8(m-1)b-4ma],\\
db/dt&=w[-24ab-4(m-2)b^2+24a+4mb],\\
dc/dt&=w[24(b-a)c+4(4c+2-m)(c-1)],\\
df/dt&=w\{-16f^2+[-24a-8(m-3)b+4(m-2)]f\\
&\quad +4(1+2c-2bc)\},\\
dw/dt&=w[\varepsilon/2+w(-16b+48f+16c+4m)].
\end{aligned} \tag{12}$$

It can be seen from (12) that for the FP coordinates a^*, b^*, c^* there is a closed system of three equations. To each set a^*, b^*, c^* there correspond two different values of the coordinate f^*. Since w^* can be determined in an elementary manner from the known values of a^*, b^*, c^*, and f^*, in the following we shall confine ourselves to indicating the first four coordinates of the FP. The coordinates of some of the FP can be found easily[2)]:

$$\begin{aligned}
&A_\pm\colon\ a^*=b^*=0, \quad c^*=1, \quad f_\pm^*=\tfrac{1}{8}\{m-2\pm[(m-2)^2+48]^{1/2}\},\\
&B_\pm\colon\ a^*=b^*=0, \quad c^*=(m-2)/4, \quad f_\pm^*=\tfrac{1}{8}[m-2\pm(m+2)],\\
&C_\pm\colon\ a^*=(4-m)/3, \quad b^*=2, \quad c^*=(2-m)/4,\\
&f_\pm^*=\tfrac{1}{8}[2-m\pm(m+2)],
\end{aligned} \tag{13}$$

$$D_\pm\colon\ a^*=b^*=\frac{m+6}{m+4}, \quad c^*=1,$$

$$f_\pm^*=\frac{1}{8(m+4)}\{-(m^2+10m+8)\pm[(m^2+10m+8)^2+16m(m+4)]^{1/2}\}.$$

Solving the system of equations $da/dt=db/dt=dc/$

TABLE I. Fixed-point coordinates obtained from the relations (14) for certain values of m.

m	a^*	b^*	c^*	f_+^*	f_-^*
2	$\frac{1+\sqrt{13}}{9}$	$\frac{5+\sqrt{13}}{3}$	0	$\frac{2+\sqrt{13}}{6}$	$\frac{2-\sqrt{13}}{6}$
	$\frac{1-\sqrt{13}}{9}$	$\frac{5-\sqrt{13}}{3}$	0	$\frac{2+\sqrt{13}}{6}$	$\frac{2-\sqrt{13}}{6}$
3	0.3083 −0.8358	2.0518 0.6841	−0.2179 −0.6381	0.5069 1.5972	−0,7193 −0,0934
4	0	2	−1	$\frac{-1+\sqrt{13}}{4}$	$\frac{-1-\sqrt{13}}{4}$
	$-5/4$	$3/4$	−1	$1+3/\sqrt{8}$	$1-3\sqrt{8}$
5	0.7863 −1.6364	−1.2079 0.7884	4.5775 −1.3182	2.7248 2.4610	−1.9464 −0.0449
6	0.6825 −2.0088	−0.8014 0.8147	3.9742 −1.6169	2.6328 2.8266	−1.4545 −0.0355
7	0.6188 −2.3728	−0.5936 0.8342	3.7339 −1.9042	2.7024 3.1699	−1.1934 −0.0291
8	0.5581 −2.7311	−0.4540 0.8495	3.6016 −2.1841	2.8163 3.4973	−1.0185 −0.0245

$dt = 0$ is equivalent to finding the roots of an eighth-order equation. Since we know four roots of this equation, the problem becomes solvable in quadratures. With the aid of simple algebraic substitutions, and also using (13), we obtain the corresponding fourth-order equation and relations determining the coordinates of the FP in terms of its roots:

$$-(m-4)(m+4)(3m-4)b^4+(m-4)(11m^2+2m-8)b^3 +(-15m^3+108m^2-196m+112)b^2 +(m-2)(9m^2-88m+76)b-2(m-2)(m^2-16m+12)=0, \tag{14}$$

$$a=\frac{b[m-(m-2)b]}{6(b-1)},$$

$$c=\frac{1}{8a}[9a^2-4ab+(m-1)b^2-2(m-1)b+ma].$$

We note that the case $m = 4$ is degenerate and there is an additional root $a^* = 0$, $b^* = 2$, $c^* = -1$ not described by the relations (14). The numerical values of the real roots b^* for the physically interesting cases $m = 2,\ldots, 8$, and also the corresponding values of the coordinates a^*, c^*, and $f_\pm^*$ at the FP are given in the table. Using the data from this table, and also (13), it is easy to find the coordinates of all the FP with $w^* \neq 0$ in the original variables; because of their cumbersomeness, we shall not give these values. Investigating the fixed points obtained, we convince ourselves that none of them is stable.

If we go over from the parameter t in (12) to the new monotonic variable

$$\tilde{t}=\int_0^t w(t')\,dt',$$

a standard analysis of the stability of the FP in the subspace $\{a, b, c, f\}$ shows that the only stable and physically accessible FP in this subspace is the point D_+. In particular, it is reached in the case of the usual relative magnitudes of the initial values of the pure and impurity vertices: $\lambda_0, g_0 \gg u_0, v_0, w_0$. The eigenvalues λ_i corresponding to this FP (for the running variable $\tilde{t}$) are equal to

$$\lambda_1=\frac{-2(m^2+10m+24)}{m+4},$$

$$\lambda_2=\frac{-4[(m^2+10m+8)^2+16m(m+4)]^{1/2}}{m+4} \tag{15}$$

$$\lambda_{3,4}=\frac{-2[m^2+4m+8\pm 2i(3m^2+24m+32)^{1/2}]}{m+4}.$$

The qualitative behavior of the phase trajectories projected on to the subspace $(s = a - b, c)$ for $m = 3$ is depicted in Fig. 2.

It is important to note that satisfying $a = b$ and $c = 1$ is equivalent to the satisfying $\lambda = g$ and $u - v = w$, implying the raising of the symmetry of the system to isotropic symmetry. The presence of the stable D_+-point in the subspace $\{a, b, c, f\}$ is evidence of the tendency of the phase trajectories of the system of Eqs. (12) to emerge on to an attracting set $\{a = b, c = 1\}$. Here the invariant charge w goes to infinity in the five-dimensional space $\{\lambda, g, u, v, w\}$, but the phase trajectories do not leave the region of stability of the Hamil-

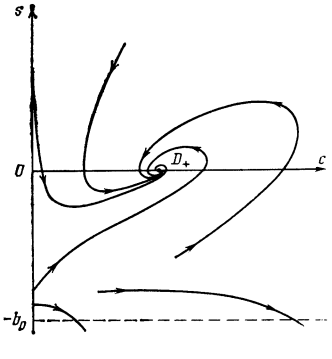

FIG. 2. Qualitative picture of phase trajectories of the system (12) in projection on to the $(s = a - b, c)$ plane, illustrating the isotropization of the solution of the GML equations. The dashed line indicates the boundary of stability of the bare Hamiltonian.

tonian. Thus we arrive at the conclusion that in the system of GML equations (4) we have a situation of the "running away to infinity" type. In the particular case of an isotropic system ($\lambda_0 = g_0$, $u_0 - v_0 = w_0$, $w_0/v_0 = -m$), which arises in the problem of the critical behavior of a magnet with an impurity of the "random anisotropy axis" type, the existence of "running away to infinity" was first pointed out in Ref. 13. The change to the variables $\{a, b, c, f, w\}$ has made it possible to elucidate the specific character of the "running away to infinity" in our case: In the impurity critical region the fluctuation interaction displays a tendency to isotropization. Of course, strictly speaking, only in the region of values $w \lesssim \varepsilon$ is it possible to verify this conclusion, which was obtained in the framework of the one-loop approximation of the renormalization group. It is interesting, therefore, to clarify whether the isotropization of the system is an effect inherent in just the one-loop approximation (4), or whether it also occurs in higher approximations.

An investigation, to this end, of the behavior of the phase trajectories of the system of GML equations (5), rewritten in the variables a, b, c, f, w, showed that the isotropization of the solutions is preserved in the two-loop approximation as well. Here, as before, the trajectories pass to the subspace with coordinates ($a = b, c = 1$); however, instead of the situation in which the invariant charge w runs away to infinity we have discovered the appearance of a completely stable FP $\tilde{D}$, situated at a finite distance from the coordinate origin. The numerical values of its coordinates $\tilde{a}^*, \tilde{f}^*, \tilde{w}^*$ depend weakly on m, and for $m = 2, 3$ are respectively equal to (to within terms of order ε)

$$\begin{aligned} &\tilde{D}(m=2):\ \tilde{a}^* \approx 1.370,\quad \tilde{f}^* \approx 0.0175,\quad \tilde{w}^* \approx 0.046,\\ &\tilde{D}(m=3):\ \tilde{a}^* \approx 1.361,\ \tilde{f}^* \approx -0.0020,\ \tilde{w}^* \approx 0.079. \end{aligned} \tag{16}$$

What physical consequences can be derived from the fact of the existence of the stable FP $\tilde{D}$ in the two-loop approximation? Of course, we cannot conclude that a sharp PT of the system to an ordinary uniform state at a definite temperature $T = T_c^*$ is possible. Although, formally, there is no running away to infinity in the two-loop approximation, the FP $\tilde{D}$ nevertheless lies beyond the limits of applicability of the ε-expansion. In addition, for the isotropic model ($a = b$, $c = 1$) it has been proved that the uniform low-temperature phase is unstable, and in the case $m \to \infty$ with long-range interaction the authors of Ref. 12 established the equivalence of the PT in the system to a transition to an Ising-spin-glass phase.

On the other hand, we suggest that the conclusion that the system becomes isotropic in the critical impurity region, which follows from the fact that the phase trajectories tend toward the subspace ($a = b, c = 1$), is evidently reliable. Indeed, comparing the coordinates of the fixed points D_+ and D in the subspace $\{a, b, c, f\}$, we see that the position of the stable FP in this subspace changed little when we went from the one-loop to the two-loop approximation of the GML equations. We note that the system of equations satisfied by the coordinates of the FP with $w^* \neq 0$ in the $(N+1)$-loop approximation can be written in the form

$$P_i^{(0)}(z) + \sum_{k=1}^{N} w^k P_i^{(k)}(z) = 0, \quad i = a,\ b,\ c,\ f,\ w, \tag{17}$$

where $z = \{a, b, c, f\}$, $P_i^{(0)}(z)$ are functions corresponding to the one-loop approximation, and $P_i^{(k)}(z)$ are functions in the $(k+1)$-loop approximation. Since, according to (16), the value of the coordinate w^* for the fixed point $\tilde{D}$ is numerically small, and $\tilde{a}^* = \tilde{b}^* \cong 1$ and $\tilde{c}^* = 1$, we arrive at the conclusion that it is most probable that the position of the FP $\tilde{D}$ changes little in calculations in higher orders as well.

CONCLUSION

To conclude we shall summarize the main results obtained above. In the case of comparatively symmetrical point defects, e.g., in the presence of noncentral ions lying on symmetry axes of the crystal, the defect matrices V_{ik}^l are diagonal. In this case the phase transition in a cubic crystal remains sharp, but the critical behavior in the presence of the defects is changed. For any number of components of the order parameter, as the PT point is approached, the fluctuating field decomposes asymptotically into a set of m noninteracting Ising models with scalar impurities. We note that it is well known that the presence of ordinary scalar impurities does not affect the critical behavior of a crystal with $m \geqslant 3$ components of the order parameter, so that defects describable by diagonal matrices that are not multiples of the unit matrix give rise to a stronger perturbation of the critical behavior than do scalar defects.

When the crystal contains point defects with arbitrarily low symmetry the phase trajectories of the system of renormalization-group equations, while not leaving the region of stability, do go beyond the limits of the region of applicability of ε-expansion, and we have a situation of the "running away to infinity" type. Here an interesting feature of the solutions of the renormalization-group equations is the special way the trajectories run away to infinity, which implies isotropization of the field of the fluctuations, both in the one-loop and in the two-loop approximation. Since in the case of the isotropic model there exist arguments that it has a transition to a spin-glass state, it is highly tempting to postulate the possibility of a PT in a cubic crystal with low-symmetry point defects to a state, with finite correlation length, of the spin-glass type at a temperature T_f^* lying in the impurity fluctuation region of temperatures.

Data from the observation of the long-lived relaxation phenomena inherent to states of the spin-glass type could serve as experimental data confirming the possibility of a phase transition of this type.

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[1)] The stable FP with coordinates $\lambda^* = g^* = 0$, $u^* = v^* = -\varepsilon/64$ describes the PT in polymers, but is not accessible in our problem. For $m > 4$ there appears one further stable FP, but it lies in the region of instability of the system.

[2)] We note that the FP C_+ in (13) corresponds to the degenerate FP of the system (4).

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