

Crystal symmetry and the structure of the relaxation terms in the antiferromagnet dynamic equations of motion

V. G. Bar'yakhtar

Institute of Metal Physics, Academy of Sciences of the Ukrainian SSR, Kiev

(Submitted 27 April 1987)

Zh. Eksp. Teor. Fiz. **94**, 196–206 (April 1988)

A method of constructing relaxation terms allowing for the symmetry of the crystal lattice is described and the hierarchy of relaxation constants is noted. Expressions are obtained for dissipative functions of antiferromagnets, the symmetry of which belongs to the orthorhombic, rhombohedral, and tetragonal systems. A comparison is made of the expressions for the damping rate of spin waves calculated on the basis of a dissipation function and a Heisenberg Hamiltonian. An expression is derived for the friction coefficient of a kink-type soliton.

1. INTRODUCTION

It is well known that the lattice symmetry affects the structure of the magnetic anisotropy and magnetostriction energies of magnetic crystals.¹ For example, the magnetic anisotropy energy of crystals belonging to the triclinic symmetry arises from terms which are quadratic in the magnetization, whereas the magnetic anisotropy energy of cubic paramagnetic crystals begins with terms which are of the fourth power in the magnetization.

It is shown in Ref. 1 that the higher the power of the magnetization in the expansion describing the magnetic anisotropy energy, the lower the coefficient in front of the corresponding term. This is due to the fact that the coefficients are determined by perturbation theory ordered in powers of v/c_0 , where v is the average velocity of electrons in an atom and c_0 is the velocity of light. Therefore, the crystal symmetry governs the hierarchy of the magnetic anisotropy terms.

However, in the existing discussions of relaxation terms in the equation of motion for the magnetic moments in a crystal the crystal symmetry has not been allowed for in any way. It is usual to describe the dissipative processes employing the relaxation term $\lambda[\mathbf{M} \times [\mathbf{M} \times \mathbf{F}]]$, proposed by Landau and Lifshitz,² or the same term rewritten in the form $\alpha[\mathbf{M} \times \dot{\mathbf{M}}]$ as suggested by Gilbert.³ (In these formulas \mathbf{M} is the magnetic moment and \mathbf{F} is the effective field.) As pointed out in Ref. 2, the relaxation constant λ is due to the relativistic interactions.

Akhiezer, Peletminskii, and the present author calculated the tensor of the high-frequency magnetic susceptibility of ferromagnets by the Green's function method⁴ and found that the relaxation term in the Landau–Lifshitz equation cannot be described by a single constant even in the linear approximation. The same conclusion was reached by Gurzhi and Tsukernik⁸ who derived the Landau–Lifshitz equation by the Bogolyubov method. In our monograph⁶ it was suggested that relaxation processes in ferromagnets can be described by the tensor of relaxation constants and the tensor itself was derived using a model of a uniaxial crystal.

It is well known that in calculations of the energy and damping of spin waves by the Green's function method both these quantities are in fact governed by the same sets of diagrams for the mass operator, since the energy is determined by the real part of the mass operator and the damping by the imaginary part. The symmetry of a crystal is manifested by the actual nature of the vertices in the graphs for the mass

operator. Some of the vertices may vanish.⁶ This is the reason for the appearance of the magnetic anisotropy energy only in the second order of perturbations in the case of cubic crystals. It is therefore clear that the ideas on the hierarchy of the anisotropy constants developed by Landau and Lifshitz in deriving the magnetic anisotropy energy of a ferromagnet can be used also to obtain the dissipation function for a ferromagnet.

The present author proposed^{7–9} expressions for describing relaxation processes due to the exchange interaction in ferromagnets and antiferromagnets and in Ref. 9 an analysis was made of the hierarchy of the relaxation constants associated with relativistic interactions and governing the dissipation function of a ferromagnet.

In this paper we derive the part of the dissipation function of an antiferromagnet due to the relativistic interactions and the corresponding relaxation terms in the equations of motion. The characteristic features of antiferromagnets are then manifested in, for example, the exchange-relativistic Dzyaloshinskii interaction. We show that in establishing the hierarchy of the dissipation function constants, we have to allow not only for the direct consequences of the symmetry governing the invariance of the dissipation function, but also for indirect consequences associated with the symmetry of the Hamiltonian. We analyze the specific examples of carbonates of transition metals with the D_{3d}^6 symmetry, orthoferrites with the D_{2h}^{16} symmetry, fluorides with the D_{4h}^{14} symmetry, and a uniaxial crystal model of antiferromagnets. In all these cases we write down the relaxation terms. We determine the damping of spin waves in the uniaxial crystal model for the easy-plane magnetic anisotropy and compare the results obtained with those obtained on the damping of spin waves by the Green's function method and by applying the standard expression for the Landau–Lifshitz relaxation term. We calculate the mobility of a 180° domain wall in an antiferromagnet.

2. DISSIPATION FUNCTION

We consider the state of an antiferromagnet close to the ground state which can be described by specifying the magnetization vector $\mathbf{m}(\mathbf{r}, t)$ which varies slowly with the distance and one antiferromagnetic vector¹¹ $\mathbf{l}(\mathbf{r}, t)$. The vectors $\mathbf{m}(\mathbf{r}, t)$ and $\mathbf{l}(\mathbf{r}, t)$ are related to the magnetic moments \mathbf{M}_1 and \mathbf{M}_2 of the sublattices

$$\mathbf{m} = (\mathbf{M}_1 + \mathbf{M}_2)/2, \quad \mathbf{l} = (\mathbf{M}_1 - \mathbf{M}_2)/2. \quad (2.1)$$

The internal energy W of such a quasiequilibrium state, known from Ref. 10, is a functional of $\mathbf{m}(\mathbf{r}, t)$ and $\mathbf{l}(\mathbf{r}, t)$:

$$W = W\{\mathbf{m}, \mathbf{l}\}. \quad (2.2)$$

The time dependence of W is⁶

$$\frac{dW}{dt} = - \int \left\{ \mathbf{H}_m \frac{\partial \mathbf{m}}{\partial t} + \mathbf{H}_l \frac{\partial \mathbf{l}}{\partial t} \right\} d^3x, \quad (2.3)$$

where the effective fields H_m and H_l are given by

$$\mathbf{H}_m(\mathbf{r}, t) = - \frac{\delta W}{\delta \mathbf{m}(\mathbf{r}, t)}, \quad \mathbf{H}_l = - \frac{\delta W}{\delta \mathbf{l}(\mathbf{r}, t)}. \quad (2.4)$$

Dissipative processes increase the entropy S of the system. Therefore, we find that

$$T \frac{dS}{dt} = \int \left\{ \mathbf{H}_m \frac{\partial \mathbf{m}}{\partial t} + \mathbf{H}_l \frac{\partial \mathbf{l}}{\partial t} \right\} d^3x. \quad (2.5)$$

It is clear from the above expression that $\partial \mathbf{m}/\partial t$ and $\partial \mathbf{l}/\partial t$ can be selected as generalized fluxes and \mathbf{H}_m and \mathbf{H}_l are generalized forces in thermodynamics of irreversible processes.^{11,12} It is convenient to combine $\partial \mathbf{m}/\partial t$ and $\partial \mathbf{l}/\partial t$ into a six-dimensional flux vector $x_\alpha = \{\partial \mathbf{m}/\partial t, \partial \mathbf{l}/\partial t\}$ and the values of \mathbf{H}_m and \mathbf{H}_l into a six-dimensional vector of generalized forces $F_\alpha = \{\mathbf{H}_m, \mathbf{H}_l\}$. We shall assume that $1 \leq \alpha \leq 6$; the Latin indices vary from 1 to 3 and label Cartesian coordinate axes. Therefore, we have

$$\frac{\partial x_k}{\partial t} = \frac{\partial m_k}{\partial t}, \quad \frac{\partial x_{k+3}}{\partial t} = \frac{\partial l_k}{\partial t}, \quad F_k = H_{m,k}, \quad F_{k+3} = H_{l,k}. \quad (2.6)$$

The Onsager equations for this system are²⁾

$$(\partial x_\alpha / \partial t) = \tilde{\gamma}_{\alpha\beta} F_\beta. \quad (2.7)$$

The transport coefficients $\tilde{\gamma}_{\alpha\beta}$ satisfy the Onsager symmetry principle:

$$\tilde{\gamma}_{\alpha\beta}(s) = \tilde{\gamma}_{\beta\alpha}(-s), \quad (2.8)$$

where we have introduced $s_\alpha \equiv (\mathbf{m}, \mathbf{l})$ or

$$s_k = m_k, \quad s_{k+3} = l_k. \quad (2.9)$$

The part $\gamma_{\alpha\beta}^A = (\tilde{\gamma}_{\alpha\beta} - \tilde{\gamma}_{\beta\alpha})/2$ of the tensor $\tilde{\gamma}_{\alpha\beta}$ anti-symmetric with respect of the indices determines the nondissipative dynamics of an antiferromagnet. Since this part has been investigated fully in Refs. 13–16, we shall not consider $\tilde{\gamma}_{\alpha\beta}^A$ in detail.

The part $\gamma_{\alpha\beta} = (\tilde{\gamma}_{\alpha\beta} + \tilde{\gamma}_{\beta\alpha})/2$ of the tensor $\tilde{\gamma}_{\alpha\beta}$ symmetric in the indices α and β determines both the relaxation terms R_α in the equations of motion (2.7) and the dissipation function Q of the system:

$$R_\alpha = \gamma_{\alpha\beta} F_\beta,$$

$$Q = 1/2 \int R_\alpha F_\alpha d^3x = 1/2 \int \gamma_{\alpha\beta} F_\alpha F_\beta d^3x. \quad (2.10)$$

We can easily see that

$$R_\alpha = \delta Q / \delta F_\alpha. \quad (2.11)$$

It is convenient to expand the transport coefficients $\gamma_{\alpha\beta}$ as a series in powers of s_α . It follows from the condition (2.8)

that this expansion contains only the terms with even powers of s_α :

$$\gamma_{\alpha\beta}(s) = \gamma_{\alpha\beta}^{(0)} + 1/2 \gamma_{\alpha\beta, \delta\epsilon}^{(2)} s_\delta s_\epsilon + \dots \quad (2.12)$$

Strictly speaking, this expansion is meaningful in the vicinity of a phase transition from the paramagnetic to the antiferromagnetic state, when the quantities \mathbf{m} and \mathbf{l} (i.e., s_α) are small. As is usually done in theoretical treatments of magnetically ordered crystals, we shall assume that the expansion of Eq. (2.12) is also valid far from a phase transition. It follows from the ideas put forward in Ref. 1 and in the Introduction that the coefficients of such an expansion can be taken to be decreasing as a power of the small parameter $(v/c_0)^2$:

$$(\gamma^{(2n+2)}/\gamma^{(2n)}) \sim (v/c_0)^2. \quad (2.13)$$

Here, $2n$ is the exponent of the parameter s in the expansion of Eq. (2.12). The number of independent components of the coefficients in the expansion of Eq. (2.12) is determined by the symmetry of the given crystal in the paramagnetic phase.

Substituting Eq. (2.12) into Eq. (2.10), we obtain

$$Q = 1/2 \int \left\{ \gamma_{\alpha\beta}^{(0)} F_\alpha F_\beta + 1/2 \gamma_{\alpha\beta, \delta\epsilon}^{(2)} F_\alpha F_\beta s_\delta s_\epsilon + \dots \right\} d^3x. \quad (2.14)$$

The dissipation function Q must naturally be invariant with respect to the symmetry group of the given crystal in the paramagnetic phase. This makes it possible to formulate the following phenomenological method for the derivation of the dissipation function Q and then of the relaxation term R_α which follows from this function. First of all, we have to find the invariant (relative to the paramagnetic group of the investigated crystal) combinations of F_α and s_α which are quadratic functions of F_α , using even powers of s_α ; this should then be followed by summation of these quantities with arbitrary coefficients. These coefficients are the relaxation constants. The higher the power exponent of the parameter s_α , the smaller should be the coefficient of the corresponding term. The series of coefficients in the expansion of the dissipation function of Eq. (2.14) should be equated to zero in accordance with the law of conservation of the components (or component) of the magnetic moment.

3. DISSIPATION FUNCTION OF CARBONATES AND ORTHOFERRITES

We shall now illustrate the general ideas of the preceding section by considering specific examples. We shall discuss transition-metal carbonates with the lattice symmetry D_{3d}^6 in the paramagnetic phase. We shall select the basic symmetry elements in the form of threefold rotation h_3 about the vertical axis z , twofold rotation h_2 about the y axis perpendicular to one of the fundamental vectors of the lattice a_1 , inversion h_{13} together with a shift by half a period along the z axis, and time reversal R .

The fields H_m and H_l transform under the influence of the symmetry elements of a crystal in the same way as \mathbf{m} and \mathbf{l} on the basis of the definitions given by Eq. (2.1) and because of invariance of the internal energy. Following Dzyaloshinskii,¹⁷ we can write down the invariance of the zeroth order in s_α and of the second order in F_α :

$$H_{m+} H_{m-}, \quad H_{mz}^2, \quad H_{l+} H_{l-}, \quad H_{lz}^2, \quad H_{l+} H_{m-} - H_{l-} H_{m+}. \quad (3.1)$$

The density q of the dissipation function considered to lowest order in s is

$$q_r = \{\lambda_{11} \mathbf{H}_{m\perp}^2 + \lambda_{13} \mathbf{H}_{mz}^2 + \lambda_{21} \mathbf{H}_{l\perp}^2 + 2\lambda_{21} [\mathbf{H}_l \mathbf{H}_m]_z\} / 2, \quad (3.2)$$

where $\mathbf{H}_l = H_x \mathbf{e}_x + H_y \mathbf{e}_y$ and $\mathbf{e}_x, \mathbf{e}_y$, and \mathbf{e}_z are unit vectors along the coordinate axes. The last term in the above expression is a complete analog of the Dzyaloshinskii energy component of the internal energy of an antiferromagnet. Equation (3.2) does not include a term $\lambda_{23} H_{lz}^2$, because the exchange part of the dissipation function [see Eq. (3.9)] contains a term $\lambda_{21} \mathbf{H}_l^2$ so that the constants λ_{21} and λ_{23} cannot be regarded as independent.

It should be noted that Eq. (3.2) does not describe the anisotropy in the basal (x, y) plane. In this approximation (representing the constant term of the expansion in the parameter s_α) the dissipation function density is constant for an arbitrary homogeneous rotation about the symmetry axis z . In other words, Eq. (3.2) for q corresponds to the model of a uniaxial crystal. We can allow for the anisotropy in the basal plane by including invariants of the type $(l_+^4 \mathbf{H}_{m+}^2 + l_-^4 \mathbf{H}_{m-}^2)$, i.e., invariants of the fourth order in s_α and of the second order in F_α .

So far we have applied symmetry considerations only in writing down the expression for the dissipation function. Consequently, all the relaxation constants λ can be regarded as being of the same order of magnitude. We now show that allowance for the symmetry of the internal energy can provide additional information on the relative values of the relaxation constants: $\lambda_{13} \ll \lambda_{11}, \lambda_{21}$.

We recall the energy density in an antiferromagnet^{17,18,6}:

$$W = \frac{1}{2} \{f(\mathbf{m}^2 + \mathbf{l}^2 + 2\mathbf{m}\mathbf{l}) + f(\mathbf{m}^2 + \mathbf{l}^2 - 2\mathbf{m}\mathbf{l}) + F(\mathbf{m}^2 - \mathbf{l}^2) + \alpha(\partial\mathbf{l}/\partial x_i)^2 + D[\mathbf{l}\mathbf{m}]_z + K_2 l_z^2 + (K_4/2i) l_z(l_+^3 - l_-^3) - \frac{1}{3} K_6(l_+^6 + l_-^6)\} - 2\mathbf{m}\mathbf{H}_0. \quad (3.3)$$

In the above expression the term $f(x^2)$ represents the exchange energy governing the magnetic moment of a sublattice and $F = (2/\chi_\perp)(\mathbf{m}^2 - \mathbf{l}^2)$ is the energy of the exchange interaction between the sublattices. In the approximation of the Landau thermodynamic potential, we have

$$f(x^2) = (1/2\chi_\parallel M_0^2)(x^2 - M_0^2)^2,$$

where M_0 is the equilibrium value of the magnetization of a sublattice in an antiferromagnet; $M_1^2 = M_2^2 = M_0^2$; χ_\parallel and χ_\perp are the transverse and longitudinal (relative to the symmetry axis) components of the magnetic susceptibility. The other quantities in Eq. (3.3) have the following meaning: α is the inhomogeneous exchange interaction; K_2, K_4 , and K_6 are the magnetic anisotropy constants (K_4 and K_6 describe the anisotropy on the basal plane); D is the Dzyaloshinskii constant. We can go over from Eq. (3.3) to the expression for the energy of a uniaxial crystal by assuming that the constants K_4 and K_6 vanish. In the exchange approximation (corresponding to the rotation group symmetry) we have to assume that not only K_4 and K_6 vanish, but also the constants K_2 and D .

The equations of motion for the vectors \mathbf{m} and \mathbf{l} are

$$\begin{aligned} \partial\mathbf{m}/\partial t &= g[\mathbf{m}\mathbf{H}_m] + g[\mathbf{l}\mathbf{H}_l] + \mathbf{R}_m, \\ \partial\mathbf{l}/\partial t &= g[\mathbf{m}\mathbf{H}_l] + g[\mathbf{l}\mathbf{H}_m] + \mathbf{R}_l, \end{aligned} \quad (3.4)$$

where $g = (|e|/2mc)$ is the gyromagnetic ratio, and \mathbf{R}_m and \mathbf{R}_l are the relaxation terms described by

$$\mathbf{R}_m = \delta Q / \delta \mathbf{H}_m, \quad \mathbf{R}_l = \delta Q / \delta \mathbf{H}_l.$$

The first two terms on the right-hand sides of the equations in the system (3.4) describe the nondissipative dynamics of an antiferromagnet.

A general property of the equations in the system (3.4) is the law of conservation of the vector representing the total magnetic moment of a sample in the exchange approximation, or its component along the anisotropy axis in the uniaxial crystal approximation. It is known that conservation of additive quantities

$$\vec{\mathcal{M}} = \int \mathbf{m} d^3x, \quad \mathcal{M}_z = \int m_z d^3x$$

implies the existence of equations of continuity for the densities of these quantities. For this reason, the first equation in the system (3.4) considered in the exchange approximation should be of the form¹⁹

$$\partial m_i / \partial t + \partial \Pi_{ik} / \partial x_k = 0, \quad (3.4a)$$

whereas in the uniaxial crystal approximation, it should be

$$\partial m_z / \partial t + \partial \Pi_{zk} / \partial x_k = 0. \quad (3.4b)$$

In these formulas the quantity Π_{ik} is the tensor representing the flux of the i th component of the magnetization m crossing a unit area perpendicular to the k th axis. The tensor consists of two parts: dynamic and dissipative. The dynamic part of Π_{ik} corresponds to the first two terms in the first of the equations in the system (3.4), whereas the dissipative part corresponds to the relaxation term \mathbf{R}_m .

These properties are manifested directly in the relaxation constants. We shall assume here the uniaxial crystal approximation. We then have

$$\begin{aligned} \mathbf{H}_m &= \left[\frac{2}{\chi_\parallel M_0^2} (\mathbf{m}^2 + \mathbf{l}^2 - M_0^2) + \frac{2}{\chi_\perp} \right] \mathbf{m} + \frac{4}{\chi_\parallel M_0^2} (\mathbf{m}\mathbf{l}) + D[\mathbf{e}_z], \\ \mathbf{H}_l &= \left[\frac{2}{\chi_\parallel M_0^2} (\mathbf{m}^2 + \mathbf{l}^2 - M_0^2) - \frac{2}{\chi_\perp} \right] \mathbf{l} \\ &+ \frac{4}{\chi_\parallel M_0^2} (\mathbf{m}\mathbf{l}) - D[\mathbf{e}_z\mathbf{m}] + K_2 \mathbf{e}_z l_z - \alpha \Delta \mathbf{l}. \end{aligned} \quad (3.5)$$

Following Eq. (3.2), we find that

$$\begin{aligned} \mathbf{R}_m &= \lambda_{ik} \left(\partial^2 \mathbf{H}_m / \partial x_i \partial x_k \right) + \lambda_{11} \mathbf{H}_{m\perp} + \lambda_{13} \mathbf{e}_z H_{mz} + \lambda_{21} [\mathbf{e}_z, \mathbf{H}_l], \\ \mathbf{R}_l &= \lambda_{21} \mathbf{H}_l + \lambda_{21} \mathbf{H}_{l\perp} - \lambda_{21} [\mathbf{e}_z, \mathbf{H}_m]. \end{aligned} \quad (3.6)$$

In writing down the formulas for \mathbf{R} we allowed for the contribution of the exchange dissipation function⁸

$$Q_e = \frac{1}{2} \int d^3x \left\{ \lambda_{ik} \frac{\partial \mathbf{H}_m}{\partial x_i} \frac{\partial \mathbf{H}_m}{\partial x_k} + \lambda_{21} \mathbf{H}_l^2 \right\}. \quad (3.7)$$

Substituting the equations from the system (3.7) into the dynamic part of the equation of motion \mathbf{m} , we find that in the nondissipative approximation the component m_z satisfies Eq. (2.4b), where

$$\Pi_{zk} = \alpha g [\mathbf{l}(\partial\mathbf{l}/\partial x_k)]_z. \quad (3.8)$$

The relaxation term \mathbf{R}_m should not alter the nature of Eq. (3.4b) for a uniaxial crystal. We can easily see that this condition is satisfied if $\lambda_{13} = 0$. In the uniaxial crystal approximation ($K_4 = K_6 = 0$) the requirement $\lambda_{13} = 0$ reflects the circumstance that the relaxation constant λ_{13} is determined by the same microscopic interactions as the constants describing the anisotropy in the basal plane. Therefore, we cannot assume that $K_4 = 0$ and $K_6 = 0$ and at the same time postulate that $\lambda_{13} \neq 0$, as one could have done on the basis of the requirement of invariance of q in the case of uniaxial crystals. An allowance for the anisotropy in the basal plane or an allowance for the magnetostriction energy, when the phonon system with its own orbital momentum is included, implies $\lambda_{13} \neq 0$. However, since $K_6 \ll K_4 \ll K_2$ and the magnon-phonon interaction is weak, it follows that $\lambda_{13} \ll \lambda_{11}$. The considerations leading to the conclusion $\lambda_{13} = 0$ in the uniaxial crystal approximation are a manifestation of the symmetry of the Hamiltonian in the properties of the relaxation constants, mentioned in the Introduction. In the exchange approximation described by $K_2 = K_4 = K_6 = D = 0$ and $\lambda_{11} = \lambda_{13} = \lambda_{21} = \lambda_d = 0$ the equation of motion for m should be Eq. (3.4a). We can readily see from Eqs. (3.4), (3.5), and (3.6) that this requirement is satisfied and the internal magnetic friction (represented by the first term in the expression for \mathbf{R}_m) does not violate the conservation of the total magnetic moment of a sample.

We should note another circumstance associated with the integrals of the equations of motion [Eq. (3.4)]. In the nondissipative approximation ($\mathbf{R}_m = \mathbf{R}_l = 0$) these equations have two local integrals of motion:

$$m^2(x, t) + I^2(x, t) = C_2(x), \quad m(x, t)l(x, t) = C_1(x). \quad (3.9)$$

Usually these two integrals are assumed to be not local but global, i.e., they do not depend on the coordinate x and can be assumed to be equal to their equilibrium values: $C_1(x) = 0$ and $C_2(x) = M_0^2$, where M_0 is the magnetic moment of a sublattice. This procedure is equivalent to the assumption that the magnetic moments of the sublattices are equal and constant, $M_1^2 = M_2^2 = M_0^2$, not only under equilibrium conditions but also under dynamic conditions. We can demonstrate that the integrals of motion $C_1(x)$ and $C_2(x)$ are lost when the dissipation terms are included. The relaxation of these integrals to their equilibrium values is governed by the relaxation constant λ_{21}^c .

We now consider orthoferrites with the symmetry group D_{2h}^{16} . We limit our treatment by employing only a two-sublattice model of the antiferromagnet. The same procedure as for a crystal with the D_{3d}^6 symmetry shows that the density of that part of the dissipation function which is due to the relativistic interactions amounts to

$$q_r = \{ \lambda_{11} H_{mx}^2 + \lambda_{12} H_{my}^2 + \lambda_{13} H_{mz}^2 + \lambda_{21} H_{ix}^2 + \lambda_{22} H_{iy}^2 + 2\lambda_{1d} H_{mx} H_{ix} + 2\lambda_{2d} H_{mz} H_{iz} \} / 2. \quad (3.10)$$

The x , y , and z axes are selected along the twofold axes. In writing down the above dissipation equation we allowed for the fact that the term $\lambda_{21}^c H_l^2$ is already included in the exchange part of q_e of the dissipation function of Eq. (3.7). Equation (3.10) readily yields the relaxation terms in the equations of motion for \mathbf{m} and \mathbf{l} . For example,

$$\begin{aligned} R_{mx} &= (\partial q / \partial H_{mx}) = \lambda_{11} H_{mx} + \lambda_{1d} H_{ix}, \\ R_{ix} &= (\partial q / \partial H_{ix}) = \lambda_{21} H_{ix} + \lambda_{2d} H_{mz}. \end{aligned} \quad (3.11)$$

Since the symmetry group D_{2h}^{16} belongs to the orthorhombic system, all the relaxation parameters in Eq. (3.10) are of the same order of magnitude.

We conclude this section by quoting the expressions for the dissipation function of transition-metal fluorides (MnF_2) with the symmetry group D_{4h}^{14} . In this case, we have

$$q_r = \{ \lambda_{11} \mathbf{H}_{m\perp}^2 + \lambda_{13} H_{mz}^2 + \lambda_{21} \mathbf{H}_{l\perp}^2 + 2\lambda_d (H_{mx} H_{ly} + H_{my} H_{lx}) \} / 2. \quad (3.12)$$

The z axis is selected along the fourfold axis, whereas the x and y axes are oriented along the twofold axes. For the same reasons as in the case of Eq. (3.2), we retained only the term $\mathbf{H}_{l\perp}^2$. In Eq. (3.12) the smallest relaxation constant is λ_{13} . The other relaxation constants are of the same order of magnitude. The relaxation terms corresponding to Eq. (3.12) are

$$\begin{aligned} \mathbf{R}_m &= \lambda_{11} \mathbf{H}_{m\perp} + \lambda_{13} \mathbf{e}_z H_{mz} + \lambda_d (\mathbf{e}_x H_{ly} + \mathbf{e}_y H_{lx}), \\ \mathbf{R}_l &= \lambda_{21} \mathbf{H}_{l\perp} + \lambda_d (\mathbf{e}_x H_{my} + \mathbf{e}_y H_{mx}). \end{aligned} \quad (3.13)$$

4. DAMPING OF SPIN WAVES AND DECELERATION OF KINK-TYPE SOLITONS

If we know the dissipation function, we can readily calculate the damping of spin waves. We can do this by finding the expression for the amplitudes \mathbf{m}_i and \mathbf{l}_i corresponding to a spin wave of frequency $\omega_i(k)$ with a wave vector k and then use

$$\Gamma_i(k) = (q/2\omega)_{m_i, l_i}. \quad (4.1)$$

An antiferromagnet with the easy-plane magnetic anisotropy is characterized by natural oscillation frequencies^{18,20}

$$\begin{aligned} \omega_1^2(k) &= s^2 k^2 + (2g)^2 H_0 (H_0 + H_D), \\ \omega_2^2(k) &= s^2 k^2 + 2g^2 H_A H_E + (2g)^2 H_D (H_0 + H_D), \end{aligned} \quad (4.2)$$

where

$$2H_D = D l_0, \quad H_E = 2l_0 / \chi_{\perp}, \quad H_A = K_2 l_0, \quad s^2 = (2gl_0)^2 \alpha / \chi_{\perp}.$$

The above formulas are derived ignoring the anisotropy in the basal plane, i.e., assuming that $K_4 = K_6 = 0$. The directions of the vectors \mathbf{l}_0 and \mathbf{m}_0 [$m_0 = \chi_{\perp} (H_0 + H_D) / 2$] is determined by an external magnetic field which is directed along the twofold axis (y axis). Namely, the vector \mathbf{l}_0 is parallel to the x axis, whereas the vector \mathbf{m}_0 directed along the y axis. Activation of the frequency of spin waves ω_1 is then governed by the external magnetic field H_0 . Inclusion of the basal-plane anisotropy gives rise to activation of the frequency ω_1 also in the absence of an external magnetic field. If $K_6 < 0$, then $\mathbf{l}_0 \parallel \mathbf{x}$ and $\mathbf{m}_0 \parallel \mathbf{y}$ and for ω_i we have

$$\omega_1^2(k) = s^2 k^2 + (2g)^2 H_0 (H_0 + H_D) + 2g^2 H_E (H_0 + H_A^2 H_A^{-1}), \quad (4.2')$$

where $H_6 = 10|K_6|l_0^5$ and $H_4 = 3K_4 l_0^3 / 2$. Inclusion of the magnetic anisotropy energy in the basal plane alters also the frequency ω_2 . The change reduces to a slight [of order H_6 / H_E^2] renormalization of the activation frequency $\omega_2(0)$ and we shall ignore it.

Using Eqs. (3.2), (3.3), (3.4), (3.7), and (4.1) we find that the damping $\Gamma_1(k)$ of spin waves of frequency ω_1 , con-

sidered in the uniaxial crystal approximation ($K_4 = K_6 = \lambda_{13} = 0$) is described by the following expressions:

$$\begin{aligned}\Gamma_1(k) &= \Gamma_{1e}(k) + \Gamma_{1r}(k), \\ \Gamma_{1e}(k) &= (k^2/2\chi_{\perp}) \{ \lambda_{1e} [1 + (2gH_0/\omega_1)^2] + \lambda_{2e} (sk/\omega_1)^2 \}, \\ \Gamma_{1r}(k) &= (2/\chi_{\perp}) \{ \lambda_{11} (gH_0/\omega_1)^2 + \lambda_{12} (gH_E^*/\omega_1)^2 (ak)^4 \\ &\quad + 2\lambda_d (g^2 H_0 H_E/\omega_1^2) (ak)^2 \},\end{aligned}\quad (4.3)$$

where

$$\lambda_{1e} = \lambda_{ik} (k_i k_k / k^2), \quad \lambda_{2e} = \lambda_{21} l_0 \alpha \chi_{\perp} / 4, \quad H_E^* = l_0 (\alpha / a^2) / 2.$$

It is clear from these relations that if $H_0 = 0$, the damping of spin waves of frequency $\omega_1 = sk$ tends to zero proportionally to k^2 :

$$\Gamma_1(k) = \frac{1}{2} (\lambda_{1e} + \lambda_{2e}) k^2 / \chi_{\perp}. \quad (4.4)$$

Therefore, the ratio Γ_1/ω_1 is described by

$$\Gamma_1/\omega_1 = \frac{1}{2} (\lambda_{1e} + \lambda_{2e}) k / s \chi_{\perp}. \quad (4.5)$$

We can see that in the long-wavelength limit ($k \rightarrow 0$) we have $\Gamma_1 \ll \omega_1$.

If we allow for the anisotropy in the basal plane ($K_6 \neq 0$) and the relaxation constant λ_{13} , then for $H_0 = 0$ the expression Γ_1 includes additional terms

$$\Delta \Gamma_{1r} = \lambda_{13} / 2\chi_{\perp} + \lambda_{21} H_0 / l_0 \approx \lambda_{13} / 2\chi_{\perp}, \quad (4.6)$$

which are independent of the wave vector. The requirement $\Gamma_1 \ll \omega_1$ leads to the condition

$$\lambda_{13} \ll g l_0 (H_0 / H_E)^{1/2}. \quad (4.7)$$

We can find the damping $\Gamma_2(k)$ of spin waves of frequency ω_2 as follows:

$$\begin{aligned}\Gamma_2(k) &= \Gamma_{2e}(k) + \Gamma_{2r}(k), \\ \Gamma_{2e} &= [\chi_{\perp} / 8 (2gl_0)^2] \{ \lambda_{1e} [\omega_2^2 + (2gH_D)^2] + \lambda_e k^2 (2gH_E)^2 \}, \\ \Gamma_{2r} &= (\chi_{\perp} / 8l_0^2) \{ \lambda_{11} H_E^2 + \lambda_{12} H_D^2 - 2\lambda_d H_D H_E \}.\end{aligned}\quad (4.8)$$

It is clear from Eqs. (4.3) and (4.8) that for $k = 0$ the damping Γ_1 of nonactivated spin waves vanishes and the damping Γ_2 of spin waves with an activation frequency differs from zero. The results of Eqs. (4.3) and (4.8) representing the dependence of the damping factors $\Gamma_1(k) \sim k^2$ and $\Gamma_2(k) \approx \Gamma_2(0)$ on the wave vector at low values of k for an antiferromagnet with the easy-plane magnetic anisotropy, are in full agreement with the calculations of these coefficients by the Green's function method, when the initial Hamiltonian of an easy-plane antiferromagnet is the Hamiltonian with the corresponding magnetic anisotropy energy (see Ref. 21). The results $\Gamma_1(0) = 0$ and $\Gamma_2(0) \neq 0$ are in agreement with the Adler theorem for systems with a continuous parameter ground-state degeneracy parameter (angle φ_0 , governing the position of the vector \mathbf{l}_0 in the basal plane).

This result ($\Gamma_1(0) = 0$, $\Gamma_2(0) \neq 0$) for an easy-plane antiferromagnet means that the relaxation of the longitudinal and transverse components of the vector \mathbf{m} depends on the symmetry. We recall that for $\mathbf{l}_0 \parallel \mathbf{x}$ and $\mathbf{m}_0 \parallel \mathbf{y}$ the normal coordinates for oscillations of frequency $\omega_1(k)$ are repre-

sented by a superposition of the components m_z and l_y , whereas the normal coordinates for the frequency $\omega_2(k)$ are represented by a superposition of the components m_x and l_z . Consequently, we have

$$\begin{aligned}m_z(k) &\sim \exp[i(\mathbf{kx} - \omega_1(k)t) - \Gamma_1(k)t], \\ m_y(k) &\sim \exp[i(\mathbf{kx} - \omega_2(k)t) - \Gamma_2(k)t].\end{aligned}$$

Hence, we can see that the relaxation time of the longitudinal component $m_z(k)$ is $\tau_1(k) = 1/\Gamma_1(k)$, whereas the relaxation time of the transverse component $m_y(k)$ is $\tau_2(k) = 1/\Gamma_2(k)$. Assuming that $k = 0$ and bearing in mind that $\omega_1(0) = \Gamma_1(0) = 0$, we can demonstrate that the homogeneous part of m_z does not vary with time [law of conservation $\mathcal{M}_z = m_z(0)V$] and the homogeneous part of m_y relaxes at a rate characterized by the time constant $\tau_2(0) = 1/\Gamma_2(0)$. Therefore, the results of a calculation²¹ of the damping times of spin waves considered in a microscopic theory can be interpreted as proof of the different nature of the relaxation of the components m_z and m_y .

A calculation of the damping $\Gamma_e(k)$ due to the exchange interaction of spin waves with one another, carried out using the Green's function method and the Heisenberg Hamiltonian for an antiferromagnet with the b.c.c. lattice at temperatures $T \ll \Theta_C$, gives²¹⁻²³

$$\Gamma_e(k) \approx 0.1 \frac{J}{\hbar s_0^{3/2}} \left(\frac{\hbar \omega_1}{\Theta_C} \right)^2 \left(\frac{T}{\Theta_C} \right)^3 \ln \left(\frac{T}{\hbar \omega_1} \right), \quad (4.9)$$

where $\hbar \omega_1 = \Theta_C(ak)$; $\Theta_C = 4(6s_0)^{1/2}J$; a is a lattice constant; s_0 is the spin of an atom; J is the integral of the exchange between the sublattices; T is the temperature in energy units; \hbar is the Planck constant. Comparing Eqs. (4.5) and (4.8), we can see that the phenomenological approach makes it possible to allow for the dependence of the damping on the wave vector with logarithmic accuracy. Truncating from below the frequency ω_1 by the frequency of the dipole oscillation $\omega_m = \chi_{\perp} g M_0$, we obtain

$$\lambda_{1e} + \lambda_{2e} \approx \frac{\chi_{\perp} J a^2}{\hbar} \left(\frac{T}{\Theta_C} \right)^3 \ln \left(\frac{T}{\chi_{\perp} g M_0} \right). \quad (4.10)$$

We can similarly relate the phenomenological relaxation constants λ_{11} , λ_{12} , and λ_d to the constants describing the interactions of atomic spins with one another. In the long-wavelength limit ($k = 0$) it follows from Ref. 21 that the damping $\Gamma_2(0)$ corresponding to $\Delta < T \ll \Theta$ is described by

$$\begin{aligned}\Gamma_2(0) &\approx \frac{d^2}{\pi J} \left(\frac{H_D}{H_E} \right)^2 \left(\frac{\mu H_D}{\Theta_C} \right)^2 \left(\frac{T}{\Theta_C} \right) \\ &\quad + \frac{1}{48\pi} \frac{\Delta^2}{\Theta_C} \left(\frac{J}{\Theta_C} \right)^2 \left(\frac{T}{\Theta_C} \right)^2 \ln \frac{T}{\Delta}.\end{aligned}$$

Here, d is the Dzyaloshinskii constant in the Hamiltonian, $\mu = g\hbar$ and

$$D = da^3 / (2\mu)^2, \quad \mu H_E = sJ, \quad \Delta^2 = 2g^2 H_A H_E + (2g)^2 H_D (H_0 + H_D).$$

Comparing the expression for $\Gamma_2(0)$ with Eq. (4.8), we can see that the constants λ_{11} , λ_{12} , and λ_d are determined by the magnetic anisotropy constant K and the Dzyaloshinskii constant D :

$$\lambda_{11}H_x + \lambda_{12}H_D^2 - 2\lambda_d H_D H_x \approx \frac{l_0^2}{\chi_\perp} \left[\frac{d^2}{\pi J} \left(\frac{H_D}{H_x} \right)^2 \left(\frac{\mu H_D}{\Theta_c} \right)^2 \left(\frac{T}{\Theta_c} \right) + \frac{1}{48\pi} \frac{\Delta^2}{\Theta_c} \left(\frac{J}{\Theta_c} \right)^2 \left(\frac{T}{\Theta_c} \right)^2 \ln \frac{T}{\Delta} \right]. \quad (4.11)$$

It should be noted that if R_m and R_l are described by the formulas

$$R_m = \lambda \{ [m[mH_m]] + [m[lH_l]] + [l[lH_m]] + [l[mH_l]] \}, \\ R_l = \lambda \{ [m[mH_l]] + [m[lH_m]] + [l[lH_l]] + [l[mH_m]] \}, \quad (4.12)$$

which corresponds to the Landau-Lifshitz relaxation terms for each of the sublattices, we find that the ratio Γ_1/ω_1 for $H_0 = 0$ and $k \rightarrow 0$ is characterized by a divergence of the $1/k$ type. Naturally, this divergence has no physical meaning and it simply indicates that the crystal symmetry must be included in writing down the relaxation terms.

We now consider deceleration of a kink-type soliton in a uniaxial antiferromagnet with the magnetic anisotropy of the easy-axis type ($K_2 < 0$). For simplicity, we ignore the Dzyaloshinskii interaction ($D = 0$). The soliton solution of the equations of motion [Eq. (3.4)] obtained ignoring dissipation ($R_m = R_l = 0$) is²⁴

$$l = l_0(0, \sin \theta, \cos \theta), \quad m = (m_x, 0, 0), \\ m_x = \frac{1}{2} l_0 (\chi_\perp |K_2|)^{1/2} \frac{v}{s} \left(1 - \frac{v^2}{s^2} \right)^{-1/2} \sin \theta, \\ \operatorname{tg} \frac{\theta}{2} = \exp \left[-\frac{x-vt}{x(v)} \right], \quad x(v) = x_0 \left(1 - \frac{v^2}{s^2} \right)^{1/2}, \\ x_0 = \left(\frac{\alpha}{|K_2|} \right)^{1/2}, \quad (4.13)$$

where v is the soliton velocity, x is the coordinate of the center of gravity, and t is the time. This solution corresponds to the hypothesis $l_0 \gg m_x$. In the expression for the effective fields H_m and H_l , corresponding to a soliton described by Eq. (4.13) there are integrals of motions C_1 and C_2 . Using Eqs. (3.5) and (4.13), we can readily calculate the energy density w in an antiferromagnet [Eq. (3.3)] as a function of C_1 and C_2 . The minimum of w corresponds to the values

$$C_1 = 0, \quad C_2 = M_0 + \chi_{10} M_0 \left[\frac{1}{\chi_\perp} - \frac{1}{2} (\alpha \theta_x^2 - |K_2| \cos^2 \theta) \right]. \quad (4.14)$$

These values of C_1 and C_2 correspond to the following expressions for H_m and H_l :

$$H_{lx} = 0, \quad H_{ly} = -H_A \sin \theta \cos^2 \theta \left(\frac{v}{s} \right)^2 \left(1 - \frac{v^2}{s^2} \right)^{-1}, \\ H_{lx} = H_A \cos \theta \sin^2 \theta \left(\frac{v}{s} \right)^2 \left(1 - \frac{v^2}{s^2} \right)^{-1}, \\ H_{mx} = \frac{2}{\chi_\perp} \sin \theta M_0 \left(\frac{|K_2|}{\chi_\perp} \right)^{1/2} \frac{v}{s} \left(1 - \frac{v^2}{s^2} \right)^{-1/2}, \quad H_{my} = H_{mz} = 0. \quad (4.15)$$

The above formulas are derived using the expressions in Eq. (4.13). It follows from them that if $v \ll s$, the effective field H_m is much higher than the field H_l . Therefore, in calculat-

ing the density of the dissipation function of a soliton it is sufficient to consider only terms with the field H_m :

$$q \approx {}^{1/2} \lambda_{11} \left(\frac{dH_{mx}}{dx} \right)^2 + {}^{1/2} \lambda_{11} H_{mx}^2$$

or

$$q = \frac{2}{\chi_\perp} |K_2| M_0^2 \left(\frac{v}{s} \right)^2 \left(\frac{\lambda_{11}}{x_0^2} \sin^2 \theta \cos^2 \theta + \lambda_{11} \sin^2 \theta \right).$$

Integrating this expression with respect to the coordinate x , we find that

$$Q = \int q dx = \gamma v^2,$$

where

$$\gamma = \frac{2}{\chi_\perp} \left(\frac{\mathcal{E}_0}{s^2} \right) \left(\lambda_{11} + \frac{\lambda_{11}^e}{3x_0^2} \right), \quad \mathcal{E}_0 = 2x_0 |K_2| M_0^2. \quad (4.16)$$

The coefficient γ represents the friction coefficient of a soliton, $F_{fr} = -\gamma v$, \mathcal{E}_0 is the energy of a soliton at rest, and $\mathcal{E}_0/s^2 \equiv m_*$ is the effective mass of a soliton.

We recall that the energy of a soliton described by the expressions in Eq. (4.13) is

$$\mathcal{E} = \mathcal{E}_0 \left(1 - \frac{v^2}{s^2} \right)^{-1/2} \approx \mathcal{E}_0 + \frac{m_* v^2}{2}. \quad (4.17)$$

The motion of a soliton in a medium with friction gives rise to an exponential reduction of its velocity. The relaxation time τ of the soliton velocity is readily found to be

$$\tau = (m_*/\gamma) = {}^{1/2} \chi_\perp (\lambda_{11} + \lambda_{11}^e/3x_0^2)^{-1}.$$

The author is deeply grateful to A. F. Andreev, I. V. Bar'yakhtar, B. A. Ivanov, and M. I. Kaganov for discussions.

¹⁾The assumption that there is only one antiferromagnetic vector means that we are limiting the discussion to two sublattice antiferromagnets.

²⁾In these equations we are ignoring the spatial dispersion of the transport coefficients, which plays an important role in describing the relaxation processes of the exchange origin, as was done in Ref. 8.

¹⁾L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, 2nd ed., Pergamon Press, Oxford (1984).

²⁾L. D. Landau and E. M. Lifshitz, *Phys. Z. Sowjetunion* **8**, 153 (1935) [see also L. D. Landau, *Collected Works* [in Russian], Nauka, Moscow (1969), p. 128].

³⁾T. L. Gilbert, *Phys. Rev.* **100**, 1243 (1955).

⁴⁾I. A. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskii, *Zh. Eksp. Teor. Fiz.* **40**, 365 (1961) [*Sov. Phys. JETP* **13**, 249 (1961)].

⁵⁾R. N. Gurzhi and V. M. Tsukernik, *Zh. Eksp. Teor. Fiz.* **43**, 977 (1962) [*Sov. Phys. JETP* **16**, 692 (1963)].

⁶⁾A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskii, *Spin Waves*, North-Holland, Amsterdam; Wiley, New York (1968); V. G. Bar'yakhtar, V. N. Krivoruchko, and D. A. Yablonskii, *Green Functions in the Theory of Magnetism* [in Russian], Naukova Dumka, Kiev (1984).

⁷⁾V. G. Bar'yakhtar, *Zh. Eksp. Teor. Fiz.* **87**, 1501 (1984) [*Sov. Phys. JETP* **60**, 863 (1984)].

⁸⁾V. G. Bar'yakhtar, *Fiz. Nizk. Temp.* **11**, 1198 (1985) [*Sov. J. Low Temp. Phys.* **11**, 662 (1985)].

⁹⁾V. G. Bar'yakhtar preprint No. ITF-86-114 R [in Russian] Institute of Metal Physics, Academy of Sciences of the Ukrainian SSR, Kiev (1986).

¹⁰⁾M. I. Kaganov and V. M. Tsukernik, *Zh. Eksp. Teor. Fiz.* **34**, 106 (1988) [*Sov. Phys. JETP* **7**, 73 (1958)]; E. A. Turov and Yu. P. Irkhin, *Izv. Akad. Nauk SSSR Ser. Fiz.* **22**, 1168 (1958).

¹¹⁾Yu. M. Gufan, *Zh. Eksp. Teor. Fiz.* **60**, 1537 (1971) [*Sov. Phys. JETP* **33**, 831 (1971)].

¹²⁾Yu. M. Gufan, A. S. Prokhorov, and E. G. Rudashevskii, preprint No.

- 61 [in Russian], Lebedev Physics Institute, Academy of Sciences of the USSR, Moscow (1979).
- ¹³I. E. Dzyaloshinskii and B. G. Kukharenko, *Zh. Eksp. Teor. Fiz.* **70**, 2360 (1976) [*Sov. Phys. JETP* **43**, 1232 (1976)].
- ¹⁴I. V. Bar'yakhtar and B. A. Ivanov, *Fiz. Nizk. Temp.* **5**, 759 (1979) [*Sov. J. Low Temp. Phys.* **5**, 361 (1979)].
- ¹⁵A. F. Andreev and V. I. Marchenko, *Usp. Fiz. Nauk* **130**, 39 (1980) [*Sov. Phys. Usp.* **23**, 21 (1980)].
- ¹⁶D. A. Yablonskii, *Fiz. Tverd. Tela (Leningrad)* **22**, 3236 (1980) [*Sov. Phys. Solid State* **22**, 1892 (1980)].
- ¹⁷I. E. Dzyaloshinskii, *Zh. Eksp. Teor. Fiz.* **32**, 1547 (1957) [*Sov. Phys. JETP* **5**, 1259 (1957)].
- ¹⁸A. S. Borovik-Romanov, *Progress in Science* [in Russian], Fizmatgiz, Moscow (1962), p. 7.
- ¹⁹E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics*, Vol. 2, 3rd ed., Pergamon Press, Oxford (1980).
- ²⁰E. A. Turov, *Physical Properties of Magnetically Ordered Crystals*, Academic Press, New York (1965).
- ²¹V. G. Bar'yakhtar, V. L. Sobolev, and A. G. Kvirikadze, *Zh. Eksp. Teor. Fiz.* **65**, 790 (1973) [*Sov. Phys. JETP* **38**, 392 (1974)].
- ²²A. B. Harris, D. Kumar, B. I. Halperin, and P. C. Hohenberg, *J. Appl. Phys.* **41**, 1361 (1970).
- ²³A. B. Harris, D. Kumar, B. I. Halperin, and P. C. Hohenberg, *Phys. Rev. B* **3**, 961 (1971).
- ²⁴A. M. Kosevich, B. A. Ivanov, and A. S. Kovalev, *Nonlinear Magnetization Waves* [in Russian], Naukova Dumka, Kiev (1983), p. 190.

Translated by A. Tybulewicz