Excitation of sound by modulated electromagnetic waves in a dense semi-infinite nonisothermal plasma

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A ponderomotive mechanism is analyzed for the excitation of sound when a modulated small-amplitude electromagnetic wave is incident normally on a nonisothermal plasma. The plasma is assumed to be dense (the dielectric constant is negative at the carrier frequency). The amplitude of the sound wave, the amplitude of the wave reflected from the plasma, and the energy flux into the plasma are found.

When an intense electromagnetic wave is incident on a plasma with a negative dielectric constant ε, the transparency region may shift into the interior of the plasma.[1] The wave amplitude required for this effect decreases as the wave frequency approaches the electron plasma frequency. If the external radiation varies in time, the electromagnetic field becomes localized in transparent regions and is transported into the interior of the plasma along with density waves. Karpman[2] has studied the conditions for the excitation of electroacoustic waves and their dynamics for the situation in which an electromagnetic field is trapped where ε has small negative values. If the dielectric constant is not close to zero, the external radiation will have to have a substantial amplitude in order to excite acoustic rarefaction waves filled with an rf field.[2] At relatively small amplitudes of the wave incident on the plasma, in a situation in which there is no transparency region near the plasma boundary, acoustic waves unrelated to transport of electromagnetic radiation may be excited. Aliev *et al.*[3] have studied a mechanism for the parametric excitation of sound in a plasma with a layer with a sharp transition to vacuum. Since the damping rate of the collisionless damping of the surface wave participating in the parametric instability exceeds the ion acoustic frequency, however, the instability threshold is high.[4] In the present paper we analyze a sound excitation process in which an electromagnetic wave with a variable amplitude is incident on a plasma with a negative dielectric constant. The wave amplitude is assumed to be small in the sense that the conditions required for the occurrence of the effects studied in Refs. 2 and 4 do not hold.

1. BASIC EQUATIONS AND BOUNDARY CONDITIONS

We assume that an unperturbed plasma bounded by a nonconducting medium fills the half-space $Z \geqslant 0$. An electromagnetic wave of variable amplitude is incident normally on the plasma from the neighboring medium (whose dielectric constant is assumed equal to unity). The electric field of the wave, which for definiteness we direct along the x axis, is

$$E_i(t,z)=\frac{1}{2}\tilde{E}_{0i}\left(t-\frac{z}{c}\right)\exp\left\{i\omega_0\left(t-\frac{z}{c}\right)\right\}+\text{c.c.} \quad (1)$$

The amplitude of the incident wave, $\tilde{E}_{0i}$, varies slowly in space and time in comparison with an exponential function. The total field in the medium, $E_v(t,z)$, consists of incident wave (1) and the wave reflected from the plasma, $E_R(t,z)$, so we have

$$E_v(t,z)=E_i(t,z)+E_R(t,z),$$

where

$$E_R(t,z)={}^1/_2\tilde{E}_R(t+z/c)\exp\{i\omega_0(t+z/c)\}+\text{c.c.} \quad (2)$$

We write the electric field of the rf wave in the plasma in the form

$$E_p(t,z)={}^1/_2\tilde{E}_p(t,z)\exp i\omega_0 t+\text{c.c.} \quad (3)$$

Taking ponderomotive effects into account, we then see that the amplitude of field (3), E_p, obeys the system of equations (see, for example, Refs. 5 and 6)

$$-2i\omega_0\frac{\partial\tilde{E}_p}{\partial t}+c^2\frac{\partial^2\tilde{E}_p}{\partial z^2}-(\omega_{pe}^2-\omega_0^2)\tilde{E}_p =\frac{\omega_{pe}^2}{\omega_0}\tilde{E}_p\left(\omega_0-i\frac{\partial}{\partial t}\right)\frac{\delta n}{n_0}, \quad (4)$$

$$\frac{\partial^2\delta n}{\partial t^2}-c_s^2\frac{\partial^2\delta n}{\partial z^2} =n_0\frac{\omega_{pe}^2}{\omega_0^2}\frac{c_s^2}{16\pi n_0T_e}\frac{\partial^2}{\partial z^2}\left|\left(1-i\frac{\partial}{\omega_0\partial t}\right)\tilde{E}_p\right|^2, \quad (5)$$

where ω_{pe} is the electron plasma frequency, $c_s=(T_e/m_i)^{1/2}$ is the sound velocity in the nonisothermal plasma, and δn and n_0 are the density variation and the equilibrium density of the plasma ($\delta n \ll n_0$). We have deliberately considered the time derivatives on the right sides of Eqs. (4) and (5), in contrast with Refs. 5 and 6. Our reason is that the amplitude of the sound wave in the plasma and thus the absorption coefficient of the plasma for the radiation are determined exclusively by the time-varying components of the external field.[2] To avoid any misunderstanding, we should thus consider those terms in the nonlinear sources in Eqs. (4) and (5) which are strongly coupled to the time variation of the rf field and the density variation. System (4), (5) must be supplemented with boundary conditions. Since it is assumed from the outset that the external radiation perturbs the plasma only slightly, the boundary of the plasma coincides with the surface of the medium bounding the plasma. We will assume below that this boundary is flat and fixed. The boundary conditions on the rf field then consist of the continuity of the electric field and of its first derivative.[7] Using (1)–(3), we can write these conditions as

$$\tilde{E}_{0i}(t)+\tilde{E}_R(t)=\tilde{E}_p(t,0),$$

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$$\frac{1}{c}\left(i\omega_0+\frac{\partial}{\partial t}\right)[\tilde{E}_R(t)-\tilde{E}_{0i}(t)]=\frac{\partial \tilde{E}_p}{\partial z}\bigg|_{z=+0}. \quad (6)$$

The boundary condition on Eq. (5) is the vanishing of the velocity of quasineutral motions of the plasma boundary:

$$\frac{\partial}{\partial z}\left[\delta n+\frac{\omega_{pe}^2}{\omega_0^2}\frac{n_0}{16\pi n_0 T_e}\left|\left(1-i\frac{\partial}{\omega_0\partial t}\right)\tilde{E}_p\right|^2\right]_{z=+0}=0. \quad (7)$$

Equations (4)–(7), along with the conditions

$$\tilde{E}_p(t, z\to+\infty)\to 0, \quad \int_0^\infty |\tilde{E}_p|^2\,dz<\infty, \quad (8)$$

thus form a complete system of equations and boundary conditions for determining the density variation, the amplitude of the electric field in the plasma, and that in the medium bounding the plasma.

2. SOLUTION OF THE BASIC EQUATIONS

We take Fourier time transforms of all the quantities which appear in Eqs. (4) and (5), using the equations

$$(\tilde{E}(z,t);\delta n(z,t))=\frac{1}{2\pi}\int_{-\infty}^{\infty}(\tilde{E}(z,\omega);\delta n(z,\omega))e^{-i\omega t}\,d\omega,$$

$$(\tilde{E}(z,\omega);\delta n(z,\omega))=\int_{-\infty}^{\infty}(\tilde{E}(z,t);\delta n(z,t))e^{i\omega t}\,dt. \quad (9)$$

The equation and boundary conditions for the Fourier components of the amplitude of the rf field are

$$\frac{d^2\tilde{E}_p(z,\omega)}{dz^2}-\varkappa_p^2(\omega)\tilde{E}_p(z,\omega)=\frac{\omega_{pe}^2}{2\pi c^2\omega_0}\int_{-\infty}^{\infty}d\omega_2(\omega_0-\omega_2)\overline{\delta n}(\omega_2,z)\tilde{E}_p(\omega-\omega_2,z), \quad (10)$$

$$\tilde{E}_{0i}(\omega)+\tilde{E}_R(\omega)=\tilde{E}_p(\omega,0), \quad (11)$$

$$i\varkappa_v(\omega)[\tilde{E}_R(\omega)-\tilde{E}_{0i}(\omega)]=\frac{\partial\tilde{E}_p(\omega,z)}{\partial z}\bigg|_{z=+0}, \quad (12)$$

where

$$\varkappa_p(\omega)=\frac{\omega_0-\omega}{c}(-\varepsilon(\omega))^{1/2}, \quad \varepsilon(\omega)=1-\frac{\omega_{pe}^2}{(\omega_0-\omega)^2},$$

$$\varkappa_v(\omega)=\frac{\omega_0-\omega}{c}, \quad \overline{\delta n}=\delta n/n_0.$$

Denoting by $F(\omega, z)$ the right side of Eq. (10), and solving the equation by the method of variation of constants, we find

$$\tilde{E}_p(\omega,z)=c_1(\omega)e^{-\varkappa_p z}+\frac{e^{\varkappa_p z}}{2\varkappa_p}\int_\infty^z F(\omega,z')e^{-\varkappa_p z'}\,dz'-\frac{e^{-\varkappa_p z}}{2\varkappa_p}\int^z F(\omega,z')e^{\varkappa_p z'}\,dz'. \quad (13)$$

In deriving expression (13) we have used the first of conditions (8). Substituting (13) into boundary conditions (11) and (12), we find an equation for the Fourier component of the amplitude of the wave reflected from the plasma:

$$\tilde{E}_R(\omega)=R_0(\omega)\tilde{E}_{0i}(\omega)+\frac{1}{\varkappa_p+i\varkappa_v}\int_\infty^0 F(\omega,z)e^{-\varkappa_p z}\,dz, \quad (14)$$

where $R_0(\omega) = (i - [\varepsilon(\omega)]^{1/2})/(i + [-\varepsilon(\omega)]^{1/2})$ is the linear reflection coefficient; we also find the constant $c_1(\omega)$. Then using the expression for $c_1(\omega)$ in Eq. (13), we find the following equation for the Fourier component of the electric field amplitude in the plasma:

$$\tilde{E}_p(\omega,z)=(1+R_0)\tilde{E}_{0i}(\omega)e^{-\varkappa_p z}+\frac{1}{2\varkappa_p}(e^{\varkappa_p z}-R_0e^{-\varkappa_p z})\int_\infty^z F(\omega,z')e^{-\varkappa_p z'}\,dz'+\frac{e^{-\varkappa_p z}}{2\varkappa_p}\int_z^0 F(\omega,z')(e^{\varkappa_p z'}-R_0e^{-\varkappa_p z'})\,dz'. \quad (15)$$

Since the nonlinear source $F(\omega, z)$ depends on the electric field in the plasma, Eq. (15) is a nonlinear integral equation. There is no need to solve it rigorously in our case. For an approximate solution it is sufficient to note that under the inequality

$$|\overline{\delta n}|_{max}\ll\frac{\omega_{pe}^2-\omega_0^2}{\omega_p^2}\equiv\mu \quad (16)$$

the integral terms in (15) are proportional to the small parameter $\lambda = |\overline{\delta n}|_{\max}/\mu$. Equation (15) can thus be solved iteratively. We retain in the resulting solution only the first nonvanishing term of the expansion in the parameter λ. We recall that in formulating this problem we assumed that inequality (16) held. To find the electric field of the reflected wave and the density variation below, we will use the linear solution of Eq. (15):

$$\tilde{E}_p(\omega,z)=D_p(\omega)\tilde{E}_{0i}(\omega)\exp\{-\varkappa_p(\omega)z\}, \quad (17)$$

where $D_p(\omega) = 1 + R_0(\omega)$. The nonlinear part of the solution of Eq. (15) was already taken into account in Eq. (14). After substituting (17) into (14), we find the following expression for the Fourier component of the field of the wave reflected from the plasma [we are using the definition of $F(\omega, z)$]:

$$\tilde{E}_R(\omega)=R_0(\omega)\tilde{E}_{0i}(\omega)+\frac{D_p(\omega)\omega_{pe}^2}{4i\pi c^2\omega_0\varkappa_v(\omega)}\int_\infty^0 dz\,e^{-\varkappa_p(\omega)z}\int_{-\infty}^{\infty}d\omega_2(\omega_0-\omega_2)\cdot D_p(\omega-\omega_2)\overline{\delta n}(\omega_2,z)\tilde{E}_{0i}(\omega-\omega_2)\exp\{-\varkappa_p(\omega-\omega_2)z\}. \quad (18)$$

To find $\tilde{E}_R(\omega)$ we thus need to find the Fourier component of the density variation $\overline{\delta n}(\omega_2, z)$, since the spectrum of the incident wave is given. The equation and boundary condition for $\overline{\delta n}(\omega_2, z)$ are

$$c_s^2\frac{d^2\overline{\delta n}}{dz^2}+\omega_2^2\overline{\delta n}=-\frac{\omega_{pe}^2}{32\pi^2 n_0 m_i\omega_0^4}\frac{\partial^2}{\partial z^2}\int_{-\infty}^{\infty}d\omega_3\tilde{E}_p(\omega_3,z)\tilde{E}_p^*(\omega_3-\omega_2) \quad (19)$$

$$\cdot(\omega_0+\omega_3)(\omega_0+\omega_3-\omega_2),$$

$$\frac{\partial}{\partial z}\left\{c_s^2\overline{\delta n}+\frac{\omega_{pe}^2}{32\pi^2 n_0 m_i \omega_0^4}\int_{-\infty}^{\infty}d\omega_3 \tilde{E}_p(\omega_3,z)\tilde{E}_p^*\right.$$

$$\left.\cdot(\omega_3-\omega_2,z)(\omega_0+\omega_3)(\omega_0+\omega_3-\omega_2)\right\}_{z=+0}=0. \quad (20)$$

We denote by $F_1(\omega_2, z)$ the second term in braces in (20), and we transform it. First, as we mentioned above, we use the linear relation (17) between the field amplitude in the plasma and the amplitude of the incident wave (the "pump"). Second, we note that the spectal width of the pump is much smaller than ω_0. Then substituting the field $\tilde{E}_p(\omega, z)$ as in (17) into $F_1(\omega_2, z)$, and expanding $D_p(\omega)$ around $\omega = 0$, we find the following expression for $F_1(\omega_2, z)$:

$$F_1(\omega_2,z)=\frac{(\omega_0+i\omega_2(-\varepsilon_0)^{-1/2})}{8\pi^2 n_0 m_i \omega_0}\int_{-\infty}^{\infty}d\omega_3 \tilde{E}_{0i}(\omega_3)\tilde{E}_{0i}^*(\omega_3-\omega_2)$$

$$\cdot\exp\{-\varkappa_p(\omega_3)z-\varkappa_p(\omega_3-\omega_2)z\}, \quad (21)$$

where $\varepsilon_0 = \varepsilon(0)$. When expression (21) is used, a solution of Eq. (19) which satisfies the boundary condition (20) is

$$\overline{\delta n}(\omega_2,z)$$

$$=-\frac{(\omega_0+i\omega_2(-\varepsilon_0)^{-1/2})}{8\pi^2 n_0 T_e \omega_0}\int_{-\infty}^{\infty}\frac{\omega_{s1}d\omega_3}{\omega_{s1}^2+\omega_2^2}\tilde{E}_{0i}(\omega_3)\tilde{E}_{0i}^*(\omega_3-\omega_2)$$

$$\cdot\left\{e^{-\omega_{s1}z/c_s}+i\frac{\omega_2}{\omega_{s1}}e^{-i\omega_2 z/c_s}\right\}, \quad (22)$$

where $\omega_{s1} = [\varkappa_p(\omega_3) + \varkappa_p(\omega_3 - \omega_2)]c_s$. The ratio ω_2/ω_{s1} cannot in general be regarded as a small parameter. The second term in braces in (22) corresponds to a sound wave which is traveling away from the plasma boundary at the sound velocity c_s. Substituting the density variation (22) into (18), and integrating over z, we find

$$\tilde{E}_R(\omega)=R_0(\omega)\left\{\tilde{E}_{0i}(\omega)-\frac{i(\omega_0-\omega)}{8\pi^3 c n_0 T_e}c_s\int_{-\infty}^{\infty}\frac{\tilde{E}_{0i}(\omega-\omega_2)}{\omega_{s1}+\omega_{s2}}\tilde{E}_{0i}(\omega_3)\right.$$

$$\left.\cdot\tilde{E}_{0i}^*(\omega_3-\omega_2)\frac{\omega_{s1}\omega_{s2}d\omega_2\,d\omega_3}{(\omega_{s1}-i\omega_2)(\omega_{s2}-i\omega_2)}\right\}, \quad (23)$$

where $\omega_{s2} = [\varkappa_p(\omega) + \varkappa_p(\omega - \omega_2)]c_s$. We can also write the Fourier component of the density variation in the sound wave which is traveling away from the plasma boundary:

$$\overline{\delta n_s}(\omega_2,z)=-i\frac{\omega_2}{8\pi^2 n_0 T_e}\int_{-\infty}^{\infty}\frac{\omega_{s1}\,d\omega_3}{\omega_{s1}^2+\omega_2^2}\tilde{E}_{0i}(\omega_3)\tilde{E}_{0i}^*(\omega_3-\omega_2)$$

$$\cdot\exp(i\omega_2 z/c_s). \quad (24)$$

Expanding ω_{s1} and ω_{s2} near the origin, we can put expressions (23) and (24) in their final form:

$$\tilde{E}_R(\omega)=R_0(\omega)\left\{\tilde{E}_{0i}(\omega)-\frac{2i\omega_{s0}c_s(\omega_0-\omega)}{32\pi^3 n_0 T_e c}\right.$$

$$\cdot\int_{-\infty}^{\infty}d\omega_2\frac{\tilde{E}_{0i}(\omega-\omega_2)}{(\omega_{s0}-i\omega_2)^2}\int_{-\infty}^{\infty}d\omega_3\tilde{E}_{0i}(\omega_3)\cdot$$

$$\left.\cdot\tilde{E}_{0i}^*(\omega_3-\omega_2)\right\}, \quad (25)$$

$$\overline{\delta n_s}(\omega_2,z)=-\frac{i}{8\pi^2 n_0 T_e}\frac{\omega_{s0}\omega_2}{(\omega_{s0}^2+\omega_2^2)}$$

$$\cdot\int_{-\infty}^{\infty}d\omega_3\tilde{E}_{0i}(\omega_3)\tilde{E}_{0i}^*(\omega_3-\omega_2)\exp(i\omega_2 z/c_s), \quad (26)$$

where $\omega_{s0} = 2\varkappa_p(0)c_s$. Finding the total energy w_s of the sound wave which penetrates into the plasma from

$$w_s=c_s n_0 T_e\int_{-\infty}^{\infty}|\overline{\delta n_s}|^2\frac{d\omega}{2\pi},$$

we see that this energy is exactly equal to the difference between the total energies of the incident wave and the reflected wave, Δw, where

$$\Delta w=\frac{c}{16\pi^2}\int_{-\infty}^{\infty}d\omega[|\tilde{E}_{0i}(\omega)|^2-|\tilde{E}_R(\omega)|^2].$$

It follows from expression (26) that when an electromagnetic wave of constant amplitude is incident on the plasma no sound wave will be excited. This fact becomes obvious when we take inverse Fourier transforms in (26). As a result we find

$$\overline{\delta n_s}\left(t-\frac{z}{c_s}\right)$$

$$=\frac{1}{4\pi n_0 T_e}\int_{-\infty}^{\infty}dt'\frac{\partial|\tilde{E}_{0i}(t',0)|^2}{\partial t'}\exp\left\{-\omega_{s0}\left|t-\frac{z}{c_s}-t'\right|\right\}.$$

We turn now to some cases of interest for applications.

3. EXCITATION OF SOUND BY A BIHARMONIC PUMP

In this case we write the Fourier components of the incident and reflected waves in the form

$$\tilde{E}_{0i}(\omega)=2\pi\tilde{E}_1\delta(\omega+\omega_1)+2\pi\tilde{E}_2\delta(\omega+\omega_2), \quad (27)$$

$$\tilde{E}_R(\omega)=2\pi R_1\tilde{E}_1\delta(\omega+\omega_1)+2\pi R_2\tilde{E}_2\delta(\omega+\omega_2). \quad (28)$$

After substituting expressions (27) and (28) into (25), we find the nonlinear reflection coefficients

$$R_1=R_0(\omega_0+\omega_1)\left\{1-\frac{(\omega_0+\omega_1)}{8\pi n_0 T_e\omega_0(-\varepsilon_0)^{1/2}}\left[|\tilde{E}_1|^2+|\tilde{E}_2|^2+\frac{\omega_{s0}^2|\tilde{E}_2|^2}{(\omega_{s0}+i\Delta)^2}\right]\right\},$$

$$R_2=R_0(\omega_0+\omega_2)\left\{1-\frac{(\omega_0+\omega_2)}{8\pi n_0 T_e\omega_0(-\varepsilon_0)^{1/2}}\left[|\tilde{E}_1|^2+|\tilde{E}_2|^2+\frac{\omega_{s0}^2|\tilde{E}_1|^2}{(\omega_{s0}-i\Delta)^2}\right]\right\}, \quad (29)$$

where $\Delta = \omega_1 - \omega_2$. The first two terms in square brackets in expressions (29) stem from a steady-state modification of the skin layer of waves with amplitudes $\tilde{E}_1$ and $\tilde{E}_2$ (Ref. 1). The magnitudes of the reflection coefficients R_1 and R_2 are written in the form

$$|R_1|=1-\frac{\omega_0-\omega_1}{\omega_0}\frac{|\tilde{E}_2|^2}{(-\varepsilon_0)^{1/2}4\pi n_0T_e}\frac{\omega_{s0}^3\Delta}{(\omega_{s0}^2+\Delta^2)^2}$$

$$|R_2|=1+\frac{\omega_0+\omega_2}{\omega_0}\frac{|\tilde{E}_1|^2}{(-\varepsilon_0)^{1/2}4\pi n_0T_e}\frac{\omega_{s0}^3\Delta}{(\omega_{s0}^2+\Delta^2)^2}. \quad (30)$$

We also write an expression for the density variation in the sound wave,

$$\overline{\delta n_s}(t,z)=-i\frac{\tilde{E}_2\tilde{E}_1^*}{4\pi n_0T_e}\frac{\omega_{s0}\Delta}{(\omega_{s0}^2+\Delta^2)}\cdot\exp\left\{i\Delta\left(\frac{z}{c_s}-t\right)\right\}+\text{c.c.}, \quad (31)$$

and an expression for the energy flux into the plasma,

$$p_s=c_s\frac{|\tilde{E}_1|^2|\tilde{E}_2|^2}{8\pi^2n_0T_e}\frac{\omega_{s0}^2\Delta^2}{(\omega_{s0}^2+\Delta^2)^2}. \quad (32)$$

The right sides of (30)–(32) are extrema at $\Delta\sim\omega_{s0}$, i.e., when the beat frequency of the rf field is equal in order of magnitude to the reciprocal of the time required for the sound wave to traverse the length of the pump skin layer. The amplitude of the density oscillations, (31), and the energy flux of the sound wave, (32), are conveniently expressed in terms of the energy fluxes of the incident waves:

$$\frac{\delta n_s}{n_0}=\frac{(p_1p_2)^{1/2}}{p_*}Q(\Delta),\qquad p_{1,2}=\frac{c}{8\pi}|\tilde{E}_{1,2}|^2,$$

$$p_s=1,6\cdot10^{-3}(T_e(\text{eV}))^{1/2}\left(\frac{m_e}{m_i}\right)^{1/2}\frac{p_1p_2}{p_*}Q^2(\Delta),$$

where $Q(\Delta)=2\omega_{s0}\Delta/(\omega_{s0}^2+\Delta^2)$, and p_* is the normalized energy flux, given by

$$p_*=1.5\cdot10^{-9}n_0(\text{cm}^{-3})T_e(\text{eV})\ \text{W/cm}^2.$$

4. EXCITATION OF SOUND WHEN AN ELECTROMAGNETIC WAVE OF RANDOM AMPLITUDE IS INCIDENT ON A PLASMA

In this case the complex amplitude of the incident wave is a random function of the time, and for the Fourier components we have[8]

$$\langle\tilde{E}(\omega)\rangle=0,\quad\langle\tilde{E}(\omega)\tilde{E}^*(\omega')\rangle=2\pi w(\omega)\delta(\omega-\omega'), \quad (33)$$

where the angle brackets mean an average over a statistical ensemble, and $w(\omega)$ is the spectral energy density, which is related to the average radiation energy density by

$$\left\langle\frac{|\tilde{E}|^2}{8\pi}\right\rangle=\frac{1}{8\pi}\int_{-\infty}^{\infty}w(\omega)\frac{d\omega}{2\pi}.$$

Using expression (25) and definitions (33) to calculate the correlation function $\langle\tilde{E}_R(\omega)\tilde{E}_R^*(\omega')\rangle$, we find the spectrum of the wave reflected from the plasma in the form

$$w_R(\omega)=w_i(\omega)\left\{1+\frac{c_s}{c}\frac{(\omega_0-\omega)}{2\pi^2n_0T_e}\int_{-\infty}^{\infty}\frac{w_i(\omega-\omega_1)}{(\omega_{s0}^2+\omega_1^2)^2}\omega_{s0}^2\omega_1\,d\omega_1\right\}. \quad (34)$$

By calculating the correlation function $\langle\overline{\delta n_s}(\omega)\overline{\delta n_s^*}(\omega')\rangle$ through the use of (26) and (33) we obtain the following expression for the spectral energy density of ion acoustic turbulence far from the plasma boundary:

$$w_s(\omega)=\frac{1}{32\pi^3n_0T_e}\frac{\omega_{s0}^2\omega^2}{(\omega_{s0}^2+\omega^2)^2}\int_{-\infty}^{\infty}d\omega_1w_i(\omega_1)w_i(\omega_1-\omega). \quad (35)$$

The average energy flux into the plasma is related to the spectral energy density $w_s(\omega)$ by

$$\langle p_s\rangle=c_s\int_{-\infty}^{\infty}w_s(\omega)\frac{d\omega}{2\pi}.$$

Equations (34) and (35) take their simplest form in the case in which the spectral width of the pump is much greater than the frequency ω_{s0}:

$$w_R(\omega)=w_i(\omega)\left\{1-\frac{c_s}{c}\frac{\omega_{s0}}{4\pi n_0T_e}(\omega_0-\omega)\frac{\partial w_i(\omega)}{\partial\omega}\right\}, \quad (36)$$

$$w_s(\omega)=\frac{1}{32\pi^3n_0T_e}\frac{\omega_{s0}^2\omega^2}{(\omega_{s0}^2+\omega^2)}\int_{-\infty}w_i^2(\omega)d\omega. \quad (37)$$

It follows from expression (37) that in this case the frequency dependence of the spectral energy density of the ion acoustic turbulence is universal.

Up to this point is has been assumed that there is no absorption of the high-frequency or low-frequency waves in the plasma. If an absorption does occur, sound excitation mechanisms associated with oscillations of the plasma temperature may be important.[9] If, however, it is assumed that the primary mechanism for the absorption of the high-frequency radiation by the plasma is collisional damping, then under the inequality

$$\nu_{ei}\ll2\omega_0c_s(-\varepsilon_0)^{1/2}/c,$$

where ν_{ei} is the rate of electron-ion collisions, the ponderomotive mechanism which we have discussed here for the excitation of sound waves in a dense plasma is the dominant mechanism.

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