Generation of quasistatic magnetic fields and stimulated magnetic scattering in a plasma with frequent collisions

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The equations of strong-collision hydrodynamics are used to develop, for a plasma located in a strong electromagnetic radiation field, a theory of plasma instability to buildup of low-frequency nonpotential waves and high-frequency longitudinal and transverse waves. A novel parametric process is observed—stimulated magnetic scattering (SMS), due to effective scattering of the pump wave by growing magnetic-field fluctuations. The SMS threshold and growth rate are investigated, as well as the spectral, angular, and polarization properties of the scattered radiation.

1. INTRODUCTION

From among the physical phenomena due to parametric instabilities, much attention is attracted by processes such as generation of quasistatic magnetic fields and stimulated scattering of electromagnetic waves. Their study is of practical interest in connection with plasma diagnostics and with absorption of radiation and transport in a plasma acted upon by strong electromagnetic fields.

Parametric interaction of electromagnetic radiation with a plasma is substantially governed by the character of the ponderomotive action, which in turn depends on the frequency of the electron collisions. Thus, in the case of negligibly small collisions, the ponderomotive interaction between an electromagnetic field and a plasma is characterized by the Miller force.[1] At sufficiently high collision frequency, the high-frequency pressure force turns out to be different and is given by the expression derived by Perel[1] and Pinskiĭ[2] (see also the later Refs. 3 and 5). The theory developed below for parametric instabilities that lead to stimulated scattering and to excitation of quasistatic magnetic fields[6] is based just on the premises concerning the force of the high-frequency action of electromagnetic radiation on a plasma under conditions when the electron mean free path and the collision time are short compared with the characteristic spatial and temporal scales of the processes.

We start with a strong-collision theory based on the use of Grad's ten-moment method of taking into account the nonlinear (quadratic in the velocity) electron viscosity.[7] This theory yields, in particular, a correct expression for the high-frequency pressure force,[2] refuting the opinion[4,5] that ponderomotive effects in a strong-collision plasma cannot be described by using transport equations and that a kinetic approach must be used. An averaging we use below an averaging method to construct a linear theory in which it is posted that a strong-collision plasma located in the field of strong electromagnetic pump radiation is unstable to the buildup of low-frequency nonpotential and high frequency perturbations, both longitudinal and transverse, of the electromagnetic field in the plasma.

Parametric instabilities in a plasma, such as stimulated Brillouin scattering (SBS), stimulated Raman scattering (SRS), and stimulated thermal scattering (STS) have now been well investigated.[8,9] Knowledge of the laws governing these processes is the basis of remote diagnostics of a plasma acted upon by powerful electromagnetic radiation. We report here a new parametric process, viz., stimulated scattering by magnetic fluctuations (SMS). Noting that this process is possible only in a dissipative medium, we point out that this premise is at variance with the conclusion of Ref. 10 (see the Appendix).

The new parametric instability observed by us corresponds to excitation of quasistatic magnetic-field perturbations and of a high-frequency transverse wave in the plasma. The physical meaning is that the electromagnetic radiation acting on the plasma excites a dissipative quasistationary current as a result of which the pump wave is effectively scattered, leading to further increase of the current. We investigate below the SMS threshold and growth rate, and also the spectral, angular, and polarization characteristics of the scattered radiation. The SMS properties differ qualitatively from the previously known properties of SBS, SRS, and STR. Study of SMS offers promise of more detailed and hence more reliable plasma diagnostics.

We study also, in the plasma-resonance region, the nonpotential aperiodic instability corresponding to excitation of a quasistatic magnetic field and high-frequency longitudinal plasma oscillations. In contrast to SMS, which evolves in a wide range of plasma densities (both at $\omega_0 \approx \omega_{Le}$ and at $\omega_0 \approx \omega_{Le}$, where ω_0 is the pump-field frequency and ω_{Le} is the Langmuir electron frequency), the aperiodic instability (an analog of which is present also in a nondissipative medium) is excited by a narrow region $|\omega_0 - \omega_{Le}| \ll \omega_0$ near the plasma resonance.

2. DISPERSION EQUATION FOR PERTURBATIONS IN A PLASMA WITH FREQUENT COLLISIONS AND LOCATED IN A MONOCHROMATIC ELECTROMAGNETIC FIELD OF FINITE WAVELENGTH

We derive in this section the principal dispersion equation of the strong-collision hydrodynamic theory of a plasma located in a high-frequency field, neglecting the ion motion. We make the simplifying assumption that all the quantities that describe the state of the plasma and of the electromagnetic field vary little in space over distances of the order of the average displacement of an electron oscillating in the high-frequency pump field. This condition limits, on the one

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hand, the characteristic scales of the investigated perturbations, and on the other hand the amplitude and frequency of the pump field.

To bring to light the principal effects of parametric interaction between high-frequency radiation and a plasma, we confine ourselves to the following transport equations for the average electron velocity **u** and the viscous-stresses tensor π_{sj}, which appear in the ten-moment approximation of Grad's method[7]:

$$\frac{\partial n_e}{\partial t}+\operatorname{div} n_e\mathbf{u}=0, \tag{2.1}$$

$$m_e n_e\left\{\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}\nabla)\mathbf{u}\right\}_s+\nabla_s n_e T_e-\frac{\partial \pi_{sj}}{\partial r_j}+\nu_{ei}m_e n_e u_s$$
$$-en_e\left(\mathbf{E}+\frac{1}{c}[\mathbf{uB}]\right)_s=0, \tag{2.2}$$

$$\frac{\partial \pi_{sj}}{\partial t}+\frac{\partial}{\partial r_k}u_k\pi_{sj}+\pi_{sl}\frac{\partial u_j}{\partial r_l}+\pi_{lj}\frac{\partial u_s}{\partial r_l}-\frac{2}{3}\delta_{sj}\pi_{lr}\frac{\partial u_r}{\partial r_l}$$
$$+a\nu_{ei}\pi_{sj}+\frac{4}{5}\nu_{ei}m_e n_e\left(u_s u_j-\frac{1}{3}\delta_{sj}u^2\right)$$
$$-n_e T_e\left(\frac{\partial u_j}{\partial r_s}+\frac{\partial u_s}{\partial r_j}-\frac{2}{3}\delta_{sj}\operatorname{div}\mathbf{u}\right)$$
$$-\frac{eB_r}{m_e c}(e_{slr}\pi_{jl}+e_{jlr}\pi_{sl})=0. \tag{2.3}$$

Here e, m_e, n_e, and T_e are the charge, mass, density, and temperature of the electrons, c the speed of light, ν_{ei} the electron-ion collison frequency, **E** and **B** the electric and magnetic field-strength vectors in the plasma, e_{ksl} a unit antisymmetric tensor, and $a=6/5[1+(2^{1/2}Z)^{-1}]$, where Z is the ion ionization multiplicity.

In view of the presence of a monochromatic pump wave, whose electric field $\mathbf{E}_0(\mathbf{r}_0,t)$ we specify in the form

$$\mathbf{E}_0(\mathbf{r},t)=\mathbf{E}_0\cos(\omega_0 t-\mathbf{k}_0\mathbf{r}), \tag{2.4}$$

we assume that the electric field in the plasma has, besides a slow time dependence, also a fast one:

$$\mathbf{E}=\langle\mathbf{E}\rangle+\tilde{\mathbf{E}}, \tag{2.5}$$

where the angle brackets denote averaging over a time interval that is long compared with the oscillation period of the pump wave (2.4), and $\tilde{E}$ is a rapidly alternating magnetic field:

$$\tilde{\mathbf{E}}={}^1/_2\mathbf{E}_1(r,t)e^{-i\omega_0 t}+\text{c.c.}, \tag{2.6}$$

with $\mathbf{E}_1$ slowly varying over the time $1/\omega_0$.

By analogy with (2.5) and (2.6), we can distinguish between slow and fast dependences of the magnetic field, of the electron density, of the electron velocity, and of the viscous-stress tensor. We obtain then, with the aid of (2.1)–(2.3) and the Maxwell equations, a set of equations for the fast and slow quantities. After deriving the equation for the slow current $\mathbf{j}=e\langle n_e\mathbf{u}\rangle$, we solve it by assuming that the electron collision time ν_{ei}^{-1} is short compared with the characteristic time of variation of the slow quantities and that the electron mean free path v_{Te}/ν_{ei} (v_{Te} is the thermal velocity of the electrons) is short compared with the characteristic distance over which the slow quantities change in space. We assume furthermore that the electron gyrofrequency $|e|\langle B\rangle/m_e c$ is low compared with ν_{ei} and neglect the contribution made to the slow current $\mathbf{j}=e\langle n_e\mathbf{u}\rangle$ by the usual drag currents obtained, for example, in Grad's five-moment approximation, which lead to small dissipative corrections of order ν_{ei}/ω_0. Taking all the foregoing into account we obtain the following expression for the density of the quasistatic current:

$$j_k=\sigma\left\{\langle E_k\rangle-\frac{1}{en_e}\nabla_k n_e T_e-\frac{e}{4m_e\omega_0^2}\nabla_k|\mathbf{E}_1|^2\right.$$
$$-\frac{e}{5am_e\omega_0^2}\frac{\partial}{\partial r_s}\left(E_{1k}E_{1s}^*\right.$$
$$\left.+E_{1k}^*E_{1s}-\frac{2}{3}\delta_{ks}|E_1|^2\right)$$
$$-\frac{e^2}{5a^2m_e^2\omega_0^2c\nu_{ei}}\frac{\partial}{\partial r_s}(E_{1k}[\mathbf{E}_1^*\langle\mathbf{B}\rangle]_s$$
$$\left.+E_{1s}[\mathbf{E}_1^*\langle\mathbf{B}\rangle]_k+\text{c.c.})\right\}, \tag{2.7}$$

where $\sigma=e^2\langle n_e\rangle/m_e\nu_{ei}$ is the static electric conductivity of the plasma. The last two terms in the curly brackets of (2.7) are proportional to $\partial\langle\pi_{ks}\rangle/\partial r_s$, i.e., are due to the viscous-stress tensor connected with the motion of the electrons in the high-frequency field.

Relation (2.7), which the analog of Ohm's law in a plasma acted upon by high-frequency radiation, corresponds at $\langle\mathbf{B}\rangle=0$ to the following expression for the ponderomotive force:

$$F_k=\frac{e^2}{4m_e\omega_0^2}\nabla_k|\mathbf{E}_1|^2$$
$$-\frac{e^2}{5am_e\omega_0^2}\frac{\partial}{\partial r_s}\left(E_{1k}E_{1s}^*+E_{1k}^*E_{1s}-\frac{2}{3}\delta_{ks}|E_1|^2\right). \tag{2.8}$$

According to (2.8), a nonpotential component of the ponderomotive force is produced in the considered case of frequent collisions in addition to the potential high-frequency-pressure force proportional to $\nabla|E_1|^2$ and produced in the collisionless limit. Expression (2.8) agrees, apart from a numerical coefficient of the order of unity, with the result of the kinetic approach of Refs. 2–5. This points to the feasibility of a simple description of the ponderomotive effects with the aid of transport equations. The opinion advanced in Refs. 4 and 5, that it is impossible in principle to obtain the result (2.8) from the transport equation, is seen to the based on using for the viscous-stress tensor an equation in which terms quadratic in **u** are neglected.[12] It is just the allowance for these terms, first made in Ref. 13, which accounts for the nonpotential ponderomotive-force component in Eq. (2.8).

The presence of a vortical current-density component in (2.7) is evidence of the feasibility of generating a quasistatic magnetic field in a plasma. The corresponding nonlinear current

$$\delta j_k\approx\frac{\omega_{Le}^2}{\omega_0^2\nu_{ei}}\frac{\partial}{\partial r_j}(V_{Ek}E_{0j}^*+V_{Ej}^*E_{0k}),$$

where ω_{Le} is the Langmuir frequency of the electrons and $\mathbf{V}_E$ is the amplitude of the electron oscillation velocity in the pump-wave field (2.4):

$$\mathbf{V}_E = e\mathbf{E}_0/m_e\omega_0,$$

exceeds by a factor ω_0/ν_{ei} the usual drag current obtained in the five-moment approximation of Grad's method. Using the Maxwell equations and neglecting the small displacement current

$$\mathrm{rot}\,\langle\mathbf{B}\rangle = \frac{4\pi}{c}\mathbf{j}, \quad \mathrm{rot}\,\langle\mathbf{E}\rangle = -\frac{1}{c}\frac{\partial\langle\mathbf{B}\rangle}{\partial t}, \tag{2.9}$$

we obtain the aid of (2.7) and (2.9) the following equation for the magnetic field:

$$\frac{\partial\langle\mathbf{B}\rangle}{\partial t} + \mathrm{rot}\left(\frac{c^2}{4\pi\sigma}\mathrm{rot}\,\langle\mathbf{B}\rangle\right) = -\frac{ec}{20am_e\omega_0^2}\mathrm{rot}\,\mathbf{J},$$

$$J_j = \frac{\partial}{\partial r_k}\Big(E_{1j}E_{1k}^* + \text{c.c.} + \frac{e}{am_e c\nu_{ei}}$$

$$\times\{E_{1k}[\mathbf{E}_1^*\langle\mathbf{B}\rangle]_j + E_{1j}[\mathbf{E}_1^*\langle\mathbf{B}\rangle]_k + \text{c.c.}\}\Big). \tag{2.10}$$

Equation (2.10) can serve as the basis for the study of generation of quasistatic magnetic fields in a strong-collision plasma.

Separating with the aid of (2.1)–(2.3) the rapidly alternating components of the electric-current density, we obtain from the Maxwell equations the following equation for the amplitude of the electromagnetic field $\mathbf{E}_1(\mathbf{r},t)$ contained in (2.10)

$$\frac{2i}{\omega_0}\frac{\partial\mathbf{E}_1}{\partial t} + \varepsilon(\omega_0)\mathbf{E}_1 - \frac{c^2}{\omega_0^2}\mathrm{rot}\,\mathrm{rot}\,\mathbf{E}_1 = i\frac{e\omega_{Le}^2}{m_e\omega_0^3 c}[\mathbf{E}_1\langle\mathbf{B}\rangle]$$

$$-\frac{iec}{m_e\omega_0^3}\{[\mathrm{rot}\,\langle\mathbf{B}\rangle\,\mathrm{rot}\,\mathbf{E}_1] + \nabla_k(\mathbf{E}_1\,\mathrm{rot}_k\langle\mathbf{B}\rangle + E_{1k}\,\mathrm{rot}\,\langle\mathbf{B}\rangle)\}, \tag{2.11}$$

where

$$\varepsilon(\omega_0) = 1 - \frac{\omega_{Le}^2}{\omega_0^2}\left(1 - i\frac{\nu_{ei}}{\omega_0}\right)$$

is the dielectric constant. We examine now the stability of a plasma located in the high-frequency field (2.4). We neglect the perturbations of the slowly varying density $\langle n_e\rangle$, confining ourselves by the same token to nonpotential perturbations.

Introducing the electromagnetic-field perturbation $\delta\mathbf{E}$ defined by the relation

$$\mathbf{E}_1 = \mathbf{E}_0 e^{i\mathbf{k}_0\mathbf{r}} + \delta\mathbf{E}(\mathbf{r}, t)$$

and representing, as usual, the spatiotemporal relations in the form

$$\langle\mathbf{B}(\mathbf{r}, t)\rangle = \delta\mathbf{B}\exp(-i\omega t + i\mathbf{k}\mathbf{r}),$$
$$\delta\mathbf{E}(\mathbf{r}, t) = \delta\mathbf{E}_+\exp\{-i\omega t + i(\mathbf{k}+\mathbf{k}_0)\mathbf{r}\}.$$

We arrive with the aid of (2.10) and (2.11) at the following system of linear equations ($|\omega|\ll\omega_0$):

$$T_\pm\delta\mathbf{E}_\pm + c^2\mathbf{k}_\pm(\mathbf{k}_\pm\delta\mathbf{E}_\pm) = \pm i\frac{\omega_{Le}^2}{c}[\mathbf{V}_E\delta\mathbf{B}]$$

$$+ ic\{\mathbf{k}_0(\mathbf{V}_E[\mathbf{k}\delta\mathbf{B}]) \pm (\mathbf{k}_\pm\mathbf{V}_E)[\mathbf{k}\delta\mathbf{B}]\}, \tag{2.12}$$

$$\left\{-i\omega + \frac{c^2k^2}{4\pi\sigma} + \frac{2}{5}\frac{(\mathbf{k}\mathbf{v}_E)^2}{a^2\nu_{ei}}\right\}\delta\mathbf{B} - \frac{2}{5a^2\nu_{ei}}[\mathbf{k}\mathbf{V}_E](\delta\mathbf{B}[\mathbf{k}\mathbf{V}_E])$$

$$= \frac{c}{5a\omega_0}\{(\mathbf{k}\mathbf{V}_E)[\mathbf{k}, \delta\mathbf{E}_+ + \delta\mathbf{E}_-] + [\mathbf{k}\mathbf{V}_E](\mathbf{k}, \delta\mathbf{E}_+ + \delta\mathbf{E}_-)\}, \tag{2.13}$$

where we use the notation: $\mathbf{k}_\pm = \mathbf{k}\pm\mathbf{k}_0$. $T_\pm = (\omega\pm\omega_0)^2\varepsilon(\omega\pm\omega_0) - c^2k_\pm^2$.

Recognizing that Eqs. (2.12) lead to the relations

$$\delta\mathbf{E}_\pm = \pm\frac{ic}{T_\pm}\left\{(\mathbf{k}_\pm\mathbf{V}_E)[\mathbf{k}\delta\mathbf{B}] + \frac{\omega_{Le}^2}{c^2}[\mathbf{V}_E\delta\mathbf{B}] \pm \mathbf{k}_0(\mathbf{V}_E[\mathbf{k}\delta\mathbf{B}])\right\}$$

$$\mp\frac{ic\mathbf{k}_\pm}{k_\pm^2}\left(\frac{1}{T_\pm} - \frac{1}{P_\pm}\right)\left\{\frac{\omega_{Le}^2}{c^2}(\mathbf{k}_\pm[\mathbf{V}_E\delta\mathbf{B}]) \pm (\mathbf{k}_\pm\mathbf{k}_0)(\mathbf{V}_E[\mathbf{k}\delta\mathbf{B}])\right.$$

$$\left.+ (\mathbf{k}_\pm\mathbf{V}_E)(\mathbf{k}_\pm[\mathbf{k}\delta\mathbf{B}])\right\}, \tag{2.14}$$

between the field-perturbation amplitudes, we obtain after substituting (2.14) in (2.13) the dispersion relation

$$\det\left\{\delta_{ij}\left[-i\omega + \frac{c^2k^2}{4\pi\sigma} + \frac{2}{5}\frac{(\mathbf{k}\mathbf{V}_E)^2}{a^2\nu_{ei}}\right.\right.$$

$$\left. + i\frac{\omega_{Le}^2 + c^2k^2}{5a\omega_0}(\mathbf{k}\mathbf{V}_E)^2\left(\frac{1}{T_+} - \frac{1}{T_-}\right)\right]$$

$$\left. - \frac{2}{5a^2\nu_{ei}}[\mathbf{k}\mathbf{V}_E]_i[\mathbf{k}\mathbf{V}_E]_j + \frac{ic^2}{5a\omega_0}(D_{ij}^{(-)} - D_{ij}^{(+)})\right\} = 0. \tag{2.15}$$

We have introduced here the notation

$$P_\pm = (\omega\pm\omega_0)^2\varepsilon(\omega\pm\omega_0),$$

$$D_{ij}^{(\pm)} = \frac{[\mathbf{k}\mathbf{V}_E]_j}{T_\pm}\left\{\left(\frac{\omega_{Le}^2}{c^2}\mp\mathbf{k}\mathbf{k}_0\right)[\mathbf{k}\mathbf{V}_E]_i \mp (\mathbf{k}\mathbf{V}_E)[\mathbf{k}\mathbf{k}_0]_i\right\}$$

$$+\left(\frac{1}{T_\pm} - \frac{1}{P_\pm}\right)\frac{[\mathbf{k}\mathbf{k}_\pm]_i[\mathbf{k}\mathbf{k}_\pm]_j}{k_\pm^2}(\mathbf{k}\mathbf{V}_E)^2$$

$$\pm\frac{(\mathbf{k}\mathbf{k}_\pm)(\mathbf{k}_0\mathbf{k}_\pm)}{k_\pm^2}[\mathbf{k}\mathbf{V}_E]_i[\mathbf{k}\mathbf{V}_E]_j$$

$$+\frac{[\mathbf{k}_\pm\mathbf{k}_0]_i[\mathbf{k}\mathbf{V}_E]_j}{k_\pm^2}(\mathbf{k}_\pm\mathbf{k}_0)(\mathbf{k}\mathbf{V}_E)$$

$$\pm(\mathbf{k}\mathbf{k}_\pm)(\mathbf{k}\mathbf{V}_E)\frac{[\mathbf{k}_\pm\mathbf{k}_0]_j[\mathbf{k}\mathbf{V}_E]_i}{k_\pm^2}$$

$$\mp\frac{\omega_{Le}^2}{c^2}(\mathbf{k}\mathbf{V}_E)\frac{[\mathbf{k}_\pm\mathbf{k}_0]_i[\mathbf{k}_\pm\mathbf{V}_E]_j}{k_\pm^2}$$

$$-\frac{\omega_{Le}^2}{c^2}\frac{[\mathbf{k}\mathbf{V}_E]_i[\mathbf{k}_\pm\mathbf{V}_E]_j}{k_\pm^2}(\mathbf{k}\mathbf{k}_\pm). \tag{2.16}$$

The dispersion equations (2.15) describe perturbations whose wave number and frequency satisfy the conditions

$$kV_E/\omega_0 \ll 1, \tag{2.17}$$

$$|\omega| \ll \nu_{ei}, \quad kv_{Te} \ll \nu_{ei}, \tag{2.18}$$

which correspond to the case of a rather weak pump field and frequent electron collisions. We report in the next two sections the results of an analysis of this dispersion equation for various pump wavelengths (values of k_0).

3. NONPOTENTIAL APERIODIC INSTABILITY NEAR PLASMA RESONANCE ($\mathbf{k}_0 = 0$)

We consider first the case of a uniform high-frequency pump field ($\mathbf{k}_0 = 0$), which describes the evolution of the instability if the wavelength of the perturbations is small compared with the pump wavelength. This situation corresponds to resonance at the plasma frequency $\omega_0 \approx \omega_{Le}$.

At $\mathbf{k}_0 = 0$, Eqs. (2.16) yield the relation

$$D_{ij}^{(\pm)}=\frac{\omega_{Le}^2}{c^2}\frac{[\mathbf{kV}_E]_i[\mathbf{kV}_E]_j}{P_\pm},$$

the use of which in (2.16) leads to the dispersion equation

$$-i\omega+\frac{c^2k^2}{4\pi\sigma}+\frac{2}{5a^2\nu_{ei}}\{(\mathbf{kV}_E)^2-[\mathbf{kV}_E]^2\}$$
$$+i\frac{\omega_{Le}^2}{5a\omega_0}\left\{[\mathbf{kV}_E]^2\left(\frac{1}{P_-}-\frac{1}{P_+}\right)\right.$$
$$\left.+(\mathbf{kV}_E)^2\left(1+\frac{c^2k^2}{\omega_{Le}^2}\right)\left(\frac{1}{T_+}-\frac{1}{T_-}\right)\right\}=0. \quad (3.1)$$

The presence of terms proportional to $1/P_\pm$ and $1/T_\pm$ distinguishes the dispersion relation (3.1) from the one resulting from the approach of Ref. 14. The reason for this difference is that in Ref. 14 were neglected the perturbations of the high-frequency electromagnetic field. This neglect cannot be regarded as justified, since the contribution of these perturbations to the dispersion relation is in no way smaller than the corresponding contribution from the low-frequency perturbations of the magnetic field. The dispersion equation corresponding to the approach in Ref. 14 is obtained directly from (2.13) by neglecting in the latter the right-hand side (the quantities $\delta\mathbf{E}_+$ and $\delta\mathbf{E}_-$), i.e., by taking into account the excited high-frequency fields.

Taking the conditions (2.18) and $\mathbf{k}_0=0$ into account, the functions $P_\pm$ and $T_\pm$ can be approximately represented in the form

$$P_\pm=\omega_0^2\left(2\delta\pm i\frac{\nu_{ei}}{\omega_0}\right),\quad T_\pm=\pm i\nu_{ei}\omega_0^2-c^2k^2, \quad (3.2)$$

where $\delta=(\omega_0-\omega_{Le})/\omega_0\ll 1$ is the frequency deviation. From (3.1) and (3.2) we get $\omega=i\gamma$, where

$$\gamma=-\frac{c^2k^2}{4\pi\sigma}+\frac{2[\mathbf{kV}_E]^2}{5a^2\nu_{ei}}\left(1+\frac{a\nu_{ei}^2}{4\delta^2\omega_0^2+\nu_{ei}^2}\right)$$
$$-\frac{2(\mathbf{kV}_E)^2}{5a^2\nu_{ei}}\left\{1+\frac{a\nu_{ei}^2(\omega_{Le}^2+c^2k^2)}{c^4k^4+\nu_{ei}^2\omega_0^2}\right\}. \quad (3.3)$$

The instability buildup is thus aperiodic ($\mathrm{Re}\,\omega=0$).

According to (3.3), the maximum instability growth rate is reached when the perturbations propagate in a direction transverse to the pump-wave electric-field strength vector ($\mathbf{k}\perp\mathbf{V}_E$). For the growth rate that is a maximum with respect to the angle we have from (3.3)

$$\gamma=k^2\left\{\frac{2V_E^2}{5a^2\nu_{ei}}\left(1+\frac{a\nu_{ei}^2}{4\delta^2\omega_0^2+\nu_{ei}^2}\right)-\frac{c^2\nu_{ei}}{\omega_0^2}\right\}, \quad (3.4)$$

and in accordance with (3.4) the most effective instability buildup occurs at $\delta=0$, when

$$\gamma=k^2\left(\frac{2}{5}\frac{a+1}{a^2}\frac{V_E^2}{\nu_{ei}}-\frac{c^2\nu_{ei}}{\omega_0^2}\right). \quad (3.5)$$

It follows from (3.5) that the threshold pump field is given by

$$\left(\frac{V_E}{c}\right)_{\rm thr}=\left(\frac{5a^2}{2(a+1)}\right)^{1/2}\frac{\nu_{ei}}{\omega_0}\approx\frac{\nu_{ei}}{\omega_0}. \quad (3.6)$$

The instability growth rate (3.5) increases with decrease in the perturbation wavelength and is a maximum at the limit $k\approx\nu_{ei}/V_{Te}$ of the validity of the theory, and its order of magnitude is

$$\gamma_m\approx\frac{2}{5}\frac{a+1}{a^2}\frac{V_E^2}{v_{Te}^2}\nu_{ei}. \quad (3.7)$$

The first inequality in (3.18) restricts the validity of Eq. (3.7) to a region in which $V_E<v_{Te}$.

The instability considered in the present section corresponds in fact to parametric excitation, under plasma resonance conditions, of a quasistatic magnetic field and high-frequency longitudinal electron oscillations. The magnetic field generated by the instability development has an intensity vector perpendicular to the plane of the vectors $\mathbf{k}$ and $\mathbf{E}_0$, and the connection between the amplitudes of the perturbation fields is given by ($\mathbf{k}\perp\mathbf{E}_0$, $\delta=0$)

$$|\delta\mathbf{E}_\pm|=\frac{V_E}{c}\frac{\omega_0}{\nu_{ei}}|\delta\mathbf{B}|=\left[\frac{5a^2}{2(a+1)}\right]^{1/2}\frac{V_E}{V_{E,\rm thr}}|\delta\mathbf{B}|. \quad (3.8)$$

The high-frequency longitudinal fields differ in phase by π from each other and by $\pi/2$ from the quasistatic magnetic field. Relation (3.8) shows that the energy density of the high-frequency longitudinal perturbations exceeds the energy density of the quasistatic magnetic field.

Note that in the derivation of the dispersion equation for the aperiodic perturbations we have neglected the contribution of the electromagnetic fields at double the pump frequency. A more accurate calculation performed by us shows that allowance for the second harmonic of the perturbation field does not influence the results.

4. STIMULATED MAGNETIC SCATTERING (SMS)

We proceed now to a discussion of the most interesting case $\mathbf{k}_0\neq 0$ with $\omega_0\gtrsim\omega_{Le}$. The main contribution to the disperison equation (2.14) is produced then for small T_+ or T_-, when $\mathrm{Re}\,T_\pm=0$, which is reached if

$$k^2=\mp 2\mathbf{kk}_0=\mp 2kk_0\cos\varphi, \quad (4.1)$$

and it suffices to consider in (4.1) only the minus sign (which corresponds to $\mathrm{Re}\,T_+=0$), since the plus sign ($\mathrm{Re}\,T_-=0$) corresponds to simple replacement of φ by $\pi-\varphi$. Then

$$T_+=\omega_0(2\omega+i\nu_{ei}\omega_{Le}^2/\omega_0^2)$$

and the dispersion equation takes the form ($\cos\theta=(\mathbf{k}\cdot\mathbf{E}_0)/kE_0$)

$$\left(-i\omega+\frac{c^2k_0^2}{\pi\sigma}\cos^2\varphi\right)^2$$
$$+\frac{4}{5a}k_0^2V_E^2\cos^2\varphi\left(-i\omega+\frac{c^2k^2}{\pi\sigma}\cos^2\varphi\right)$$
$$\times\left\{\frac{2}{a\nu_{ei}}(3\cos^2\theta-1)+\frac{\omega_{Le}^2+4c^2k_0^2\cos^2\varphi}{\omega_{Le}^2\nu_{ei}-2i\omega\omega_0^2}(2\cos^2\theta-\sin^2\varphi)\right\}$$
$$+\frac{32k_0^4V_E^4\cos^4\varphi\cos^2\theta(2\cos^2\theta-1)}{25a^3\nu_{ei}}$$
$$\times\left(\frac{2}{a\nu_{ei}}+\frac{\omega_{Le}^2+4c^2k_0^2\cos^2\varphi}{\omega_{Le}^2\nu_{ei}-2i\omega\omega_0^2}\right)=0, \quad (4.2)$$

with $\omega=\mathrm{Re}\,\omega+i\gamma$ in (4.2), as usual.

It can be shown with the aid of (4.2) that the maximum growth rate is realized at $\theta=\pi/2$, i.e., for perturbations with a wave vector transverse to the pump electric field strength vector $\mathbf{E}_0$. In this case $\mathrm{Re}\,\omega = 0$, and we have for the growth rate

$$\gamma=\frac{4c^2k_0^2}{\omega_{Le}^2}\nu_{ei}\cos^2\varphi\left[\frac{V_E^2\omega_{Le}^2}{5ac^2\nu_{ei}^2}\left(\frac{2}{a}+\sin^2\varphi+\frac{c^2k_0^2}{\omega_{Le}^2}\sin^2 2\varphi\right)-1\right]$$
$$\times\left\{1+2\frac{c^2k_0^2\omega_0^2}{\omega_{Le}^4}\sin^2 2\varphi\left[\sin^2\varphi+\frac{2\omega_{Le}^2}{a(\omega_{Le}^2+4c^2k_0^2\cos^2\varphi)}\right]\right\}^{-1}. \qquad (4.3)$$

According to (4.3), instability sets at an excess above the threshold defined by the relation

$$\left(\frac{V_E}{c}\right)_{\rm thr}=5^{1/2}\nu_{ei}a\left[2\omega_{Le}^2+ac^2k_0^2\left(1+\frac{\omega_{Le}^2}{4c^2k_0^2}\right)^2\right]^{-1/2}. \qquad (4.4)$$

i.e., in order of magnitude we have $V_{E,\rm thr}\approx c\nu_{ei}/\omega_0$.

In accordance with the condition $\tau<\nu_{ei}$ and with Eqs. (4.3) and (4.4) we find that in the region of sufficiently dense plasma $\omega_0\approx\omega_{\rm Le}$ the expression (4.3) for the instability growth rate is valid only near the instability threshold. Depending on the excess above threshold and on the parameter a, the angle $\varphi_{\max}$ at which instability builds up most effectively is in the range $180^\circ \gtrsim \varphi_{\max} 135^\circ$. The ratio of the perturbation-field amplitudes is in this case of the order of $|\delta\mathbf{E}_+|/|\delta\mathbf{B}|\gtrsim 1$.

For a tenuous plasma $\omega_0\gg\omega_{\rm Le}$, when the threshold value of the field is given according to (4.4) by

$$(V_E/c)_{\rm thr}=5^{1/2}a\nu_{ei}/\omega_0,$$

it is possible to assess the instability buildup in a wide range of the excess above threshold, up to $V_E\gg V_{E,\rm thr}$. The corresponding instability growth rate that follows from (4.3) is

$$\gamma=\frac{\nu_{ei}\omega_{Le}^2}{2\omega_0^2}\left(\frac{1}{5a}\frac{\omega_0^2}{\nu_{ei}^2}\frac{V_E^2}{c^2}\sin^2 2\varphi-1\right). \qquad (4.5)$$

It follows from (4.5) that the maximum growth rate corresponds to $\varphi_{\max} = 135^\circ$ and is equal to

$$\gamma_{max}=\frac{\omega_{Le}^2}{10a\nu_{ei}}\frac{V_E^2}{c^2}=\frac{2\pi\sigma}{5a}\frac{V_E^2}{c^2}. \qquad (4.6)$$

According to (4.6), instability development in a tenuous-plasma region is characterized by a linear dependence of the growth rate on the pump-energy flux density.

Under conditions of optimal instability buildup, corresponding to realization of relation (4.6), we have the following connection between the amplitudes of perturbation fields in a rarefield plasma:

$$|\delta\mathbf{E}_+|=5\cdot 2^{1/2}a\frac{c}{V_E}\frac{\nu_{ei}\omega_0}{\omega_{Le}^2}|\delta\mathbf{B}|. \qquad (4.7)$$

In contrast to the instability discussed in Sec. 3, no phase shift is produced between the high-frequency field and the quasistatic magnetic field.

The nonpotential aperiodic parameter instability considered here corresponds to the onset of a field $\delta\mathbf{E}_+$ of a transverse electromagnetic magnetic wave, in fact the field of a scattered wave. The pump wave is scattered here by the perturbations produced by this wave in a quasistatic magnetic field. This is why we call this process stimulated magnetic scattering (SMS). According to (4.1), besides the scattered-wave field $\delta\mathbf{E}_+$ there is produced in SMS a scattered field $\delta\mathbf{E}_-$ corresponding to optimal instability buildup at angles $0\lesssim\varphi_{\max}\lesssim 45^\circ$ ($\varphi_{\max}\approx 45^\circ$ for $\omega_0\gg\omega_{\rm Le}$).

Thus, in SMS the angles between the scattered-wave vectors $\mathbf{k}_\pm = \mathbf{k}\pm\mathbf{k}_0$ and the vector $\mathbf{k}_0$ are close to 90° for scattering in a tenuous plasma ($\omega_0\gg\omega_{\rm Le}$) and in the range 0–90° for scattering in a sufficiently dense plasma ($\omega_0\approx\omega_{\rm Le}$). In an inhomogeneous (say, a laser) plasma this means that SMS leads to a rather broad angular distribution of the scattered ratiation. This makes it possible to relate the SMS phenomenon to the process of diffuse scattering of radiation by a laser plasma.[15,16]

Turning to Eq. (2.14) and taking (4.1) into account, we easily verify that $(\delta\mathbf{E}_+\cdot\mathbf{E}_0) = 0$. This means that SMS rotates the electromagnetic-wave polarizan wave by 90° (the vectors $\delta\mathbf{E}_\pm$ lie in the plane passing through $\mathbf{k}$ and $\mathbf{k}_0$ and perpendicular to $\mathbf{E}_0$).

Summarizing the contents of this section, we emphasize that the revealed parameteric instability corresponds to stimulated scattering of the electromagnetic pump wave by quasistatic excitations of the magnetic field. This scattering is characterized by large scattering angles and by a 90° rotation of the electromagnetic-wave polarization vector, and is not subject to a frequency shift. The aggregate of these features distinguishes the described mechanism of nonlinear scattering in a dissipative plasma from the heretofore known ones (SBS, SRS, STS).

Note that we started above with a fully ionized plasma as the model. Yet the results of the present paper can be used for a weakly ionized plasma by assuming $a = 1$ and taking ν_{ei} to mean the frequency of the collisions between electrons and neutral atoms.

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APPENDIX

The SMS phenomenon investigated in the present paper is a unique nonlinear process in which only transverse fields participate. As noted in the Introduction, this process is possible only in a dissipative medium. We prove this statement by using the hydrodynamic equations for a cold nondissipative plasma:

$$\frac{\partial\mathbf{u}}{\partial t}+(\mathbf{u}\nabla)\mathbf{u}=\frac{e}{m_e}\left(\mathbf{E}+\frac{1}{c}[\mathbf{uB}]\right). \qquad (A.1)$$

Taking into account the identity

$$(\mathbf{u}\nabla)\mathbf{u}=\nabla\frac{\mathbf{u}^2}{2}-[\mathbf{u}\,\mathrm{rot}\,\mathbf{u}] \qquad (A.2)$$

and the Maxwell equation

$$\mathrm{rot}\,\mathbf{E}=-c^{-1}\partial\mathbf{B}/\partial t \qquad (A.3)$$

we obtain after taking the curls of both sides of (A.1)

$$\partial\boldsymbol{\Omega}/\partial t=\mathrm{rot}\,[\mathbf{u}\boldsymbol{\Omega}], \qquad (A.4)$$

where $\boldsymbol{\Omega} = \mathrm{curl}\,\mathbf{u} + (e/m_0c)\mathbf{B}$ is the generalized curl. It is easily seen from (A.4) that if $\boldsymbol{\Omega}(t=0)=0$ at the initial instant, we have also $(\partial\boldsymbol{\Omega}/\partial t)_{t=0} = 0$. By successively differentiating (A.4) we verify that all the higher-order derivatives also vanish:

$$(\partial^n\boldsymbol{\Omega}/\partial t^n)_{t=0}=0,\quad n=2,3,\ldots \qquad (A.5)$$

If the function $\Omega(\mathbf{r},t)$ is analytic, this means that it is equal to zero also at $t>0$. This premise is a generalization, to the case of a cold nondissipative plasma located in an electromagnetic field, of the known Lagrange theorem of conservation of the curl of an ideal liquid.[17]

Since the plasma has in the initial state only the pump waves (2.4), it follows that

$$\Omega(\mathbf{r},t=0)=\left(\mathrm{rot}\,\mathbf{u}+\frac{e}{m_e c}\mathbf{B}\right)_{t=0}=0. \qquad (A.6)$$

According to the generalized Lagrange theorem, no motion with a nonzero generalized curl can take place in the plasma during the subsequent evolution in time.

Relation (A.6) excludes the possibility of nonlinear interaction of waves with participation of only transverse waves. In fact, calculating the curl of the current

$$\mathrm{rot}\,\mathbf{j}=en_e\,\mathrm{rot}\,\mathbf{u}+e[\nabla n_e\mathbf{u}], \qquad (A7)$$

we see that nonlinear interaction of waves in a nondissipative (cold) plasma is possible only if the motion of the medium is accompanied by perturbations of its density.

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