Positron cooling in a magnetized electron beam

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Relaxation of a positron beam in an electron gas moving in a magnetic field is investigated. The case when the electron longitudinal temperature is low compared with the transverse is considered. Expressions are obtained for the friction force and for the scattering coefficients of positron momenta in the case of a low-density electron beam, when the time of passage through the electron-cooling system is shorter than the plasma-oscillation period. It is shown that in a strong magnetic field the effective cross section of cooling the longitudinal motion of the positrons is determined by their longitudinal velocity relative to the electron gas. The proximity of the electron and positron Larmor-rotation frequencies, in turn, increases greatly the intensity of the cooling of the positron transverse degree of freedom. These circumstances, taken jointly, can be used to enhance by many times of the cooling of high-temperature positron beams.

Beam cooling by electrons, a method that was reported more than 20 years ago and increased the proton- and anti-proton-beam intensities to a new level, is universally known.[1] The experimental verification of the cooling effect led to the possibility of producing colliding proton–anti-proton beams, one of the main sources of information in high-energy physics.[2–7]

A timely subject of study is the possibility of electron cooling of other particle species. We investigate positron-beam cooling, which can be used for effective injection of moderately relativistic positrons into a storage ring. A possible interesting and highly promising possibility is the use of electron cooling of intense anti-hydrogen beams for a high precision experimental comparison of their properties with those of hydrogen beams.[6,8,9] No less interesting may be the use, as a research tool, of a monochromatic beam of exotic positronium atoms produced by electron cooling of positrons.[10,11]

1. QUALITATIVE DESCRIPTION OF POSITRON COOLING BY AN ELECTRON GAS IN A LONGITUDINAL MAGNETIC FIELD

Before proceeding to a direct calculation of the deceleration and diffusion of positrons in an electron beam, we examine qualitatively the picture of electron-positron collisions in a strong magnetic field, in a reference frame comoving with the electron beam. An electron beam is assumed to move along the Z axis of a Cartesian XYZ frame and the vector of a uniform magnetic field $\mathbf{B}$ to be directed along the same axis.

It is known that in Coulomb interaction the momentum and energy exchange of the colliding particles diverges logarithmically in the region of large impact parameters, and cutoff should take place at a certain parameter ρ_0 beyond which the interaction is effectively decreased. The value of this parameter is determined by the effective interaction time τ_f viz., $\rho_0 = V\tau_f$ where V is the relative velocity of the particles and $\tau_f \sim \min(\omega_e^{-1}, l/\gamma\beta_\parallel c)$. Here $\omega_e = (4\pi e^2 n/m)^{1/2}$ is the plasma frequency (e, m, and n are the charge, mass, and density of the electrons), l is the length of the electron-cooled section, $\beta_\parallel c$ the electron velocity in the lab, and γ the relativistic factor.

If the particles are in a sufficiently strong field, such that their Larmor period $T = 2\pi/\Omega$ is much shorter than τ_f, there always exists an impact-parameter region $\rho_0 > r > uT$ (u is the relative velocity of the Larmor orbits) such that the collisions are slow relative to the Larmor rotation and span over many cycles. It is necessary then to take into account the influence of the magnetic field on the particle collision process.

Let us estimate the contribution of the slow collisions to the rates of the squared longitudinal (p_z^2) and transverse ($p_\perp^2$) momenta of a positron moving at a longitudinal velocity $u = p_z/m$ relative to the cold electron gas and having a Larmor radius $\rho = p_\perp/m\Omega$.

The electron transverse momentum is changed in a time t by the resonant force $F = e^2 e^{i\Omega t}/\rho^2$, exerted on it by the positron by an amount on the order of $e^2 t e^{i\Omega t}/\rho^2$ (resonant interaction of the particles is ineffective if $r > \rho$), and the number of electrons effectively participating in the interaction is of the order of $2\pi n\rho^3$. Hence

$$\frac{dp_\perp{}^2}{dt} \sim -2\pi n\rho^3 \frac{d}{dt}\left(\frac{e^2 t}{\rho^2}\right)^2 = -4\pi e^4 n \frac{t}{\rho}. \qquad (1.1)$$

Recognizing that the decisive contribution to the longitudinal-momentum exchange between the positron and the electrons is made by the uncompleted collisions with impact parameters $r \gtrsim ut$,[4] it is easy to estimate the rate of change of p_z^2 under the influence of the longitudinal friction force:

$$\frac{dp_z{}^2}{dt} \sim -\left(\frac{e^2}{u\rho}\right)^2 \frac{d}{dt}(2\pi n\rho u^2 t^2) = -4\pi e^4 n \frac{t}{\rho}. \qquad (1.2)$$

The electron-positron interaction time is actually restricted either by the time of flight $t_c = l/\gamma\beta_\parallel c$ through the electron-cooling system, or by the collision time $t_s = \rho/u$, or else by the plasma screening times $t_{\text{scr}}^\parallel \sim \omega_e^{-1}$ and $t_{\text{scr}}^\perp \sim \Omega\omega$ for the longitudinal or transverse interactions ($\omega_e^2/2\Omega$ is the Larmor-oscillation frequency shift by the Coulomb interaction of the electrons). We consequently have in Eqs. (1.1) and (1.2)

$$t_{max} \sim \min\,(l/\gamma\beta_\parallel c,\ \rho/u,\ t_{\text{scr}}). \qquad (1.3)$$

It follows from (1.1)–(1.3) that in a sufficiently strong magnetic field the contribution of the slow collisions to the friction power is inversely proportional to the longitudinal

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component of the positron velocity relative to the electron gas. At the same time, the contribution of the fast collisions, $r<uT$, is known to be inversely proportional to the total relative velocity of the particles.

Consequently, the positron-cooling decrement should increase in strong magnetic fields at low longitudinal positron velocities relative to the electron beam ($v_\parallel \ll v_\perp$).

2. KINETIC EQUATION

We proceed now directly to calculate the positron cooling in an electron beam in a magnetic field. We use a representation in which the physical quantities that evolve in accordance with exact equations are averaged over the initial microscopic conditions with a distribution $\mathscr{D}(\Gamma, 0)$.[3] We examine the evolution of the positron density in six-dimensional phase space; this distribution satisfies, in accord with the laws of motion, the Liouville equation

$$\frac{\partial}{\partial t}f+\frac{\partial}{\partial \mathbf{r}}\mathbf{v}f+\frac{\partial}{\partial \mathbf{p}}\mathbf{F}f=0.$$

Averaging this equation, we get

$$\frac{\partial}{\partial t}\bar{f}+\frac{\partial}{\partial \mathbf{r}}\mathbf{v}\bar{f}+\frac{\partial}{\partial \mathbf{p}}\mathbf{F}_p\bar{f}+\frac{\partial}{\partial \mathbf{p}}\bar{\mathbf{F}}_s\bar{f}=-\frac{\partial}{\partial \mathbf{p}}\overline{\delta\mathbf{F}_s\delta f}, \qquad (2.1)$$

where $\bar{f}$ is the positron-distribution probability density, also called the single-particle distribution function, $\mathbf{F}_p$ is the regular part of the force exerted by the external fields, $\mathbf{F}_s$ is the regular part of the force due to the positron-electron interaction, $\delta f=f-\bar{f}$, $\delta\mathbf{F}_s=\mathbf{F}_s-\bar{\mathbf{F}}_s$; the superior bar denotes averaging over the initial electron velocities and coordinates. An expression for the right-hand side of (2.1) can be obtained by assuming that the force $\mathbf{F}_s$ is weak enough to be able to account for its effect on the positron motion by perturbation theory. We get then in first-order approximation

$$\delta f=-\int_0^t \frac{\partial}{\partial \mathbf{p}_{t-\tau}}\{\delta\mathbf{F}_s(t-\tau)\bar{f}(t-\tau)\}d\tau$$

and the right-hand side of (2.1) takes the form

$$-\frac{\partial}{\partial \mathbf{p}}\overline{\delta f\delta\mathbf{F}_s}$$

$$=\frac{\partial}{\partial p_i}\left[\int_0^t \frac{\partial p_k(t)}{\partial p_j(t-\tau)}\overline{\delta F_s^{\,i}(t)\delta^j F_s(t-\tau)}\frac{\partial}{\partial p_k(t)}\bar{f}(t-\tau)d\tau\right].$$

If the single-particle distribution function f depends only on conservative integrals of the unperturbed motion, the collision integral is found to be

$$\mathrm{St}\bar{f}\equiv-\frac{\partial}{\partial \mathbf{p}}\overline{\delta\mathbf{F}_s\delta f}=\frac{\partial}{\partial p_i}D_{ij}\frac{\partial}{\partial p_j}\bar{f},$$

$$D_{ij}=\int_0^t \frac{\partial p_i(t)}{\partial p_k(t-\tau)}\overline{\delta F_s^{\,i}(t)\delta F_s^{\,k}(t-\tau)}d\tau.$$

It follows then from the kinetic equation that the rates of change of the integrals of motion p_z^2 and $p_x^2+p_y^2\equiv p_\perp^2$ of the "probing" particles interacting with the electron beam in the magnetic field are

$$\frac{dp_z^2}{dt}=2(\mathbf{F}_{fr}+\mathbf{F}_{fl})\mathbf{p}_z+d_z, \qquad (2.2)$$

$$\frac{dp_\perp^2}{dt}=2(\mathbf{F}_{fr}+\mathbf{F}_{fl})\mathbf{p}_\perp+d_\perp, \qquad (2.3)$$

where

$$\mathbf{F}_{fr}=\bar{\mathbf{F}}_s,\quad d_z=2D_{zz},\quad d_\perp=2(D_{xx}+D_{yy}),\quad F_{fl}^{\,i}=\frac{\partial}{\partial p_j}D_{ij}.$$

We shall refer hereafter to $\mathbf{F}_{fr}$, $\mathbf{F}_{fl}$, d_z, and $d_\perp$ as the friction force, the fluctuation force, and the diffusion coefficients, respectively. Knowledge of these quantities, the analysis of which is the subject of the present paper, provides a rather complete picture of the rate of positron relaxation to some stationary state whose properties will be discussed below.

In a nonrelativistic approximation of the weak interaction and neglecting electron-electron collisions, the expressions for the friction force $\mathbf{F}_{fr}$ and for the force correlation tensor $\overline{\delta F_s^{\,i}(t)\delta F_s^{\,j}(t-\tau)}$ can be written in the form[3]:

$$\mathbf{F}_{fr}=ze\mathbf{E}[\mathbf{r}(t)]=ze\int i\mathbf{k}\langle\varphi_k(t)\rangle e^{i\mathbf{kr}(t)}\,d\mathbf{k},$$

$$\overline{\delta F_s^{\,i}(t)\delta F_s^{\,j}(t-\tau)}$$

$$=(2\pi)^3z^2e^2n\int k_ik_j\langle\varphi_k^{\,a}(t)\varphi_k^{\,a}(t-\tau)\rangle$$

$$\times\exp\{i\mathbf{k}[\mathbf{r}(t)-\mathbf{r}(t-\tau)]\}d\mathbf{k}. \qquad (2.4)$$

Here

$$\langle\varphi_k(t)\rangle=\frac{ze}{2\pi^2k^2}\int d\omega\frac{L(\omega,\mathbf{k})}{\varepsilon_e(\omega,\mathbf{k})}e^{-i\omega t}$$

is the spatial Fourier transform of the potential induced by a probing particle having a charge ze in the electron beam ($z=-1$ for positrons);

$$\varphi_k^{\,a}(t)=\frac{e}{2\pi^2k^2}\int d\omega\frac{L^a(\omega,\mathbf{k})}{\varepsilon_e(\omega,\mathbf{k})}e^{-i\omega t}$$

is the same for an individual electron;

$$L\equiv\frac{1}{2\pi}\int_0^\infty d\tau\exp(i\mathbf{kr}(\tau)+i\omega\tau);$$

$\mathbf{r}(t)$ and $\mathbf{r}^a(t)$ are the trajectories, unperturbed by the interaction, of the particles in the magnetic field; $\varepsilon_e(\omega,\mathbf{k})$ is the dielectric constant of a one-component electron plasma[3,12];

$$\varepsilon_e(\omega,\mathbf{k})=1+\omega_e^2\int_0^\infty\tau\,d\tau\left(\frac{k_\parallel^2}{k^2}+\frac{k_\perp^2\sin\Omega\tau}{k^2\Omega\tau}\right)$$

$$\times\left\langle J_0\left(2k_\perp r_\perp^{\,a}\sin\frac{\Omega\tau}{2}\right)\exp\{i(\omega-k_\parallel v_\parallel^{\,a})\tau\}\right\rangle,$$

the angle brackets denote averaging over the distributions of the longitudinal and transverse electron velocities $v_\parallel^a$ and $v_\perp^a$.

We restrict hereafter the analysis of the expressions for the friction force and for the diffusion coefficients to the case of a short ($\omega_e t\ll 1$) electron-electron and positron-electron interaction time, assuming the two interactions to be "turned on" simultaneously. The electron-electron interaction is actually turned on somewhat earlier, but this is immaterial for a stable electron beam, so that a simpler situation can be theoretically considered.

3. SHORT INTERACTION TIMES (INTERACTION WITHOUT SCREENING)

The expressions for $\langle\varphi_k(t)\rangle$ and $\varphi_k^{\,a}(t)$ take in this case the form

$$\langle\varphi_k(t)\rangle=\frac{ze\omega_e^2}{2\pi^2k^2}\int_0^t\tau\,d\tau\lambda_k(\tau)\exp\{-i\mathbf{kr}(t-\tau)\},\qquad(3.1)$$

$$\varphi_k^a(t)=\frac{e}{2\pi^2k^2}\exp(-i\mathbf{kr}^a(t)),\qquad(3.2)$$

where

$$\lambda_k(\tau)=\left(\frac{k_\parallel^2}{k^2}+\frac{k_\perp^2\sin\Omega\tau}{k^2\Omega\tau}\right)\times\left\langle J_0\left(2k_\perp r_\perp^a\sin\frac{\Omega\tau}{2}\right)\exp(-ik_\parallel v_\parallel^a\tau)\right\rangle.$$

Note that $\varphi_k^a(t)$ corresponds to the Coulomb potential of a free electron moving in a magnetic field, and $\langle\varphi_k(t)\rangle$ to a potential produced by the electron "cloud" induced by a charge moving in the plasma at the instant $t\ll\omega_e^{-1}$. Taking (2.4), (3.1), and (3.2) into account, we get

$$\mathbf{F}_{fr}=\frac{(ze\omega_e)^2}{2\pi^2}\int\frac{i\mathbf{k}\,d\mathbf{k}}{k^2}\int_0^t\tau\,d\tau\left\{\left(\frac{k_\parallel^2}{k^2}+\frac{k_\perp^2\sin\Omega\tau}{k^2\Omega\tau}\right)\times\left\langle J_0\left(2k_\perp r_\perp^a\sin\frac{\Omega\tau}{2}\right)\times\exp\{ik_\parallel\tau(v_\parallel-v_\parallel^a)\}\right\rangle\exp\{i\mathbf{k}_\perp[\mathbf{r}_\perp(t)-\mathbf{r}_\perp(t-\tau)]\}\right\},\qquad(3.3)$$

$$\overline{\delta F_e^i(t)\delta F_e^j(t-\tau)}=\frac{2z^2e^4n}{\pi}\int\frac{k_ik_j\,d\mathbf{k}}{k^4}\times\exp\{i\mathbf{k}_\perp[\mathbf{r}_\perp(t)-\mathbf{r}_\perp(t-\tau)]\}\times\left\langle J_0\left(2k_\perp r_\perp^a\sin\frac{\Omega\tau}{2}\right)\exp\{ik_\parallel\tau(v_\parallel-v_\parallel^a)\}\right\rangle,\qquad(3.4)$$

$$\mathbf{F}_{fl}=-\frac{m(ze\omega_e)^2}{M2\pi^2}\int\frac{i\mathbf{k}\,d\mathbf{k}}{k^2}\int_0^t\tau\,d\tau\left\{\left(\frac{k_\parallel^2}{k^2}+\frac{k_\perp^2\sin\tilde\Omega\tau}{k^2\tilde\Omega\tau}\right)\times\left\langle J_0\left(2k_\perp r_\perp^a\sin\frac{\Omega\tau}{2}\right)\times\exp\{ik_\parallel\tau(v_\parallel-v_\parallel^a)\}\right\rangle\exp\{i\mathbf{k}_\perp[\mathbf{r}_\perp(t)-\mathbf{r}_\perp(t-\tau)]\}\right\}.\qquad(3.5)$$

Here M and $v_\parallel$ are the mass of the probing particles and its velocity along the magnetic field, m and $v_\parallel^a$ are the same for the electrons, $J_0(x)$ is a Bessel function of zero index, and $\tilde\Omega=mz\Omega/M$.

It is interesting to note that the ratio of the friction force to the fluctuation force turns out to be of the order of M/m, so that for heavy particles ($M\gg m$) the action of this force can be neglected. For positrons, on the other hand, in the unscreened-interaction approximation these forces are simple equal. Putting $M=m$ and $\tilde\Omega=-\Omega$ we obtain with the aid of (3.3)–(3.5)

$$d_\parallel\equiv d_z=\frac{4e^4n}{\pi}\int\frac{k_\parallel^2\,d\mathbf{k}}{k^4}\int_0^t d\tau\left\langle J_0\left(2k_\perp r_\perp^a\sin\frac{\Omega\tau}{2}\right)\times\exp\{ik_\parallel\tau(v_\parallel-v_\parallel^a)\}\right\rangle\times\exp\{i\mathbf{k}_\perp[\mathbf{r}_\perp(t)-\mathbf{r}_\perp(t-\tau)]\}\equiv\frac{4e^4n}{\pi}\int\frac{k_\parallel^2\,d\mathbf{k}}{k^4}\int_0^t\xi(\tau)\,d\tau,$$

$$d_\perp=\frac{4e^4n}{\pi}\int\frac{k_\perp^2\,d\mathbf{k}}{k^4}\int_0^t\xi(\tau)\cos\Omega\tau\,d\tau,$$

$$2F^zp_z\equiv2(F_{fr}^z+F_{fl}^z)p_z=4F_{fr}^zp_z\equiv2F^\parallel p_\parallel=\frac{8e^4n}{\pi}\int\frac{ik_\parallel v_\parallel\,d\mathbf{k}}{k^2}\int_0^t\left(\frac{k_\parallel^2}{k^2}+\frac{k_\perp^2\sin\Omega\tau}{k^2\Omega\tau}\right)\xi(\tau)\tau\,d\tau,$$

$$2\mathbf{F}^\perp\mathbf{p}_\perp=\frac{8e^4n}{\pi}\int\frac{i(k^xv_x+k^yv_y)\,d\mathbf{k}}{k^2}\times\int_0^t\left(\frac{k_\parallel^2}{k^2}+\frac{k_\perp^2\sin\Omega\tau}{k^2\Omega\tau}\right)\xi(\tau)\tau\,d\tau,$$

$$\xi(\tau)\equiv\left\langle J_0\left(2k_\perp r_\perp^a\sin\frac{\Omega\tau}{2}\right)\exp\{ik_\parallel\tau(v_\parallel-v_\parallel^a)\}\right\rangle\times\exp\{i\mathbf{k}_\perp[\mathbf{r}_\perp(t)-\mathbf{r}_\perp(t-\tau)]\}.$$

Integrating the foregoing expressions with respect to $d\varphi$ and $dk_\parallel$, we get

$$d_\parallel=4\pi e^4n\left\langle\int dk_\perp\int_0^t[1-k_\perp|u|\tau]\times J_0\left(2k_\perp r_\perp^a\sin\frac{\Omega\tau}{2}\right)J_0\left(2k_\perp\rho\sin\frac{\Omega\tau}{2}\right)\times\exp(-k_\perp|u|\tau)\,d\tau\right\rangle,\qquad(3.6)$$

$$d_\perp=4\pi e^4n\left\langle\int dk_\perp\int_0^t[1+k_\perp|u|\tau]\times J_0\left(2k_\perp r_\perp^a\sin\frac{\Omega\tau}{2}\right)J_0\left(2k_\perp\rho\sin\frac{\Omega\tau}{2}\right)\times\exp(-k_\perp|u|\tau)\cos\Omega\tau\,d\tau\right\rangle.\qquad(3.7)$$

$$2F^\parallel p_\parallel=2v_\parallel\frac{\partial}{\partial v_\parallel}d_\parallel+\frac{8\pi e^4nv_\parallel}{\Omega}\times\left\langle\int k_\perp^2\,dk_\perp\int_0^t u\tau\sin\Omega\tau J_0\left(2k_\perp r_\perp^a\sin\frac{\Omega\tau}{2}\right)\times J_0\left(2k_\perp\rho\sin\frac{\Omega\tau}{2}\right)\exp(-k_\perp|u|\tau)\,d\tau\right\rangle,\qquad(3.8)$$

$$2\mathbf{F}^\perp\mathbf{p}_\perp=8\pi e^4n\left\langle\int dk_\perp\int_0^t\left[\left(1+\frac{\sin\Omega\tau}{\Omega\tau}\right)-\left(1-\frac{\sin\Omega\tau}{\Omega\tau}\right)k_\perp|u|\tau\right]\times\exp(-k_\perp|u|\tau)J_0\left(2k_\perp r_\perp^a\sin\frac{\Omega\tau}{2}\right)\right.$$

$$\frac{d}{d\tau}\left[J_0\left(2k_\perp\rho\sin\frac{\Omega\tau}{2}\right)\right]\tau\,d\tau\Big\rangle. \tag{3.9}$$

Here ρ is the positron Larmor radius and $u\equiv v_\parallel - v_\parallel^a$.

The entire region of impact parameters of the electron-positron collisions can obviously be subdivided into a region of fast collisions, for which the cyclic character of the positron and electron motion in the magnetic field is immaterial, and a region of slow collisions, when the particles manage to execute many Larmor rotations during the collision time. The integration intervals corresponding to the first and second regions are $k_{\max}\gtrsim k_\perp \gtrsim \Omega/|u|$, and $k_\perp<\Omega/|u|$, respectively.

In the fast-collision region the integrals with respect to time in Eqs. (3.6)–(3.9) converge over intervals $\tau\sim(k_\perp|u|)^{-1}<\Omega^{-1}$. Putting in this case $\sin\Omega\tau\to\Omega\tau$ and returning to the integral representation of the Bessel functions of zero order, we obtain for the friction force, for the fluctuating force, and for the diffusion tensor expressions that correspond to collisions of charged particles in the absence of a magnetic field:

$$\mathbf{F}_{fr}=\mathbf{F}_{fl}=-\left\langle\frac{4\pi e^4 n\mathbf{V}}{m|V|^3}\ln\frac{r_{\max}}{r_{\min}}\right\rangle,$$

$$d_{ij}=\int_0^t\langle\delta F_s^i(t)\,\delta F_s^j(t-\tau)\,d\tau\rangle \tag{3.10}$$

$$=\left\langle 4\pi e^4 n\left(\frac{V^2\delta_{ij}-V_iV_j}{V^3}\ln\frac{r_{\max}}{r_{\min}}-\frac{1}{2}\frac{V^2\delta_{ij}-3V_iV_j}{V^3}\right)\right\rangle.$$

Here $\mathbf{V}=\mathbf{v}-\mathbf{v}^a$ is the velocity of the positrons relative to the electrons,

$$r_{\max}=\min(Vt,\ |u|/\Omega)$$

$r_{\min}$ is the minimum impact distance at which perturbation theory with allowance for a finite interaction time t is applicable. Expressions (3.10) differ only by the upper and lower parameters of the Coulomb logarithm from those usually cited (see, e.g., Ref. 12), which have been determined for completed collisions in the absence of a magnetic field.

In the slow-collision region $k_\perp<\Omega/|u|$, and also under the condition $\Omega t\gg1$ (in the opposite case the influence of the magnetic field on the collision kinetics becomes insignificant), the integrands in (3.6)–(3.9) can be averaged over the period of the Larmor rotations of the electrons and positrons in the magnetic field. The difference between the expressions obtained in this manner and those in (3.11) and (2.12) turns out to be relatively small in the parameter region $k_\perp\ll\Omega/|u|$. Integrating next with respect to time and substituting the results in (2.2) and (2.3), we obtain the following expressions for the rates of change of the squares of the longitudinal and transverse momenta of a positron that experiences, on moving in the plasma, electron collisions that are slow relative to the Larmor rotation:

$$\frac{dp_\parallel^2}{dt}=4\pi e^4 nt\left(1+2v_\parallel\frac{\partial}{\partial v_\parallel}\right)\left\langle\int^{\Omega/|u|}dk_\perp\exp(-k_\perp|u|t)\times\sum_{-\infty}^{\infty}J_l^2(k_\perp\rho)J_l^2(k_\perp r_\perp^a)\right\rangle \tag{3.11}$$

$$\frac{dp_\perp^2}{dt}=4\pi e^4 n\left\langle\int^{\Omega/|u|}\frac{dk_\perp}{k_\perp|u|}[2-(2+k_\perp|u|t)\exp(-k_\perp|u|t)]\times\sum_0^{\infty}(2l+1)[J_l^2(k_\perp\rho)J_{l+1}^2(k_\perp r_\perp^a)-J_l^2(k_\perp r_\perp^a)J_{l+1}^2(k_\perp\rho)]\right\rangle. \tag{3.12}$$

The upper integration limit in (3.11) can be set equal to ∞, since the characteristic convergence interval of the integral is $k_\perp^x\lesssim1/|u|t\ll\Omega/|u|$. The same can be done in (3.12) provided that the positron transverse velocities exceed the longitudinal substantially. The procedure for extending the integration in (3.11) and (3.12) into the region of infinitesimal impact parameters is correct within the framework of perturbation theory if the resultant expressions are such that the changes of the squared transverse and longitudinal positron momenta during the characteristic time $\tau^x\sim\min(t,\rho/|u|)$ of formation or action of the friction and diffusion turn out to be relatively small.

If $\rho^2\gg\langle(r_\perp^a)^2\rangle$ it suffices to retain in (3.11) and (3.12) the terms with $l=0$, putting $J_0(k_\perp r_\perp^a)\approx1$.

The result is

$$\frac{dp_\parallel^2}{dt}\approx 8e^4n\left(1+2v_\parallel\frac{\partial}{\partial v_\parallel}\right)\left\langle\frac{K[x(1+x^2)^{-1/2}]}{|u|(1+x^2)^{1/2}}\right\rangle, \tag{3.13}$$

$$\frac{dp_\perp^2}{dt}=-8e^4n\left\langle\frac{K[x(1+x^2)^{-1/2}]}{|u|(1+x^2)^{1/2}}\right\rangle\quad\text{for } v_\perp\gg|u|,$$

$$\frac{dp_\perp^2}{dt}=-\pi e^4n\left\langle\frac{v_\perp^2}{|u|^3}\right\rangle\quad\text{for } v_\perp\ll|u|. \tag{3.14}$$

Here $x=2\rho/|u|t$, and $K[x(1+x^2)^{-1/2}]$ is a complete elliptic integral of the first kind and is approximated with good accuracy by the expression[13]

$$K[x(1+x^2)^{-1/2}]\approx\pi/2-\ln\{1-x(1+x^2)^{-1/2}\}.$$

It was assumed so far that the Larmor frequencies of the electrons and positrons are equal; this is strictly speaking not true, in view of the difference between their energies. Nonetheless, the foregoing results remain in force provided that the characteristic difference $\Delta\Omega$ between the Larmor frequencies of the electrons and positrons is relatively small, viz., $\Delta\Omega\tau^{in}\ll1$, where $\tau^{in}\approx\min(t,\rho/|u|,r_\perp^a/|u|)$ is the electron-positron interaction time.

In the opposite case, the expressions for the cooling rates of the squares of the longitudinal and transverse positron momenta take the form

$$\frac{dp_\parallel^2}{dt}\approx4\pi e^4tn\left(1+2v_\parallel\frac{\partial}{\partial v_\parallel}\right)\left\langle\int dk_\perp\exp(-k_\perp|u|t)J_0^2(k_\perp\rho)J_0^2(k_\perp r_\perp^a)\right\rangle$$

$$\approx4\pi e^4n\left(1+2v_\parallel\frac{\partial}{\partial v_\parallel}\right)\begin{cases}\left\langle\dfrac{1}{|u|}\right\rangle, & |u|t\gg R_{\max}\\[2mm] \left\langle\dfrac{t}{\pi R_{\max}}\ln\dfrac{R_{\max}}{|u|t}\right\rangle, & R_{\max}\gg|u|t\gg R_{\min}.\\[2mm] \left\langle\dfrac{t}{\pi R_{\max}}\ln\dfrac{R_{\max}}{R_{\min}}\right\rangle, & R_{\min}\gg|u|t\end{cases}$$

Here

$$R_{max}=\max(\rho, r_\perp{}^a), \quad R_{min}=\min(\rho, r_\perp{}^a),$$

$$\frac{dp_\perp{}^2}{dt}\approx -4\pi e^4 n\left\langle\int dk_\perp \int_0^t d\tau \exp(-k_\perp |u| \tau)\right.$$

$$\times \cos\Delta\Omega\tau\,[1+k_\perp |u| \tau]$$

$$\left.\times [J_0{}^2(k_\perp r_\perp{}^a) J_1{}^2(k_\perp\rho) - J_0{}^2(k_\perp\rho) J_1{}^2(k_\perp r_\perp{}^a)]\right\rangle$$

$$\approx -4\pi e^4 n\left\langle\int \frac{(k_\perp |u|)^3 dk_\perp}{[(k_\perp |u|)^2+(\Delta\Omega)^2]^2}[J_0{}^2(k_\perp r_\perp{}^a) J_1{}^2(k_\perp\rho)\right.$$

$$\left.- J_0{}^2(k_\perp\rho) J_1{}^2(k_\perp r_\perp{}^a)]\right\rangle$$

$$\approx -4\pi e^4 n \begin{cases} \dfrac{1}{3\pi\rho|\Delta\Omega|}, & \rho\gg\dfrac{|u|}{|\Delta\Omega|}\gg r_\perp{}^a \\ \left\langle\dfrac{|u|}{\pi^2\rho r_\perp{}^a(\Delta\Omega)^2}\right\rangle, & \rho\gg r_\perp{}^a\gg\dfrac{|u|}{|\Delta\Omega|} \end{cases} \tag{3.16}$$

Comparison of expressions (3.13) and (3.14) with (3.15) and (3.16) shows that the difference between the Larmor-rotation frequencies of the electrons and positrons alters primarily the kinetics of the cooling of the transverse degrees of freedom of the latter. This is an obvious consequence of the violation of the resonant character of the interaction of the electron and positron transverse degrees of freedom.

As applied to electron cooling of positrons in a storage ring, the case $t_c\omega_e \ll 1$ considered here corresponds to a relatively short cooling section. The electron-positron interaction is turned on periodically for a time t_c, the period being equal to that of the positron motion. With the particle motion in the storage ring correctly organized, this interaction cools the hot positrons.

If the longitudinal magnetic field is strong enough and the kinetics of this process is governed by slow collisions, and if there is no coupling between the longitudinal and transverse motions in the cooling section, the positron transverse degree of freedom will be cooled to the transverse temperatue of the electron beam. This follows from relation (3.12): $dp_\perp^2/dt\to 0$ and $\rho^2\to r_\perp^a$.

As to the equilibrium longitudinal positron temperature $T_\parallel{}^+$, it follows from (3.13) that its order of magnitude is

$$T_\parallel{}^+\approx\max\left(T_\parallel{}^-, \frac{T_\perp^-}{\Omega^2 t_c^2}\right)$$

and is in general not equal to the longitudinal temperature of the electron beam.

To implement the considered acceleration of the positron relaxation in a magnetic field it is necessary that the magnetized electron beam have a small longitudinal-velocity spread. It is known that the longitudinal temperatures of the electron beams used for cooling and obtained from electrostatic guns are so low that they are limited in fact only to the Coulomb-interaction energy fluctuations, i.e., to a value of order $e^2n^{1/3}$. The transverse temperature remains in this case equal to the cathode temperature. This strong anisotropy of the distribution in velocity in the electron-beam acceleration section is maintained furthermore by the strong magnetic field and by the mutual repulsion of the electrons, which hinders energy exchange between the longitudinal and transverse degrees of freedom.[2,3,14]

If $m\langle u^2\rangle \gg e^2/\rho$, the positrons are cooled in such a beam in the manner described in this paper. This condition is violated in a sufficiently strong magnetic field. A quantitative investigation of this situation calls for a more detailed examination of the collision kinetics. It is clear, however, that even in this case the magnetic field accelerates strongly the relaxation of the positron transverse degree of freedom, since the electron-positron (attractive) interaction does not hinder approach of the particles to within distances $r\sim\rho$ at which effective resonant exchange of the particles' transverse degrees of freedom takes place.

We note also that a large initial spread of the longitudinal positron velocity does not prevent in principle the use of a magnetic field for the described enhancement of the cooling growth rates, for the "sweeping" method[2,3] can be used in this situation.

[1]G.I. Budker, Atom. Energ. **22**, 346 (1967).
[2]Ya. S. Derbenev and A. N. Skrinsky, Sov. Phys. Rev. Vol 3, p. 165, 1981.
[3]Ya. S. Derbenev, Doctoral thesis, Novosibirsk, 1978.
[4]Ya. S. Derbenev and A. N. Skrinskii, Fiz. Plazmy **4**, 492 (1978) [Sov. J. Plasma Phys. **4**, 273 (1978)].
[5]V. V. Parkhomchuk, Doctoral thesis, Novosibirsk, 1978.
[6]G. I. Budker and A. N. Skrinskiĭ, Usp. Fiz. Nauk **124**, 561 (1978) [Sov. Phys. Usp. **21**, 277 (1978)].
[7]V. I. Kudelaĭnen, V. V. Parkhomchuk, and D. V. Pestrikov, Zh. Tekh. Fiz. **53**, 870 (1978) [Sov. Phys. Tech. Phys. **28**, 556 (1978).
[8]H. Herr, D. Mohnl, and A. Winnacker, Proc. of a Workshop on Physics at LEAR with slow-energy cooled antiprotons. Vol. 15, Plenum, 1984, p. 659.
[9]A. S. Artamonov, Y. S. Derbenev, and E. L. Saldin, Preprint No. 79, Inst. Nucl. Phys., Siberian Div., USSR Acad. Sci., 1984.
[10]V. I. Gol'danskiĭ, Physical Chemistry of Positrons and Positronium [in Russian], Nauka, 1968.
[11]S. M. Surko, M. V. Leventhal, and W. S. Crane, Rev. Sci. Instr. **57**, 1862 (1986).
[12]E. M. Lifshitz and L. P. Pitaevskiĭ, *Physical Kinetics*, Pergamon, 1981.
[13]M. Abramowitz and I. A. Stegun, eds., *Handbook of Mathematical Functions*, Dover, 1964.
[14]V. I. Kudelaĭnen, V. A. Lebedev, I. I. Meshkof *et al.*, Zh. Eksp. Teor. Fiz. **82**, 2056 (1982) [Sov. Phys. JETP **55**, 1191 (1982).

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