Inevitable symmetry lowering in a domain wall near a reordering phase transition

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(Submitted 18 June 1987)
Zh. Eksp. Teor. Fiz. **94**, 121–129 (June 1988)

As the line of the (first- or second-order) transition between ordered phases is approached, there is an inevitable lowering of the symmetry within the domain wall. In the cases considered, this symmetry lowering occurs by a second-order phase transition. Calculations are carried out for examples of a reordering governed by the second component of a multicomponent order parameter. The Landau potential for describing phase transitions in a domain wall is calculated. The regions in which high- and low-symmetry domain walls exist on the phase diagram are determined. The line of phase transitions between different domain-wall structures is found for a three-component order parameter.

If the width of a domain wall is substantially greater than the interatomic distances, the wall can be characterized by the symmetry of small but macroscopic regions of the crystal inside the wall. The symmetry of a domain wall is determined by geometric and energy considerations. This is an important point, since when ordering describable by a multicomponent order parameter (or by several order parameters) occurs, the configuration of the distribution of this parameter in the interior of neighboring domains always permits several possible configurations of the order parameter in the domain wall. A change in the conditions at the reservoir may cause a change in structure and thus in the symmetry of a domain wall, without any change in the structure of the domains. A change of this sort in the symmetry of a domain wall occurs as a phase transition in the wall.

A temperature-induced phase transition in a domain wall was first observed[1] in $DyFeO_3$. A transition induced by an external magnetic field in a domain wall has been observed in $CuCl_2 \cdot 2H_2O$ (Ref. 2) and $(C_2H_5NH_3)_2CuCl_4$ (Ref. 3).

A classification of domain walls by symmetry class which was proposed by Bar'yakhtar *et al.*[4] was used by Bogdanov *et al.*[3] to construct a theory for phase transitions in a domain wall. By taking the approach of Ref. 4, Bogdanov *et al.*[3] were able to predict the possibility of phase transitions in domain walls. Furthermore, it can be established on the basis of the classification of Ref. 4 that a transition between certain symmetry classes of domain walls cannot go as a second-order transition.

To evaluate the field which causes a phase transition within a domain wall, Bogdanov *et al.*[2,3] equated the energies of walls of different symmetries, which depend on the external field. The field dependence of the wall energy was calculated far from the transition field in Refs. 2 and 3. If the phase transition in a wall occurs as a second-order phase transition (or as a first-order transition which is approximately a second-order transition), however, the dependence of the order parameter on the coordinates within the wall will depend strongly on the external conditions near the transition, as we will show below, so the field dependence of the wall energy will also change. The approach of Refs. 2 and 3 will therefore be inappropriate for determining the point of the transition, for calculating the temperature dependence of the order parameter within the wall, and for calculating the singularities in the thermodynamic quantities upon the transition.

Galkina *et al.*[5] have found the temperature of the second-order transitions in a domain wall for the case of two interacting order parameters as a branch point of the equations of state. They also gave a general form of the branched solution for this case. A determination of the temperature dependence of the amplitude of the solution describing a phase transition, the anomalies during the transition, the order in which the phases occur on the phase diagram, and the type of transition between the phases of a domain wall, however, requires the construction of a Landau potential for this transition.

In this paper we analyze the branching of the solutions of the equations of state corresponding to a phase transition in a domain wall for the case in which the order parameter describing the transition in the crystal is a multicomponent parameter. We also find the symmetry and the structure as well as the conditions for the stability of walls of different symmetries. We show that the wall structures which are allowed are dictated by the initial Landau potential describing the transitions in the crystal.

1. PHASE TRANSITION IN A DOMAIN WALL WHEN ORDERING DESCRIBABLE BY A TWO-COMPONENT ORDER PARAMETER OCCURS

Formally, the only difference between a description of a transition in a domain wall in the case of a multicomponent order parameter and that in the case of several order parameters[5] is that there are relations among the coefficients of the potential, which are determined by the symmetry of the order parameter. This difference, however, leads to qualitative differences in the solutions. Let us consider the very simple Landau potential which describes transitions induced by a two-component order parameter (η_1, η_2):

$$F=\int_{-\infty}^{\infty}\Big\{\frac{1}{2}(\dot\eta_1^2+\dot\eta_2^2)+{}^1/_2\alpha_1(\eta_1^2+\eta_2^2)+{}^1/_4\alpha_2(\eta_1^2+\eta_2^2)^2 + {}^1/_6\alpha_3(\eta_1^2+\eta_2^2)^3+{}^1/_2\gamma_1\eta_1^2\eta_2^2+{}^1/_2\gamma_2\eta_1^2\eta_2^2(\eta_1^2+\eta_2^2)\Big\}S\,dz, \qquad (1)$$

where $\dot\eta = \partial\eta/\eta z$, and S is the area of the domain wall. This potential corresponds to the situation that there are three second-order operations among the symmetry operations of

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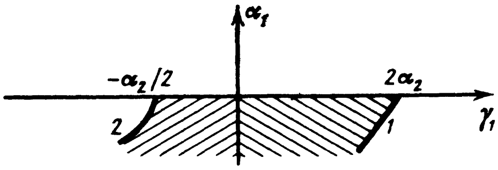

FIG. 1. Phase diagram for potential (1). The region $\alpha_1 < 0$, $\gamma_1 > 0$—phase II ($\eta_1 \neq 0$, $\eta_2 = 0$); the region $\alpha_1 < 0$, $\gamma_1 < 0$—phase III ($\eta_1 = \eta_2 \neq 0$); hatched regions—domain wall of symmetry C_1. 1) Line of transition in a wall in phase II, (16); 2) line of a transition in a wall in phase III, (17).

the high-symmetry phase of the crystal:

$$\hat{G}_1(\eta_1, \eta_2) = (\eta_2, \eta_1), \quad \hat{G}_2(\eta_1, \eta_2) = (-\eta_1, \eta_2), \quad \hat{G}_3(\eta_1, \eta_2) = (\eta_1, -\eta_2).$$

The order parameter satisfies the Lifshitz condition.

The potential (1) can be used to study a transition from the high-symmetry phase I ($\eta_1 = \eta_2 = 0$), which is stable at $\alpha_1 > 0$, to low-symmetry phases II ($\eta_1 \neq 0$, $\eta_2 = 0$; $\alpha_1 < 0$, $\gamma_1 > 0$) and III ($\eta_1 = \eta_2 \neq 0$; $\alpha_1 < 0$, $\gamma_1 < 0$) (Fig. 1). Assuming $\alpha_2, \alpha_3, \gamma_2 > 0$, we will consider only the second-order transitions I→II and I→III.

The factors which make it necessary to retain the terms of sixth degree in the order parameter in (1) for a description of the transition in a domain wall will be discussed below.

In the space $\mathbf{E}_2$ of the components of the order parameter, the high-symmetry phase I has symmetry C_{4v}. Phases II and III have the same symmetry, C_s. Any pair of possible domains of the phases (II and III) corresponds to a domain wall, whose symmetry in the space $\mathbf{E}_2$ may be either C_s or C_1.

The equations of state which follow from (1),

$$\begin{aligned}\ddot{\eta}_1 &= \alpha_1\eta_1 + \alpha_2\eta_1(\eta_1^2+\eta_2^2) + \alpha_3\eta_1(\eta_1^2+\eta_2^2)^2 + \gamma_1\eta_1\eta_2^2 + \gamma_2(2\eta_1^3\eta_2^2 + \eta_1\eta_2^4),\\ \ddot{\eta}_2 &= \alpha_1\eta_2 + \alpha_2\eta_2(\eta_1^2+\eta_2^2) + \alpha_3\eta_2(\eta_1^2+\eta_2^2)^2 + \gamma_1\eta_1^2\eta_2 + \gamma_2(2\eta_1^2\eta_2^3 + \eta_1^4\eta_2),\end{aligned} \quad (2)$$

make it possible to describe a domain wall with either C_s or C_1 symmetry. In phase II, the distribution of the order parameter in a wall of symmetry C_s (a Zhirnov wall[6]) is described by the solution[7]

$$\eta_2 = 0, \quad \eta_1^2 = \varphi_0^2(t) = \bar{\varphi}_0^2 \,\mathrm{th}^2 t/[1 + (\bar{\varphi}_0^2/\tau^2)\mathrm{ch}^{-2} t], \quad (3)$$

where

$$\bar{\varphi}_0^2 = [-\alpha_2 + (\alpha_2^2 - 4\alpha_1\alpha_3)^{1/2}]/2\alpha_3 \approx |\alpha_1|(1-\varepsilon)/\alpha_2,$$
$$\tau^2 = [\alpha_2 + 2(\alpha_2^2 - 4\alpha_1\alpha_3)^{1/2}]/2\alpha_3, \qquad t = \bar{\varphi}_0[4(\alpha_2^2 - 4\alpha_1\alpha_3)]^{1/4} z/2,$$

In the region close to the line of second-order transitions, the quantity

$$\varepsilon = |\alpha_1|\alpha_3/\alpha_2^2 \quad (4)$$

is a small parameter. To first order in ε, a solution of (3) is

$$\varphi_0^2(t) \approx \frac{|\alpha_1|}{\alpha_2} \mathrm{th}^2 t \left\{1 - \varepsilon\left(1 + \frac{2}{3}\mathrm{ch}^{-2} t\right)\right\}, \quad (5)$$

where

$$t \approx z(|\alpha_1|/2)^{1/2}(1 + \varepsilon/2).$$

If we place the origin of coordinates at the center of the wall, a solution of the equations of state describing a Zhirnov wall is antisymmetric under the substitution $t \to -t$; i.e., it disrupts the symmetry of potential (1) [the Lifshitz condition can be written as the requirement that the symmetry operator $\hat{G}_4(\eta_1, \eta_2, t) = (\eta_1, \eta_2, -t)$ exist in the high-symmetry phase of the crystal[8]].

According to the classification of Ref. 4, branching solutions may in principle have both symmetric and antisymmetric parts. We show below that for (1) the solution η_1 is always antisymmetric, while η_2 is symmetric, under the substitution $t \to -t$.

We now consider the case in which a solution of symmetry C_1 branches off from (5):

$$\eta_1 = \varphi_0(t) + u(t), \quad \eta_2 = v(t), \quad u(\pm\infty) = v(\pm\infty) = 0.$$

The equations for $u(t)$ and $v(t)$ are

$$u'' - 2(2 - 3\,\mathrm{ch}^{-2} t - \varepsilon[6\,\mathrm{ch}^{-2} t - 7\,\mathrm{ch}^{-4} t])u = 2\varkappa v^2 \,\mathrm{th}\, t\left\{3 + \varepsilon\left[\frac{22p-33}{14} - (2p+1)\mathrm{ch}^{-2} t\right]\right\}, \quad (6)$$

$$\begin{aligned} v'' - 2\{y - (y+1)\mathrm{ch}^{-2} t &- \varepsilon[(5-p) + (2p-4)\mathrm{ch}^{-2} t - (p+2)\mathrm{ch}^{-4} t]\}v \\ &= 4\varkappa uv\,\mathrm{th}\, t\left\{3 + \varepsilon\left[\frac{22p-33}{14} - (2p+1)\mathrm{ch}^{-2} t\right]\right\} \\ &+ 2\varkappa^2 v^3\{1 + \varepsilon[(2p-1) - 2p\,\mathrm{ch}^{-2} t]\}, \end{aligned} \quad (7)$$

where

$$p = 1 + \gamma_2/\alpha_3, \quad y = \gamma_1/\alpha_2, \quad \varkappa = (\alpha_2/|\alpha_1|)^{1/2},$$
$$u' = \partial u/\partial t, \quad v' = \partial v/\partial t.$$

As soon as y becomes equal to the first eigenvalue of the equation

$$\psi'' - 2\{\lambda_n - (y+1)\mathrm{ch}^{-2} t - \varepsilon[(5-p) + (2p-4)\mathrm{ch}^{-2} t - (p+2)\mathrm{ch}^{-4} t]\}\psi = 0, \quad (8)$$

a nonzero solution branches off from the solution $u = v = 0$ of Eqs. (6) and (7) (Ref. 9).

The branching condition is, to first order in ε,

$$y_0 = 2 - 3(p-5)\varepsilon/7, \quad (9)$$

and the leading term of the solution should be sought in the form[9]

$$v = \xi\psi(t), \quad (10)$$

where ξ is to be determined, and

$$\psi(t) = \mathrm{ch}^{-2} t\{1 - (\varepsilon/7)[(p+2)\mathrm{ch}^{-2} t + 2(p-5)\ln \mathrm{ch}\, t]\} \quad (11)$$

is the eigenfunction of Eq. (8) which corresponds to λ_0. Equation (8) has the unique solution (11), which satisfies the boundary conditions which have been imposed. This solution is symmetric under the substitution $t \to -t$.

The leading term for $u(t)$ is found from (10) and (6) to be

$$u(t) = \{-\varkappa\,\mathrm{th}\, t\,\mathrm{ch}^{-2} t + \varepsilon u_1(t)\}\xi^2, \quad (12)$$

where $u_1(t)$ satisfies the equation

$$u_1''-2(2-3\,\mathrm{ch}^{-2}t)u_1=2\varkappa\,\mathrm{th}\,t\,\mathrm{ch}^{-4}t\left\{\frac{22p+51}{14}-\frac{20p+68}{7}\mathrm{ch}^{-2}t-\frac{12(p-5)}{7}\ln\mathrm{ch}\,t\right\}. \qquad (13)$$

The calculation of the Landau potential for a transition in a domain wall is described in the Appendix. As a result we have

$$F=S\left(\frac{|\alpha_1|}{2}\right)^{1/2}\left\{\frac{4}{15}\mu\xi^2+K\frac{\alpha_3(p-5)}{\alpha_2}\xi^4\right\}, \qquad (14)$$

where $\mu=y=y_0$ is the displacement from the branch point, and

$$K=128/2205\approx 0.058.$$

Minimizing (14) with respect to ξ, we find the equation of state $\xi^2=0$, $\mu\geqslant 0$ (a domain wall of symmetry C_s):

$$\xi^2=-\frac{147}{64}\frac{\alpha_2}{\alpha_3(p-5)}\mu, \quad \mu<0 \quad \text{(wall of symmetry } C_1\text{)}. \qquad (15)$$

The correction to the surface energy of the wall in phase II is thus

$$F=-\frac{49}{160}S\left(\frac{|\alpha_1|}{2}\right)^{1/2}\frac{\alpha_2}{\alpha_3(p-5)}\mu^2.$$

Assuming $\mu=b(T-T_c)$, we find the jump in the heat capacity associated with the phase transition in the wall:

$$\Delta C_v=\frac{49}{80}S\left(\frac{|\alpha_1|}{2}\right)^{1/2}\frac{\alpha_2 b^2}{\alpha_3(p-5)}T_c.$$

An equation for the line defining the region in which the walls of different symmetries exist on the phase diagram follows from (9). This equation is (Fig. 1)

$$\frac{\gamma_1}{\alpha_2}=2-\frac{3}{7}(p-5)\frac{|\alpha_1|\alpha_3}{\alpha_2^2}, \quad \alpha_1<0. \qquad (16)$$

It is easy to see that the problem of the transition in a wall in phase III through a 45° rotation in the space $\mathbf{E}_2$ reduces to the problem discussed above. In this case we need to replace α_2 by $\alpha_2+\gamma_1$, γ_1 and γ_2 by $-2\gamma_1$ and $2\gamma_2$, and α_3 by $\alpha_3+3\gamma_2/2$ in Eqs. (5)–(16). The equation of the branching line takes the form

$$(\alpha_2+\gamma_1)(\alpha_2+2\gamma_1)=-6(\alpha_3+2\gamma_2)|\alpha_1|/7. \qquad (17)$$

The coefficient of ξ^4 in (14) is proportional to α_3, the coefficient of $(\eta_1^2+\eta_2^2+\eta_3)^3$ in (1). In order to describe a second-order transition in a wall we thus need to retain terms of sixth order in the Landau potential describing transitions in the volume of the crystal. The coefficient of ξ^4 is equal to zero in the case $\alpha_3=0$ because of the symmetry of the functional (1).

2. GENERALIZATION TO THE CASE OF A MULTICOMPONENT ORDER PARAMETER

If the change in the symmetry in the interior is described by an m-component order parameter ($m>2$), the symmetry of the wall far from the line of transitions between phases will be described by the multidimensional groups $\mathscr{L}$ in $\mathbf{E}_m$ (Refs. 10 and 11). In the space $\mathbf{E}_m$ we can carry out a symmetry classification of domain walls similar to that which was carried out in Ref. 4. We find that domain walls whose symmetry groups are not subgroups of each other are possible. Only first-order transitions can occur between such domain walls. Also possible are phase transitions for which walls of high symmetry become unstable. Under various external conditions (in different parts of the phase diagram), different types of low-symmetry walls will arise. The phase diagrams may thus have triple points or even N-phase points of transitions among several types of walls.

To illustrate the arguments we consider the particular case in which a transition in the interior of a crystal is described by a three-component order parameter.[10] Taking account of only plane domain walls, oriented perpendicular to one of the crystal axes, which are possible in a crystal, we write the Landau potential as

$$F=\int_{-\infty}^{\infty}\left\{\frac{1}{2}(\dot\eta_1^2+\dot\eta_2^2+\dot\eta_3^2)+\frac{1}{2}\alpha_1(\eta_1^2+\eta_2^2+\eta_3^2)+\frac{1}{4}\alpha_2(\eta_1^2+\eta_2^2+\eta_3^2)^2+\frac{1}{6}\alpha_3(\eta_1^2+\eta_2^2+\eta_3^2)^3+\frac{1}{2}\gamma_1[\eta_1^2(\eta_2^2+\eta_3^2)+\eta_2^2\eta_3^2]+\frac{1}{2}\gamma_3\eta_1^2\eta_2^2\eta_3^2+\frac{1}{2}\gamma_2[\eta_1^2(\eta_2^2+\eta_3^2)+\eta_2^2\eta_3^2](\eta_1^2+\eta_2^2+\eta_3^2)\right\}S\,dz. \qquad (18)$$

This potential can describe three one-parameter, low-symmetry phases.[10,11] One of them has symmetry C_{4v} in $\mathbf{E}_3$, with $\eta_1=\eta_2=0$ and $\eta_3\neq 0$. This phase is thermodynamically stable in the space $(\alpha_1\gamma_1)$, where $\alpha_1<0$ for $\gamma_1>0$ and

$$\alpha_1<\frac{\alpha_2}{\gamma_2}\gamma_1-\frac{\alpha_3}{\gamma_2^2}\gamma_1^2 \quad \text{at} \quad \gamma_1<0$$

(Fig. 2). In the phase of symmetry C_{4v}, various types of symmetries are possible for domain walls.[4] The highest-symmetry walls have symmetry C_{4v} in $\mathbf{E}_3$ and the structure of Zhirnov walls. Domain walls also arise between (η00) and

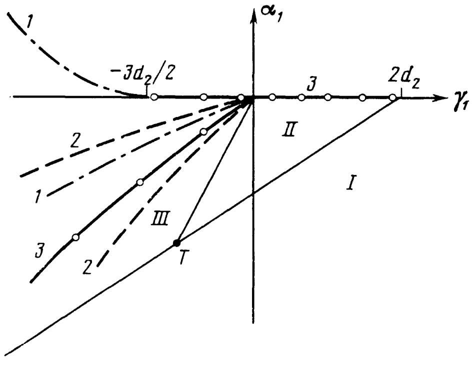

FIG. 2. Phase diagram in the (α_1, γ_1) plane corresponding to (18). 1, 2, 3—stability boundaries of phases whose symmetries are described by $\eta_1=\eta_2=\eta_3$; $\eta_1=\eta_2$, $\eta_3=0$; $\eta_1=\eta_2=0$, $\eta_3\neq 0$, respectively. I) Stability region of high-symmetry domain walls; II, III) stability regions of walls whose symmetry is determined by $\xi_1\neq 0$, $\xi_2=0$ and $\xi_1=\xi_2\neq 0$, respectively.

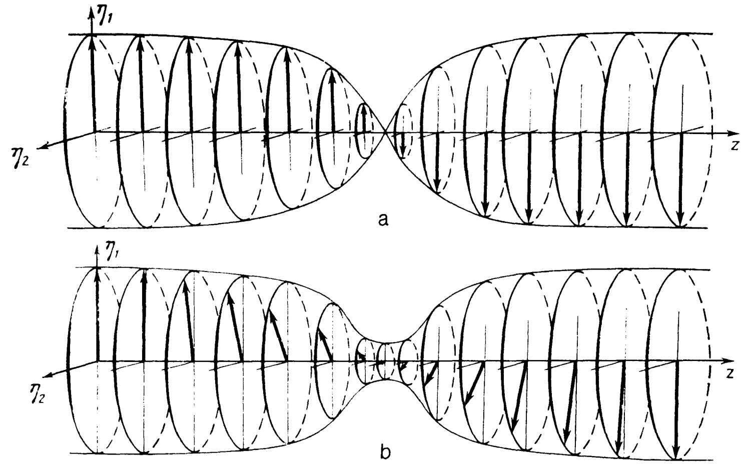

FIG. 3. Transition from a change in only the magnitude of the order parameter in a domain wall (a) to simultaneous changes in the magnitude and the rotation (b).

$(-\eta 00)$ domains. However, there is a region on the phase diagram in which high-symmetry domain walls between such domains are absolutely unstable. To determine these regions, we consider a branching off, from the solution of the equations of state which describes a Zhirnov wall, $\eta_1 = \varphi_0(t)$, $\eta_2 = \eta_3 = 0$, of a solution of a more general type, $\eta_1 = \varphi_0(t) + u(t)$, $\eta_2 = v(t)$, $\eta_3 = \omega(t)$, under the condition $u(\pm\infty) = v(\pm\infty) = \omega(\pm\infty) = 0$.

The linearized equations for determining u, v, and ω are

$$u'' - 2(2 - 3\,\mathrm{ch}^{-2}t - \varepsilon[6\,\mathrm{ch}^{-2}t - 7\,\mathrm{ch}^{-4}t])u = 0,$$
$$v'' - 2\{y - (y+1)\,\mathrm{ch}^{-2}t - \varepsilon[(5-p) + (2p-4)\,\mathrm{ch}^{-2}t - (p+2)\,\mathrm{ch}^{-4}t]\}v = 0,$$
$$\omega'' - 2\{y - (y+1)\,\mathrm{ch}^{-2}t - \varepsilon[(5-p) + (2p-4)\,\mathrm{ch}^{-2}t - (p+2)\,\mathrm{ch}^{-4}t]\}\omega = 0. \quad (19)$$

It can be seen from (19) that the equations for v, ω branch at the same values of y as in (7). The solutions which branch off are[9]

$$v = \xi_1 \psi(t), \qquad \omega = \xi_2 \psi(t), \quad (20)$$

where $\psi(t)$ is defined in (11). By analogy with (14), we find a Landau potential which describes transitions in the domain wall in this case:

$$F = S\left(\frac{|\alpha_1|}{2}\right)^{1/2}\left\{\frac{4}{15}\mu(\xi_1^2+\xi_2^2) + K\frac{\alpha_3(p-5)}{\alpha_2}(\xi_1^2+\xi_2^2)^2 + \frac{32}{315}\left(9\frac{\gamma_1}{|\alpha_1|} + \frac{\gamma_2+\gamma_3}{\alpha_2}\right)\xi_1^2\xi_2^2\right\}. \quad (21)$$

The sign of the last term determines the particular structure of the domain wall. If $\alpha_1 < 9\gamma_1\alpha_2/(\gamma_2+\gamma_3)$, then a domain-wall structure between $(\eta, 0, 0)$ and $(-\eta, 0, 0)$ domains is stable; in a first approximation in ξ, this structure is of the form $(\varphi_0(t), \xi\psi(t), 0)$ while in the case $\alpha_1 > 9\gamma_1\alpha_2/(\gamma_2+\gamma_3)$ the wall structure is of the form $(\varphi_0(t), \xi\psi(t), \xi\psi(t))$.

If $5.75\alpha_3 > \gamma_2 > 4\alpha_3$, then the coordinates of the point T at which the line $\alpha_1 = 9\gamma_1\alpha_2/(\gamma_2+\gamma_3)$ intersects the branching line (16) are

$$\gamma_1^T = -14\alpha_2(\gamma_2+\gamma_3)/5(23\alpha_3 - 4\gamma_2),$$
$$\alpha_1^T = -126\alpha_2^2/5(23\alpha_3 - 4\gamma_2). \quad (22)$$

For other values of γ_2, these lines do not intersect, and there is no point T in the (α_1, γ_1) plane. We are left with only regions of stability of all types of domain walls. The coordinates of the point T given in (22) obviously go outside the region in which it is legitimate to use the approximation $\varepsilon \ll 1$, which we have used here. Consequently, the results in (22) are only qualitative. Figure 2 shows the complete phase diagram corresponding to (18). The regions of stability of the various types of walls between $(\eta, 0, 0)$ and $(-\eta, 0, 0)$ domains are shown. Walls in which we have $\xi_1 \neq 0$, $\xi_2 = 0$ and walls in which we have $\xi_1 = \xi_2 \neq 0$ have symmetry C_s in the space $\mathbf{E}_3$, and their symmetry groups are not related by a group-subgroup relation. Consequently, a transition between such walls is always of first order. The line on which they lose their stability in Fig. 2 coincides with the line on which their energies are equal only because we limited the calculation of potential (21) to fourth degree in ξ_1 and ξ_2 (Ref. 11).

3. DISCUSSION

These calculations thus show that as the conditions in the reservoir approach the conditions corresponding to an order-order transition in the interior of a crystal there will inevitably be a phase transition in domain walls which involves a lowering of the symmetry of the walls. In the space of the components at the order parameter, $\mathbf{E}_m$, the transition from the highest-symmetry wall to a low-symmetry wall is a transition from a wall in which the order parameter varies only in magnitude (Fig. 3a) to a wall in which the change in magnitude is accompanied by a rotation of the order parameter (Fig. 3b). For ferromagnets, if we allow for the relative magnitudes of the anisotropic and exchange terms in F, we see that we have $p - 5 = \gamma_2/\alpha_3 - 4 < 0$, and this transition can occur only as a first-order transition in a domain wall.

Finally, we note that the branching of the solutions of the equations of state of a domain wall which we have studied here makes it possible to describe a possible new type of defects of domain walls in a region of the phase diagram with $\xi \neq 0$. This type of planar wall defect takes the form of lines which separate regions of the wall in which the relations $\xi = \pm\xi_0$ hold, e.g., far from this line. Along the centers of these lines the symmetry of the wall is higher than in the neighboring regions, in total analogy with the circumstance that the symmetry at the center of a Zhirnov domain wall corresponds to a high-symmetry phase of a crystal. The nature of the transition from $\xi = +\xi_0$ to $\xi = -\xi_0$ on the straight sections of these lines is described by the potentials (14) and (21), supplemented with gradient (along x) terms. The potential (14) leads to the antisymmetric (in x) change $\xi = \xi(x)$ [the solution of the equation of state for $\xi(x)$ is similar to (5)]. In the case of an m-component order parameter ($m > 2$), we find obvious possibilities of a more complex structure of domain-wall defects of a new type, described by a solution of type (5), (14). In other words, both symmetric and antisymmetric components $\xi_i(x)$ ($i = 1...m - 1$) are possible on these lines.

APPENDIX: CALCULATION OF THE LANDAU POTENTIAL DESCRIBING A TRANSITION IN A DOMAIN WALL

After the substitution $\eta_1 = \varphi_0 + u$, $\eta_2 = v$ and a renormalization of the coordinates, functional (1) becomes

$$F=F(\varphi_0)+S\left(\frac{|\alpha_1|}{2}\right)^{1/2}\int_{-\infty}^{\infty}\left\{\frac{1}{2}(u'^2+v'^2)\right.$$

$$+[2-3\,\mathrm{ch}^{-2}t-\varepsilon(6\,\mathrm{ch}^{-2}t-7\,\mathrm{ch}^{-4}t)]u^2+\{y-(y+1)\,\mathrm{ch}^{-2}t$$

$$-\varepsilon[(5-p)+(2p-4)\,\mathrm{ch}^{-2}t-(p+2)\,\mathrm{ch}^{-4}t]\}v^2$$

$$\left.+R_1(t)uv^2+R_2(t)v^4\right\}dt,$$

where

$$R_1(t)=2\left(\frac{\alpha_2}{|\alpha_1|}\right)^{1/2}\mathrm{th}\,t\left\{3+\varepsilon\left[\frac{22p-33}{14}-(2p+1)\,\mathrm{ch}^{-2}t\right]\right\},$$

$$R_2(t)=\frac{1}{2}\frac{\alpha_2}{|\alpha_1|}\{1+\varepsilon[(2p-1)-2p\,\mathrm{ch}^{-2}t]\}$$

and $F(\varphi_0)$ is the free energy corresponding to a Zhirnov domain wall. We will omit this term below.

We set $y = y_0 + \mu$, integrate the terms with u'^2 and v'^2 by parts, and use Eqs. (6) and (7). We easily see that in this case the coefficient of ξ^4 is

$$S\left(\frac{|\alpha_1|}{2}\right)^{1/2}\int_{-\infty}^{\infty}\left\{\frac{1}{2}R_1(t)uv^2+R_2(t)v^4\right\}dt.$$

The calculation of the second term in this integral is carried out by substituting expressions (10) and (11) directly into this expression. In the evaluation of the first term, on the other hand, we run into the complication that we do not have an explicit expression for u_1. A solution of (13) does exist, however, since we have $(\Phi(t), \psi(t)) = 0$ by virtue of the odd parity of $\Phi(t)$. Here $\Phi(t)$ is the right side of (13), and

$$(\Phi,\psi)=\int_{-\infty}^{\infty}\Phi\psi\,dt$$

is the scalar product in $L_2(-\infty, \infty)$. The solution of (13) is thus of the form $u_1 = \hat{\Gamma}\Phi(t)$, where the operator $\hat{\Gamma}$ is the inverse of the contraction of the operator $\hat{L}_0 = \partial^2/\partial t^2 - 2(2\text{–}3\mathrm{ch}^{-2}t)$.

In first order in ε, u_1 appears only in the integral

$$J=(\mathrm{th}\,t\,\mathrm{ch}^{-4}t,\ u_1)=(\mathrm{th}\,t\,\mathrm{ch}^{-4}t,\ \hat{\Gamma}\Phi(t)).$$

Making use of the Hermitian nature of $\hat{\Gamma}$, we find

$$J=(\hat{\Gamma}\,\mathrm{th}\,t\,\mathrm{ch}^{-4}t,\ \Phi(t)).$$

A direct calculation verifies

$$\hat{\Gamma}\,\mathrm{th}\,t\,\mathrm{ch}^{-4}t=-\mathrm{th}\,t\,\mathrm{ch}^{-2}t/6\,,$$

and the evaluation of integral J can be completed.

Finally, we find expression (14) for the Landau potential describing a transition in a domain wall.

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Translated by Dave Parsons