Dynamic damping of domain walls in weak ferromagnets

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A microscopic theory is developed for the energy dissipation of a moving domain wall of a weak ferromagnet by interaction between the wall and thermal magnons. Different dynamic regimes are considered—translational wall motion and propagation of flexure waves along the wall. The special roles of small system-energy terms that disturb the soliton character of the domain wall, and of the fact that the wall does not reflect the magnons with which it interacts, are pointed out. A phenomenological equation that describes consistently the domain-wall dynamics and relaxation is proposed on the basis of the microscopic calculations.

1. INTRODUCTION

Dynamic damping of domain walls (DW) determines the character of the phenomena accompanying rapid magnetization reversal of high-grade magnets, the motion of solitary domains and DW, and others. Study of these phenomena is of interest for technical applications.[1] Analysis of problems encountered in DW relaxation is important also when it comes to use the highly developed theory of magnetic solitons[2] in physical applications of magnetism. The problems posed and solved in natural fashion involve in this case the interaction of a DW (magnetic soliton) with a magnon heat reservoir, or the allowance for the loss of total integrability in real magnets.

Particular interest attaches to a comparison of the theory of DW damping with experimental data on the viscous-friction coefficient (mobility) of DW in translational motion, and on the damping of flexural waves in domain walls.[1,3] Experimental data on the damping of DW in a wide temperature range, however, have been obtained so far for two-sublattice weak ferromagnets (WFM) such as orthoferrites [and it can be assumed that DW damping in orthoferrites is determined by intrinsic relaxation processes (see Ref. 3)], whereas a theoretical analysis was carried out only for ferromagnets.[4,5] In addition to interpreting the experimental data, analysis of DW relaxation in WFM is also of general interest for soliton theory in real nonlinear systems. The point is that the dynamics of WFM such as orthoferrites are described, albeit approximately, by the sine-Gordon equation (see Ref. 3), which has been thoroughly investigated and has become in a certain sense the standard exactly integrable equation.[6]

The present paper is devoted to an analysis and relaxation of DW based on a macroscopic approach to translational motion and propagation of flexural waves (FW) of DW in WFM such as orthoferrites. The viscous-friction coefficient of the DW and the damping decrement of the FW are found. A general phenomenological equation that accords with microscopic calculations is derived for the DW coordinate.

1. THE MODEL

To describe the dynamic properties of a two-sublattice rhombic WFM we start with the standard equation for the energy[7]

$$W=M_0^2\int d\mathbf{r}\left\{\frac{\delta}{2}\mathbf{m}^2+\frac{\alpha}{2}(\nabla\mathbf{l})^2+\frac{\beta_1}{2}l_x^2+\frac{\beta_2}{2}l_z^2+\frac{1}{4}(b_1l_x^4+b_2l_x^2l_z^2+b_3l_z^4)+d_1m_xl_z-d_3m_zl_x\right\}. \quad (1)$$

Here $\mathbf{m}=(\mathbf{M}_1+\mathbf{M}_2)/2$ and $\mathbf{l}\,(\mathbf{M}_1-\mathbf{M}_2)/2$ are respectively the weak-ferromagnet and the antiferromagnetism vectors; $\mathbf{M}_1$ and $\mathbf{M}_2$ are the sublattice-magnetization vectors $M_0=|\mathbf{M}_1|=|\mathbf{M}_2|$; α and δ are the exchange constants; β_i and b_i are respectively the second- and fourth-order anisotropy constants; d_1 and d_3 are the Dzyaloshinskiĭ-interaction constants; x, y, and z coincide with the **a**, **b**, and **c** axes of the WFM.

It is convenient to begin with the equations for the normalized antiferromagnetism vector $\mathbf{l}(\mathbf{l}^2=1$; see Refs. 8 and 3 for details). The dynamics of the angle variables for the vector $\mathbf{l}(l_x+il_y=\sin\theta e^{i\varphi},\ l_z=\cos\theta)$ is determined by a Lagrangian of the form

$$\mathcal{L}=\mathcal{L}^{(0)}+\mathcal{L}^{(i)}, \quad (2)$$

where the two terms are of different order of magnitude,

$$\mathcal{L}^{(0)}=M_0^2\int d\mathbf{r}\left\{\frac{\alpha}{2c^2}(\dot\theta^2+\sin^2\theta\dot\varphi^2)-\frac{\alpha}{2}[(\nabla\theta)^2+\sin^2\theta(\nabla\varphi)^2]-\frac{1}{2}(\beta_1\sin^2\theta\sin^2\varphi+\beta_3\cos^2\theta)\right\}, \quad (3)$$

$$\mathcal{L}^{(i)}=M_0^2\int d\mathbf{r}\left\{D\dot\theta\sin^3\theta\sin^2\varphi-\frac{1}{4}\sin^4\theta(b_2+b_3\sin^2\varphi+b_1\sin^4\varphi)\right\}. \quad (4)$$

Here $\dot\theta=\partial\theta/\partial t$, $c=1/2gM_0(\alpha\delta)^{1/2}$ is the spin-wave velocity, $g=2|\mu_0|/\hbar$, μ_0 is the Bohr magneton, and $D=6(d_1-d_3)/g\delta M_0$. Note that the nontrivial term with $\dot\theta$ appears when $d_1\neq d_3$, i.e., when the Dzyaloshinskiĭ interaction is not antisymmetric.[9–11] Usually $|d_1-d_3|\ll d_{1,3}$ and

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$b_i \ll \beta_i$ (Ref. 7), therefore $\mathscr{L}^{(i)} \ll \mathscr{L}^{(0)}$. The terms in $\mathscr{L}^{(i)}$, however, are basic for the analysis of the damping of the magnetic solitons that describe the DW. We shall ascertain later on that the basic difference between the models with $\mathscr{L} = \mathscr{L}^{(0)}$ $(b, D = 0)$ and with $\mathscr{L} = \mathscr{L}^{(0)} + \mathscr{L}^{(i)}$ appears literally in all the aspects of the problem. We shall therefore consider these models separately, and name them respectively "idealized (for $\mathscr{L} = \mathscr{L}^{(0)}$) and "generalized" (when $\mathscr{L}^{(i)}$ is taken into account). Analysis of the idealized model is important not only from the procedural but also from the physical viewpoint, since the values of the constants b_i/β_i and $D\omega_0$ are usually small ($\lesssim 10^{-2}$) (ω_0 is the WFM magnon frequency).

We shall assume that $\beta_1 > 0$ and $\beta_2 > 0$, and that in the WFM ground state **l** and **m** are parallel to the axes **a** and **c**, respectively. The equations corresponding to the idealized WFM model have two particular classes of solutions of the form $\theta = \pi/2$, $\varphi = \varphi(\mathbf{r}, t)$ and $\varphi = \pi/2$, $\theta = \theta(\mathbf{r}, t)$ (solutions of type *ac* and *ab*, see Ref. 3). For $\beta_2 > \beta_1$, as is usually realized in orthoferrites, the *ac*-type solutions are stable (see Refs. 3 and 12). We consider henceforth just this case; the situation with the sign of $(\beta_2 - \beta_1)$ reversed are analyzed similarly.

The dynamics of the angle variable φ in the *ac*-type solution is described by the Lorentz-invariant equation

$$2\alpha\left(\frac{\partial^2\varphi}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}\right) = \beta_1 \sin 2\varphi. \tag{5}$$

The simplest solution of (5) describes an *ac*-type 180-deg moving DW:

$$\operatorname{tg}\frac{\varphi_0}{2} = \exp(\pm\xi), \qquad \xi = \frac{y - vt - y_s}{y_0(1 - v^2/c^2)^{1/2}}, \tag{6}$$

where $y_0 = (\alpha/\beta)^{1/2}$ is the DW thickness at $v = 0$, and y_s is an arbitrary constant. For the generalized model we have at $v = 0$

$$\operatorname{tg}\varphi_0 = (1+p)^{-1/2}/\operatorname{sh}\xi, \tag{7}$$

where $p = b_1/2\beta_1$, with $p \ll 1$ for $b \ll \beta$. For $v \neq 0$, however, allowance for the term with D can alter the DW symmetry,[13] and there is no known exact solution that describes the DW.

2. SPIN WAVES ON A DW BACKGROUND

A microscopic analysis of the DW relaxation calls for knowledge of the spectrum of the spin waves on a DW background. Putting

$$\theta = \theta_0 + \vartheta(\mathbf{r}, t), \quad \varphi = \varphi_0 + \psi(\mathbf{r}, t), \quad \theta_0 = \pi/2, \quad \vartheta, \ \psi \ll 1 \tag{8}$$

and substituting (8) in (3) and (4), we represent the Lagrangian of the magnons in a WFM with DW in the form

$$\mathscr{L} = \mathscr{L}_2 + \mathscr{L}_3 + \mathscr{L}_4 + \ldots, \tag{9}$$

where $\mathscr{L}_n$ contains the variables ϑ and ψ raised to a total power n, and $\mathscr{L}_1 = 0$ for any WFM model by virtue of the equations for θ_0 and φ_0. We represent the Lagrangians $\mathscr{L}_2$, $\mathscr{L}_3$, etc. in the form $\mathscr{L}_n = \mathscr{L}_n^{(0)} + \mathscr{L}_n^{(i)}$, where $\mathscr{L}^{(i)} \propto b_i$ or D.

For $v = 0$ it is easy to calculate all the $\mathscr{L}_n$ from (7), but for $v \neq 0$ only $\mathscr{L}_n^{(0)}$ can be expressed exactly. We confine ourselves to the case of low velocities, assuming the small parameters b_i/β, $D\omega_0$ and v/c to be independent.

We begin the actual analysis with the case of the idealized model, for which we have at $v = 0$

$$\mathscr{L}_2^{(0)} = \frac{\beta_1 M_0^2}{2}\int d\mathbf{r}\left\{\frac{1}{\omega_0^2}(\dot\vartheta^2 + \dot\psi^2) - \psi\hat{L}\psi - \vartheta(\hat{L} + \sigma)\vartheta\right\}, \tag{10}$$

where $\omega_0 = c/y_0$, $\sigma = (\beta_2 - \beta_1)/\beta_1$, and the operator $\hat{L}$ has the form of a Schrödinger operator with a nonreflecting potential and a known complete set of eigenfunctions:

$$\hat{L} = -y_0^2\nabla^2 + 1 - 2/\operatorname{ch}^2\xi, \quad \xi = (y - y_s)/y_0, \quad \hat{L}f_j = \lambda_j f_j, \tag{11}$$
$$f_k = (\Omega^{1/2} b_k)^{-1}(\operatorname{th}\xi - ik_y y_0)\exp(i\mathbf{kr}), \quad \lambda_k = 1 + k^2 y_0^2;$$

$$f_\varkappa = (2y_0 S)^{-1/2}(1/\operatorname{ch}\xi)\exp(i\boldsymbol{\varkappa}\mathbf{r}_\perp), \quad \lambda_\varkappa = \varkappa^2 y_0^2. \tag{12}$$

Here Ω is the volume of the crystal, S the DW area, $b_k = (1 + k_y^2 y_0^2)^{1/2}$, $\boldsymbol{\varkappa}$ a two-dimensional vector in the DW plane, $k^2 = \mathbf{k}^2$, and $\varkappa^2 = \boldsymbol{\varkappa}^2$.

For a correct description of the interaction between a moving DW and magnon heat reservoir we proceed as follows. We assume that the DW is immobile at an arbitrary point y_s and expand the functions ϑ and ψ is the eigenfunctions of the operator (12):

$$\psi = \sum_1 q_1 f_1(\mathbf{r}, y_s), \qquad \vartheta = \sum_1 Q_1 f_1(\mathbf{r}, y_s),$$
$$1 = \mathbf{k}_1 \quad \text{or} \quad \boldsymbol{\varkappa}_1. \tag{13}$$

If the DW moves uniformly, the parameter $y_s = vt$ depends on the time. In this case, however, the functions f_k and $f_\varkappa$ remain a complete and orthonormal system, and we can use as before an expansion in the form (13). This choice of the magnon states corresponds to a natural physical condition that the magnon gas (heat reservoir) be at rest far from the DW, and that the magnons near the DW be adibatically "attuned" to the instantaneous DW position.

For a moving DW y_s, and hence $f_1(\mathbf{r}, y_s)$, depends explicitly on the time. The kinetic part of $\mathscr{L}_2^{(0)}$ does not reduce therefore to the form $\Sigma_1 \dot q_1 \dot q_{-1}$, and contains additional terms bilinear in q_1, $\dot q_1$ and Q_1, $\dot Q_1$. The canonical momenta conjugate to q_1 and Q_1 are therefore not proportional to $\dot q_{-1}$ and $\dot Q_{-1}$:

$$p_1 = m\left(\dot q_{-1} + \sum_2 \dot q_2\langle f_1 \dot f_2\rangle\right), \quad P_1 = m\left(\dot Q_{-1} + \sum_2 Q_2\langle f_1 \dot f_2\rangle\right),$$
$$m = \beta_1 M_0^2/\omega_0^2, \tag{14}$$

[where $\langle\ldots\rangle = \int(\ldots)d\mathbf{r}$], which makes $H_2^{(0)}$ nondiagonal for $v \neq 0$ (see Sec. 4 below). If, however, $v = 0$, then $H_2^{(0)} = H_0$ and has a canonical form, and in terms of the magnon-field creation and annihilation operators ψ and ϑ (henceforth ψ and ϑ magnons)

$$q_1 = \left(\frac{\hbar}{2m\omega_1}\right)^{1/2}(a_1 + a_{-1}^+), \qquad Q_1 = \left(\frac{\hbar}{2m\Omega_1}\right)^{1/2}(A_1 + A_{-1}^+), \tag{15}$$

$$p_1 = i\left(\frac{\hbar m\omega_1}{2}\right)^{1/2}(a_1^+ - a_{-1}), \qquad P_1 = i\left(\frac{\hbar m\Omega_1}{2}\right)^{1/2}(A_1^+ - A_{-1}) \tag{16}$$

the Hamiltonian $H_2^{(0)} = H_1$ is diagonal:

$$H_0 = \sum_1(\hbar\omega_1 a_1^+ a_1 + \hbar\Omega_1 A_1^+ A_1), \tag{17}$$

and describes free magnons against the background of an immobile DW. The spectrum of a WFM with DW contains four magnon modes, two with wave functions f_k and frequencies

$$\omega_k=\omega_0(1+k^2y_0^2)^{1/2}, \quad \Omega_k=\omega_0(1+\sigma+k^2y_0^2)^{1/2}, \tag{18}$$

corresponding to bulk (intradomain) excitations, and two with wave functions $f_\varkappa$ having frequencies

$$\omega_\varkappa=\omega_0 y_0|\varkappa|=c|\varkappa|, \quad \Omega_\varkappa=\omega_0(\sigma+\varkappa^2y_0^2)^{1/2}, \tag{19}$$

and corresponding to excitations localized on the DW (intraboundary excitations).

It is known (see Ref. 3 that the ψ-magnon localized mode describes FW propagating along the DW, and the operators $a_\varkappa^+$ and $a_\varkappa$ have the meaning of FW-quantum creation and annihilation operators. This permits the FW damping decrement $\gamma(\varkappa)$ to be calculated as the imaginary part of the mass operaor of localized ψ magnons having a momentum $\hbar\varkappa$ (see Refs. 14 and 15). A contribution to $\gamma(\varkappa)$ can be made by three- and four-magnon processes described by Hamiltonians H_3 and H_4.

In the idealized model the three-magnon Hamiltonian $H_3=\mathscr{L}_3=-(\mathscr{L}_3^{(0)}+\mathscr{L}_3^{(i)})$ takes the form

$$H_3^{(0)}=-\frac{\beta_1 M_0^2}{2}\int d\mathbf{r}\left\{\sin 2\varphi_0\left(\vartheta^2+\frac{2}{3}\psi^2\right)\psi +2y_0\sin\varphi_0\left(\vartheta^2\frac{\partial\psi}{\partial y}\right)\right\}, \tag{20}$$

or in terms of the magnon creation and annihilation operators

$$H_3^{(0)}=\sum_{123}\{P_{123}(a_1+a_{-1}^+)(a_2+a_{-2}^+)(a_3+a_{-3}^+) +R_{1,23}(a_1+a_{-1}^+)(A_2+A_{-2}^+)(A_3+A_{-3}^+)\}\Delta(\mathbf{1}_\perp+\mathbf{2}_\perp+\mathbf{3}_\perp), \tag{21}$$

where $\mathbf{1}_\perp=\mathbf{k}_\perp$ or $\varkappa$, $\mathbf{k}_\perp=(k_x,0,k_z)$,

$$P_{123}=\frac{\hbar\omega_0^2}{3}\left(\frac{\hbar}{2m\omega_1\omega_2\omega_3}\right)^{1/2}\left\langle\frac{f_1f_2f_3\,\mathrm{sh}\,\xi}{\mathrm{ch}^2\xi}\right\rangle\exp(iQ_{123}vt), \tag{22}$$

$$R_{1,23}=2\hbar\omega_0^2\left(\frac{\hbar}{2m\omega_1\Omega_2\Omega_3}\right)^{1/2}\left\langle\left(f_1\,\mathrm{sh}\,\xi-y_0\frac{df_1}{dy}\right)\frac{f_2f_3}{\mathrm{ch}^2\xi}\right\rangle \cdot\exp(iQ_{123}vt). \tag{23}$$

We have left out of these equations the terms proportional to v,

$$\langle\ldots\rangle=\int_{-\infty}^{+\infty}(\ldots)d\xi.$$

For each of the interaction processes described by Eq. (21), the DW acquires in the direction of its normal a total momentum $\hbar Q_{123}$, where $Q_{k1k2k3}=k_{1y}+k_{2y}\,k_{3y}$, $Q_{k1k2k3}=k_{1y}+k_{2y}$, etc., and the momentum of the DW plane is conserved.

Note that the amplitude of a process in which three surface ψ magnons participate is strictly zero by virtue of (22). (A similar process is possible with θ magnons.) This holds true for any WFM model if the DW does not stick to an inhomogeneity. If the DW sticks, such a process is possible and should make the main contribution to the FW damping, especially at low temperatures (a similar problem was analyzed for a ferromagnet by Janak[16]). In a homogeneous magnet, the contribution of the three-magnon processes with participation of activated magnons may have to compete with four-magnon processes $H_4^0=\mathscr{L}_4^{(0)}$ (see Ref. 3) with participation of only FW magnons (see below), and the FW damping decrement $\gamma^{(0)}(\varkappa)$ of the idealized model takes the form

$$\gamma^{(0)}(\varkappa)=\gamma_3^{(0)}(\varkappa)+\gamma_4^{(0)}(\varkappa). \tag{24}$$

Note that if the DW moves, the Hamiltonian $H_3^{(0)}$ depends explicitly on the time via the factor $\exp(iQ_{123}vt)$ [see Eqs. (22) and (23)]. The same holds also for the off-diagonal part of the two-magnon Hamiltonian H_2^{nd}; the existence of this part was already noted above.[1)] The explicit time dependence of the terms in H_1, H_4, and H_2^{nd} at $v\neq 0$ cause a change of the magnon-gas energy: the DW transfers an energy $\hbar Qv$ to the magnons in a single event. (In this approach, the DW enters in the magnon Hamiltonian $H=H_2+H_3+\ldots$ as an external time-dependent classical field.) Since the "DW + magnons" system is closed, the quantity dE/dt, where E is the energy of the magnons, is naturally connected with the DW damping. In particular, the dynamic-damping force F acting on a uniformly moving DW can be expressed in terms of dE/dt (see Refs. 4 and 5):

$$F=-(dE/dt)(1/v). \tag{25}$$

We have thus two approaches to the calculation of the DW relaxation characteristics. In one approach we investigate the damping rate of the DW FW, while the FW magnon is treated together with all others on the basis of the general Hamiltonian. In the second approach the DW acts as an external field and leads to inelastic processes in the magnon gas, and the DW damping is determined by the value of dE/dt. we shall compare below the results of analyses based on the two approaches.

3. FW RELAXATION IN THE IDEALIZED MODEL

As noted above, the three-magnon Hamiltonian $H_3^{(0)}$ of the idealized model contains a large number of terms that describe processes of the *aaa* or *AAA* type. Ten processes vital in the discussion that follows are shown schematically in Fig. 1.

We begin with an analysis of the FW damping decrement $\gamma_3^{(0)}$. In the Born approximation, $\gamma_3^{(0)}$ is determined by the four process, 2, 3, 7, and 10 of Fig. 1:

$$\gamma_3^{(0)}=\gamma^{(2)}+\gamma^{(3)}+\gamma^{(7)}+\gamma^{(10)},$$
$$\gamma^{(m)}=\frac{2\pi}{\hbar^2}\sum_{1,2}|\Phi^{(m)}(12\varkappa)|^2(n_1-n_2)\delta(\omega_1+\omega_\varkappa-\omega_2), \tag{26}$$

where $n=n(\hbar\omega_1)$ or $n(\hbar\Omega_1)$ are the magnon occupation numbers. The amplitudes of the processes can be easily determined form Eqs. (21)–(23) and the wave functions (12). We present one of them for $v=0$:

$$\Phi^{(2)}(k_1k_2\varkappa)=6P_{-k_1k_2\varkappa}$$

$$=\frac{i\pi\omega_0^2(\hbar^3Sy_0)^{1/2}(b_1^2-b_2^2)\Delta(\mathbf{k}_{1\perp}-\mathbf{k}_{2\perp}-\varkappa)}{2b_1b_2(2\Omega)^{3/2}(m\omega_{k_1}\omega_{k_2}\omega_\varkappa)^{1/2}}. \quad (27)$$

Transforming in (26) from summation to integration, we can calculate the dependence of $\gamma^{(m)}$ on $\varkappa$ and on the temperature T in the limits of high and low temperatures, $T>\varepsilon_0$ and $T<\varepsilon_0$, where $\varepsilon_0=\hbar\omega_0$ is the magnon activation energy ($\varepsilon_0\sim$ 16–18 K for orthoferrites).

For $T>\varepsilon_0$, corresponding to nitrogen and room temperatures, the damping decrement $\gamma_3^{(0)}$ is given by

$$\gamma_3^{(0)}(\varkappa,T)=\frac{\hbar}{my_0^3}\begin{cases}{}^1/_2(\varkappa y_0/\pi)^2(T/\varepsilon_0)^2, & \varkappa y_0\ll\varepsilon_0/T, \quad \{2,7\}\\ (4/\pi^2)(\varkappa y_0T/\varepsilon_0), & \varepsilon_0/T\ll\varkappa y_0\ll1, \quad \{2,3,7,10\};\\ {}^5/_{64}(1/\varkappa y_0)(T/\varepsilon_0), & 1\ll\varkappa y_0, \quad \{3\}\end{cases} \quad (28)$$

and for $T<\varepsilon_0$, particularly helium temperatures,

$$\gamma_3^{(0)}(\varkappa,T)=\frac{\hbar}{my_0^3}\begin{cases}(1/2\pi^3)(\varkappa y_0)^2(T/\varepsilon_0)\exp(-\varepsilon_0/T), & \varkappa y_0\ll T/\varepsilon_0, \quad \{2\}\\ (32\varkappa y_0)^{-1}(T/\varepsilon_0)\exp\left(-\frac{\varepsilon_0}{4T\varkappa y_0}\right), & T/\varepsilon_0\ll\varkappa y_0\ll\varepsilon_0/T, \quad \{3\};\\ {}^5/_{128}(1/\varkappa y_0)(T/\varepsilon_0), & \varepsilon_0/T\ll\varkappa y_0, \quad \{3\}\end{cases} \quad (29)$$

the numbers in the curly brackets denote the processes that make the main contribution to $\gamma_3^{(0)}$. Note that $\gamma_3^{(0)}=\lambda\varkappa^2\to0$ as $\varkappa\to0$.

As noted above, for $T\ll\varepsilon_0$ account must be taken of the four-magnon processes with FW interaction. They are descibed in the Born approximation by the Hamiltonian $H_4^{(0)}=-\mathscr{L}_4^{(0)}$,

$$H_4^{(0)}=\sum_{1234}\Psi_4(12,34)a_1^+a_2^+a_3a_4,$$

$$\Psi_4(12,34)\equiv\Psi_4(\varkappa_1\varkappa_2,\varkappa_3\varkappa_4)$$

$$=\frac{m\omega_0^2}{5Sy_0}\left(\frac{\hbar}{2m}\right)^2\frac{\Delta(\varkappa_1+\varkappa_2-\varkappa_3-\varkappa_4)}{(\omega_1\omega_2\omega_3\omega_4)^{1/2}}. \quad (30)$$

The amplitude Ψ_4 diverges as $\varkappa_j\to0$, but this divergence is offset by the contribution of the three-magnon processes in the perturbation theory of order higher than the Born approximation. The effective vertex that takes into account the diagrams of Fig. 2 is given by

$$\Psi^{(e)}(12,34)=-\frac{\hbar^2\Delta(\varkappa_1+\varkappa_2-\varkappa_3-\varkappa_4)}{8mSy_0\omega_0^2}f(\mathbf{n}_j)(\omega_1\omega_2\omega_3\omega_4)^{1/2},$$

$$f(\mathbf{n}_j)=(1-\mathbf{n}_1\mathbf{n}_2)(1-\mathbf{n}_3\mathbf{n}_4)+(1-\mathbf{n}_1\mathbf{n}_3)(1-\mathbf{n}_2\mathbf{n}_4)$$

$$+(1-\mathbf{n}_1\mathbf{n}_4)(1-\mathbf{n}_2\mathbf{n}_3), \quad (31)$$

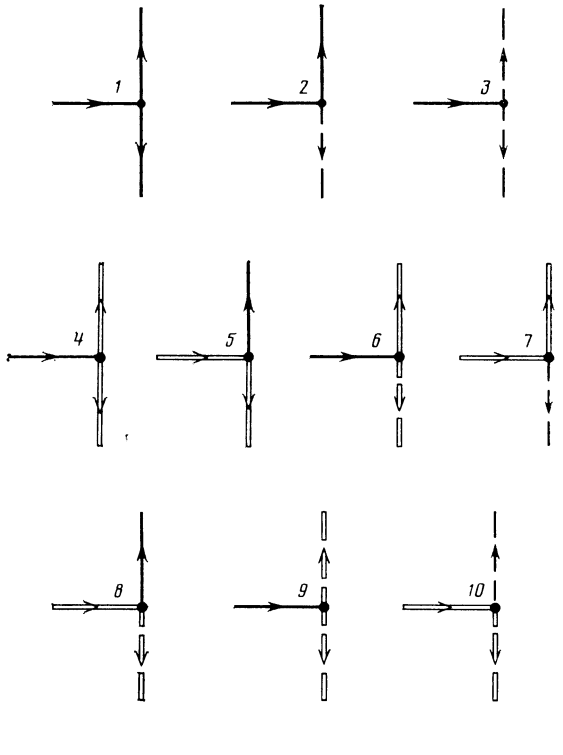

FIG. 1. Three-magnon processes that contribute to the DW damping strength in the Born approximation: single lines—ψ magnons, double—ϑ magnons, solid—bulk magnons, dashed—magnons localized on DW.

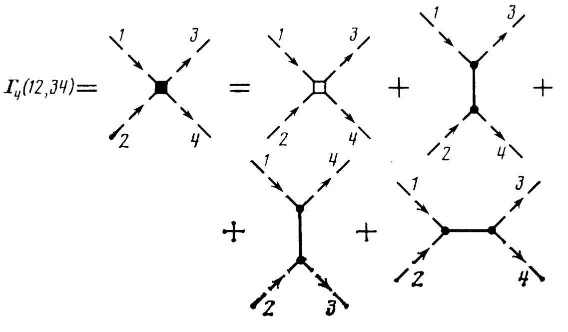

FIG. 2. Effective four-magnon interaction amplitude $\Gamma(\varkappa_1,\varkappa_2,\varkappa_3,\varkappa_4)=\Gamma(12,34)$: light square—"initial" vertex (27), circles—three-magnon vertices, see Fig. 1.

where $\mathbf{n}_j = \varkappa_j/|\varkappa_j|$. Calculating the decrement $\gamma_4^{(0)}(\varkappa, T)$ on the basis of (31), we obtain in the interval $\varkappa y_0 \ll 1$, $T \ll \varepsilon_0$ of interest to us

$$\gamma_4^{(0)}(\varkappa, T) = \frac{32}{105\pi^2} \frac{\hbar^2}{m^2\omega_0 y_0^6} (\varkappa y_0)^3 \left(\frac{T}{\varepsilon_0}\right)^4. \tag{32}$$

At extremely small $|\varkappa|$ the value $\gamma_3^{(0)} \propto \varkappa^2$ predominates over $\gamma_4^{(0)} \propto |\varkappa|^3$, but even for sufficiently small $\varkappa \gtrsim \varkappa_c$ ($\varkappa_c \to 0$ as $T \to 0$), where

$$\varkappa_c y_0 = \frac{\beta_1 M_0^2 y_0^3}{\varepsilon_0} \left(\frac{\varepsilon_0}{T}\right)^3 \exp\left(-\frac{\varepsilon_0}{T}\right),$$

we have $\gamma_4^{(0)} > \gamma_3^{(0)}$, and in the idealized model $\gamma^{(0)}(\varkappa) \propto |\varkappa|^3$.

Let us emphasize the main result of the present section: in the idealized WFM model all the processes considered lead to a $\gamma(\varkappa)$ that tends to zero as $\varkappa \to 0$. This behavior is typical of nonactivated Goldstone excitations and is the result of the vanishing of the amplitude on the mass shell of the process as $\varkappa \to 0$ [the Adler principle, see Ref. 15. This principle is valid for the amplitudes of the idealized model (see Eqs. (17) and (31)].

First, however, there are no grounds for regarding FW in an isolated DW as Goldstone excitations, and second, the result $\gamma(\varkappa) \to 0$ as $\varkappa \to 0$ contradicts both physical intuition (relaxation should occur also at $\varkappa = 0$, corresponding to translational motion of a planar DW), and the calculation of Sec. 4.

4. DAMPING OF TRANSLATIONAL MOTION OF A DW IN THE IDEALIZED WFM MODEL

We calculate the energy lost by a moving DW per unit time by transfer of energy to the magnon gas through inelastic two- and three-magnon processes.

Two-magnon processes. At arbitrary DW velocity, the two-magnon Hamiltonian of the idealized WFM model can be written in the form

$$H_2^{(0)} = H_0 + \hat{T} + \hat{R},$$

where H_0 corresponds to the case $v = 0$ and is diagonal [see Eqs. (15) and (17)], and the off-diagonal terms $\hat{T}$ and $\hat{R}$ are small to the extent that the DW velocity v is low. The term $\hat{T} \propto v$ appears because the momentum satisfies $p_1 = m\dot{q}_{-1}$ for $v \neq 0$ [see Eq. (14)], and

$$\hat{T} = v \sum_{1,2} \left\langle f_1 \cdot \frac{df_2}{dy_s} \right\rangle (p_1 q_2 + P_1 Q_2). \tag{33}$$

The term $\hat{R} \propto v^2$ stems from the decrease of the DW thickness as it moves $y_0 \to y_0(v) = y_0(1 - v^2/c^2)^{1/2}$, and ψ and ϑ are expanded in terms of the set (12) which contains y_0 rather than $y_0(v)$.

The amplitudes in the off-diagonal part of H_2^0 (the terms $\hat{T}$ and $\hat{R}$) are proportional respectively to the small parameters v/c and $(v/c)^2$. The value of dE/dt and the damping force $F(v)$ can therefore be calculated by using standard thermodynamic perturbation theory in the parameter v/c. It is easy to verify that $F_2(v)$ is determined in the Born approximation only by the bulk-magnon scattering:

$$F_2^B = \frac{\pi}{\hbar} \sum_{1,2} q_{12} \{ |T_{12}'|^2 (n_2 - n_1) \delta(\omega_2 - \omega_1 + q_{12} v) + |T_{12}''|^2 (N_2 - N_1) \delta(\Omega_2 - \Omega_1 + q_{12} v) \}, \tag{34}$$

where $q_{12} = k_{1y} - k_{2y}$, and n_1 and N_1 are the Bose distribution functions of the ψ and ϑ magnons, respectively. Since the amplitudes T_{12}' and T_{12}'' are proportional to v, and furthermore, on account of the δ function, $n_2 - n_1$ and $N_2 - N_1$ are also proportional to v, it may turn out that $F_2 \propto v^3$ and this is the contribution of principal order in the parameter v/c to the damping force. The amplitudes in T_{12}, however, vanish at $k_{1y} = \pm k_{2y}$, i.e, $T_{12} \propto (k_{1y}^2 - k_{2y}^2)$. This quantity, by virtue of the condition $\mathbf{k}_{1\perp} = \mathbf{k}_{2\perp}$, is proportional to $\omega_1^2 - \omega_2^2$ (or $\Omega_1^2 - \Omega_2^2$). The amplitudes T_{12}' and T_{12}'' acquire therefore an additional factor v/c and the integral in (34) is found to be proportional to v^5 rather than v^3. A contribution of the same order in v, however, is made also by the next higher perturbation-theory orders for the Hamiltonian $\hat{T}$, and also the Hamiltonian $\hat{R}$. The situation is likewise similar for a ferromagnet,[4,5] and also in the three-dimensional sine-Gordon and φ^4 models (Ref. 17). Analysis has shown that the damping force can be calculated, with account taken of the vital orders of perturbation theory, by using Eq. (34) in which the amplitudes are replaced by effective amplitudes $V_{12} \propto v^2$ (see Ref. 17 for details). It has turned out there for WFM the effective amplitude vanishes on the mass shell of the process (see the preliminary communication, Ref. 18), i.e., in the idealized WFM model described by the Lagrangian $\mathscr{L}^{(0)}$ the two-particle processes fail to contribute not only to the viscous friction coefficient $\eta (F = \eta v)$, which determines the DW mobility, but also to the nonlinear damping force, and it is necessary to resort to processes in which three (or more) quasiparticles take part.

Three-magnon processes. Contributions to the damping of moving DW are made by all ten processes of Fig. 1, and the amplitudes of these processes are easily calculated (see Ref. 18). We present only a formula for the amplitude of a process in which three bulk ψ magnons participate:

$$\Phi_1(k_1 k_2 k_3) = 3P_{-k_1 k_2 k_3} = \frac{\pi m \omega_0^2 y_0 S}{4 b_1 b_2 b_3 \Omega^{3/2}} \left(\frac{\hbar}{2m}\right)^{3/2} \frac{(b_1 + b_2 + b_3)(b_1 - b_2 + b_3)(b_1 + b_2 - b_3)(-b_1 + b_2 + b_3)}{(\omega_1 \omega_2 \omega_3)^{1/2} \operatorname{ch}(\pi Q y_0/2)} e^{iQvt} \cdot \Delta(\mathbf{k}_{1\perp} - \mathbf{k}_{2\perp} - \mathbf{k}_{3\perp}), \quad Q = k_{1y} - k_{2y} - k_{3y}. \tag{35}$$

It is easily seen from this that in the one-dimensional case, when $\omega_1 = \omega_0 b_1 = \omega_0(1 + k_{1y}^2 y_0^2)^{1/2}$, the amplitude Φ_1 vanishes on the mass shell (at $\omega_1 = \omega_2 + \omega_3$). This result is understandable: when only the dynamics of the angle φ is taken into account, the WFM model reduces to an exactly integrable sine-Gordon equation in which there is no dissipation. In the φ^4 model, which is not exactly integrable, the corresponding amplitude is in the homogeneous case also $\Phi_1 \neq 0$ and leads to dissociation of the DW. In WFM, even in the one-dimensional case, processes (in which both ψ and ϑ magnons participate, e.g., the processes $a_1 A_2^+ A_3$) occur and lead to DW relaxation. It can therefore be stated that even the idealized WFM model is not exactly integrable in the homogeneous case and has, in analogy with the φ^4 model, only the property that it reflects no magnons (the amplitude for magnon scattering by a DW is zero on the mass shell).

The damping force F acting on a unit of a DW can be represented as a sum of the ten terms F_1 to F_{10}. For the DW viscosity coefficient $\eta_3(T)$, where $\eta = \lim_{v \to 0} [F(y)/v]$, we can also write

$$\eta_3(T)=\sum_{j=1}^{10}\eta_3^{(j)}(T),$$

$$\eta_3^{(j)}(T)=\frac{2\pi\zeta_j}{TS}\sum_{123}|\Phi_j(123)|^2Q^2(n_1+1)n_2n_3\delta(\omega_1-\omega_2-\omega_3). \quad (36)$$

Here $\hbar Q$ is the momentum transfer, $\zeta_j=1$ for F_2, F_5 to F_8, and F_{10} and $\zeta_j=4$ for F_1, F_3, F_4, and F_9.

At low temperatures ($T\ll\varepsilon_0$), all the $\eta_3^{(j)}$ are exponentially small, and the main contributions are determined by the process 3,

$$\eta_3(T)\approx\eta_3^{(3)}(T)=\frac{(2\pi)^{1/2}\varepsilon^2}{2^{11}mc^3y_0^4}\left(\frac{T}{\varepsilon_0}\right)^2 e^{-\varepsilon_0/T}, \quad T<\varepsilon_0, \quad (37)$$

while at high temperatures ($T>\varepsilon_0$)

$$\eta_3(T)=\frac{T^2}{\beta_1 M_0^2 c y_0^6}\left\{0.36+8\cdot10^{-3}\zeta(\sigma)\left(\frac{T}{\varepsilon_0}\right)+3.6\cdot10^{-3}\left(\frac{T}{\varepsilon_0}\ln\frac{T}{\varepsilon_0}\right)\right\}, \quad (38)$$

where $\zeta(\sigma)\approx1$ for $\sigma\sim1$, a value typical of most orthoferrites at room temperature [for yttrium orthoferrite ($YFeO_3$), in particular, $\sigma\approx1.46$]. Since $\varepsilon_0\sim16-18$ K for orthoferrites, at room temperature the values of all three terms of (38) are comparable. Analysis of the idealized WFM model predicts thus a transition from an $\eta\propto T^2$ dependence to an $\eta\propto T^3$ dependence near room temperature.

Let us summarize the analysis of DW relaxation in the idealized WFM model. The damping decrement of the DW flexural waves has a "Goldstone" dependence on the wave vector $\varkappa$, $\gamma=\lambda\varkappa^2$ as $\varkappa\to0$. The viscous-friction coefficient of a moving DW differs from zero and is determined by three-magnon processes. The temperature dependences of the relaxation constants $\gamma(\varkappa, T)$ and $\eta_3(T)$ are substantially different, let alone the fact that a comparison of the result $\eta\neq0$ with $\gamma(\varkappa)=0$ is impossible at $\varkappa=0$.

5. RELAXATION INTHE GENERALIZED WFM MODEL

If we start with the generalized model, for which $\mathscr{L}=\mathscr{L}^{(0)}+\mathscr{L}^{(i)}$ [see Eqs. (3) and (4)], two-magnon off-diagonal terms appear even at $v=0$, and the three-magnon Lagrangian acquires additions. The corresponding additions $\mathscr{L}_2^{(i)}$ and $\mathscr{L}_3^{(i)}$ can be easily determined and contain terms proportional to D and b_i. The smallness of the coefficients $D\omega_0$ and b/β makes it possible to regard $\mathscr{L}^{(i)}$ as a perturbation, so that the additional terms in the system Hamiltonian are $H_{2,3}^{(i)}=-\mathscr{L}_{2,3}^{(i)}$. Expanding ϑ and ψ in $\mathscr{L}_{2,3}^{(i)}$ in terms of the set (12), we obtain the WFM Hamiltonian in the form $H=H_2+H_3+\dots$, where $H_{2,3}=H_{2,3}^{(0)}+H_{2,3}^{(i)}$,

$$H_2^{(i)}=\sum_{12}\{D_{12}(A_1+A_{-1}^+)(a_2^+-a_{-2})+B_{12}'(a_1+a_{-1}^+)(a_2+a_{-2}^+)+B_{12}''(A_1+A_{-1}^+)\cdot(A_2+A_{-2}^+)\}\exp\{i(k_{1y}-k_{2y})vt\}, \quad (39)$$

$$H_3^{(i)}=\sum_{123}\{D_{123}(a_1+a_{-1}^+)(a_2^+-a_{-2})(A_3+A_{-3}^+)+B_{123}'(a_1+a_{-1}^+)\cdot(a_2+a_{-2}^+)(a_3+a_{-3}^+)+B_{123}''(a_1+a_{-1}^+)(A_2+A_{-2}^+)(A_3+A_{-3}^+)\} \quad (40)$$

The amplitudes D_{12} and D_{123} are due to the first terms in (4), which are associated with the inequality of the constants d_1 and d_3

$$D_{12}=-i\left(\frac{\hbar D}{2\beta_1}\right)\omega_0^2\left(\frac{\omega_2}{\Omega_1}\right)^{1/2}\left\langle f_1f_2^*\frac{\operatorname{sh}\xi}{\operatorname{ch}^2\xi}\right\rangle, \quad (41)$$

$$D_{123}=i\left(\frac{D}{\beta_1}\right)m\omega_0^2\left(\frac{\hbar}{2m}\right)^{3/2}\left(\frac{\omega_2}{\omega_1\Omega_3}\right)^{1/2}\cdot\left\langle f_1f_2^*f_3\left(1-\frac{2}{\operatorname{ch}^2\xi}\right)\right\rangle. \quad (42)$$

The amplitudes B' and B'' in (39) and (40) take into account the fourth-order anisotropy. They depend differently on the magnon frequencies, $B_{12}'\propto(\omega_1\omega_2)^{-1/2}$ and $B_{12}''\propto(\Omega_1\Omega_2)^{-1/2}$, so that the viscosity coefficient has a diferent temperature dependence. We proceed now to calculate the dissipative properties of the DW. The damping of FW is determined in the Born approximation by equations similar to (26), in which $\Phi(12\varkappa)$ must be replaced by the total amplitudes of the processes, including the amplitudes $H_3^{(1)}$. It is important that the amplitudes $D_{12\varkappa}$ and $B_{12\varkappa}$, in contrast to $\Phi_{12\varkappa}$, do not vanish on the mass shells of the corresponding processes as $\varkappa\to0$. For example, the amplitude of the $a_\varkappa a_1A_2^+$ process for $\omega_1=\Omega_2+\omega_\varkappa$ and $\varkappa\to0$ is equal to

$$\frac{i\pi D}{16\beta_1}\frac{\omega_0^2\hbar^{3/2}(Sx_0)^{1/2}}{\Omega b_1b_2(m\omega_\varkappa)^{1/2}}\frac{\Delta(\varkappa+\mathbf{k}_{2\perp}-\mathbf{k}_{1\perp})}{\operatorname{sh}(\pi q/2)}[1-(q_1^2-q_2^2)^2], \quad (43)$$

where $q_1^2=\sigma+q_2^2$, $q=q_1-q_2$, and $q_j=k_{jy}y_0$. The damping rate therefore remains finite as $\varkappa\to0$, viz., $\gamma_3^{(i)}\to\gamma_0\neq0$. As a result, while the contribution of the Hamiltonian $H_3^{(i)}$ does contain the small parameters $D\omega_0$ and b/β, it is important and must be taken into account, but it suffices here to calculate γ_0.

In addition to the Born approximation in $H_3^{(i)}$, contributions of the same order in $D\omega_0$ and b/β are made by terms due to allowance for $H_3^{(0)}$ and $H_2^{(0)}$ in the next orders of perturbation theory (see the graphs in Fig. 3). Instead of the unwieldy calculation of a tremendous number of terms of this type, it is convenient to carry out a unitary transformation that diagonalizes the two-magnon Hamiltonian $H_2=H_0+H_2^{(i)}$ at $v=0$.

We consider only that part of $H^{(i)}$ that corresponds to the first terms in $\mathscr{L}^{(i)}$ of Eq. 40) and is due to $d_1\neq d_3$; the analysis of the fourth-order anisotropy contribution is similar. The effective amplitudes $D'(\mathbf{k}_1\varkappa\mathbf{k}_2)$ and $D''(\mathbf{k}_1\varkappa\mathbf{k}_2)$ of the processes vital to the FW relaxation, such as $b_1b_\varkappa B_2^+$ and $b_1^+b_\varkappa B_2$ (b_1 and B_2 are the operators obtained after the unitary transformation) in the limit $\varkappa\to0$ take the form

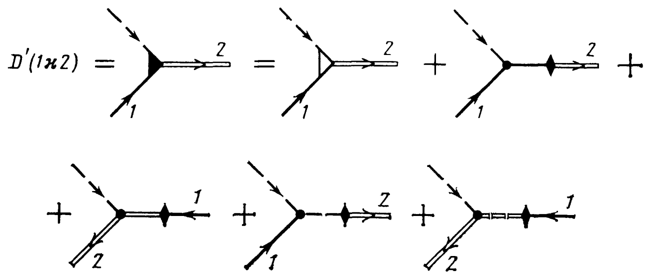

FIG. 3. Effective amplitude $D'(\mathbf{k}_1\varkappa\mathbf{k}_2)$ of three-magnon interaction in the generalized model; light triangle—"initial" amplitude D_{123} [Eq. (42)], circle—three magnon vertex P_{123} or R_{123} [Eqs. (22) and (23)] diamond—two-magnon vertex D_{12} (Eq. (44)].

$$D'(\mathbf{k}_1\varkappa\mathbf{k}_2)=\frac{i\pi D^2\hbar^3\omega_0{}^2y_0\Delta(\mathbf{k}_{1\perp}-\mathbf{k}_{2\perp})}{12\beta_1{}^2(gM_0)^2(mSy_0\omega_\varkappa)^{1/2}}$$

$$\cdot\frac{(q_1-q_2)(1+q_1{}^2+q_2{}^2+q_1q_2)}{(1+q_1{}^2)^{1/2}(1+q_2{}^2)^{1/2}\,\mathrm{ch}[\pi(q_1-q_2)/2]},$$

$$D''(\mathbf{k}_1\varkappa\mathbf{k}_2)=[D'(\mathbf{k}_1\varkappa\mathbf{k}_2)]^*. \tag{44}$$

These amplitudes, like the "initial" amplitude in $H_3^{(i)}$, diverge like $\omega_\varkappa^{-1/2}$ as $\varkappa\to 0$, i.e., they do not satisfy the Adler principle. Writing down the expression for the FW damping decrement due to $d_1\neq d_3 (D\neq 0)$ in the limit as $\varkappa\to 0$:

$$\gamma_{0D}=\frac{4\pi\omega_\varkappa}{\hbar T}\sum_{12}|D'(\mathbf{k}_1\varkappa\mathbf{k}_2)|^2 n_1(n_1+1)\delta(\omega_1-\Omega_2), \tag{45}$$

we find that $\gamma_{0D}\neq 0$ as $\varkappa\to 0$. The contribution to $H_3^{(i)}$ from the second group of processes governed by b_i leads similarly to a finite value $\gamma_{0B}\neq 0$ as $\gamma\to 0$. Prior to the actual calculation of γ_{0D} and γ_{0B} it is convenient to discuss the dynamic damping of a forward-moving DW.

Viscous friction of DW. The main contribution to the friction coefficient η is made by two-magnon processes described by the Hamiltonian $H_2^{(i)}$. Since the coefficient η is determined in the idealized model by three-magnon processes, the corresponding contributions do not interfere and

$$\eta=\eta_3+\eta_2, \tag{46}$$

where η_3 is determined by the equations of Sec. 4 and η_2 by the two-magnon processes governed by $H_2^{(i)}$.

In the Hamiltonian $H_2^{(i)}$, the difference between d_1 and d_3 leads to conversions of magnons of different modes of type $A_1a_2^+$ and $A_1^+a_1$, while the terms connected with b_i determine magnon scattering processes of type $a_1^+a_2$ or $A_1^+A_2$.

By virtue of the damping, the contribution of the scattering processes is determined by Eq. (34) in which T'_{12} and T''_{12} are replaced by B'_{12} and B''_{12}. An expression for the contribution of the conversion processes can also be obtained from (34) by replacing $|T'_{12}|^2$ with $|D_{12}|^2$ and n_1-n_2 with n_1-N_2. The equations for the two-magnon contributions to η (η_D and η_B) are also obtained in elementary fashion, e.g.,

$$\eta_D=\frac{1}{\hbar T}\sum_{12}|D_{12}|^2(q_1-q_2)^2 n_1(n_1+1)\delta(\omega_1-\Omega_2). \tag{47}$$

Comparison of this equation with expression (45) for γ_{0D} shows readily a definite similarity of their structures. If, however, the expressions for D_{12} and $D'_{1\varkappa 2}$ are compared as $\varkappa\to 0$, it is found that

$$\lim_{\varkappa\to 0}[\omega_\varkappa{}^{1/2}D'(1\varkappa 2)]=\frac{i}{2}(q_1-q_2)\left(\frac{\hbar}{my_0S}\right)D_{12}. \tag{48}$$

Comparing (47) and (45) and taking (48) into account, it is easy to verify that the damping rate γ_{0D} and the DW viscous-friction coefficient η_D are equal, apart from a trivial dimensional factor: $\gamma_{0D}=\eta_D\,y_0/2m$. A similar relation holds also for the contributions made to η_2 by magnon scattering and to γ_0 by processes of type $a_\varkappa a_1^+a_2$ and $a_\varkappa A_1^+A_2$. All these contributions and the total dissipative characteristics satisfy a common relation

$$\gamma(\varkappa\to 0)=\gamma_0=\frac{y_0}{2m}\eta. \tag{49}$$

This answer is physically understandable, for in the long-wave limit ($\varkappa\to 0$) propagation of flexural waves in DW and translational motion of DW are identical physical phenomena. It can be shown that to every n-magnon inelastic DW–magnon intereaction $a_1^+\ldots a_mA_1^+\ldots A_{n-m}\exp(iQvt)$, which make a contribution η_n to the DW damping force there corresponds an $(n+1)$-magnon interaction $a_\varkappa a_1^+\ldots a_mA_1^+\ldots A_{n-m}$ of the magnons with one another, and the contribution of the latter interaction to γ_0 is connected with η_n by relation (49). This can explain the apparent contradiction of the results for $\gamma(\varkappa)$ and η in the idealized model, noted at the end of Sec. 4. Actually, two-magnon processes make no contribution to the damping of a planar DW (exact integrability in the one-dimensional case), so the three-magnon FW damping rate $\gamma_3^{(0)}(\varkappa)$, while different from zero if $\varkappa\neq 0$, does not vanish as $\varkappa\to 0$. A nonzero γ_0 is obtained in the idealized model when account is taken of processes in which four bulk magnons participate, and its value is connected with η_3 by relation (49).

Relation (49) suggests thus, first, a consistent phenomenological description of DW dynamics (see below), and second, permits the calculation of γ_0 to be replaced by a much simpler calculation of the dynamic-damping coefficient η.

Let us present equations for the coefficient η_2. At low temperatures ($T<\varepsilon_0$) the main contribution to η_2 are made by ψ-magnon scattering process having the lowest activation:

$$\eta_2\approx\eta_B=\frac{16\hbar p^2}{\pi^2y_0{}^4}\left(\frac{T}{\varepsilon_0}\right)\exp\left(-\frac{\varepsilon_0}{T}\right), \tag{50}$$

and at high temperatures ($T>\varepsilon_0$) by magnon-transformation processes

$$\eta_2\approx\eta_D=\left(\frac{D\omega_0}{\beta_1}\right)^2\frac{\hbar}{72y_0{}^4}A_1(\sigma)\left(\frac{T}{\varepsilon_0}\right)^2, \tag{51}$$

where $A_1(\sigma)\sim 1$ for $\sigma\sim 1$ is an unwieldy numerical coefficient. Estimates for orthoferrites show (see also Refs. 11 and 18) that the value of η_2 at $T\sim 300$ K are approximately an order of magnitude larger than the three-magnon contribution η_3 [Eq. (38)]. The value η_2 of Eq. (51) agrees well with experiment for orthoferrites.[19]

CONCLUSION

Note that although the analysis was carried out for orthoferrites, the situation is in many respects analogous for other WFM. The idealized model is universal, and the equations for $\gamma(\varkappa)$ and η_3 for all WFM are the same. As to the generalized models, contributions to $\mathscr{L}^{(i)}$ come from the uniaxial anisotropy of next higher order (of type bl_z^4) for all WFM, or by anisotropy in the basal plane for unmiaxial WFM, and also by the deviation from antisymmetry of the Dzyaloshinskiĭ interaction (the invariants $m_x l_y + m_y l_x$ for WFM such as NiF_2 and MnF_2; $im_z[(l_x + il_y)^3 - (l_x - il_y)^3]$ for rhombohedral WFM for WFM such as $MnCO_3$, $FeBO_3$, etc.). Analysis has shown that the dependence of two-and three-magnon amplitudes on the magnon frequencies is the same for $\mathscr{L}_B^{(i)}$ and $\mathscr{L}_D^{(i)}$ as for an orthoferrite. As a result, the temperature dependence of the friction coefficients η_B and η_D is identical with (50) and (51) for any orthoferrite. A relation $\eta \propto T^2$ or $\eta \propto T^3$ should therefore be observed at high temperatures for any WFM if two- or three-magnon processes predominate, respectively.

Let us discuss the following important circumstances. The finite $\gamma(\varkappa)$ as $\varkappa \to 0$ means that the FW of the DW are not weakly damped modes if $\varkappa$ is small enough, $[\gamma(\varkappa)/\omega(\varkappa)] \to \infty$ as $\varkappa \to 0$. The reason, in our opinion, is that the FW of the DW are not Goldstone excitations in the strict meaning of this word, since in magnets with a solitary DW there exists at $T \neq 0$ a preferred reference frame connected with the magnon heat reservoir.

The microscopic calculation developed here permits development of a phenomenological theory of DW motion, in which the DW position is determined by the coordinate of its center: $u = u(\mathbf{r}_\perp, t)$. The dynamics of u can be described by the Lagrangian

$$L\{u\} = \int d\mathbf{r}_\perp \left\{ \frac{\sigma}{2} \left[\frac{1}{c^2} \dot{u}^2 - (\nabla_\perp u)^2 \right] + 2m_0 H u \right\}, \qquad (52)$$

where σ is the DW surface energy, with $\sigma = 2M_0^2(\alpha\beta_1)^{1/2}$ for an ac-type DW, and $\nabla_\perp = \mathbf{e}_x(\partial/\partial x) + \mathbf{e}_y(\partial/\partial y)$. Equation (52) takes also into account the external magnetic field $\mathbf{H}$ capable of displacing the DW, the modulus Hm_0 of the scalar product in H, and the weakly ferromagnetic moment in the domains; positive values of u correspond to motion of the DW towards a domain with $\mathbf{m}_0 \cdot \mathbf{H} > 0$.

The dissipative function $Q\{\dot{u}\}$ should be chosen in accordance with the microscopic calculations in the form

$$Q\{\dot{u}\} = (\sigma/c^2) \int d\mathbf{r}_\perp \{\gamma_0 \dot{u}^2 + \lambda(\nabla_\perp \dot{u})^2\}. \qquad (53)$$

Here γ_0 and λ are the constants calculated above, which determine $\gamma(\varkappa) = \gamma_0 + \lambda\varkappa^2$ [see (49) and also (28) and (29)]. Note that according to the theory λ has a universal form for all WFM, while γ_0 depends substantially on the constants of the Lagrangian $\mathscr{L}^{(i)}$. The DW equations of motion are obtained by the usual variation $\delta L/\delta u - \delta Q/\delta\dot{u} = 0$.

The dynamic equations for u determine the spectrum and the damping of FW having a two-dimensional wave vector $\varkappa$, $\omega_\varkappa = c|\varkappa|$, $\gamma = \gamma_0 + \lambda\varkappa^2$, and also the homogeneous-DW motion induced by the field $\mathbf{H}$. The DW mobility $\mu = v/H$ is related as $H \to 0$ to γ_0 by $\mu = m_0 c^2/\sigma\gamma_0$, from which, taking (49) into account ($m_0 = 2dM_0/\delta$, $m = \beta_1 M_0^2 y_0^2/c^2$), we obtain the standard equation $\mu = 4dm_0/\delta\eta$, see Ref. 3. Relation (49) thus makes it possible to reconcile the dissipative characteristics of DW in various modes of their motion.

The description on the basis of (52) and (53) differs from the standard one[3] by the last term in the dissipative function Q. Note that γ_0 is small, $\gamma_0/\lambda = \varkappa_*^2 \ll y_0^{-2}$, where $(y_0\varkappa_*)^2 \sim \max(D\omega_0/\beta_1, p)$ or $(y_0\varkappa_*) = (\beta/\delta)^{1/2}(a/y_0)^3$ (a is the lattice constant) for the cases $\eta_2 > \eta_3$ and $\eta_3 > \eta_2$, respectively. Consequently, the term with λ dominates at fairly small values $\varkappa > \varkappa_* \ll 1/y_0$. No such strong dispersion of the FW damping occurs in the standard phenomenological description of DW dynamics. The limits of validity of the dynamic theory, which are connected with a transition to the nonanalytic $\gamma(\varkappa)$ dependences, $\gamma \propto |\varkappa|^3$ for $\varkappa > \varkappa_c$ [see (33)] or $\gamma \propto |\varkappa|$ [see (39), and (30)], can also be determined within the scope of the microscopic method.

Bar'yakhtar[20–22] has recently generalized the equations of magnitization dynamics. On the basis of his approach, the dispersion of the FW damping is obtained in a natural manner. In his theory, however, $\lambda \sim \gamma_0 y_0^2$ and $\varkappa_* \sim 1/y_0$. The reason is apparently the following: this theory takes full account of the dynamic symmetry of the exchange and relativistic interactions, but disregards the "latent" symmetry due to proximity of the system of being exactly integrable (to the smallness of the constants in $\mathscr{L}^{(i)}$). The results of the two approaches agree qualitatively only if $b_i \approx \beta_i$ and $D\omega_0 \sim \beta_1$, i.e., $d_1 - d_3 \approx d_1$. The microscopic approach is therefore essential not only for the prediction of the values and temperature dependences of the relaxation constants, but also for a description of arbitrary magnets with allowance for the proximity of the models that describe them to exact integrability.

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[1)]Note that diagonalization of $H_2 = H_0 + H_2^{nd}$ is possible at $v = 0$ in an arbitrary magnet and reduces to solution of a stationary Schrödinger equation. If $v \neq 0$, the terms describing the inelastic processes cannot be "disposed off" by an arbitrary unitary transformation. Diagonalization for $v \neq 0$ is possible only in exactly integrable systems.

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