Resonant emission of electromagnetic waves by plasma solitons

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The ability of plasma-wave solitons to radiate electromagnetic waves at the frequency of the natural oscillations of the field is considered. It is shown that this radiation is the main energy dissipation channel for strong plasma turbulence in a magnetoactive plasma. An interpretation is proposed for the artificial radio emission produced when the ionosphere is acted upon by beams of strong electromagnetic waves. The use of this phenomenon for plasma turbulence, particularly in the outer-space plasma near the earth, is discussed.

Emission of electromagnetic waves by plasma turbulence is one of the main sources of information on the dynamics of the processes in the plasma. An important role is played in the dynamics of strong plasma turbulence by solitons. Electromagnetic radiation from clusters of plasma oscillations at harmonics of the fundamental frequency were treated in a number of earlier investigations.[1–4] The mechanism of this radiation is nonlinear, and the amplitude of the excited wave turns out to be proportional to a certain degree (not lower than the second) of the electric field in the soliton. Note that no radiation is produced in an isotropic plasma at the soliton carrier frequency, since no electromagnetic waves can propagate in such a frequency. A typical situation in a magnetoactive plasma is one in which the soliton carrier frequency coincides with the frequency of one of the propagating electromagnetic waves. It is known, for example, that in a weakly magnetized plasma with $\omega_H \ll \omega_p, \omega$ (ω_p, ω_H are the plasma and gyro frequencies of the electrons, ω is the electromagnetic-field frequency) are present two high-frequency waves; extraordinary, continuously converted into the upper-hybrid frequency along the dispersion curve, and ordinary. In an upper–hybrid soliton resulting from compensation of the nonlinearity and disperison, the electric-field oscillation frequency is lower than the resonant frequency. Propagation is therefore possible of both extraordinary and ordinary waves with spatial scales greatly exceeding the soliton size. If the electromagnetic-wave polarization is in this case such that they have, just as the soliton, a longitudinal (potential) electric-field component, a soliton having a dipole moment will effectively emit these waves.[5] A similar situation obtains also at lower-hybrid frequencies.

We consider in the present paper new important properties of solitons in a magnetoactive plasma, viz., their ability to emit electromagnetic waves at the frequencies of the natural oscillations of the electric field in the soliton. We investigate first this effect in a homogeneous weakly magnitized plasma, and then study its singularities in a inhomogeneous medium, mainly as applied to the interpretation of the experimentally observed artificial ionosphere radiation (AIR) produced when strong waves act on the F-layer. We conclude by estimating the role of this process in the earth's magnetosphere.

1. To investigate the radiation effect we use an equation for the slow complex amplitude $\mathbf{E}$ of the electric field; the equation is averaged over a frequency ω close to the upper-hybrid one:

$$-2i\omega\frac{\partial \mathbf{E}}{\partial t}+\omega^2\hat{\varepsilon}\mathbf{E}+3v_T^2\nabla\,\mathrm{div}\,\mathbf{E}=c^2\,\mathrm{rot}\,\mathrm{rot}\,\mathbf{E}. \quad (1)$$

Here $\hat{\varepsilon}$ is the dielectric tensor of the cold plasma. Its components, in view of the nonlinear deformation of the density profile, depend also on the electric-field amplitude; v_T is the thermal velocity of the electrons. Since the magnetization is stipulated to be weak, $\omega_H^2 \ll \omega^2$ and in the applications of interest to us the nonlinear-formation scale is significantly larger than the electron Larmor radius ($k\rho \ll 1$), the term with thermal spatial dispersion is of the same form as in an anisotropic plasma (see, e.g., Ref. 6).

Since purely potential waves in a magnetoactive plasma are not rigorous solutions of the Maxwell equations, we seek the solution of Eq. (1) in the form of a sum of fields of small-scale plasma waves (solitons), $\mathbf{E}^{(1)} = -\nabla\varphi$, and a large-scale increment $\mathbf{E}^{(0)}$ connected with the emitted electromagnetic waves. The potential φ satisfies the usual equation for plasma waves

$$\Delta\left(-2i\omega\frac{\partial\varphi}{\partial t}+3v_T^2\Delta\varphi\right)=-\omega^2\,\mathrm{div}\,(\hat{\varepsilon}\nabla\varphi), \quad (2)$$

if $|\mathbf{E}^{(1)}| \gg |\mathbf{E}^{(0)}|$. The perturbations of the plasma parameters depend then only on the field $\mathbf{E}^{(1)}$. Separating the vortical part in (1), we obtain an equation for the field of the emitted electromagnetic waves:

$$-2i\omega\frac{\partial \mathbf{E}^{(0)}}{\partial t}+\omega^2\hat{\varepsilon}_0\mathbf{E}^{(0)}-c^2\,\mathrm{rot}\,\mathrm{rot}\,\mathbf{E}^{(0)}=-\omega^2(\hat{\varepsilon}_0\mathbf{E}^{(1)})_v\,. \quad (3)$$

Here $\hat{\varepsilon}_0$ is the unperturbed dielectric tensor, and the subscript "v" denotes separation of the vortical part. Since the electromagnetic waves are significantly longer than the soliton size, we neglect in the derivation of (3) the influence of spatial dispersion and of the deformation of the density profile on the electromagnet-wave propagation. Introducing in lieu of $\hat{\varepsilon}_0$ the polarizability tensor $\hat{P}_0 = (\hat{\varepsilon}_0 - \hat{I})/4\pi$ ($\hat{I}$ is a unit tensor), Eq. (3) takes the form

$$\left(-2i\omega\frac{\partial}{\partial t}+\omega^2\hat{\varepsilon}_0-c^2\,\mathrm{rot}\,\mathrm{rot}\right)\mathbf{E}^{(0)}=-4\pi\omega^2(\hat{p}_0\mathbf{E}^{(1)})_v\,. \quad (4)$$

Equation (4) reveals the mechanism that produces the radiation. Owing to the anisotropy of the medium, a vortical polarization current results from the self-consistent small-scale distribution of the field and of the plasma density, and it is this current which excites the electromagnetic waves.[1)] The anisotropic character of the polarizability of the medi-

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um leads simultaneously to the appearance of a potential component of the electromagnetic-wave field, and this component matches together the fields of the soliton and of the emitted wave.

2. We consider first of all the emisison of electromagnetic waves by a one-dimensional soliton in a plasma that is homogeneous in the absence of a high-frequency field. We choose the coordinate frame such that the soliton field depends on x, and the vector $\mathbf{H}_0$ of the external magnetic field lies in (x, z) plane and makes an angle α with the x axis. For the simplest case of local cubic nonlinearity $\delta N/N_0 = -|E_x^{(1)}|^2/E_p^2$, ($\delta N/N_0$ is the relative perturbation of the plasma density and E_p is the characteristic nonlinearity field),[2] the field distribution in the soliton takes the form

$$E_x^{(1)}=\psi(x)\exp(-i\Omega t),\quad \psi(x)=A_0/\mathrm{ch}\,(\omega A_0 x/6^{1/2}v_T E_p). \tag{5}$$

The soliton is then characterized by a negative frequency shift proportional to the square of the amplitude A_0: $\Omega/\omega = A_0^2/4E_p^2$. In the one-dimensional case it is natural to assume that ω is the resonant frequency for the given α.

In the one-dimensional geometry, Eq. (4) in terms of the components along the y and z axes takes the form

$$\begin{aligned}&\frac{d^2\varepsilon_y}{d\zeta^2}+\left(-u\cos^2\alpha+\frac{u\omega}{2\Omega}\sin^2\alpha-2\frac{\Omega}{\omega}\right)\varepsilon_y\\&+iu^{1/2}\cos\alpha\left(1+\frac{u\omega}{2\Omega}\sin^2\alpha\right)\varepsilon_z=-iu^{1/2}\psi\sin\alpha,\\&\frac{d^2\varepsilon_z}{d\zeta^2}+\left(-u\cos^2\alpha+u\sin^2\alpha+\frac{u^2\omega\sin^2 2\alpha}{8\Omega}\right)\varepsilon_z\\&-iu^{1/2}\cos\alpha\left(1+\frac{u\omega}{2\Omega}\sin^2\alpha\right)\varepsilon_y=-\frac{u\psi}{2}\sin^2\alpha,\end{aligned} \tag{6}$$

where $\mathbf{E}^{(0)} = \varepsilon\exp(-i\Omega t)$, $u = \omega_H^2/\omega^2$, $\zeta = k_0 x = \omega x/c$. The left-hand side of (6) describes the propagation of the ordinary and extraordinary waves at a frequency shifted relative to the plasma resonance; the soliton field in the right-hand side is a small-scale source that excites electromagnetic waves that are emitted from it in different directions.

The system (6) can be simplified somewhat in the case of solitons of sufficiently low amplitude, when $A_0^2/E_p^2 \sim \Omega/\omega \ll u\sin^2\alpha$, i.e., precisely in the region where both normal waves can be excited. At such detunings from the upper-hybrid frequency, the scales of the extraordinary and ordinary waves differ substantially, and furthermore their polarization is such that the principal transverse field components in them are ε_y and ε_z, respectively. Using this, we can uncouple equations (6) and rewrite the system in the form

$$\begin{aligned}&\frac{d^2\varepsilon_y}{d\zeta^2}+\frac{u\omega\sin^2\alpha}{2\Omega}\varepsilon_y=-iu^{1/2}\psi\sin\alpha,\\&\frac{d^2\varepsilon_z}{d\zeta^2}+\left(u-\frac{2\Omega}{\omega\sin^2\alpha}\right)\varepsilon_z=-u\psi\sin\alpha\cos\alpha.\end{aligned} \tag{7}$$

The coefficients $n_1^2 = u\omega\sin^2(\alpha/2\Omega)$ and $n_2^2 = u - 2\Omega/\omega\sin^2\alpha$ in the system (7) approximate the dependence of the refractive indices of the extraordinary and ordinary waves at small deviations from the upper-hybrid frequency.

Integrating (7) in the dipole approximation and matching the fields $\mathbf{E}^{(1)}$ and $\mathbf{E}^{(0)}$ on the soliton wings, we determine the energy flux into the outgoing electromagnetic waves:

into the extraordinary wave

$$S_H=\frac{3\pi u v_T^2E_p^2\sin^2\alpha}{16cn_1}=\frac{3^{1/2}\pi\omega_H v_T\sin\alpha}{8c}W, \tag{8}$$

into the ordinary wave

$$S_0=\frac{3\pi u^2v_T^2\sin\alpha\cos^2\alpha}{16cn_2}=\frac{3\pi u^2v_T^2E_p^2\sin\alpha\cos^2\alpha}{16cu^{1/2}}. \tag{9}$$

Here

$$W=\int_{-\infty}^{\infty}\psi^2\,dx/8\pi=6^{1/2}v_TE_pA_0/4\pi\omega$$

is the high-frequency energy of the soliton.

If the soliton parameters change adiabatically slowly with time, the energy W is given by

$$dW/dt=-2(S_e+S_o). \tag{10}$$

The energy flux $S_{o,e}$ is proportional to the square of the dipole moment

$$p=-\int_{-\infty}^{\infty}\psi\,dx/4\pi=-3^{1/2}v_TE_p/2\cdot 2^{1/2}\omega,$$

which does not depend on the soliton parameters and is inversely proportional to the refractive index of the radiated electromagnetic wave. Since the refractive indices of the ordinary and extraordinary waves have different dependences on the frequency shift, and hence on the amplitude and energy of the soliton, emission into either wave can predominate. For solitons of sufficiently large amplitude with

$$\Omega/\omega\sim A_0^2/E_p^2\gg u^2\cos^4\alpha, \tag{11}$$

we have $S_e \gg S_o$ and the radiation into the extraordinary wave predominates. The radiative lifetime of the soliton

$$\frac{W}{dW/dt}=\tau_H=4c/3^{1/2}\pi^2v_T\omega_H\sin\alpha \tag{12}$$

is in this case a constant that depends only on the plasma parameters. This, however, is not evidence that any one-dimensional packet of plasma waves, including a linear one, will have such a lifetime, but is the consequence of the inverse proportionality of the soliton amplitude to its width.

In the opposite limiting case, the radiation is mainly into the ordinary wave. The energy flux is independent here of the soliton amplitude and the soliton amplitude decreases like $W = W_0 - 2S_o t$ (W_0 is the initial soliton energy) and becomes equal to zero after a time

$$\tau_o=2\cdot 2^{1/2}cA_0/3^{1/2}\pi^2\omega_H u v_T E_p\sin\alpha\cos^2\alpha. \tag{13}$$

The employed adiabatic approximation is valid if the shortest of the times τ_e and τ_o exceeds the characteristic time $t\sim 1/\Omega$ of the variation of the field phase in the soliton. In addition, the scale of the extraordinary electromagnetic wave must exceed substantially the soliton size. These conditions determine the lower bound of the soliton amplitude. At $v_T/c > u^{3/2}\cos^4\alpha$ the adiabaticity is violated before the radiation into the ordinary wave becomes substantial. Comparison of the times and scales leads in this case to the same result, and the applicability condition takes the form

$$\Omega/\omega \sim A_0^2/E_p^2 > (3u)^{1/2}\pi^2 v_T/4c. \quad (14)$$

If $c_T/c < u^{3/2}\cos^4\alpha$, it is easy to compare τ_0 with $1/\Omega$. The validity condition is in this case

$$A_0^3/E_p^3 > 3^{1/2}\pi^2 u^{3/2} v_T \sin\alpha \cos^2\alpha/2\cdot 2^{1/2}c. \quad (15)$$

Since the Landau-damping decrement decreases exponentially with decrease of the wave number, and hence of the soliton amplitude, the radiative lifetime τ can be significantly shorter than the time of collisionless damping of the soliton by the particles:

$$\tau \sim \frac{4c}{3^{1/2}\pi^2 v_T \omega_H} \ll \frac{A_0^3}{E_p^3\omega_p}\exp\left(\frac{3}{2}+\frac{E_p^2}{2A_0^2}\right). \quad (16)$$

The emission of electromagnetic waves is thus an additional channel for energy loss in a strong plasma turbulence; this channel is in a number of cases more effective than collisionless damping by the particles.

3. We consider now the soliton radiation in a smoothly inhomogeneous medium. Let the plasma density increase with increase of x. The characteristic scale L of its variation exceeds significantly the vacuum electromagnetic wavelength, $L \gg \lambda$. Under these conditions the extraordinary wave excited by the soliton turns out to be trapped between the points of resonance $v = \omega_p^2/\omega^2 = 1 - u\sin^2\alpha$ and reflection $v = 1 + u^{1/2}$, and the emitted extraordinary wave propagates freely into the vacuum. It is the latter wave in which we are interested below.

The problem of radiation by a soliton is similar in this case to the problem of radiation from a dipole layer near an ideally reflecting wall. At $x > x_c$ (x_c is the location of the soliton) a standing field structure with zero energy flux is established, while at $x < x_c$ the field constitutes a traveling wave in a rarefied plasma. Clearly, the soliton will start to emit more or less, depending on whether it is at the maximum or minimum of the standing-wave field. If the source frequency is changed (if the frequency shift in the soliton increases in absolute value), the reflection point and the entire field structure in the standing wave are shifted towards the less dense plasma, and the soliton is alternately located at a maximum and a minimum of the field. The energy flux is therefore an oscillating function of the frequency shift: the refractive index in the expression for this flux is replaced by a form factor connected with the restructuring of the radiated field with change of source frequency. If the density is linear in the coordinate, the expression for the energy flux is

$$S = \frac{3\pi u^2 v_T^2 E_p^2 \sin\alpha\cos^2\alpha}{4c}(k_0L\sin^2\alpha)^{2/3}V^2 \times\left[\frac{k_0L(2\Omega/\omega - u\sin^2\alpha)}{(k_0L\sin^2\alpha)^{1/3}}\right]. \quad (17)$$

The flux is proportional to the square of the Airy function V at the location of the soliton. Similarly, knowing the field distribution in the standing wave, we can find the energy flux also for another profile.

Real solitons are not one-dimensional. Collapse of plasma waves causes all the dimensions of the turbulence unit cell to become shorter than the radiated electromagnetic wavelength. Since the dynamics of non-one-dimensional turbulence has not been sufficiently well investigated to date, the problem of the radiation from a three-dimensional structure meets with great difficulties. The qualitative difference from the one-dimensional situation is that the soliton, being a dipole source for the electromagnetic wave, will have a directivity pattern and will radiate electromagnetic energy in a homogeneous plasma in all directions. In a planar smoothly inhomogeneous medium, the presence of the dielectric constant "bends" the rays into a less dense plasma, and the oscillatory dependence of the radiation intensity on the frequency shift in the soliton remains the same as before.

4. The considered radiation of the electromagnetic waves appears in active experiments in the near-earth plasma,[7,8] and also in the earth's magnetosphere when turbulence is excited by particle streams. In experiments on modification of the ionosphere, one observes AIR produced by the action of a beam of powerful short radio waves on the F layer. The radiation is generated by applying waves with ordinary polarization as they are reflected from the ionosphere, corresponds to an ordinary wave, and undergoes a spectral broadening $\Delta f = 20$–50 kHz towards the red side of $f_{\text{P.W.}}$ ($f_{\text{P.W.}} = 5$–1 MHz is the pump-wave frequency). Detailed investigations of the AIR properties have shown that its spectrum exhibits regularly maxima at $\Delta f = 15$ and 35 kHz. The most probable radiation sources are the elementary cells of strong plasma turbulence—solitons excited near the reflection point (in the first few maxima of the Airy function) on account of modulational instability. Estimating from (17) the width of the spectrum to be $\Delta f = f_{\text{P.W.}}\, u\sin^2\alpha$ and recognizing that $u = 0.04$, $\sin^2\alpha \approx 0.1$, at the latitude in which the experiments were performed, we get $\Delta f \sim 25$ kHz ($f_{\text{P.W.}} = 5.7\cdot 10^6$). The positions of the maxima in the AIR spectrum agree well with this model if the solitons are sufficiently uniformly located in the first few maxima of the Airy function.[3)] The curvature radius of the rays in the region of turbulence excitation, estimated from the geometric-optics equations, amounts in the experiments of Ref. 8 to $R = L(1 - v_0) \sim 2 - 5$ km. Since R is much shorter than the distance from the earth to the F layer and than the transverse dimension of the turbulence-excitation region, the one-dimensional model turns out to be adequate for the description of experiments on AIR excitation.

Electromagnetic-wave emission can serve also as a diagnostic tool for the investigation of plasma turbulence in the earth's magnetosphere. Artificial-satellite experiments in the region of the plasmopause[11,12] revealed intense potential waves polarized perpendicular to the geomagnetic field, at close to upper-hybrid frequencies. The region occupied by these waves has a source of radiation with ordinary and extraordinary polarization, which propagated over a distance much larger than the size of the region occupied by the turbulence. The soliton radiative lifetime $\tau = 4c/3^{1/2}\pi^2 v_T\omega_H$ calculated for these conditions is 10^{-2} s ($v_T \sim 2\cdot 10^8$ cm/s, $\omega_H \sim 5$ kHz) and is significantly shorter than the time of collisionless damping of the turbulence by the particles. This confirms the conclusion that electromagnetic-wave emission is the principal channel of energy loss by strong turbulence in the magnetosphere plasma.

[1)]The right-hand side of Eq. (4) has the same structure as the dipole-radiation source obained in Ref. 4 for an isotropic plasma. In an isotropic medium this source does not lead to radiation at the soliton oscillation frequency, in view of the absence of a suitable propagating electromagnetic wave.

[2)]The contribution of the perturbations of a quasistationary magnetic field to the nonlinear frequency shift is, as a rule, much smaller than the contribution of the relative perturbations of the density. Thus, for the case of stationary striction nonlinearity of the radiation, the correction is a small quantity of order v_T^2/c^2.

[3)]Another mechanism based on weak turbulence[9] does not explain the fine structure of the AIR spectra.

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