Polarization of relativistic photoelectrons

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We investigate the mechanism responsible for the preferential orientation of electron spin when neutral atoms are photoionized. We qualitatively estimate the contribution of processes with and without spin flip to the total cross section, and we study the polarization of the resulting photoions. Neutral atoms in both *s*- and *p*-states are considered, and it is shown that the self-polarization mechanism is quite different for these two states.

The basic properties of relativistic photoelectrons were first studied by Sauter,[1] and independently by Sommerfeld.[2] Their results are widely known, and have become a standard fixture of the academic literature.[3,4] The book by Bethe and Salpeter[5] and the paper of Fano and McVoy[6] provide a detailed bibliography and analysis of work on the photoelectric effect up to 1950.

All of this work is basically concerned with the angular distribution of the photoelectrons, without regard for polarization effects. The Sauter-Sommerfeld theory treated the simplest case, namely ionization of a hydrogen-like atom in the ground state; the final state is described by Sommerfeld-Maue functions.[2] Calculations were carried out assuming $Z\alpha/v$ to be a small parameter (we use the same notation and system of units here as in Ref. 3) in the lowest-order (nonzero) approximation. The high-energy limit[10,11] and higher-order approximations[7–9] in $Z\alpha$ were considered somewhat later, and subsequent work[12–20] involved detailed theoretical and experimental investigations of screening effects and the angular distribution of photoelectrons.

The first study of the polarization properties of photoelectrons[21] established a fundamental result: in the high-energy limit, the photoelectron ejected by the interaction of a *K*-shell electron with a circularly polarized photon is preferentially polarized in the direction of the photon spin. The theory was further developed in follow-up work,[22–25] including an especially complete and detailed investigation of the nonrelativistic case (Ref. 25). The preferential polarization of photoelectrons came to be called the Fano effect. Later work then brought the theory to its final, finished form; a survey of the literature on the Fano effect appears in Ref. 32.

One characteristic of all of the theoretical studies of photoelectron polarization, however, was the practice of summing over initial photoelectron spins, thereby losing any possibility of tracking the mechanism responsible for the preferred photoelectron orientation, or the effect of initial electron spin orientation on the cross section for the process. In the present paper, we propose to study this mechanism.

We expand upon the Sauter-Sommerfeld theory[1–4] by taking the spin states of the initial and final electron into account. The wave functions of the final electron state are represented in the Sommerfeld-Maue approximation[2,3]:

$$\Psi_2=N_2\bar{u}(\mathbf{p})\exp(-i\mathbf{pr})\left[1+\frac{i}{2E}\gamma_0\boldsymbol{\gamma}\nabla\right]\Phi(i\nu,1;ix), \quad (1)$$

$$N_2=\left(\frac{2\pi\nu}{1-e^{-2\pi\nu}}\right)^{1/2}, \quad \nu=\frac{Z\alpha}{v}, \quad x=\mathbf{pr}+pr.$$

Here γ_0 and $\boldsymbol{\gamma}$ are Dirac matrices, E, $\mathbf{p}$, and $\mathbf{v}$ are the energy, momentum, and velocity of the photoelectron, $\Phi(\alpha,\beta,x)$ is the confluent hypergeometric function, and $u(\mathbf{p})$ is the bispinor specifying the state of a free electron with momentum $\mathbf{p}$:

$$u(\mathbf{p})=\left(\frac{m}{\gamma+1}\right)^{1/2}\begin{pmatrix}\gamma+1\\ \gamma\boldsymbol{\sigma}\mathbf{v}\end{pmatrix}w, \quad \gamma=(1-v^2)^{-1/2}. \quad (2)$$

In Eq. (2), m is the rest mass of the electron, γ is the relativistic Lorentz factor, and $\boldsymbol{\sigma}$ represents the Pauli spin matrices. The arbitrary two-component spinor w in Eq. (2) characterizes the spin state of the final electron. This latter spin state can be conveniently specified as follows. Every two-component spinor w is an eigenfunction of the operator $\boldsymbol{\sigma}\mathbf{l}$,

$$\boldsymbol{\sigma}\mathbf{l}w=\zeta w, \quad (3)$$

where $\mathbf{l}$ is some constant unit vector, and $\zeta=\pm 1$. The vector $\mathbf{l}$ obviously characterizes the spin direction: for $\zeta=1$, the spin is oriented parallel to $\mathbf{l}$, and for $\zeta=-1$, it is antiparallel. Normalizing w by requiring that $w^+w=1$, we find from (3) that $w=w_\zeta(\mathbf{l})$ satisfies

$$w_{\zeta'}^+(\mathbf{l})w_\zeta(\mathbf{l})=\delta_{\zeta,\zeta'}, \quad w_\zeta(\mathbf{l})w_\zeta^+(\mathbf{l})=[1+\zeta\boldsymbol{\sigma}\mathbf{l}]/2, \quad (4)$$

which makes it easy to calculate cross sections with the spin states explicitly separated out.

According to Ref. 3, the initial state of the electron can be described by the relativistic wave functions of a hydrogen-like atom. We examine the photoelectric effect for the case in which the initial electron state has $j=1/2$. The orbital angular momentum will then be either $l=0$ or $l=1$.

1. ORBITAL MOMENTUM $l=0$

To first order in $Z\alpha$, the wave functions of the discrete spectrum of a hydrogen-like atom[3] are

$$\Psi_1(\mathbf{r})=N_1e^{-x/2}\left[g(x)+i\frac{Z\alpha}{2nr}\gamma_0\boldsymbol{\gamma}\mathbf{r}f(x)\right]u_0,$$

$$g(x)=(n-1)\Phi(2-n,3;x)-(n+1)\Phi(1-n,3;x), \quad (5)$$

$$f(x)=(n-1)\Phi(2-n,3;x)+(n+1)\Phi(1-n,3;x),$$

$$N_1=\left(\frac{Z^3\alpha^3m^3}{4\pi n^3}\right)^{1/2}, \quad x=\frac{2Z\alpha mr}{n}, \quad u_0=(2m)^{1/2}\begin{pmatrix}1\\0\end{pmatrix}w_0,$$

where $n=1,2,3...$ is the principal quantum number, and the two-component spinor w_0 describes the initial electron spin orientation.

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The transition matrix element for the photoelectric effect takes the form[4]

$$M=\int \overline{\Psi}_2(\mathbf{r})\boldsymbol{\gamma}\mathbf{e}\exp(i\mathbf{kr})\Psi_1(\mathbf{r})\,d\mathbf{r}, \qquad (6)$$

where $\mathbf{k}$ and $\mathbf{e}$ are the momentum and polarization of the incident photon. Substituting (1) and (5) into (6), we obtain integrals of hypergeometric functions; these are calculated in Ref. 33. Some straightforard manipulations then give the matrix element for the photoelectric effect for an initial state with $l=0$ and arbitarary n:

$$M=\frac{8\pi^{1/2}(Z\alpha m)^{5/2}}{(k-p)^2n^{3/2}}\bar{u}(\mathbf{p})[a\boldsymbol{\gamma}\mathbf{e}+(\boldsymbol{\gamma}\mathbf{e})\gamma_0(\boldsymbol{\gamma}\mathbf{b})+(\boldsymbol{\gamma}\mathbf{c})\gamma_0(\boldsymbol{\gamma}\mathbf{e})]u_0,$$

$$a=\frac{1}{(\mathbf{k}-\mathbf{p})^2}+\frac{E}{m(\mathbf{k}^2-\mathbf{p}^2)}, \qquad (7)$$

$$\mathbf{b}=\frac{\mathbf{p}-\mathbf{k}}{2m(\mathbf{k}-\mathbf{p})^2},\quad \mathbf{c}=\frac{\mathbf{k}-\mathbf{p}}{2m(\mathbf{k}^2-\mathbf{p}^2)}.$$

For $n=1$, this is the same expression as in Ref. 4. It implies that the polarization properties and angular distribution of the photoelectrons are independent of the actual level from which they are ejected. The cross section for the photoelectric effect falls off as n^{-3} with increasing n. Electrons sitting in the ground state, then, are the most susceptible to γ-rays. This is an entirely reasonable result if we bear in mind that with increasing n, electrons further and further from the nucleus can be considered closer and closer to being free, and first-order quantum electrodynamics tells us that for a free particle, a vertex with one photon line and two electron lines is forbidden.

Invoking the conservation laws and the explicit form of the bispinors $u(\mathbf{p})$ and u_0, we obtain from Eqs. (7) a differential cross section for photoionization of the atom:

$$d\sigma=f[4\pi\gamma^4(\gamma-1)^4\mu^4(\gamma^2-1)^{1/2}]^{-1}|w^+Rw_0|^2d\Omega,$$
$$f=\pi Z^5\alpha^4r_e^2n^{-3},\quad R=\gamma[\gamma+3-\gamma(\gamma-1)\mu]\mathbf{ev}+i\boldsymbol{\sigma}[\mathbf{eq}], \qquad (8)$$
$$\mathbf{q}=(\gamma-1)[(\gamma+1)(\gamma\mu-1)\mathbf{n}+\gamma(\gamma\mu+1)\mathbf{v}],$$
$$\mu=1-\mathbf{nv},\quad \mathbf{n}=\mathbf{k}/\omega,\quad \omega=|\mathbf{k}|,$$

which provides a complete description of the polarization effects (r_e is the classical radius of the electron).

Recalling that the atoms are unpolarized in their initial state, we average (8) over initial spins ζ to obtain

$$d\sigma=f[4\pi\gamma^4\mu^4(\gamma^2-1)^{1/2}]^{-1}(2\gamma^2\mu-\gamma^2\mu^2-1)[s_0+\zeta'\xi\mathbf{ls}]d\Omega,$$
$$s_0=\gamma(\gamma+1)(\gamma-1)^2\mu+2(\gamma+1)[2-\gamma(\gamma-1)\mu](\mathbf{e}^+\mathbf{e}_\perp)(\mathbf{e}\mathbf{e}_\perp),$$
$$\mathbf{s}=2(\gamma^2-1)\mathbf{n}+\gamma(\gamma-1)[\gamma(\gamma-1)\mu-2]\mathbf{v}, \qquad (9)$$
$$\mathbf{e}_\perp=[\mathbf{v}-\mathbf{n}(\mathbf{nv})][v^2-(\mathbf{nv})^2]^{-1/2},\quad \xi=i\mathbf{n}[\mathbf{ee}^+].$$

Here ξ is the degree of circular polarization of the incident photon,[4] and ζ' is the spin quantum number in the final state. Summing (9) over final spin states ζ' yields the photoionization differential cross section for unpolarized electrons and an arbitrarily polarized incident photon:

$$d\sigma=\frac{f(2\gamma^2\mu-\gamma^2\mu^2-1)s_0\,d\Omega}{2\pi\gamma^4(\gamma-1)^4\mu^4(\gamma^2-1)^{1/2}}, \qquad (10)$$

and for a linearly polarized photon, Eq. (10) gives the expression derived by Sauter.

The equations (9) lead to a physically obvious inference: the cross section will depend most sensitively on the final spin ζ' when $|\xi|=1$, that is, if the photon inducing photoionization is circularly polarized; this result was first obtained by McVoy.[21] If the causative photon is linearly polarized ($\xi=0$), there will be no ζ'-dependence of the photoelectric effect cross section. From here on, we assume the photon to have right-hand circular polarization ($\xi=1$).

The total photoionization cross section, when the ejected photoelectron is polarized, may be calculated by integrating (8) or (9) over photoelectron angles; it depends on whether the spin vector $\mathbf{l}$ is or is not coupled to the electron velocity vector $\mathbf{v}$.

Consider an electron with given helicity, i.e., with $\mathbf{l}=\mathbf{v}/v$. Integrating (9) over all angles, we then obtain

$$\sigma=f[3(\gamma-1)^4]^{-1}\{(\gamma^2-1)^{1/2}[3\gamma^2-2\gamma+4+3(2-\gamma)\varphi(\gamma)]$$
$$+\zeta'[3\gamma^3-2\gamma^2-3\gamma-4+3(1+2\gamma-\gamma^2)\varphi(\gamma)]\},$$
$$\varphi(\gamma)=\ln[\gamma+(\gamma^2-1)^{1/2}](\gamma^2-1)^{-1/2}. \qquad (11)$$

Summing (11) over ζ' produces the familiar Sauter cross section[1]

$$\sigma=[2f(\gamma^2-1)^{1/2}/3(\gamma-1)^4][3\gamma^2-2\gamma+4+3(2-\gamma)\varphi(\gamma)], \qquad (12)$$

which can also be derived from (10) by integrating over all angles, and is thereby independent of the polarization of the incident photon. In the nonrelativistic approximation $v\ll 1$, we have from (11)

$$\sigma=\frac{2^7f}{3v^7}\left(1+\frac{v^2}{2}+\zeta'\frac{3}{20}v^3+\ldots\right), \qquad (13)$$

and we thus find that in the nonrelativistic limit, the total cross section is a weak function of the photoelectron spin, which then shows practically no preferential polarization. As we remarked earlier, the Sauter-Sommerfeld theory, and therefore Eq. (13), are only valid for $v\gg Z\alpha$. The conclusion remains unchanged,[28] however, if higher-order terms in $Z\alpha$ are included in the expansion.

For relativistic electrons ($\gamma\gg 1$), the equations (11) yield

$$\sigma=\frac{f}{\gamma}\left[1+\zeta'+\frac{11}{12\gamma^2}(1-\zeta')+\ldots\right], \qquad (14)$$

which implies that the cross section for electrons with spin $\zeta'=-1$ is negligible compared with that for electrons with $\zeta'=1$. Right-hand circularly polarized photons at relativistic energies therefore generate electrons with right-handed helicity (and practically none with left-handed helicity). The existence of such phenomena as "helicity transfer" is qualitatively fairly obvious in quantum electrodynamics,[34] but it is far from trivial to show that the polarization of relativistic photoelectrons predicted by the theory is complete. The complete transfer of helicity to a K-shell electron in the relativistic photoelectric effect was first established by Nagel.[28]

In considering outgoing electrons of a given helicity, however, it is impossible to classify transitions on the basis of the behavior of the electron spin, since it makes no sense to introduce helicity for electrons in the final state. We therefore now consider those electrons whose spin vector $\mathbf{l}$ is constant and independent of the velocity $\mathbf{v}$. Integrating (8) over all angles, we find that the cross section depends most sensitively on the spin quantum numbers ζ and ζ' when $|\mathbf{nl}|=1$, confirming the results obtained by Cherepkov[27]. This should be fairly obvious, since the incoming photon defines a physically preferred direction **n**. Putting **l** = **n**, we can then pursue our analysis in more depth and separately examine transitions with and without spin flip. With **l** = **n**, (8) gives the total cross section

$$\sigma=\frac{2f}{3(\gamma-1)^3(\gamma^2-1)^{1/2}}[(F_0+\zeta F_1)\delta_{\zeta,\zeta'}+2(\gamma-1)(1-\zeta)F_2\delta_{\zeta,-\zeta'}],$$

$$F_0=\gamma^3+3\gamma^2-2\gamma+8+3(\gamma^2-\gamma+2)\varphi(\gamma),\tag{15}$$

$$F_1=\gamma^3+3\gamma^2+8\gamma-6-3(\gamma^2-\gamma+2)\varphi(\gamma),\quad F_2=\gamma^2+2-3\gamma\varphi(\gamma).$$

If in (15) we average over the initial spin ζ, then in the nonrelativistic- and relativistic-electron limits we obtain

$$\sigma=\frac{2^7f}{3v^7}\left[1+(1+\zeta')\frac{v^2}{2}+\ldots\right],\quad Z\alpha<v\ll1,$$

$$\sigma=f\gamma^{-1}[1+\zeta'+(1-\zeta')\gamma^{-2}\ln 2\gamma+\ldots],\quad \gamma\gg1.\tag{16}$$

This means that in the relativistic limit, the photoelectrons produced are practically all oriented with their spin in the direction of the incident photon spin, with almost none in the opposite direction. A comparison of (14) and (16) shows that the production cross section for electrons with spin oriented parallel to that of the photon is the same (for $\gamma\gg1$) as that for right-handed helicity. This is physically understandable, since in the relativistic limit the differential cross section has a pronounced maximum as a function of **v** when **v**‖**n**. The complete polarization of the photoelectron spin in the direction of the incident photon spin for *K*-shell electrons in the relativistic limit was first predicted by McVoy.[21]

If we sum over final spins in (15), with $\gamma\gg1$, we obtain

$$\sigma=(2f/3\gamma)(3-\zeta).\tag{17}$$

This means that the photoionization cross section of an atom with initial electron spin $\zeta=-1$ is twice that for $\zeta=+1$. The net result is that there will be an excess among those neutral atoms that remain of atoms whose electron spin is oriented parallel to the spin of the incident photon. A similar result obtains for the spins of newly created positive ions: for $\gamma\gg1$, there will be twice as many ions with spin antiparallel to that of the incident photon as parallel (if the neutral atom started off with zero total spin). A similar effect at nonrelativistic energies has been considered by Ovsyannikov.[35]

The conclusion we draw from (15) is that the only possible spin-flip transitions are those that go from the state $\zeta=-1$ to $\zeta'=1$. Spin flip is strictly forbidden in the state $\zeta=1$. In the limit $\gamma\gg1$, (15) yields

$$\sigma=(2f/3\gamma)\{[1+\zeta+(1-\zeta)\gamma^{-2}\ln 2\gamma]\delta_{\zeta,\zeta'}+2(1-\zeta)\delta_{\zeta,-\zeta'}\}.\tag{18}$$

This means that a spin-flip transition from $\zeta=-1$ to $\zeta'=1$ is the most likely, being twice as probable as a non-flip transition from $\zeta=1$ to $\zeta'=1$. The non-flip photoionization cross section from the initial state $\zeta=-1$ is markedly reduced. Spin-flip transitions thus play a major role in the present case.

2. ORBITAL MOMENTUM $l=1$

We now examine the photoionization of an atom with an electron in a $j=1/2$, $l=1$ state. As in the preceding example, the principal quantum level from which photoionization takes place is considered to be arbitrary. To first order in $Z\alpha$ the wave function for the initial state of a hydrogen-like atom takes the form

$$\Psi_1(\mathbf{r})=\tilde N_1\left[\frac{i}{r}\gamma_0\boldsymbol{\gamma}\mathbf{r}\tilde g(x)-\frac{Z\alpha}{2n}\tilde f(x)\right]e^{-x/2}\tilde u_0,\tag{19}$$

$$\tilde g(x)=\frac{x}{3}\Phi(2-n,4;x),\quad \tilde N_1=\left[\frac{Z^3\alpha^3m^3(n^2-1)}{4\pi n^3}\right]^{1/2},$$

$$\tilde u_0=(2m)^{1/2}\begin{pmatrix}0\\1\end{pmatrix}w_0,$$

$$\tilde f(x)=\Phi(2-n,3;x)+\Phi(1-n,3;x),\quad x=2Z\alpha mrn^{-1}.$$

The transition matrix element is

$$M=\frac{\pi^{1/2}(Z\alpha)^{7/2}}{6m^{3/2}\gamma^3(\gamma-1)^3\mu^3}\left(\frac{n^2-1}{n^5}\right)^{1/2}$$

$$\times\bar u(\mathbf{p})[a\boldsymbol{\gamma}\mathbf{e}+(\boldsymbol{\gamma}\mathbf{e})\gamma_0(\boldsymbol{\gamma}\mathbf{b})+(\boldsymbol{\gamma}\mathbf{c})\gamma_0(\boldsymbol{\gamma}\mathbf{e})+\boldsymbol{\gamma}\mathbf{d}]\tilde u_0,$$

$$a=2\gamma(\gamma-1)\mu(2-3\gamma^2\mu),\tag{20}$$

$$\mathbf{b}=4\gamma^3(\gamma-1)\mu^2\mathbf{n}+4(1-\mu\gamma^2)\tilde{\mathbf{q}},\quad \mathbf{c}=3\gamma^2(\gamma-1)\mu^2\tilde{\mathbf{q}},$$

$$\mathbf{d}=2\mu\gamma\tilde{\mathbf{q}}[(\boldsymbol{\Sigma}\mathbf{e})(\boldsymbol{\Sigma}\tilde{\mathbf{q}})+i(\gamma-1)\mu\gamma\boldsymbol{\Sigma}[\mathbf{ne}]]$$

$$-2\mu^2\gamma^2(\gamma-1)(\boldsymbol{\Sigma}\mathbf{e})\boldsymbol{\Sigma},\quad \tilde{\mathbf{q}}=(\gamma-1)\mathbf{n}-\gamma\mathbf{v},$$

where Σ represents the Dirac matrices, and we have taken the conservation laws into account. It follows from (20) that at large *n*, as in the case $l=0$, the photoionization probability goes approximately as n^{-3}, in agreement with Eq. (70.8) of Ref. 5, which was obtained in the plane-wave approximation. We also note that in the ultrarelativistic limit, the cross section will have an even sharper maximum when **n**‖**v** ($\sim\mu^{-6}$) than was the case in (8), where the cross section was of order μ^{-4}.

We take the spin vector **l** parallel to **n**, and thereby obtain for the photoionization cross section

$$d\sigma=\tilde f[144\gamma^6(\gamma-1)^6\mu^6(\gamma^2-1)^{1/2}]^{-1}|w^+\tilde R w_0|^2d\Omega,$$

$$\tilde f=Z^7\alpha^6(n^2-1)n^{-5}r_e^2,\quad \tilde R=R_0+\boldsymbol{\sigma}\mathbf{R},$$

$$R_0=i\gamma(\gamma-1)[\gamma^2(\gamma-1)\mu^2+4\gamma^2\mu-4]\mathbf{v}[\mathbf{en}],$$

$$\mathbf{R}=\gamma(\gamma-1)[\gamma^2(\gamma-1)\mu^2-4\gamma\mu-4](\mathbf{ev})\mathbf{n}\tag{21}$$

$$-4\gamma^2[\gamma(\gamma-1)\mu-2](\mathbf{ev})\mathbf{v}$$

$$+(\gamma-1)[\gamma^3(\gamma-1)\mu^3-\gamma^2(2\gamma^2+3\gamma+7)\mu^2+8\gamma^2\mu-4]\mathbf{e}.$$

Making use of (4), it is then straightforward to derive the final form of the cross section for specified initial and final spin orientations. The expression, however, is so cumbersome that it makes little sense to present it here.

As in the previous section, spin effects show up most strongly when an atom is ionized by a circularly polarized photon. Assuming right-circular polarization, the expression for the total cross section becomes

$$\sigma=\pi\tilde f[270(\gamma-1)^5(\gamma^2-1)^{1/2}]^{-1}[(\tilde F_0+\zeta\tilde F_1)\delta_{\zeta,\zeta'}$$

$$+(\tilde F_2+\zeta\tilde F_3)\delta_{\zeta,-\zeta'}],$$

$$\tilde F_0=16\gamma^5+16\gamma^4-5\gamma^3-15\gamma^2-311\gamma+59$$

$$+15(\gamma^4+\gamma^3+13\gamma^2-5\gamma+6)\varphi(\gamma),$$

$$\tilde F_1=16\gamma^5+16\gamma^4-67\gamma^3+63\gamma^2-129\gamma+101$$

$$-15(\gamma-1)(\gamma-2)(\gamma^2+4\gamma-1)\varphi(\gamma),\tag{22}$$

$$F_2 = 14\gamma^5 - 16\gamma^4 + 125\gamma^3 + 195\gamma^2 + 161\gamma - 239$$
$$- 15(2\gamma^4 + 7\gamma^3 + 13\gamma^2 - 11\gamma + 5)\varphi(\gamma),$$
$$F_3 = -14\gamma^5 + 16\gamma^4 - 77\gamma^3 - 147\gamma^2 + 271\gamma - 49$$
$$+ 15(\gamma - 1)(2\gamma^3 + 9\gamma^2 + 6\gamma + 11)\varphi(\gamma).$$

If we average (22) over the initial spin, we obtain

$$\sigma = \pi f[108(\gamma-1)^4(\gamma^2-1)^{1/2}]^{-1}\{3(\gamma+1)[2(\gamma^3+5\gamma+6) - (\gamma^2+6\gamma+1)\varphi(\gamma)] + \zeta'[6\gamma^4+6\gamma^3+8\gamma^2+50\gamma - 30 - 3(3\gamma^3+11\gamma^2-3\gamma+13)\varphi(\gamma)]\}. \quad (23)$$

Finally, in the ultrarelativistic limit,

$$\sigma = \pi f[1 + \zeta' + (1-\zeta')\gamma^{-2}\ln 2\gamma](18\gamma)^{-1}. \quad (24)$$

As was the case in states with $l = 0$, therefore, the photoelectrons that are produced are completely polarized in the direction of the incident photon spin. The difference now, however, is that in the nonrelativistic limit $v \ll 1$,

$$\sigma = (2^7\pi f/27v^9)(3-\zeta'), \quad (25)$$

and the spin exerts a considerable influence. According to Eqs. (13) and (16), this was not the case in states with $l = 0$, where the probability of photoelectron production with spin opposite that of the incident photon was twice the probability for parallel spin. The first discussion of the strong electron-spin dependence of the $l = 1$ cross section in the nonrelativistic limit can be found in Refs. 26 and 27; the explicit equation (25), however, is not to be found there. For some value of γ, therefore (calculations indicate $\gamma = 1.483$), the cross section (23) will cease to depend on the spin ζ of the ejected electron. References 26 and 27 also contain the first dicussion of the fact that the electron polarization in the $l = 1$ state changes sign at a certain energy.

If we sum over the final spin in (22), we get the photoionization cross section as a function of the initial electron spin:

$$\sigma = \pi f(\gamma^2-1)^{1/2}[270(\gamma-1)^5]^{-1}\{15[2(\gamma^3+5\gamma+6) - (\gamma^2+6\gamma+1)\varphi(\gamma)] + \zeta[2(\gamma^3+16\gamma^2-71\gamma-26) + 15(\gamma+3)^2\varphi(\gamma)]\}. \quad (26)$$

The limiting cases for Eq. (26) are

$$\sigma = (2^7\pi f/27v^9)(3+\zeta), \quad v \ll 1,$$
$$\sigma = (\pi f/135\gamma)(15+\zeta), \quad \gamma \gg 1. \quad (27)$$

It is evidently easier to ionize an atom whose initial electron spin is parallel to the indicent photon spin, and accordingly, this results in the preponderance of one spin over the other in the positive ions that are produced. For $\gamma \gg 1$, (22) finally gives

$$\sigma = \frac{\pi f}{135\gamma}\left[\left(16\frac{1+\zeta}{2} + 15\frac{1-\zeta}{2}\frac{\ln 2\gamma}{\gamma^2}\right)\delta_{\zeta,\zeta'} + \left(14\frac{1-\zeta}{2} + 24\frac{1+\zeta}{2}\frac{1}{\gamma^2}\right)\delta_{\zeta,-\zeta'}\right]. \quad (28)$$

We see from (22) that there are no forbidden spin transitions, but when $\gamma \gg 1$, transitions to states with $\zeta' = -1$ are strongly inhibited. It is also significant here that non-flip transitions are more likely than spin-flip transitions, in contrast to the situation for states with $l = 0$ [see Eq. (18)].

Our analysis thus reveals a multifaceted and physically nontrivial picture of spin transitions in the photoelectric effect.

[1] F. Sauter, Ann. Phys. **403**, 454 (1931).
[2] A. Sommerfeld, *Atomic Structure and Spectral Lines*, Methuen, London (1934) [Russ. transl. Fizmatgiz, Moscow (1954)].
[3] A. I. Akhiezer and V. B. Berestetskii, *Kvantovaya elektrodinamika*, Nauka, Moscow (1969) [*Quantum Electrodynamics*, Interscience, New York (1965)].
[4] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Kvantovaya elektrodinamika*, Nauka, Moscow (1980).
[5] H. Bethe and E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms*, Pergamon (1957) [Russ. transl. Fizmatgiz, Moscow (1960)].
[6] U. Fano and K. W. McVoy, Phys. Rev. **116**, 1147 (1959).
[7] M. Gavrila, Phys. Rev. **113**, 514 (1960).
[8] M. Gavrila, Nuovo Cim. **15**, 691 (1960).
[9] M. Gavrila, Zh. Eksp. Teor. Fiz. **38**, 309 (1960).
[10] R. H. Pratt, Phys. Rev. **117**, 1017 (1960).
[11] R. H. Pratt, Phys. Rev. **119**, 1619 (1960).
[12] V. G. Gorshkov and A. I. Mikhailov, Zh. Eksp. Teor. Fiz. **43**, 991 (1962) [Sov. Phys. JETP **16**, 701 (1963)].
[13] V. G. Gorshkov and A. I. Mikhailov, Zh. Eksp. Teor. Fiz. **44**, 2142 (1963) [Sov. Phys. JETP **17**, 1439 (1963)].
[14] V. G. Gorshkov, A. I. Mikhailov, and V. S. Polikanov, Zh. Eksp. Teor. Fiz. **52**, 1570 (1967) [Sov. Phys. JETP **25**, 1045 (1967)].
[15] A. I. Mikhailov, Zh. Eksp. Teor. Fiz. **55**, 625 (1968).
[16] V. G. Gorshkov, A. I. Mikhailov, and V. S. Polikanov, Nucl. Phys. **55**, 273 (1964).
[17] V. G. Gorshkov and A. I. Mikhailov, Izv. Akad. Nauk SSSR, ser. fiz. **28**, 1169 (1964).
[18] T. A. Weber and C. J. Mullin, Phys. Rev. **12**, 615 (1962).
[19] A. A. Rimskii-Korsakov and V. V. Smirnov, Izv. Akad. Nauk SSSR, ser. fiz. **26**, 1169 (1962).
[20] A. A. Rimskii-Korsakov and V. V. Smirnov, Zh. Eksp. Teor. Fiz. **42**, 67 (1962) [Sov. Phys. JETP **15**, 47 (1962)].
[21] K. W. McVoy, Phys. Rev. **108**, 365 (1957).
[22] U. Fano, Phys. Rev. **116**, 1156 (1959).
[23] U. Fano, K. W. McVoy, and J. R. Albers, Phys. Rev. **116**, 1159 (1959).
[24] K. W. McVoy and U. Fano, Phys. Rev. **116**, 1168 (1959).
[25] U. Fano, Phys. Rev. **178**, 131 (1969).
[26] N. A. Cherepkov, Zh. Eksp. Teor. Fiz. **65**, 933 (1973) [Sov. Phys. JETP **38**, 463 (1974)].
[27] N. A. Cherepkov, Phys. Lett. **A40**, 119 (1972).
[28] B. Nagel, Ark. Fys. **18**, 1 (1960).
[29] B. Nagel and P. Olsson, Ark. Fys. **18**, 29 (1960).
[30] S. Hultberg, B. Nagel, and P. Ossson, Ark. Fys. **20**, 555 (1962).
[31] B. Nagel, Ark. Fys. **24**, 151 (1963).
[32] N. B. Delone and M. V. Fedorov, Usp. Fiz. Nauk **127**, 651 (1979) [Sov. Phys. Uspekhi **22**, 252 (1979)].
[33] B. G. Bagrov, G. F. Kopytov, V. B. Tlyachev, and N. I. Fedosov, VINITI Doc. No. 8440-B86 (1986).
[34] V. N. Baier, V. S. Fadin, V. A. Khoze, and E. A. Kuraev, Phys. Rep. **78**, 293 (1981).
[35] V. D. Ovsyannikov, Zh. Eksp. Teor. Fiz. **88**, 1168 (1985) [Sov. Phys. JETP **61**, 687 (1985)].

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