Effect of multiple scattering on parametric x radiation

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The influence of multiple scattering of charged particles in a crystal on the characteristics of the generated x rays is considered. The spectral-angular and angular distributions of the x rays and of the bremsstrahlung diffracted by atomic planes and emitted in the diffraction peak are obtained. These distributions are given for Laue and Bragg diffraction geometries and for the Laue-Bragg transitions. It is shown that the theory is in satisfactory agreement with the known experimental results.

1. INTRODUCTION

Parametric x radiation (PXR) constitutes Čerenkov x-rays excited in a crystal by a uniformly moving charged particle and emitted in a region of frequencies ω and angles ϑ for which, in view of the photon diffraction by the crystal lattice, the real part n′ of the x-ray refractive index becomes larger than unity.[1,2] It follows from the Čerenkov condition

$$1-vn'(\omega,\ \vartheta)\cos\vartheta=0 \tag{1}$$

(v is the particle velocity, $\hbar = c = 1$) that for Čerenkov photons the coherent emission length $l=\omega^{-1}(1-vn'\cos\vartheta)^{-1}$ is infinite. PXR is therefore formed over the entire thickness L of the crystal. The foregoing is valid only of the photon absorption in the medium is neglected. In a crystal of thickness L much larger than the photon absorption depth $L_a = (2\omega n'')^{-1}$, the radiation is formed on a particle path of the order of L_a ($n = n' + in''$ is the complex refractive index). Consequently, as the crystal thickness increases, the PXR intensity eventually no longer increases with L and turns out to be directly proportional to L_a (Ref. 3) (an exception to this situation is the extremely asymmetric diffraction case considered in Ref. 2).

Another factor that likewise limits the longitudinal dimensions of the region in which Čerenkov radiation is formed is multiple scattering (MS) of a charged particle by the atoms of the material. Just as in the case of radiation generation in an amorphous medium (see, e.g., Refs. 1 and 4), the PXR characteristics depend substantially on the relation between the crystal thickness L and the coherent length of the bremsstrahlung (BS) $L^{\rm BS} = (\omega q)^{-1/2}$, where $q=\overline{\theta_s^2}/4$, $\overline{\theta_s^2}$ is the mean square angle of MS of the charged particle per unit path in the substance. If $L \ll L^{\rm BS}$ the influence of the MS on the PXR reduces in fact to the appearance of an increment to the generated x radiation, due to the bremsstrahlung mechanism of the emission. In the opposite case, when $L > L^{\rm BS}$, the MS changes in addition the parameters of the PXR itself.

The influence of the MS on the PXR and the contribution of the bremsstrahlung to the intensity of the diffraction maximum of the x-ray emission were phenomenologically taken into account in Refs. 5–8. A quantitative approach to the analysis of the influence of MS of a charged particle on the PXR characteristics was indicated in a brief communication.[9] The present paper is devoted to a detailed quantitative treatment of the spectral-angular, angular, integral, and polarization characteristics of x radiation generated in a crystal, with account taken of MS. We obtain for the radiation a spectral-angular distribution that is valid in practice for arbitrary relations between the lengths L, $L^{\rm BS}$, and L_a. For the $L \ll L^{\rm BS}$ case we obtain simple equations that refine the phenomenological relations of Refs. 5–8. We conclude with an analysis, with the aid of the obtained integral characteristics, of the experimental data[10–12] on x radiation emitted by passage of electrons of energy ~ 1 GeV through diamond and silicon single crystals. We show that the theory explains satisfactorily the experimental data of Refs. 10–12.

2. X RAYS GENERATED BY PASSAGE THROUGH A PLANE-PARALLEL PLATE

1. We obtain the spectral-angular and polarization distributions of x rays generated by passage of ultrarelativistic charged particles through a plane-parallel single-crystal plate, with allowance for MS of the particles by the crystal-lattice atoms. It is known[1] that PXR is emitted in the direction of the velocity vector ($\mathbf{v}_0$) of the particles incident on the crystal (diffraction maximum of forward radiation), and also along directions determined by the vectors $\mathbf{k}_B = \omega_B \mathbf{v}_0 + \boldsymbol{\tau}$ (diffraction maxima of lateral radiation; see also Ref. 6). Here $\boldsymbol{\tau}$ are the reciprocal-lattice vectors that define families of crystallographic planes by which the emitted x rays are diffracted, and $\omega_{\rm B} = \tau^2/2|\mathbf{v}_0\cdot\boldsymbol{\tau}|$ are the Bragg frequencies. In the direction of the lateral diffraction maximum specified by the unit vector $\mathbf{k}_{\rm B}/\omega_{\rm B} = \mathbf{v}_0 + \boldsymbol{\tau}/\omega_{\rm B}$ ($v_0 \approx 1$), radiation is emitted near different Bragg frequencies corresponding to different values of $|\boldsymbol{\tau}| = \tau = 2\pi I/d$ ($I = 1,2,3,\ldots, d$ is the distance between planes). In the equations that follow we leave out the symbol indicating summation over all possible Bragg frequencies emitted into a given reflection.

The probability density of emitting a photon with wave vector $\mathbf{k}$ and polarization s is[1–3]

$$N_{\mathbf{k}s}=\frac{e^2\omega}{(2\pi)^2}\left|\int\limits_{-\infty}^{+\infty} dt\ \mathbf{v}\mathbf{E}_{\mathbf{k}s}^{(-)}(\mathbf{r},\omega)\exp[-i\omega t]\right|^2, \tag{2}$$

where $e^2 = 1/137$, $\omega = |\mathbf{k}|$, $\mathbf{r}\equiv\mathbf{r}(t)$ is the radius vector of the particle, $\mathbf{v}=d\mathbf{r}/dt$, $\mathbf{E}_{\mathbf{k}s}^{(-)}(\mathbf{r},\omega)$ is the exact solution of the homogeneous Maxwell equations in the case of scattering, by a crystal plate, of a plane electromagnetic wave $\mathbf{e}_s \exp(i\mathbf{k}\cdot\mathbf{r})$ with polarization vector $\mathbf{e}_s$ and having the asymptotic form of an "incident plane wave plus a converging spherical wave." We set the time of entry of the particle into the crystal at $t = 0$; the instant of emergence of the particle

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from a crystal plate of thickness L is then $L_0 = L/\gamma_0$, where $\gamma_0 = \mathbf{v}_0 \cdot \mathbf{N} > 0$ and $\mathbf{N}$ is a unit vector along the normal to the crystal-plate surface. (The fluctuations, due to MS, of the time of fight of the particle through the plate can be neglected.) Expressing the integral with respect to time is the right-hand side of (2) in the form of three integrals from $-\infty$ to 0, from 0 to L_0, and from L_0 to ∞, we write down the spectral-angular distribution of the radiation in the form of a sum of six terms:

$$N_{\mathbf{k}s} = \sum_{\substack{i,j=1\\(i\leq j)}}^{3} N_{\mathbf{k}s}^{ij}, \qquad (3)$$

each of which corresponds to radiation emitted as the particle moves in the regions of space i and j; $i(j) = 1$—vacuum ahead of the plate ($t\in[-\infty,0]$), $i(j) = 2$—space occupied by the plate ($t\in[0,L_0]$), $i(j) = 3$—vacuum past the plate ($t\in[L_0,\infty]$). Thus for example, the term $N_{\mathbf{k}s}^{12}$ describes the radiation produced as a result of interference of the electromagnetic fields generated by motion of the charged particle in the vacuum ahead of the plate ($i = 1$) and inside the plate ($j = 2$). Contributions to the diffraction maximum of the forward radiation are made in the general case by all the terms of the sum (3), but contributions to the lateral diffraction maximum is made only by the three terms $N_{\mathbf{k}s}^{11}$, $N_{\mathbf{k}s}^{12}$ and $N_{\mathbf{k}s}^{22}$, since the electric field generated when the particle moves past the crystal ($j = 3$) is not diffracted by the crystal and consequently does not participate in the formation of the diffraction peak.

The superior bar in the right-hand side of (2) corresponds to averaging of the squared modulus over all possible particle trajectories in the crystal. Representing the particle velocity vector in the form $\mathbf{v} = \mathbf{v}_0 \cos\theta + v_0\boldsymbol{\theta}$ ($\theta \equiv |\boldsymbol{\theta}|$ is the particle MS angle, $\theta \ll 1$, $\boldsymbol{\theta}$ is a two-dimensional vector, $\boldsymbol{\theta}\perp\mathbf{v}_0$ and $\boldsymbol{\theta} = 0$ at $t\leqslant 0$), we average the right-hand side of (2) with the aid of the distribution functions $W_1(\mathbf{r},\boldsymbol{\theta},t)$ and $W_2(\boldsymbol{\rho},\boldsymbol{\theta},\boldsymbol{\theta}',\tau)$, where $W_1(\mathbf{r},\boldsymbol{\theta},t)$ is the probability density of observing the particle in the vicinity of the "point" $(\mathbf{r},\boldsymbol{\theta})$ at the instant of time t, $W_2(\boldsymbol{\rho},\boldsymbol{\theta},\boldsymbol{\theta}',\boldsymbol{\tau})$ is the density of the conditional probability of observing the particle in the vicinity of the point $(\boldsymbol{\rho},\boldsymbol{\theta}')$ at the instant τ if it was located at $(0,\theta)$ at the instant $\tau = 0$, $\boldsymbol{\rho} = \mathbf{r}(t') - \mathbf{r}(t)$, $\tau = t' - t$. In the case of Laue-diffraction geometry ($\gamma_1 = \mathbf{k}_B\mathbf{N}/\omega_B > 0$) and of radiation in the direction of the lateral diffraction maximum, we obtain for the terms of the sum $N_{\mathbf{k}s} = N_{\mathbf{k}s}^{11} + N_{\mathbf{k}s}^{12} + N_{\mathbf{k}s}^{22}$ the expressions

$$N_{\mathbf{k}s}{}^{11} = \frac{e^2\omega}{4\pi^2}(\mathbf{e}_{\tau s}\mathbf{v}_0)^2 l_0{}^2 |A|^2, \qquad (4)$$

$$N_{\mathbf{k}s}{}^{12} = \frac{e^2\omega}{2\pi^2}\operatorname{Im}\Big\{(\mathbf{e}_{\tau s}\mathbf{v}_0) l_0 A\int_0^{L_0} dt \int d\boldsymbol{\theta}\, F_s(\boldsymbol{\theta}) \sum_{\mu=1}^{2} \xi_{\mu s}{}^{*2}$$
$$\times \exp(-i\omega n_{\mu s}{}^{*} L_0)\, U_1(\boldsymbol{\theta}, t, \mathbf{k}_{\tau\mu s}, \omega)\Big\}, \qquad (5)$$

$$N_{\mathbf{k}s}{}^{22} = \frac{e^2\omega}{2\pi^2}\operatorname{Re}\int_0^{L_0} dt \int_0^{L_0-t} d\tau \iint d\boldsymbol{\theta}\, d\boldsymbol{\theta}'\, F_s(\boldsymbol{\theta}) F_s(\boldsymbol{\theta}')$$
$$\times\Big\{\sum_{\mu=1}^{2}|\xi_{\mu s}|^2 \tilde{U}_1\Big(\boldsymbol{\theta}, t, \frac{2i\operatorname{Im} n_{\mu s}}{\gamma_0}, \omega\Big) U_2(\boldsymbol{\theta}, \boldsymbol{\theta}', \tau, \mathbf{k}_{\tau\mu s}^{*}, \omega)$$
$$+\xi_{1s}\xi_{2s}{}^{*}\Big[U_2(\boldsymbol{\theta}, \boldsymbol{\theta}', \tau, \mathbf{k}_{\tau 2s}^{*}, \omega)\tilde{U}_1\Big(\boldsymbol{\theta}, t, \frac{n_{1s}-n_{2s}^{*}}{\gamma_0}, \omega\Big)$$
$$+U_2(\boldsymbol{\theta}, \boldsymbol{\theta}', \tau, -\mathbf{k}_{\tau 2s}^{*}, -\omega)$$
$$\times \tilde{U}_1\Big(\boldsymbol{\theta}, t, \frac{n_{1s}-n_{2s}^{*}}{\gamma_0}, -\omega\Big)\exp(i\omega L_0(n_{1s}-n_{2s}^{*}))\Big]\Big\}, \qquad (6)$$

in which $\mathbf{e}_{\tau s}$ is the polarization vector of a photon having a momentum $\mathbf{k} - \boldsymbol{\tau}$ and propagating at a small angle to the vector $\mathbf{v}_0$ (see Ref. 1, pp. 81 and 82),

$$l_0 = \omega^{-1}(1 - v_0\cos\vartheta)^{-1} = 2/\omega(\gamma^{-2} + \vartheta^2) \qquad (7)$$

is the vacuum coherence length of the radiation, ϑ is the polar angle of the emitted photon, measured from the direction of the vector $\mathbf{k}_B$ ($\vartheta \ll 1$); $\gamma = (1 - v_0^2)^{-1/2}$ is the Lorentz factor of the particle,

$$A = \sum_{\mu=1}^{2} \xi_{\mu s}\exp(i\omega n_{\mu s} L_0), \quad \xi_{\mu s} = (-1)^{\mu} C_s \chi_{\tau}/2(n_{2s} - n_{1s}), \qquad (8)$$

$n_{\mu s} = 1 + \delta_{\mu s} + \alpha_1/2$ is the refractive index of the x rays in the crystal plate in the case of two-wave photon diffraction by the atomic lattice,

$$\delta_{1(2)s} = \frac{1}{4\gamma_1}\{-\alpha_1\gamma_1 + \chi_0(\gamma_1 + \gamma_0)$$
$$\pm[(\alpha_1\gamma_1 + \chi_0(\gamma_0 - \gamma_1))^2 + 4\gamma_0\gamma_1 r_s]^{1/2}\}, \qquad (9)$$

$r_s = \chi_{\tau}\chi_{-\tau} C_s^2$, $C_s = \mathbf{e}_s\mathbf{e}_{\tau s}$; χ_0, χ_{τ}, $\chi_{-\tau}$ are the complex polarizabilities of the crystal[13] (see also Refs. 1–3), $\alpha_1 = [(\mathbf{k} - \boldsymbol{\tau})^2 - \mathbf{k}^2]/\omega^2$ is a parameter determining the deviation from the exact Bragg condition: $\alpha_1 = 0$; $F_s(\boldsymbol{\theta}) = \mathbf{v}\mathbf{e}_{\tau s}$, $\mathbf{k}_{\tau\mu s}$ is the wave vector of a photon propagating inside the crystal plate ($\mathbf{k}_{\tau\mu s} = \mathbf{k} - \boldsymbol{\tau} + \mathbf{N}\omega\delta_{\mu s}\gamma_0{}^{-1}$),

$$U_1(\boldsymbol{\theta}, t, \mathbf{a}, \omega) = \int d\mathbf{r}\, W_1(\mathbf{r}, \boldsymbol{\theta}, t)\exp(-i\omega t + i\mathbf{a}\mathbf{r}),$$
$$\tilde{U}_1(\boldsymbol{\theta}, t, \mathbf{a}, \omega) = \int d\mathbf{r}\, W_1(\mathbf{r}, \boldsymbol{\theta}, t)\exp(i\omega a(L - z)),$$
$$U_2(\boldsymbol{\theta}, \boldsymbol{\theta}', \tau, \mathbf{a}, \omega) = \int d\boldsymbol{\rho}\, W_2(\boldsymbol{\rho}, \boldsymbol{\theta}, \boldsymbol{\theta}', \tau)\exp(-i\omega\tau + i\mathbf{a}\boldsymbol{\rho}).$$

2. The terms $N_{\mathbf{k}s}^{11}$ and $N_{\mathbf{k}s}^{12}$ due to the electromagnetic field generated by the motion of the particle in vacuum to the crystal will be analyzed below. We shall dwell here on a detailed examination of the distribution $N_{\mathbf{k}s}^{22}$ that describes the characteristics of the radiation generated by the particle motion inside the crystal, including PXR and BS, the photons of which are diffracted by atomic planes and are emitted along the vector $\mathbf{k}_B$ into a lateral diffraction peak.

If the vector $\mathbf{v}_0$ is not directed along any of the principal crystallographic axes (planes), the distribution functions $W_1(\mathbf{r},\boldsymbol{\theta},t)$ and $W_2(\boldsymbol{\rho},\boldsymbol{\theta},\boldsymbol{\theta}',\tau)$ are obviously the usual particle distribution functions in the coordinates $\mathbf{r}$ and the angles $\boldsymbol{\theta}$ used tp calculate the BS intensity of ultrarelativistic electrons in amorphous media (see, e.g., Refs. 1 and 4).

From the usual distribution functions we obtain after integrating in (6) over the particle scattering angles $\boldsymbol{\theta}$ and $\boldsymbol{\theta}'$,

$$N_{\mathbf{k}s}{}^{22} = \frac{e^2\omega}{2\pi^2} R_L^s \operatorname{Re}\int_0^{L_0} dt \int_0^{L_0-t} d\tau[(\mathbf{e}_{\tau s}\mathbf{v}_0)^2 Q_1(1+\Delta) + 2qtQ_2]$$
$$\times\Big(\sum_{\mu=1}^{2} F_{\mu} - \psi\Big), \qquad (10)$$

where

$$R_L^s=C_s^2\left|\frac{\chi_\tau}{2(n_{2s}-n_{1s})}\right|^2 \qquad (11)$$

is a factor that determines the effectiveness of the x-ray photon diffraction,

$$Q_1=\operatorname{ch}^{-1}\eta\tau(1+\eta t\operatorname{th}\eta\tau)^{-1}\exp\left[\frac{i\omega\tau\vartheta^2}{2}-\frac{\eta\vartheta^2\operatorname{th}\eta\tau}{4q(1+\eta t\operatorname{th}\eta\tau)}\right]$$

$$Q=\frac{Q_1}{\operatorname{ch}\eta\tau(1+\eta t\operatorname{th}\eta\tau)}\qquad \Delta=\frac{1-\operatorname{ch}\eta\tau(1+\eta t\operatorname{th}\eta\tau)^2}{\operatorname{ch}\eta\tau(1+\eta t\operatorname{th}\eta\tau)^2} \qquad (12)$$

$$F_\mu=\exp\left[-\frac{i\tau}{l_{\mu s}}-\frac{L_0-t-\tau/2}{L_a^{\mu s}}\right], \qquad (13)$$

$$\psi=\exp\left[-\frac{i\tau}{l_{2s}}+\frac{\tau}{2L_a^{2s}}-i\omega(n_{2s}^*-n_{1s})(L_0-t)\right]+\exp\left[-\frac{i\tau}{l_{2s}}-\frac{\tau}{2L_a^{2s}}+i\omega(n_{2s}-n_{1s}^*)t\right], \qquad (14)$$

$\eta=(2i\omega q)^{1/2}=(1+i)/L^{\rm BS}$, $L_a^{\mu s}=(2\omega\operatorname{Im}\delta_{\mu s})^{-1}$ is the depth of x-ray absorption in the μth dispersion mode,

$$l_{\mu s}=2\omega^{-1}[\gamma^{-2}+\vartheta^2+2(1-n_{\mu s}')]^{-1} \qquad (15)$$

is the coherent radiation length, and $n_{\mu s}=n'_{\mu s}+in''_{\mu s}$.

The functions Q_1, Q_2, and Δ, which depend on the coherent bremsstrahlung length $L^{\rm BS}$, describe the influence of the MS on the spectral and angular characteristics of the generated x rays. As $L^{\rm BS}\to\infty$ $(q\to 0)$ (radiation excited by high-energy electrons ($L^{\rm BS}\sim\gamma$) or by heavy charged particles), the functions Q_1 and Q_2 tend to unity, while Δ tends to zero. Thus, the first term in the square brackets in the right-hand side of (1), $(\mathbf{e}_{\tau s}\cdot\mathbf{v}_0)Q_1(1+\Delta)$, is due to radiation emitted by uniform and linear motion of the charged particle in the crystal. Since $(\mathbf{e}_{\tau s}\mathbf{v}_0)^2\sim\vartheta^2$, the x rays emitted into the diffraction peak and excited by a linearly and uniformly moving charged particle are described by an angular distribution with a characteristic minimum along the vector $\mathbf{k}_{\rm B}$, just as ordinary transition x radiation is described by an angular distribution along the particle velocity vector $\mathbf{v}_0$. Corresponding to the bremsstrahlung in the square brackets of the right-hand side of (1) is the term $2qtQ_2$, which vanishes as q tends to zero (when the MS of the charged particle is "turned off"). In contrast to the radiation excited in uniform and linear motion of a particle, bremsstrahlung is described by an angular distribution with a maximum in the direction of the vector $\mathbf{k}_{\rm B}$ (polar angle $\vartheta=0$).

3. PARAMETRIC X RADIATION

In our present notation, the Čerenkov radiation mode corresponds to the index $\mu=1$: it is precisely for the refractive index n_{1s} that the condition (1) can be met, or, equivalently, $l_{1s}=\infty$. Therefore in the integrand of expression (10) the term corresponding to PXR is $(\mathbf{e}_{\tau s}\mathbf{v}_0)^2F_1Q_1(1+\Delta)$, which contains the exponential F_1 with a phase equal to $-i\tau/l_{1s}$. If the crystal thickness along the charged-particle incidence direction is much smaller than the coherence length of the bremsstrahlung ($L_0\ll L^{\rm BS}$), which is the case of "weak" MS, the functions Q_1 and Δ can be expanded in powers of the small parameters $\tau/L^{\rm BS}$ and $t/L^{\rm BS}$. Assuming, furthermore, that $L_0\ll L_a^{1s}$ ("weak" absorption), we obtain for the PXR spectral-angular distribution the expression

$$N_{ks}^{\rm PXR}=\frac{e^2\omega}{2\pi}(\mathbf{e}_{\tau s}\mathbf{v}_0)^2R_L^sL_0\delta(l_{1s}^{-1}), \qquad (16)$$

in which

$$l_{1s}=2\omega^{-1}[\gamma^{-2}+\vartheta^2+2(1-n_{1s}')+{}^{24}/_5qL_0]^{-1} \qquad (17)$$

is the coherent emission length in the case of weak MS. Thus, if the condition $L_0\ll L^{\rm BS}$ is met, MS alters insignificantly the phase of the emitted radiation and leads, as a consequence, to the appearance of an additional term $(24/5)qL_0$ in the expression for the coherent length (17). It is impossible to obtain a compact expression for the spectral-angular distribution of the radiation when the condition $L_0\ll L^{\rm BS}$ is met for an arbitrary ratio of the lengths L_0 and L_a^{1s}. However, knowing the dependences of the angular distribution of the radiation on the lengths L_1 and L_a^{1s} with MS neglected ($q=0$), and finding the correction for MS to the phase in (13) when the double inequality $L_a^{1s}\ll L_0\lesssim L^{\rm BS}$ is satisfied, we can represent the PXR angular distribution in the case of weak MS in the form

$$\frac{dN_s^{\rm PXR}}{d\boldsymbol{\vartheta}}=\frac{e^2\omega_{\rm B}L_{\rm eff}C_s^2|\chi_\tau|^2(\mathbf{e}_{\tau s}\mathbf{v}_0)^2}{4\pi\sin^2\theta_{\rm B}[(\vartheta^2+\vartheta_{\rm ph}^2)^2+\beta r_s']}, \qquad (18)$$

where

$$C_s^2=\begin{cases}1, & s=\sigma,\\ \cos^2 2\theta_{\rm B}, & s=\pi\end{cases},\quad (\mathbf{e}_{\tau s}\mathbf{v}_0)^2=\begin{cases}\vartheta_y^2, & s=\sigma\\ \vartheta_x^2, & s=\pi.\end{cases}$$

We have introduced here the polarization vectors $\mathbf{e}_\sigma\|[\mathbf{k}\boldsymbol{\tau}]$, $\mathbf{e}_\pi\|[\mathbf{k}\mathbf{e}_\sigma]$ and $\mathbf{e}_{\tau\sigma}=\mathbf{e}_\sigma$, $\mathbf{e}_{\tau\pi}\|[(\mathbf{k}-\boldsymbol{\tau})\mathbf{e}_\sigma]$, and also the two-dimensional photon-emission vector $\boldsymbol{\vartheta}=\vartheta_x\mathbf{n}_x+\vartheta_yn_y$ with $\boldsymbol{\vartheta}\perp\mathbf{k}_{\rm B}$, $|\boldsymbol{\vartheta}|=\vartheta$, $\mathbf{n}_y\|[\mathbf{v}_0\boldsymbol{\tau}]$, $\mathbf{n}_x\|[\mathbf{n}_y\mathbf{k}_{\rm B}]$ (see Refs. 6 and 7); here $\theta_{\rm B}$ is the Bragg angle ($\sin\theta_{\rm B}=|\mathbf{v}_0\boldsymbol{\tau}/\tau|$),

$$\vartheta_{\rm ph}=(\gamma^{-2}-\chi_0'+\eta L_0\theta_s^2/2)^{1/2} \qquad (19)$$

is the effective emission angle, $\chi_0=\chi_0'+i\chi_0''$,

$$\eta=\begin{cases}2,4, & L_0\ll L_a^{1s}\\ 6, & L_0\gg L_a^{1s}\end{cases},$$

$$L_{\rm eff}=L_a^{1s}(1-\exp[-L_0/L_a^{1s}]), \qquad (20)$$

$$L_a^{1s}=\frac{(\vartheta^2+\vartheta_{\rm ph}^2)^2+\beta r_s'}{\beta\chi_0''\omega_{\rm B}[(\vartheta^2+\vartheta_{\rm ph}^2+\delta_s)^2+r_s'-\delta_s^2]}, \qquad (21)$$

$r_s=r_s'+ir_s''$, $\delta_s=r_s''/2\chi_0''$, $\beta=\gamma_0/\gamma_1$, with the quantities χ_0, χ_τ, and r_s calculated for the a frequency equal to $\omega_{\rm B}$ ($\chi_0'(\omega_{\rm B}=-\omega_L^2/\omega_{\rm B}^2)$. The quantity η in the expression for the effective emission angle $\vartheta_{\rm ph}$ is a function of L_0 and varies in the range from 2.4 to 6. It is just this dependence that distinguishes the distribution (18) from the analogous distribution, with $\eta=2$, obtained in Ref. 6 on the basis of a phenomenological allowance for the MS. (Reference 6, which gives $\eta=2$, makes use of a kinematic approximation which would call for putting $r_s=0$ in Eq. (18)). The values of η in (18) were obtained for two limiting cases $L_0\ll L_a^{1s}$ and $L_0\gg L_a^{1s}$, so that at crystal thicknesses $L_0\sim L_a^{1s}$ the distribution (18) can be used only for semiquantitative estimates of the PXR properties. Integrating (18) over the emission angles $\boldsymbol{\vartheta}$, we obtain an expression for the total number of photons having a polarization s and emitted into a lateral diffraction peak in the angle region $\vartheta\lesssim\vartheta_d$:

$$N_s^{\rm PXR}=\frac{e^2\omega_{\rm B}L_{\rm eff}C_s^2|\chi_\tau|^2}{8\sin^2\theta_{\rm B}}\left\{\frac{1}{2}\ln\left[\frac{(\vartheta_0^2+\vartheta_d^2)^2+a_s}{\vartheta_0^4+a_s}\right]\right.$$

$$-\frac{\vartheta_0^2}{a_s^{1/2}}\left[\operatorname{arctg}\frac{\vartheta_d^2+\vartheta_0^2}{a_s^{1/2}}-\operatorname{arctg}\frac{\vartheta_0^2}{a_s^{1/2}}\right]\Bigg\},$$

$$\vartheta_0^2=\begin{cases}\vartheta_{\rm ph}^2, & L_0\ll L_a^{1s}\\ \vartheta_{\rm ph}^2+\delta_s, & L_0\gg L_a^{1s}\end{cases},\quad a_s=\begin{cases}\beta r_s', & L_0\ll L_a^{1s}\\ r_s'-\delta_s^2, & L_0\gg L_a^{1s}\end{cases}\tag{22}$$

where ϑ_d is the angle subtended by the radiation detector, and the photon absorption length, which enters in the expression for $L_{\rm eff}$, is equal to $L_a=(\beta\omega_{\rm B}\chi_0^{11})^{-1}$.

When the inequality $L_0>L^{\rm BS}$ is valid (case of "strong" MS), the integrands Q_1 and Q_2 in the right-hand side of (10) cut off the values of the integral with respect to the variable τ at times $\tau\approx L^{\rm BS}$. Thus, in a thick crystal ($L_0>L^{\rm BS}$) the longitudinal region of formation of both BS and PXR turns out to be equal to the coherence length $L^{\rm BS}=(\omega q)^{-1/2}$.

4. X-RAY BREMSSTRAHLUNG IN A CRYSTAL

1. It was noted above that the term $2qtQ_2$ in the integrand of the right-hand side of (10) corresponds to BS, and that the phase of the exponential factor F_1 contains the coherence length l_{1s} of the Čerenkov radiation. We see that, owing to diffraction, a situation is possible in which BS photons produced in a crystal propagate with a phase velocity lower than that of the generating charged particle. Bremsstrahlung by a charged particle faster than the radiation it generates ($v_0>v_{\rm ph}$) will henceforth be called superluminal bremsstrahlung (SBS). SBS was first taken into account phenomenologically in Ref. 5, and was later considered, likewise on the basis of a phenomenologial description, in Refs. 7 and 8. It is possible to analyze the SBS quantitatively by starting from the spectral-angular distribution (10), in the integrand of which the SBS corresponds to the term $2qtQ_2F_1$. In particular, using assumptions similar to those made in the derivation of the distribution (18), we obtain the angular distribution of SBS in the case of weak MS ($L_0\ll L_{\rm BS}$):

$$\frac{dN_s^{\rm SBS}}{d\vartheta}=\frac{e^2\omega_{\rm B}\overline{\theta}_s^2L_0L_{\rm eff}C_s^2|\chi_\tau|^2}{16\pi\sin^2\theta_{\rm B}[(\vartheta^2+\vartheta_{\rm ph}^2)^2+\beta r_s']}\tag{23}$$

in which

$$L_{\rm eff}=2L_a^{1s}\left\{1-\frac{L_a^{1s}}{L_0}\left[1-\exp\left(-\frac{L_0}{L_a^{1s}}\right)\right]\right\},\tag{24}$$

and the quantities $\vartheta_{\rm ph}$ and L_a^{1s} are determined as before by Eqs. (19) and (21), but the function η in (19) has now the following asymptotic values:

$$\eta=\begin{cases}2,5, & L_0\ll L_a^{1s}\\ 4, & L_0\gg L_a^{1s}\end{cases}.$$

The reason for the last circumstance is that in the case $L_0\ll L^{\rm BS}$ the functions $Q_1(1+\Delta)$ and Q_2 in (10) lead to different corrections for the phase in $F_1\propto\exp(-i\tau/l_{1s}$. We call attention also to the different dependences of the effective lengths (20) and (24) on L_0 (cf. Ref. 8), which are due to the different dependences of the terms $(\mathbf{e}_{\tau s}\mathbf{v}_0)^2Q_1(1+\Delta)$ and $2qtQ_2$ in the right hand side of the distribution (10) on the variable t: as $q\to 0$ we have respectively $(\mathbf{e}_{\tau 0}\cdot\mathbf{v}_0)^2$ and $2qt$.

Integrating the distribution (23) over the angles ϑ, we obtain the total number of SBS photons emitted into the lateral diffraction peak:

$$N_s^{\rm SBS}=\frac{e^2\omega_{\rm B}L_0\overline{\theta}_s^2L_{\rm eff}C_s^2|\chi_\tau|^2}{16a_s^{1/2}\sin^2\theta_{\rm B}}\left(\operatorname{arctg}\frac{\vartheta_d^2+\vartheta_0^2}{a_s^{1/2}}-\operatorname{arctg}\frac{\vartheta_0^2}{a_s^{1/2}}\right),\tag{25}$$

where a_s and ϑ_0 are defined as in (22).

The photon absorption length, which enters in (25), is $L_a=(\beta\omega_{\rm B}\chi_0'')^{-1}$. Let us compare the characteristics of SBS and PXR. According to (18), at an angle $\vartheta=0$ (in the direction of the vector $\mathbf{k}_{\rm B}$), the angular distribution of the PXR has a minimum. In contrast to the PXR, the angular distribution of the SBS reaches a maximum at $\vartheta=0$ (similar positions are occupied by the minima and maxima of the distributions of the x-ray transition radiation and the bremsstrahlung), therefore the emission at small angles is determined in fact by the SBS. The polar angle ϑ for which $N_{\vartheta s}^{\rm PXR}\approx N_{\vartheta s}^{\rm SBS}$, is equal to $\vartheta_q\approx\frac{1}{2}(\overline{\theta}_s^2L_0)^{1/2}$. Thus, in a sufficiently thick crystal, such that $\vartheta_q\sim\vartheta_{\rm ph}$, the angular distribution of the summary radiation (PXR + SBS) contains no funnel-like ($\theta_{\rm B}\neq\pi/4$) or two–hump ($\theta_{\rm B}\approx\pi/4$) structure typical of the PXR angular distribution. Comparison of expressions (22) and (25) shows that the polarization characterizations of SBS and PXR are the same when the recorded radiation is emitted into a cone oriented along the vector $\mathbf{k}_{\rm B}$, are described by the factor C_s^2, and are determined by the polarization of the x rays when they are reflected from the atomic planes through by an angle $2\theta_{\rm B}$. If, however, the SBS and PXR are recorde behind a collimator in the form of a narrow slit oriented in the radiation-diffraction plane or in a direction perpendicular to it (along the vector $\mathbf{n}_x$ or $\mathbf{n}_y$), the polarizations of the SBS and PXR will be different. The degree of polarization of the SBS is

$$P^{\rm SBS}=(1-\cos^2 2\theta_{\rm B})(1+\cos^2 2\theta_{\rm B})^{-1},\ \text{a}\ P^{\rm PXR}=1.$$

2. Besides the SBS, there is generated in the crystal also BS which, like the SBS, is diffracted by the atomic lattice, but its refractive index inside the crystal is less than unity. This is BS in a non-Čerenkov radiation mode ($\mu=2$), and also in a Čerenkov mode ($\mu=1$) but at frequencies ω and angles ϑ for which the refractive index is $n_{1s}<1$. The BS excited in both dispersion modes, as well as the radiation due to interference of electromagnetic field excited when the charged particle motion and its MS in the crystal are uniform and linear, are described in the integrand in the right-hand side of the distribution (10) by the expression

$$[(\mathbf{e}_{\tau s}\mathbf{v}_0)^2Q_1\Delta+2qtQ_2]\left(\sum_{\mu=1}^{2}F_\mu-\psi\right),\tag{26}$$

which tends to zero when the MS is "turned off" ($q\to 0$). If $n_{\mu s}<1$, the lateral diffraction peak is formed at frequencies and angles for which the function (11), which determines the effectiveness of the x-ray reflection from the atomic planes, reaches a maximum. This condition can be written in the form

$$\chi_0'(\beta-1)+\alpha_1=0.\tag{27}$$

The radiation that corresponds in (10) to terms that vanish as $q\to 0$, and is generated in the frequency and angle range given by Eq. (27), will hereafter be called diffractive bremsstrahlung (DBS). Using assumptions similar to those used in the derivation of the distribution (18), we obtain the DBS distribution over the emission angles in the case of weak MS ($L_0 \ll L^{\rm BS}$) and weak photon absorption in the crystal ($L_0 \ll L_a^{\mu s}$):

$$\frac{dN_s^{\rm DBS}}{d\vartheta}=\frac{e^2\overline{\theta_s^2}L_0}{8\pi\sin^2\theta_{\rm B}}\left\{\sum_{\mu=1}^{2}\frac{(r_s'/\beta)^{1/2}}{(\vartheta^2+\vartheta_{\mu s}^2)^2}\left[1-\frac{8(\mathbf{e}_{\tau s}\mathbf{v}_0)^2}{(\vartheta^2+\vartheta_{\mu s}^2)}+\frac{8\vartheta^2(\mathbf{e}_{\tau s}\mathbf{v}_0)^2}{(\vartheta^2+\vartheta_{\mu s}^2)^2}\right]-\frac{1}{\beta(\vartheta^2+\vartheta_{1s}^2)}\left[\frac{1}{2}-\frac{3(\mathbf{e}_{\tau s}\mathbf{v}_0)^2}{(\vartheta^2+\vartheta_{1s}^2)}+\frac{2\vartheta^2(\mathbf{e}_{\tau s}\mathbf{v}_0)^2}{(\vartheta^2+\vartheta_{1s}^2)^2}\right]\right\}, \tag{28}$$

where

$$\vartheta_{1(2)s}=\gamma^{-2}-\chi_0'\mp(\beta r_s')^{1/2}.$$

Comparing (28) with the distribution (23), we see that the ratio of the SBS and DBS intensities is of the order of $L_0\omega_{\rm B}|\chi_\tau|/2(\gamma_0\gamma_1)^{1/2}$. Thus, for a crystal of thickness $L_0 \lesssim 2(\gamma_0\gamma_1)^{1/2}/\omega_{\rm B}|\chi_\tau|$ it is necessary to take into the DBS in addition to the SBS. (If $L_a^{\mu s}<L_0<L^{\rm BS}$, the ratio of the SBS to the DBS is of the order of $L_a^{1s}\omega_L^2/\omega_{\rm B}\gg 1$.)

The total number of DBS photons emitted into the lateral diffraction peak is

$$N_s^{\rm DTR}=\frac{e^2\overline{\theta_s^2}L_0}{32\sin^2\theta_{\rm B}}\left\{\sum_{\mu=1}^{2}\frac{4(r_s'/\beta)^{1/2}}{3\vartheta_{\mu s}^2}\left[1-\frac{1+3(\vartheta_d/\vartheta_{\mu s})^4}{[1+(\vartheta_d/\vartheta_{\mu s})^2]^3}\right]-\frac{2}{\beta}\frac{(\vartheta_d/\vartheta_{1s})^2}{[1+(\vartheta_d/\vartheta_{1s})^2]^2}\right\}. \tag{29}$$

Note that the DBS intensity can exceed considerably the intensity of either the SBS and of the PXR in the particle energy region $E<E_{\rm thr}=m|\chi_0'|^{-1/2}$ (see Refs. 1 and 6; m is the particle rest mass), for in this case the DBS, according to (29), ceases to depend on E, whereas the PXR and the SBS decrease with e as the functions $(E/E_{\rm thr})^4$ and $(E/E_{\rm thr})^2$, respectively.

In the energy region $E<E_{\rm thr}$ the crystal thickness along the direction of the vector $\mathbf{v}_0(L_0)$ can be larger than the coherence length of x-ray transition radiation (XTR):

$$l^{\rm XTR}=2/\omega(\gamma^{-2}+\vartheta^2-\chi_0'), \tag{30}$$

but smaller than the optical length $l_{\rm opt}=2/\omega|\chi_0'|$, i.e., the plate is already thick enough and the RTR can be neglected compared with the radiation generated inside the plate [the distribution (10)], but the refraction of the radiation in the substance still changes the phase little, and $L_0<L_{\rm opt}$. In this case the term ψ in the right-hand side of (10), due to the interference between the Čerenkov and non-Čerenkov radiation modes, cancels completely the first members of the expansion of the sum

$$\sum_{\mu=1}^{2}F_\mu$$

in terms of the small parameters $|\delta_{\mu s}|\tau$. As a result, in the case of weak MS we obtain, taking the interference term of (14) into account, the followiong expressions for DBS:

$$\frac{dN_s^{\rm DTR}}{d\vartheta}=\frac{e^2\omega_{\rm B}^2\overline{\theta_s^2}L_0^3C_s^2|\chi_\tau|^2}{96\pi\sin^2\theta_{\rm B}(\gamma^{-2}+\vartheta^2)}\left\{1-\frac{6(\mathbf{e}_{\tau s}\mathbf{v}_0)^2}{(\gamma^{-2}+\vartheta^2)}+\frac{4\vartheta^2(\mathbf{e}_{\tau s}\mathbf{v}_0)^2}{(\gamma^{-2}+\vartheta^2)^2}\right\}, \tag{31}$$

$$N_s^{\rm DTR}=\frac{e^2\omega_{\rm B}^2\overline{\theta_s^2}L_0^3C_s^2|\chi_\tau|^2}{96\sin^2\theta_{\rm B}}\frac{\vartheta_d^2\gamma^2}{(1+\vartheta_d^2\gamma^2)^2}. \tag{32}$$

According to (32), at angles $\vartheta_d<\gamma^{-1}$ the quantity $N_s^{\rm DBS}$ does not depend on the Lorentz factor γ of the particle (recall that $\overline{\theta_s^2}\sim\gamma^{-2}$).

5. RADIATION DUE TO PARTICLE MOTION FROM THE VACUUM INTO THE CRYSTAL

When analyzing the intensity of the radiation into the diffraction peak in the case of a thin crystalline plate satisfying the inequality $L_0<l^{\rm XTR}$, it is necessary to consider, besides the radiation in the medium, the radiation due to the electromagnetic field produced by the charged particle as it moves in the vacuum prior to incidence on the plate. According to (4) and (5), the spectral-angular distributions of the radiation generated by the motion of the charged particle in the vacuum and due to the interference of the electromagnetic fields excited by the particle motion in the vacuum and in the crystal are described by the expressions

$$N_{\mathbf{k}s}{}^{11}=R_L^s\frac{e^2\omega}{(2\pi)^2}(\mathbf{e}_{\tau s}\mathbf{v}_0)^2 l_0^2\left|\sum_{\mu=1}^{2}(-1)^\mu\exp[i\omega n_{\mu s}L_0]\right|^2, \tag{33}$$

$$N_{\mathbf{k}s}{}^{12}=R_L^s\frac{e^2\omega}{2\pi^2}(\mathbf{e}_{\tau s}\mathbf{v}_0)^2 l_0\,{\rm Im}\int_0^{L_0}dt\frac{\exp[-\eta({\rm th}\,\eta t-\eta t)\vartheta^2(4q)^{-1}]}{{\rm ch}^2\eta t}$$
$$\times\{\exp[-it/l_{2s}+t/L_a^{2s}](\exp[-L_0/L_a^{2s}]-\exp[i\omega L_0(n_{1s}-n_{2s}^*)])+\exp[-it/l_{1s}+t/L_a^{1s}](\exp[-L_0/L_\pi^{1s}]-\exp[i\omega L_0(n_{1s}^*-n_{2s})]\}, \tag{34}$$

which were derived, just as (10), with the aid of the usual distribution functions $W_1(\mathbf{r},\theta,t)$ and $W_2(\rho,\theta,\theta',\tau)$. Comparing (33), (34), and (0), we see that at incidence angles $\vartheta\sim\vartheta_{\rm ph}$ corresponding to the diffraction peak the ratio of N_{ks}^{22} and N_{ks}^{11} is of the order of $(L_{\rm eff}/l^{\rm XTR})^2$, and the ratio of N_{ks}^{22} to N_{ks}^{12} is of the order of $L_{\rm eff}/l^{\rm XTR}$, where $L_{\rm eff}\approx\min\{L_0,L_a\}$. In a sufficiently thick crystal, therefore, the distributions (33) and (34) do not take part in the formation of a lateral diffraction peak. On the contrary, in the case $L_0\lesssim l^{\rm RTR}$ the main contribution to the diffraction-peak intensity is made by the term N_{ks}^{11}, which describes the resonant transition radiation (RR) (Refs. 14, 15) in an extremely thin ($L_0<l^{\rm RTR}$) crystalline plate:

$$N_{ks}^{\rm RR}=N_{\mathbf{k}s}{}^{11}=\frac{e^2}{\pi}\frac{(\mathbf{e}_{\tau s}\mathbf{v}_0)^2C_s^2|\chi_\tau|^2}{(\vartheta^2+\gamma^{-2})^2}L_0\delta(\alpha_1). \tag{35}$$

6. BRAGG-DIFFRACTION GEOMETRY AND LAUE-BRAGG TRANSITION

1. We have considered above the characteristics of a lateral diffraction peak in the case of Laue-diffraction geometry, in which the vector $\mathbf{k}_{\rm B}$ is directed towards the vacuum space past the crystalline plate ($\gamma_1=\mathbf{k}_{\rm B}\mathbf{N}/\omega_{\rm B}>0$, $\gamma_0=\mathbf{v}_0\mathbf{N}>0$). We consider now the case of Bragg-diffraction geometry, viz., the vector $\mathbf{k}_{\rm B}$ is directed into the vacuum space ahead of the plate ($\gamma_1<0$). The spectral-angular distributions of the radiation for the Bragg-diffraction geometry can be obtained from the distributions (10), (33), and (34) by replacing in them the coefficient R_L^s by $R_{\rm B}^s$, which characterizes the effectiveness of x-ray reflection from atom-

ic planes in the case of Bragg diffraction

$$R_B^s = \tilde{R}_B^s \exp[2\omega L_0 \operatorname{Im} \delta_{2s}], \tag{36}$$

where

$$\tilde{R}_B^s = C_s^2 \left| \frac{\gamma_1 \chi_\tau}{(2\gamma_1\delta_{2s} - \chi_0\gamma_0)\exp[i\omega(n_{1s}-n_{2s})L_0] - (2\gamma_1\delta_{1s} - \chi_0\gamma_0)} \right|^2 \tag{37}$$

If the inequality $(\gamma^{-2} - \chi_0') > (|\beta| r_s')^{1/2} (\beta = \gamma_0/\gamma_1)$ holds then, in contrast to the Laue-diffraction geometry, the Čerenkov and non-Čerenkov radiation modes correspond to $\mu = 2$ and $\mu = 1$, respectively [see Eq. (9)], and furthermore $\operatorname{Im} \delta_{2s} < 0$ and $\operatorname{Im} \delta_{1s} > 0$. Consequently, the non-Čerenkov mode ($\mu = 1$) is totally absorbed in a thick plate. The spectral-angular distribution of the radiation emitted inside the crystal in the Čerenkov mode takes, according to (10) and (36), the form

$$N_{\mathbf{k}s} = \tilde{R}_B^s \frac{e^2\omega}{2\pi^2} \operatorname{Re} \int_0^{L_0} dt \int_2^{L_0 - t} d\tau [(\mathbf{e}_{\tau s}\mathbf{v}_0)^2 Q_1(1+\Delta) + 2qtQ_2] F_2, \tag{38}$$

where

$$F_2 = \exp[-i\tau/l_{2s} - \tau/2L_a^{2s} - t/L_a^{2s}], \quad L_a^{2s} = (2\omega|\operatorname{Im} \delta_{2s}|)^{-1}.$$

For the angular distributions of the PXR and SBS we get from (38) in the case of weak MS the following expressions:

$$\frac{dN_s^{\rm PXR}}{d\vartheta} = \frac{e^2\omega_B L_{\rm eff}^{\rm PXR} C_s^2 |\chi_\tau|^2 (\mathbf{e}_{\tau s}\mathbf{v}_0)^2}{4\pi \sin^2\theta_B} F(\vartheta), \tag{39}$$

$$\frac{dN_s^{\rm SBS}}{d\vartheta} = \frac{e^2\omega_B \overline{\theta_s^2} (L_{\rm eff}^{\rm SBS})^2 C_s^2 |\chi_\tau|^2}{16\pi \sin^2\theta_B} F(\vartheta), \tag{40}$$

in which

$$L_{\rm eff}^{\rm PXR} = L_a^{2s}\left(1 - \exp\left[-\frac{L_0}{L_a^{2s}}\right]\right),$$

$$(L_{\rm eff}^{\rm SBS})^2 = 2(L_a^{2s})^2\left[1 - \left(1 + \frac{L_0}{L_a^{1s}}\right)\exp\left[-\frac{L_0}{L_a^{2s}}\right]\right], \tag{41}$$

$$L_a^{2s} = \frac{(\vartheta^2 + \vartheta_{\rm ph}^2)^2 - |\beta| r_s'}{|\beta|\omega_B\chi_0''[(\vartheta^2+\vartheta_{\rm ph}^2+\delta_s)^2 + r_s' - \delta_s^2]}, \tag{42}$$

$$F(\vartheta) = \frac{(\vartheta^2+\vartheta_{\rm ph}^2)^2 - |\beta| r_s'}{|(\vartheta^2+\vartheta_{\rm ph}^2)^2 - |\beta| r_s' \exp[-\omega L_0(\xi + i\xi')]|^2}, \tag{43}$$

$$\xi = \frac{1+|\beta|}{2|\beta|\omega_B L_a^{2s}}, \quad \xi' = \frac{1}{2}\left(\vartheta^2 + \vartheta_{\rm ph}^2 - \frac{|\beta| r_s'}{\vartheta^2 + \vartheta_{\rm ph}^2}\right).$$

The effective photon-emission angle in (42) and (43) is

$$\vartheta_{\rm ph} = \begin{cases} \left(\gamma^{-2} - \chi_0' + \eta\dfrac{\overline{\theta_s^2}}{2} L_0\right)^{1/2}, & L_0 \ll L_a^{2s}, \\ \left(\gamma^{-2} - \chi_0' + \eta\dfrac{\overline{\theta_s^2}}{2} L_a^{2s}\right)^{1/2}, & L_0 \gg L_a^{2s}, \end{cases} \tag{44}$$

where

$$\eta = \begin{cases} 2{,}4, & L_0 \ll L_a^{2s} \\ 18, & L_0 \gg L_a^{2s} \end{cases} \quad \text{in the case of PXR and}$$

$$\eta = \begin{cases} 5/2, & L_0 \ll L_a^{2s} \\ 20, & L_0 \gg L_a^{2s} \end{cases} \quad \text{in the case of SBS.}$$

The distributions (39) and (40) are valid under the conditions $L_0 \ll L^{\rm BS}$ or else $L_a^{2s} \ll L^{\rm BS}$ and $L_a^{2s} \ll L_0$. In the latter case the ratio of the lengths L_0 and $L^{\rm BS}$ can be arbitrary. This circumstance, as well as the deviation of $L_{\rm eff}^{\rm SBS}$ from the effective length (24) (in particular, the fact that under the condition $L_0 \gg L_a^{2s}$ the distribution (40) is directly proportional to $(L_a)^2$ whereas the distribution (23) is directly proportional to the product $L_0 L_a$), and also the dependence of the effective emission angle (44) on L_a^{2s}, are all due to the obvious circumstance that the PXR and SBS are formed on a particle-trajectory segment near the start of the particle path in the crystal for the Bragg geometry (in the interval $[0, L_a]$), and at the end of the path in the Laue geometry (the interval $[L_0 - L_a, L_0]$).

2. If the charged particle moves inside the crystal parallel to the crystal-vacuum interface and the Bragg angle is $\theta_B \approx \pi/4$, the diffracted radiation is emitted along the normal $\mathbf{N}$ to the crystal surface (vector $\mathbf{k}_B \| \mathbf{N}$). This geometry is called in diffractometry the Laue-Bragg transition or the case of extremely asymmetric diffraction.[16] A theoretical analysis of the PXR characteristic in the case of the Laue-Bragg transition, with a phenomenological description of MS and a comparison of the theory with the experimental data, were reported recently in Refs. 2 and 7. We present here the results of a quantitative analysis of the characteristics of the radiation generated in a Laue-Bragg transition. The spectral-angular distribution of the radition emitted by a particle moving inside the crystal is given by

$$N_{\mathbf{k}s} = R_{L-B}^s \frac{e^2\omega}{2\pi^2} \operatorname{Re} \int_0^L dt \int_0^{L-t} d\tau \left[[(\mathbf{e}_{\tau s}\mathbf{v}_0) + (\mathbf{e}_{\tau s}\mathbf{N}) q\eta_0^2 t]^2 (1+\Delta) + \frac{2qt}{\operatorname{ch}\eta\tau(1+\eta t \operatorname{th}\eta\tau)} \right\} Q_{L-B} F_{L-B}, \tag{45}$$

where

$$R_{L-B}^s = \frac{C_s^2|\chi_\tau|^2}{|\chi_0 - \alpha_1|^2}\exp[-\omega\chi_0'' z], \quad F_{L-B} = \exp[-i\tau/l],$$

$$Q_{L-B} = Q_1 \exp\left[\frac{\eta_0^2 t^2 (2\vartheta_x \eta \operatorname{th}\eta\tau + \eta_0^2 qt)}{4(1+\eta t \operatorname{th}\eta\tau)} + \frac{\eta_0^4 q t^3}{12}\right],$$

$\eta_0 = (\omega\chi_0'')^{1/2}$, $l = 2\omega^{-1}(\gamma^{-2} + \vartheta^2 - \alpha_1)^{-1}$, L is the particle path length in the crystal ($\mathbf{v}_0 \perp \mathbf{N}$), and z is the distance from the particle trajectory to the crystal-vacuum interface (along the $\mathbf{N}$ direction). The term $(\mathbf{e}_{\tau s}\mathbf{v}_0)^2 Q_{a-B} F_{a-B} (1+\Delta)$ in the integrand in the right-hand side of (45) describes the PXR, and the remaining terms the BS and the radiation due to the interference between the BS and the PXR.

In the case of weak MS ($L \ll L^{\rm BS}$) we obtain for the angular distribution and for the integral number of the PXR photons equations similar to those given in Refs. 2 and 7, but with an effective photon emission angle $\vartheta_{\rm ph} = (\gamma^{-2} - \chi_0' + 1, \overline{2\theta_s^2} L)^{1/2}$. To obtain the angular distribution of the SBS in the case $L \ll L^{\rm BS}$ expression (23) must be multiplied by $\exp(-\omega\chi_0'' z)$ and the substitutions $L_{\rm eff} \to L$, $\gamma_0 \to 1$, $\beta r_s \to 0$.

We note in conclusion that the distributions (10, (33), (34), and (45) were obtained for the case $L_0 \ll L_{\max} = L^{\rm BS}(\omega_B L_a)^{1/2}$. Actually, however, this inequality does not restrict the region of validity of the equations obtained above, since $L_{\max} \ll L^{\rm BS}$.

BLE I.

№	Exper-iment	E, GeV	E_{thr}, GeV	Crystal (hkl)	θ_B, deg	geometry, γ_0/γ_1	$L_0=L/\gamma_0$, cm	L^{BS}, cm	ω_B^{exp}, keV	ω_B^{theor}, keV	Observed quantity*	Theory	Rem
	1	2	3	4	5	6	7	8	9	10	11	12	
1	[10]	4.5	0.11	C (220)	35	$\frac{0.82}{0.82}$	$2.4\cdot10^{-2}$	0.11	$9{,}2\pm1{,}1$	8,57	$(6.61\pm0.8)\cdot10^{-7}$	$1.2\cdot10^{-6}$ $7{,}83\cdot10^{-7}$	According to Ref. [1 » (22
2			0.13	C (220)	30	$\frac{0.87}{0{,}87}$	$2.3\cdot10^{-2}$	0,10	$11{,}3\pm1{,}3$	9.83	$(1.0\pm0{,}17)\cdot10^{-6}$	$9{,}3\cdot10^{-7}$ $9{,}8\cdot10^{-7}$	» [1 » (22
3			0.11	C (220)	35	$\frac{0{,}82}{0.82}$	0,12	0,11	$8{,}9\pm1$	8.57	$(1.1\pm0.1)\cdot10^{-6}$	$1.38\cdot10^{-5}$ $2.33\cdot10^{-6}$	» [1 Eq. (22
4			0.23	C (440)	35	$\frac{0.82}{0.82}$	0.12	0,08	21 ± 1.8	17.14	$(2.62\pm1{,}2)\cdot10^{-7}$	$6{,}17\cdot10^{-7}$	» (22
5	[11]	0.9	5.99	Si (220)	0.51	$\frac{1}{1}$	$3\cdot10^{-3}$	$2{,}8\cdot10^{-3}$	350	362,8	0,3	0.34 0.28	» (22 The sam ance for p ientation
6	[12]	0,9	0.32	(220) Si	9.5	$\frac{0.986}{0.986}$	$3.8\cdot10^{-2}$	$1.2\cdot10^{-2}$	$17{,}8\pm0{,}4$ $21{,}4\pm0{,}4$	19,6	$(5{,}67\pm1.03)\cdot10^{-6}$	$4{,}81\cdot10^{-6}$	» (22
7			0.20	(111)	9,5	$\frac{0.9}{0.71}$	$4.17\cdot10^{-2}$	$1{,}55\cdot10^{-2}$	12	11,98	$(1{,}55\pm0.3)\cdot10^{-6}$	$5.09\cdot10^{-6}$	» (22

Lines 1–4, 6, and 7 show the total radiation yield [photon/e^-] (lines 1–4—angular width of photon detector $\vartheta_d = 7.98\cdot10^{-3}$ rad; lines 6 and 7—$\vartheta_d = 4\cdot10^{-4}$ rad), line 5—spectral-a
nsity [photon/($e^-\cdot$MeV$\cdot$sr)] ($\vartheta_d = 3\cdot10^{-4}$ rad).

7. COMPARISON OF THEORY WITH EXPERIMENT

We proceed to analyze the experiments of Refs. 10–12. The principal experimetal parameters are listed in Table I. Columns 4, 9 and 11 of the table indicate the type of crystal and the reflection plane (hkl), the experimental value of the Bragg frequency ω_B^{exp}, and the integral characteristics, measured in Refs. 10–12, of the radiation emitted to the lateral diffraction peak. It can be seen that the condition that the MS be small ($L_0 = L/\gamma_0 \ll L^{BS}$) was met in the experiments of Refs. 10–12 only in the measurements described in lines 1 and 2 of the table. As expected, in this case the theoretical values calculated from Eq. (22) are in satisfactory agreement with the measurement results. We note only a substantial discrepancy in line 1 of the table between the measured total PXR yield ($N^{PXR} = N_\sigma^{PXR} + N_\pi^{PXR}$) and the theoretical value given in Ref. 10. We do not know the cause of this discrepancy. The total yield $7.83 \cdot 10^{-7}$ photon/e^-, calculated from Eq. (22), is in satisfactory agreement with the experimental $(6.61 \pm 0.8) \cdot 10^{-7}$ photon/e^-. (In the cases described in rows 1–4 of the table, the contributions of the SBS and DBS to the diffraction maximum is quite small (on the order of 0.1%) and was disregarded by us).

In the experiments described in lines 3 and 4 of the table, the condition that the MS be small is no longer met: the length L^{BS} is shorter than $L/\gamma_0 = 0.12$ cm. Equation (22) can be used in this case only for a semiquantitative estimate of the total yield of the generated radiation. The theoretical values listed in the table and calculated from Eq. (22) turn out to be approximately double the observed ones. We have thus also in this case a qualitative agreement between theory and experiment. (We emphasize that in the case of Ref. 10, described in line 3 of the table, the theoretical value of the total radiation yield, $1.38 \cdot 10^{-3}$ photon/e^-, is an order of magnitude larger than the experimental $(1.1 \pm 0.1) \cdot 10^{-6}$ photon/e^-.)

An interesting experimental situation was realized in the measurement described in line 5. In this case,[11] owing to emission of hard x rays ($\omega_B \approx 350$ keV), the electron energy $E = 0.9$ GeV turns out to be much lower than the threshold $E_{thr} = m|\chi_0'|^{-1/2} = 6$ GeV. As a result, the PXR intensity is greatly suppressed,[2,6] and a substantial contribution to the diffraction maximum is made by the SBS and DBS. Under the experimental conditions of Ref. 11, the crystal thickness is subject to the double inequality $l^{XTR} < L_0 < L_{opt}$. Equation (29) is thus no longer valid and must be replaced by expression (32), which was derived with account taken of the interference between the Čerenkov and non-Čerenkov modes. The effective crystal thickness used in Ref. 11 was $L_0 \approx L^{BS}$. Nonetheless, the theoretical value of the spectral-angular density of the generated radiation, calculated from Eqs. (22), (25), and (32), is in satisfactory agreement with the experimental value 0.3 photon/$e^-\cdot$MeV$\cdot$sr. The "weights" of the PXR, SBS, and DBS in the observed radiation maximum turn out to be here 13.5%, 23.5%, and 53%, respectively. The width $\Delta\omega^{exp} = 25$ keV of the diffraction maximum observed in Ref. 11 is determined by the angular dimensions of the photon collimator ($2\vartheta_d = 6 \cdot 10^{-4}$ rad) and is also in satisfactory agreement with the theoretical $\Delta\omega^{theor} = 2\vartheta_d \vartheta_B \cot \omega_B = 23.6$ keV. (The experimental and theoretical values of the Bragg frequency differ in Ref. 11 by 12.8 keV. This difference can be attributed, for example, to a misorientation $\Delta\vartheta \sim 3 \cdot 10^{-4}$ rad of the angles in this experiment. If it is assumed that such an angle misorientation actually obtains in Ref. 11, we obtain for the spectral-angular density a value 0.28 photon/$e^-\cdot$MeV$\cdot$sr).

In the experiment of Ref. 12, the length L_0 is several times larger thant L^{BS}. Therefore the good agreement in row 6 of the table between the experimental and theoretical total radiation yields (calculated form Eqs. (22), (25), and (29)) is apparently fortuitous. In line 7 of the table, the total radiation yield agrees with an estimate obtained from Eqs. (22) and (25) (the DBS can be neglected in this case).

Thus, an analysis carried out with the aid of the simple equations (22), (25), (29), and (32), show that the experimental data of Refs. 10–12 are satisfactorily explained by the theory of omission of x-ray photons with allowance for multiple scattering of the electrons in the substance.

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