E. M. F. induced by entrainment current in a magnetic field

É. Normantas and G. E. Pikus

Institute of Semiconductor Physics, Academy of Sciences of the Lithuanian SSR and A. F. Ioffe Physicotechnical Institute, Academy of Sciences of the USSR

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The e.m.f. produced by the entrainment current in an extended specimen is calculated. It is shown that the current components due to electron heating do not produce an e.m.f. on electrodes outside the illuminated region in the absence of a magnetic field. The magnitude and, in some cases, the sign of the e.m.f. in a magnetic field depend on the location and configuration of the measuring electrodes.

1. The effect of electron entrainment by an electromagnetic wave was first discovered in Refs. 1, 2. It was shown[1,3–8] that an entrainment current includes components caused by the Hall current, produced by intersecting variable electrical and magnetic fields, by the spatial dispersion in high-frequency conductivity, and by inhomogeneous electron heating. The first two components make a contribution to the current, proportional to the wave vector $\mathbf{q}$ of the light. The last is proportional to $\nabla|\mathbf{E}|^2$, where $\mathbf{E}$ is the field of the electromagnetic wave. This component becomes significant and can exceed the first two components when the energy relaxation time τ_ε, substantially exceeds the momentum relaxation time τ. If in fact the specimen is located in the field, not of a travelling, but rather of a standing wave, for example in the field of a resonator, then only the component of the current $\mathbf{j}$ proportional to $\nabla||\mathbf{E}|^2$, is retained.

However, as shown below, in the usual conditions for the observation of the entrainment effect, specifically in e.m.f. measurements in open homogeneous specimens when the contacts are not illuminated, the current component caused by heating does not contribute in isotropic semiconductors to the e.m.f. being measured. If the electrodes are located on the illuminated surface, then it is impossible to separate the internal e.m.f., associated with electron heating, and the e.m.f. on the contacts, which is dependent on their geometry and material.

It is possible to measure the contribution to the entrainment current due to heating, by placing the specimen in a transverse magnetic field. When this is done, however, the e.m.f. measured will appear substantially different depending on whether one uses flat electrodes which contact the entire lateral surface, or point electrodes. Moreover, in the latter case, the magnitude, and when the absorption coefficients are small, the sign of the e.m.f. also, depend on the position of the electrodes. As far as we know, no previous attention has been paid to these important particulars, which are present in all experiments in which initial heating of electrons appears.

The entrainment current in a magnetic field was calculated in Ref. 9 and in our previous work.[10] In contrast to Ref. 9, we took into account the energy dependence of the relaxation time τ, and also the contribution from the longitudinal field, arising as a consequence of the violation of neutrality by entrainment currents in a magnetic field. It was also shown in Ref. 10 that the entrainment current in a magnetic field includes a number of components not previously noted. Currents which arise as a consequence of electron heating by microwaves in a magnetic field were observed[11,12] even before the discovery of the entrainment effect. However, in the work reported in these papers the exposed region also took in the electrodes, which made it impossible to separate the internal and the contact e.m.f.

2. The general expression for the entrainment current $\mathbf{j}_0$ in an isotropic, nongyrotropic crystal can be written in the form

$$\mathbf{j}_0=k_1^{(0)}\mathbf{q}|\mathbf{E}|^2+k_2^{(0)}\nabla|\mathbf{E}|^2+k_3^{(0)}[\boldsymbol{\varkappa}\nabla|\mathbf{E}|^2]. \quad (1)$$

Here $\mathbf{E}$ and $\mathbf{q}$ are the electric field strength and the wave vector of the electromagnetic wave, and

$$\boldsymbol{\varkappa}=i[\mathbf{E}\mathbf{E}^*]/|\mathbf{E}|^2=(\mathbf{q}/q)P_{\text{circ.}}$$

where $P_{\text{circ.}}$ is the degree of circular polarization of the radiation. It was further taken into account in (1) that $\mathbf{q}\perp\mathbf{E}$, and $\boldsymbol{\varkappa}\|\mathbf{q}$.

It is usually assumed that the e.m.f. between nonilluminated electrodes at the ends of a spread-out specimen (see Fig. 1) of length L is determined by the formula

$$\Delta V=\frac{\bar{j}L}{\sigma}=\frac{1}{\sigma s}\int_0^L dz\int ds\, j(\mathbf{r},z), \quad (2)$$

where s is the cross section of the specimen, and $\bar{j}$ is the average current density. As will be shown below, this expression, under the given conditions, indeed determines the e.m.f. induced by the first component of the current in (1). However, this is not so for the remaining components. For the calculation of the electrical field in the specimen, it is necessary to solve the equation

$$\nabla^2\varphi-\sigma^{-1}\operatorname{div}\mathbf{j}_0=0, \quad (3)$$

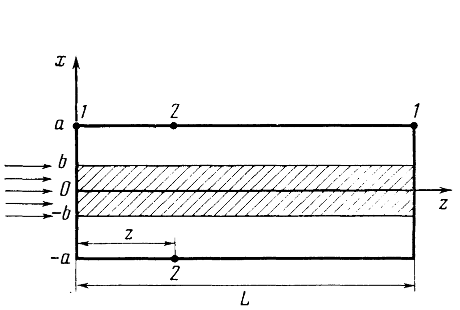

FIG. 1. Diagram of the illumination of the specimen and the arrangement of electrodes: 1—when measuring longitudinal e.m.f., 2—when measuring transverse e.m.f..

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which satisfies the boundary conditions

$$(\partial\varphi/\partial\mathbf{n})_{rp}=\sigma^{-1}j_{0n}, \tag{4}$$

where $\mathbf{n}$ is the normal to the surface, j_{0n} is the normal component of the current in (1), and σ is the conductivity in a constant field. For the second component of the current in (1), due to electron heating, Eq. (3) and the boundary conditions (4) yield the relations

$$\nabla^2\chi=0, \quad (\partial\chi/\partial\mathbf{n})_{rp}=0, \tag{5}$$

where $\chi=\varphi-k_2^{(0)}\sigma^{-1}|\mathbf{E}|^2$. It is apparent that the solution to these equations is $\chi\equiv0$, and consequently, outside of the illuminated region, where $|\mathbf{E}|^2=0$, we have $\varphi\equiv0$ as well. Thus, this component of the current under the present conditions does not change. A similar conclusion applies to the third component in (1), if the light is incident normally on an end face, and does not illuminate the side. Indeed, in this instance div $\mathbf{j}_0=0$, and $j_{0n}=0$, and the obvious solution to (3) and (4) is $\varphi\equiv0$. In this way, the second component of the current in (1) is balanced by the field, arising only in the illuminated region, and the third component gives rise to closed currents, which do not create a field.

The first component of the current in (1) is indeed the entrainment current usually measured. The magnitude of the e.m.f. being measured in a general case depends, apart from the intensity of the light, on the relation between the coefficient of absorption α and the dimensions of the specimen, and also on the extent of the illuminated area. It follows from the solution to the equations (3) and (4) that in a cylindrical specimen of radius R and length L, when a spot of radius r is illuminated on the end face at $z=0$, the e.m.f. between the points on the lateral surface $z_1=0$, $z_2=L$ is determined by the expression

$$\Delta V=\varphi(R,L)-\varphi(R,0)=\frac{j(0)}{\alpha\sigma}\left(\frac{r}{R}\right)^2(1-e^{-\alpha L})(1+S), \tag{6}$$

where

$$S=\frac{2\alpha R^2}{r}\sum_{k=1}^{\infty}\frac{J_1(r\lambda_k/R)}{[\lambda_k^2-(\alpha R)^2]J_0(\lambda_k)}\left[\frac{\operatorname{th}(\lambda_k L/2R)}{\operatorname{th}(\alpha L/2)}-\frac{\alpha R}{\lambda_k}\right].$$

Here $j(0)=k_1^{(0)}q|\mathbf{E}_0|^2|_{z=0}$, $\mathbf{E}_0=\mathbf{E}(0)$, and the λ_k are the root of the equation $J_1(\lambda_k)=0$, where J_n is the usual Bessel function. In (6) it is assumed that the light propagates normally to the illuminated surface, and for simplification, reflection of the light from the back side is not considered. It is evident that one can disregard the term S when $R^2\ll rL$ and $\alpha R\ll(r/R)^{1/2}$, and in this case Eq. (2) is valid.

When a wide, rectangular specimen of length L and height $2a$ is illuminated by a spot in the center of the $z=0$ end face of width $2b$, we have similarly

$$\Delta V=\varphi(a,L)-\varphi(a,0)=\frac{j(0)}{\alpha\sigma}\frac{b}{a}(1-e^{-\alpha L})(1+S), \tag{7}$$

where

$$S=\frac{2\alpha a^2}{b}\sum_{k=1}^{\infty}\frac{\sin(\lambda_k b/a)}{[\lambda_k^2-(\alpha a)^2]\cos\lambda_k}\left[\frac{\operatorname{th}(\lambda_k L/2a)}{\operatorname{th}(\alpha L/2)}-\frac{\alpha a}{\lambda_k}\right]$$

and $\lambda_k\,\pi k$. It is evident that in this case Eq. (2) is applicable when $L\gg a^2/b$ and $\alpha a\ll(b/a)^{1/2}$.

3. In a magnetic field $\mathbf{H}_0$, the general expression for a current $\mathbf{j}_H$ linear in $\mathbf{H}_0$ may be written in the form

$$\begin{aligned}\mathbf{j}_H={}&k_1^{(1)}[\boldsymbol{\Omega}\mathbf{q}]|\mathbf{E}|^2+k_2^{(1)}[\boldsymbol{\Omega}\nabla|\mathbf{E}|^2]+k_3^{(1)}\boldsymbol{\varkappa}(\boldsymbol{\Omega}\nabla|\mathbf{E}|^2)\\&+k_4^{(1)}(\boldsymbol{\Omega}\boldsymbol{\varkappa})\nabla|\mathbf{E}|^2+k_5^{(1)}\boldsymbol{\Omega}(\boldsymbol{\varkappa}\nabla|\mathbf{E}|^2)+k_6^{(1)}\boldsymbol{\Omega}(\boldsymbol{\varkappa}\mathbf{q})|\mathbf{E}|^2\\&+k_7^{(1)}\boldsymbol{\varkappa}(\boldsymbol{\Omega}\mathbf{q})|\mathbf{E}|^2+k_8^{(1)}\{\operatorname{Re}[\mathbf{E}\mathbf{q}](\boldsymbol{\Omega}\mathbf{E}^*)-{}^1/_2[\boldsymbol{\Omega}\mathbf{q}]|\mathbf{E}|^2\}.\end{aligned} \tag{8}$$

Here $\boldsymbol{\Omega}=e\mathbf{H}_0/mc$, and m is the effective mass. The first two terms in (8) (and also in part the third and fourth) are Hall components from the corresponding contributions to the current $\mathbf{j}_0$ in (1).

We first examine the contribution from the second term, which is caused by heating. In addition to the term proportional to $k_2^{(1)}$, it is necessary here to take into account also the Hall current, produced by the field $-\nabla\varphi=-k_2^{(0)}\sigma^{-1}\nabla|\mathbf{E}|^2$, which balances the second component of the current $\mathbf{j}_0$ in (1), i.e.,

$$\mathbf{j}_H=-\beta k_2^{(0)}\sigma^{-1}[\boldsymbol{\Omega}\nabla|\mathbf{E}|^2],$$

which reduces to replacing the coefficient $\tilde{k}_2^{(1)}$ with the coefficient $k_2^{(1)}$ in (8). The coefficients $k_2^{(1)}$ and $\tilde{k}_2^{(1)}$ are calculated in Ref. 10. Here we will only note that when τ does not depend on the energy ε, $\tilde{k}_1^{(1)}\equiv0$. We calculate the e.m.f. produced by this current, which occurs between electrodes located on the sides. As before, we assume that on a flat sample, a strip is illuminated in the center of the end face $z=0$ with width $2b$, i.e., $|\mathbf{E}|^2=|\mathbf{E}_0|^2e^{-\alpha z}\theta(|x|-b)$, where $\theta(\xi)=1$ for $\xi<0$ and $\theta(\xi)=0$ for $\xi>0$, and the magnetic field is directed along axis y, perpendicular to x and z. Then the potential difference between the electrodes situated opposite one another at the points $x=a$ and $x=-a$ on the surface depends on their z coordinates, and is defined by the expression

$$\delta V(z)=\varphi(a,z)-\varphi(-a,z)=\frac{4\Omega\tilde{k}_2^{(1)}|\mathbf{E}_0|^2}{\sigma}\sum_{k=0}^{\infty}\frac{(-1)^k}{\lambda_k}\sin\frac{\lambda_k b}{a}\cdot\frac{e^{-\alpha L}\operatorname{ch}(\lambda_k z/a)-\operatorname{ch}[\lambda_k(L-z)/a]}{\operatorname{sh}(\lambda_k L/a)}, \tag{9}$$

where $\lambda_k=\pi(1/2+k)$.

From (9) it can be seen that $\delta V(z)$ essentially depends on where the elctrodes are located, and quickly diminishes with increasing distance from the points $z=0$ and $z=L$ to a distance on the order of a, going through zero at the point $z\approx L/2$ (see Fig. 2).

In the limit $\alpha L\ll1$ in agreement with (9), for $L>a$,

$$\delta V(0)=-\delta V(L)=-\frac{4}{\pi\sigma}\tilde{k}_2^{(1)}|\mathbf{E}_0|^2\Omega\ln\operatorname{tg}\left[\frac{\pi}{4}\left(1+\frac{b}{a}\right)\right]. \tag{10}$$

For $\alpha L\gg1$ and $L>a$, the potential difference is determined by that same expression (10), and $\delta V(L)$ decreases as $\exp(-\pi L/2a)$. If extended electrodes are attached to the sides, then the condition (4) changes on these surfaces to

$$\varphi|_{x=\pm a}=\text{const}, \tag{11}$$

and the potential difference $\delta V=\varphi(a)-\varphi(-a)$ is determined by the fact that the full current to the electrode is equal to zero, i.e.,

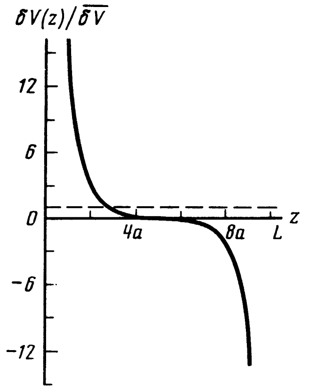

FIG. 2. Distribution of transverse potential difference along the surface of the specimen when measuring the component of the current $\sim k_2^{(1)}$, caused by electron heating in a magnetic field, $L/a = 10$, $\alpha L = 0.2$, $b/a = 0.9$. The values for $\delta V(z)/\overline{\delta V}$ when $z = 0$ and $z = L$ are equal to $1.2\cdot 10^2$ and $-0.96\cdot 10^2$, respectively. The value for $\delta V(z) = \overline{\delta V}$ is indicated by the dashed line.

$$\int_0^L \frac{\partial\varphi}{\partial x}\bigg|_{x=\pm a} dz=0. \tag{12}$$

Although even in this instance the potential distribution remains inhomogeneous, the potential difference between widely separated electrodes does not depend on the correlation of a, b, L and α, but is determined by a simple expression, similar to (2):

$$\overline{\delta V} = \frac{2a}{\sigma}\bar{j}_{HX} = \frac{1}{\sigma L}\int_{-a}^{a}\int_0^L j_{HX}(z,x)\,dx\,dz$$

$$= -\Omega\frac{2b\tilde{k}_2^{(1)}|\mathbf{E}_0|^2}{\sigma L}(1-e^{-\alpha L}). \tag{13}$$

From a comparison of (10) and (13), it can be seen that $\delta V(0)$ for $L > a$ and $\alpha L < 1$ can exceed $\overline{\delta V}$ by a large amount.

As for the first component of the current in (8), under the very same illumination conditions, the transverse e.m.f. essentially depends on the correlation of a, L and α. For $H = H_y$,

$$\delta V(z)=2k_1^{(1)}\Omega q|\mathbf{E}_0|^2 be^{-\alpha z}\sigma^{-1}(1-S_1), \tag{14}$$

where

$$S_1=\frac{2\alpha a^2}{b}\sum_{k=0}^{\infty}(-1)^k\frac{\sin(\lambda_k b/a)}{\lambda_k[\lambda_k^2-(\alpha a)^2]}$$

$$\cdot\left\{\frac{\{\mathrm{ch}[\lambda_k(L-z)/a]-e^{-\alpha L}\,\mathrm{ch}(\lambda_k z/a)\}e^{\alpha z}}{\mathrm{sh}(\lambda_k L/a)}-\frac{\alpha a}{\lambda_k}\right\},$$

and $\lambda_k = \pi(1/2 + k)$. For $\alpha L \ll 1$ and $\alpha a^2/b \ll 1$ we have $S_1 \ll 1$, and δV does not depend on z. When there are extended electrodes on the sides, δV is determined by a formula analogous to (13), regardless of how a, b, L and α are related.

In addition to the e.m.f. created by the current $\mathbf{j}_{H1} = k_1^{(1)}[\mathbf{\Omega q}]|\mathbf{E}|^2$, it is necessary to take into account also the Hall e.m.f. created by the field $-\nabla\varphi_0$, which balances the first component in (1) $-\mathbf{j}_{01} = k_1^{(0)}\mathbf{q}|\mathbf{E}|^2$. The Hall current $\mathbf{j}_X = -\beta[\mathbf{\Omega}\nabla\varphi_0]$ when $\alpha L \ll 1$ and $\alpha a \ll 1$ gives rise to a transverse e.m.f.

$$\Delta\varphi_\perp(z)=-2j_{0\perp}\Omega\beta b/\sigma^2=-2\Omega b\beta k_1^{(0)}q|\mathbf{E}_0|^2/\sigma^2. \tag{15}$$

In this way, in this case the resultant e.m.f. is determined by the quantity

$$\tilde{k}_1^{(1)}=k_1^{(1)}-\beta k_1^{(0)}/\sigma.$$

Note that for $\tau(\varepsilon) = \mathrm{const}$, the quantity $\tilde{k}_1^{(1)}$, as with $\tilde{k}_2^{(1)}$, vanishes. If extended electrodes are attached to the sides which short out the j_z component of the current, then the intensity of the field $\mathscr{E}_0 = -\nabla\varphi_0$ in the specimen decreases, and the transverse e.m.f. created by the Hall current of the field $\mathscr{E}_0$ also essentially decreases, and depends on the relationship of a, b, L and α. So, for $\alpha L \ll 1$ and $L > a$

$$\Delta\varphi_\perp=-(2ba/L)F(b/a)k_1^{(0)}\beta\Omega q|\mathbf{E}_0|^2/\sigma^2 \tag{16}$$

$[1.48 \geqslant F(z) \geqslant 1.09$ for $0 \leqslant z \leqslant 1]$, i.e., the e.m.f. is smaller by a factor a/L with point electrodes.

The third component of the current in (8), for $\mathbf{H}\|\mathbf{x}$, produces a transverse e.m.f.

$$\delta V(z)=-\frac{4k_3^{(1)}\varkappa\Omega|\mathbf{E}_0|^2}{\sigma}\sum_{k=0}^{\infty}(-1)^k\frac{\lambda_k\sin(\lambda_k b/a)}{\lambda_k^2-(\alpha a)^2}$$

$$\cdot\left\{\frac{e^{-\alpha L}\,\mathrm{ch}(\lambda_k z/a)-\mathrm{ch}[\lambda_k(L-z)/a]}{\mathrm{sh}(\lambda_k L/a)}+\frac{\alpha a}{\lambda_k}e^{-\alpha z}\right\}, \tag{17}$$

where $\lambda_k = \pi(1/2 + k)$. It is evident that in this case $\delta V(0)$ and $\delta V(L)$ have different signs, and $|\delta V(z)|$ decreases with distance from the ends of the specimen to a distance of order a. At the same time, a longitudinal e.m.f.

$$\Delta V(a)=\varphi(a,L)-\varphi(a,0)=\frac{2k_3^{(1)}\varkappa\Omega|\mathbf{E}_0|^2}{\sigma}$$

$$\sum_{k=0}^{\infty}(-1)^k\frac{\lambda_k\sin(\lambda_k b/a)}{\lambda_k^2-(\alpha a)^2}\left[\mathrm{th}\frac{\lambda_k L}{a}-\frac{\alpha a}{\lambda_k}\right](1-e^{-\alpha L}) \tag{18}$$

appears.

When there are extended electrodes on the sides, the third component of the current in (8), does not produce an e.m.f. between the electrodes with any relationship between a, b, L and α. For $\mathbf{H}\|\mathbf{z}$, the e.m.f. created by this component of the current is determined by an expression which differs from (7) in having $-\alpha k_3^{(1)}\varkappa\Omega$ instead of $k_1^{(0)}$.

The e.m.f. created by the remaining components in (8) are determined by relations similar to those given above. Thus, for $\mathbf{H}\|\mathbf{z}$ the fourth component, like the second component in (2), produces no e.m.f. between the electrodes situated in a nonilluminated region. For $\mathbf{H}\|\mathbf{z}$ the e.m.f. due to the fifth component is determined by a relation similar to (7), but for $\mathbf{H}\|\mathbf{x}$, by one similar to (14), with, either $-\alpha k_5^{(1)}\varkappa\Omega_z$ or $-\alpha k_5^{(1)}\varkappa\Omega_x$, instead of $k_1^{(0)}q$ or $k_1^{(1)}\Omega q$. For the sixth component, the e.m.f. is determined by the same expression with the substitution of either $k_6^{(1)}\varkappa\Omega_z$ or $k_6^{(1)}\varkappa\Omega_x$ for $k_1^{(0)}$ or $k_1^{(1)}\Omega$. The e.m.f. produced by the seventh component for $\mathbf{H}\|\mathbf{z}$ is determined by an expression similar to (7), with $k_7^{(1)}\varkappa\Omega_z$ substituted for $k_1^{(0)}$. For the eighth component with $\mathbf{H}\|\mathbf{E}\|\mathbf{y}$, the e.m.f. is determined by an expression analogous to (14), with $k_8^{(1)}/2$ substituted for $k_1^{(1)}$.

4. The coefficients $k_1^{(0)}$ and $k_1^{(1)}$ in (1) and (7) have been calculated in Ref. 10 for an isotropic, parabolic spectrum in two limiting cases: $\omega\tau \ll 1$ and $\omega\tau \gg 1$, where ω is the frequency of the exciting light.

We estimate the order of magnitude of these coefficients. For $\omega\tau \ll 1$ we have $k_1^{(0)} \approx e^3N\tau^2/m^2\omega$, where N is the electron density, $k_3^{(0)}$ is small by an additional factor of $\omega^2\tau^2$, and $k_2^{(0)} \sim k_1^{(0)}\omega\tau_\varepsilon$. The constants $k_1^{(1)}$ and $k_8^{(1)}$ are of order $e^3N\tau^3/m^2\omega$, $k_6^{(1)}$ and $k_7^{(1)}$ are small by an additional factor of $\sim\omega\tau$, and we have $k_3^{(1)}$, $k_5^{(1)} \sim (\omega\tau)^2$. We also have $k_2^{(1)} \sim k_1^{(1)}\omega\tau_\varepsilon$, $k_4^{(1)} \sim k_1^{(1)}\omega^2\tau\tau_\varepsilon$. For $\omega\tau \gg 1$ $k_1^{(0)}$ and $k_3^{(0)}$ are also $\approx e^3N/m^2\omega^3$, $k_2^{(0)} \sim k_1^{(0)}\omega\tau_\varepsilon$, $k_1^{(1)}$, $k_3^{(1)}$ and $k_5^{(1)}$ are of the order $e^3N\tau/m^2\omega^3$, $k_6^{(1)}$ and $k_7^{(1)}$ are small by the additional factor $(\omega\tau)^{-1}$, $k_8^{(1)} \sim (\omega\tau)^{-2}$, $k_2^{(1)} \sim k_1^{(1)}\ \omega\tau_\varepsilon$, $k_4^{(1)} \sim k_1^{(1)}\tau\varepsilon/\tau$.

It is evident that of all the components of (7) containing $\nabla|\mathbf{E}|^2$, only the second and fourth contain a contribution from electron heating, and therefore only they can be relatively larger for $\tau_\varepsilon \gg \tau$, and the most favorable condition for observing them is $\omega\tau \gg 1$.

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