Transient effects at a superconductor–normal metal interface in the presence of a current

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An investigation is made of the solutions of the time-dependent Ginzburg–Landau equations near the boundary between a superconductor and a normal metal under current flow conditions. The current-voltage characteristics of *SN* and *SIN* junctions subjected to high-frequency illumination are derived. The results are compared with experimental data.

1. INTRODUCTION

Siyuan Han *et al.*[1] reported recently their discovery of an anomalous Josephson effect at a point contact between niobium and a superconductor containing heavy fermions. The effect was unusual because it persisted up to temperatures ~ 8 K, although the heavy-fermion compounds investigated were characterized by $T_c \lesssim 1$ K. The observed effect consisted of the presence of kinks in the current-voltage characteristics, resembling the Shapiro steps. The distance between the kinks satisfy the usual Josephson relationship $\hbar\omega = 2$ eV. In the absence of illumination the current-voltage characteristics were nonlinear and exhibited a critical current J_c (the differential resistance in the range $J < J_c$ was low). The temperature dependence of J_c resembled the temperature dependence of the critical current of Josephson junctions.

A similar effect was observed later[2] for a Ta–Mo contact at $T \sim 3.5$ K, which was considerably higher than $T_{c\text{Mo}} \sim 0.9$ K. This led to the hypothesis that the anomalous effect is not due to exotic properties of heavy-fermion compounds, but is common to all conductors. The effect was observed in Refs. 1 and 2 at temperatures much higher than the superconducting transition temperatures of Mo and UBe_{13}, so that the properties of the superconducting state of these substances were unimportant. Therefore, we shall consider a contact between a superconductor and a normal metal characterized by $T_c = 0$.

The anomalous Josephson effect had been observed also earlier.[3,4] As pointed out, this effect can be due to the formation of a conventional Josephson weak link inside a superconducting point due to a reduction of its thickness or to chipping. The authors of Refs. 1 and 2 were of the opinion that this was avoided in their experiments. Consequently, we shall ignore this possibility, although it can explain the experimental results.

The phenomenon in question is clearly associated with the proximity effect since the superconducting order parameter is induced in a normal metal near its interface with the superconductor. The authors of Refs. 1 and 2 are of the opinion that the application of a potential to a contact creates a transient Josephson effect between the superconductor and that part of the normal metal where the superconducting order parameter is induced. However, this interpretation of the effect seems to us too simplistic. In the case of the conventional transient Josephson effect the time dependence of the order parameters at the superconducting edges is governed only by their potential: $\psi_{1,2} \sim \exp(-i\mu_{1,2} t)$. The difference between the phases ψ_1 and ψ_2 is a linear function of time which necessarily leads to a time-dependent voltage across the contact. In the case of the proximity effect the phase of the order parameter induced in a normal metal is governed by its phase in the superconductor. The voltage across a contact may then be independent of time.

We shall try to account for the experimental results of Refs. 1 and 2 using the time-dependent Ginzburg–Landau equations. We shall consider the cases of superconductor–insulating layer–normal metal (*SIN*) and superconductor–normal metal (*SN*) contacts. We shall obtain the solutions of the equations and also the current-voltage characteristics of the contacts in the presence of a constant current and high-frequency illumination.

2. MODEL

Since the microscopic system of transport equations for a superconductor is extremely complicated the task of investigating the superconducting state near an interface between a superconductor and a normal metal by means of this system is very difficult, and quite possibly insuperable. We are thus faced with the selection of a model which describes satisfactorily the physical phenomena which occur at the *SN* interface. We shall use the time-dependent Ginzburg–Landau equations. These equations were first derived in Ref. 5 for superconducting alloys with a high concentration of paramagnetic impurities when there is no superconducting gap. In this case the equations are valid at any temperature.

We shall assume that near the *SN* interface the concentration of paramagnetic impurities is high both in the superconductor and in the normal metal. In the case of such a contact the time-dependent Ginzburg–Landau equations are undoubtedly valid. In the experiments reports in Refs. 1 and 2 there were no paramagnetic impurities. The effect was observed at temperatures well below the critical value and at voltages $V \ll \Delta$, when there are no quasiparticle excitations in the superconductor and the phenomena associated with the proximity effect occur in the normal metal. The order parameter of the normal metal decreases rapidly away from the interface, so that the situation is effectively of the zero-gap type. Therefore, the use of the equations for zero-gap superconductors seems to be physically justified.

The time-dependent Ginzburg–Landau equations for a high concentration of paramagnetic impurities are[5]

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$$\frac{\partial\Delta}{\partial t}+\frac{\tau_\varepsilon}{3}\left[-\pi^2(T_c^2-T^2)+\frac{|\Delta|^2}{2}\right]$$

$$-\frac{\pi}{e^2m^2v_F}\nabla(\sigma\nabla\Delta)+2ie\,\Delta\varphi=0, \qquad (1)$$

$$\mathbf{j}=-\sigma\nabla\varphi+\frac{i\sigma\tau_\varepsilon}{2e}(\Delta\nabla\Delta^*-\Delta^*\nabla\Delta)$$

(the notation is taken from the original paper). For simplicity, we shall assume that the situation is one-dimensional. Since a consistent description of the experimental results requires three-dimensional equations, qualitative results can be obtained by solving the one-dimensional problem. The relationship between the measured experimental quantities and the one-dimensional problem will be discussed later.

We shall rewrite the system (1) in the following dimensionless form:

$$u\left(\frac{\partial\psi}{\partial t}+i\mu\psi\right)=\psi''+(\theta(x)-|\psi|^2)\psi, \quad j=\mathrm{Im}(\psi^*\psi')-\mu', \qquad (2)$$

where

$$\theta(x)=\begin{cases}1, & x<0 \text{ (in the superconductor)}\\ -\tau^2, & x>0 \text{ (in the normal metal)}\end{cases}$$

and $\tau^2 = T^2/(T_c^2 - T^2)$. In writing down the system (2) it is assumed that the critical temperature is T_c for the superconductor and zero for the normal metal, but all the other parameters of both are the same. In the presence of a high concentration of paramagnetic impurities, we have $u = 12$ (Ref. 5). In our system of units, the critical current in the superconductor is $j_c = 2/3^{3/2} \approx 0.385$.

The system of equations (2) should be supplemented by a boundary condition at $x = 0$. If between the superconductor and the normal metal there is no insulating spacer (i.e., if we are dealing with an *SN* contact), then the boundary conditions specify continuity of ψ, ψ', and μ at $x = 0$. The boundary conditions in the case of an *SIN* contact will be derived in Sec. 4.

3. SUPERCONDUCTOR–NORMAL METAL CONTACT

It is known that when the current flows through a narrow superconducting channel, a resistive state in the form of a phase-slip center may exist. A phase-slip center appears in a certain range of currents $j_1 < j < j_2$ and for $u = 12$, we have $j_1 \approx 0.284$ and $j_2 \approx 0.291$ (Ref. 6). We shall now consider the possibility that a similar time-dependent solution exists near a superconductor–normal metal (*SN*) interface.

We shall assume that near the *SN* interface the critical temperature varies slowly with distance. Then a phase-slip center exists for an arbitrary current $j < j_2$. In fact, $j_{1,2}(\theta) \propto \theta^{3/2}$, so that in the case of a slow variation of $\theta(x)$, there is always a fairly wide range where $j_1(\theta) < j < j_2(\theta)$. However, in the case when T_c changes abruptly at the interface, it is difficult to determine *a priori* whether a time-dependent solution exists. Time-dependent solutions can exist only in that range where the nonlinear terms of the system (2) are large. Hence, it seems justified to seek time-dependent solutions in the case of an *SN* contact also at $T = 0$ when the order parameter penetrates deepest into the normal metal.

We carried out numerical integration of the system (2) for $u = 12$ in order to find the transient solutions near the boundary. It was found that the static solution was stable right up to the current $j = j_2(u)$. No periodic time-dependent solutions near the contact were found. Arbitrary initial perturbations reduced to a single steady-state solution although in the case of a similar calculation for a pure superconducting channel it was found that there were phase-slip centers in the range of currents $j_1 < j < j_2$.[6] We recall that the experimental situation corresponded to the case of $j \ll j_2$.

For the sake of completeness, we shall give numerical results for currents of the order of the critical value. It was found that for $j \sim 0.1$–0.2 the electric field penetrates into the superconductor. If $j = j_2(u)$, the domain wall separating the superconducting and normal phases becomes detached from the contact. If $j > j_2(u)$, the wall moves into the superconductor at a constant velocity leaving behind a region characterized by $\psi = 0$ (Ref. 7). In other words, the superconductivity is destroyed also in the region where $z < 0$.

In the case of an *SN* contact the chemical potential is continuous at the boundary and its absolute value increases without limit inside the normal metal. Therefore, we shall define here the voltage at a contact to be

$$V=-\lim_{x\to+\infty}[jx+\mu(x)]. \qquad (3)$$

Figure 1 shows a numerically calculated dc current-voltage characteristic. The negative values of the voltage V correspond to penetration of the electric field into the superconductor at currents of the order of the critical value. It is found that the current-voltage characteristic of an *SN* contact subjected to illumination (when the current oscillates at a constant frequency near its average value j^0) does not have kinks.

We shall consider the situation when $j \ll 1$ and $T = 0$. In this case a dc current-voltage characteristic can be found analytically. For $j = 0$ the solution of the system (2) is of the form

$$\psi=\begin{cases}\mathrm{th}(\ln(1+2^{1/2})-x/2^{1/2}), & x<0\\ 2^{1/2}/(x+2), & x>0\end{cases}. \qquad (4)$$

When a small current flows across the contact, the solution changes significantly only in the range of positive values

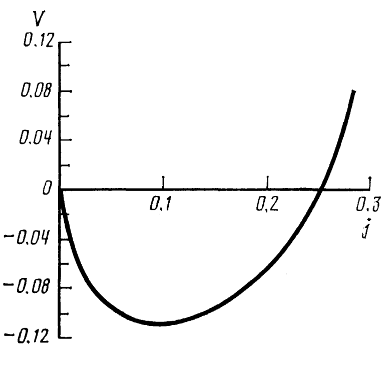

FIG. 1. Current-voltage characteristics of an *SN* contact under constant-current conditions ($u = 12$).

$x \gg 1$ [$\psi(x)$ is small]. Therefore, Eq. (4) remains valid near the interface.

We shall transform the variables in Eq. (2) as follows:

$$x = x_1 j^{-1/3} - 2, \qquad \psi = \psi_1 j^{1/3}, \qquad \mu = \mu_1 j^{2/3}, \qquad t = t_1 j^{-2/3}. \tag{5}$$

The system (2) expressed in terms of the new variables is

$$u(\partial\psi_1/\partial t_1 + i\mu_1\psi_1) = \partial^2\psi_1/\partial x_1^2 + [\theta(x_1 - 2j^{1/3}) j^{-2/3} - |\psi_1|^2]\psi_1,$$

$$1 = -\frac{\partial\mu_1}{\partial x_1} + \mathrm{Im}\left(\psi_1^* \frac{\partial\psi_1}{\partial x_1}\right). \tag{6}$$

We can see that if $x > 0$ ($x_1 > 2j^{1/3}$), there is no current in the system (6). At low values $2j^{1/3} < x_1 \ll 1$, the solution of Eq. (6) should reduce asymptotically to Eq. (4), which can be rewritten in the form

$$\psi_1 = 2^{1/2}/x_1, \qquad \mu_1 = 0 \quad \text{for} \quad 2j^{1/3} < x_1. \tag{7}$$

In the case of high values of $x_1 \ll 1$, the solution has the asymptotic form

$$\psi_1 \sim \exp(-2^{1/2}(1-i)u^{1/2}x_1^{3/2}/3). \tag{8}$$

At distances $x_1 \sim 1$ there is a change from the asymptotic form of Eq. (7) to Eq. (8) and the superconducting current is transformed into the normal current. In terms of the old variables this corresponds to distances $x \sim j^{1/3}$. The results of a numerical solution of the system (6) subject to Eq. (7) are plotted in Fig. 2. At large values of x_1, we find that $\mu_1 \approx C - x_1$. If $u = 12$, then $C \approx 1.46$. Going back to the old variables, we find that μ is described by

$$\mu(x) = j^{2/3}\mu_1(x^{1/3} + 2j^{1/3}). \tag{9}$$

The voltage across the contact is

$$V(j) = -Cj^{2/3} + 2j, \qquad j \ll 1. \tag{10}$$

We recall that the voltage V is understood here to be the quantity defined by Eq. (3). A comparison of Eq. (10) with the results of numerical calculations shows that the above expression is quite accurate right up to currents $j \sim 0.1$–0.2.

4. SUPERCONDUCTOR–INSULATOR–NORMAL METAL CONTACT

The experiments reported in Refs. 1 and 2 were carried out on a point contact when the coupling between the normal metal and the superconductor was weak. We can distinguish then two cases: an insulating (oxide) spacer or the presence of a certain number of microcircuits of size small compared with ξ. Qualitatively, these two cases yield the same results. We shall consider specifically the case with an insulating spacer (representing a region with a low conductivity).

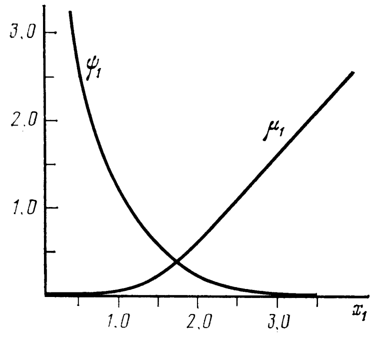

FIG. 2. Solution of the system of equations (6) together with Eq. (7) obtained for $u = 12$.

The boundary conditions can be derived by going back to the dimensional equations of the system (1) and assuming now that the conductivity σ is a function of the coordinates. The size of the insulating spacer is taken to be $a \ll \xi$. Outside the spacer we have $\sigma = \sigma_0$, whereas inside it we find that $\sigma \ll \sigma_0$. We shall use S and N to identify the quantities near the spacer but in the superconductor and in the normal metal, respectively.

Since $a \ll \xi$, it follows from the first equation of the system (1) that

$$\sigma\Delta' = \text{const} = \sigma_0\Delta_N' = \sigma_0\Delta_S', \tag{11}$$

which gives

$$\Delta_N - \Delta_S = \sigma_0\Delta_N' \int_{-1/2a}^{1/2a} \frac{dx}{\sigma}. \tag{12a}$$

Similarly, integrating the equation for the current, we obtain

$$\varphi_N - \varphi_S = \sigma_0\varphi_N' \int_{-1/2a}^{1/2a} \frac{dx}{\sigma}. \tag{12b}$$

The integral in the above equations is simply the total resistance R of the insulating layer.

The boundary conditions of Eqs. (11) and (12) can be rewritten in a dimensionless form as follows:

$$\psi_S' = \psi_N' = D(\psi_N - \psi_S), \qquad \mu_S' = \mu_N' = D(\mu_N - \mu_S). \tag{13}$$

The dimensionless constant D is related to the resistance as follows:

$$D = \xi/\sigma_0 R.$$

We can readily see that the boundary condition for the order parameter gives rise to the Josephson expression for the superconducting current across the contact $j = j_c \sin(x_1 - x_2)$, where j_c depends in the usual way on the contact resistance.

It is found that under certain conditions we can obtain an approximate analytic solution of the system (2) subject to the boundary conditions of Eq. (13). We shall assume that $D \ll 1$ (low-transparency barrier). As pointed out above, this hypothesis clearly corresponds to the experimental situation. Then, in the superconductor near the interface there is a practically unperturbed usual superconducting state characterized by $|\psi| = \text{const}$. The chemical potential, which is zero in the superconductor changes abruptly at the interface to the value $\mu(+0) = V$. In the low-transparency case when $D \ll 1$ the chemical potential does not change significantly over distances $x \sim 1$, where the order parameter differs from zero. We can therefore assume that $\mu = -V$ is constant when $x > 0$. At a moderately low temperature (all the assumptions will be expressed quantitatively below), in the case of a normal metal ($x > 0$) the term $|\psi|^2\psi$ in the system (2) is small compared with $\tau^2\psi$ and can be omitted.

The condition of validity of the above assumptions is of the form

$$D \ll 2\tau^2/u^{1/2}. \qquad (14)$$

In this case the equation for the order parameter can be written as follows:

$$u(\dot\psi - iV\psi) = \psi'' - \tau^2\psi, \quad \psi'(+0) = -D. \qquad (15)$$

The only solution of Eq. (15) is time-independent. If $V \neq 0$, then in the normal metal near the contact there are both superconducting and normal components of the current. The current-voltage characteristic is now nonlinear:

$$j^0 = j_n^0 + j_s^0, \quad j_n^0(V) = DV,$$

$$j_s^0(V) = \frac{D^2}{\tau\sqrt{2}} \frac{[(1+(uV/\tau^2)^2)^{1/2}-1]^{1/2}}{[1+(uV/\tau^2)^2]^{1/2}}. \qquad (16)$$

A similar result for a film of a normal metal was obtained in Ref. 8. At low voltages we have $j_s = VD^2u/2\tau^3$ and then at $V_c = 3^{1/2}\tau^2/u$ the superconducting current has a maximum $j_c = D^2/2^{3/2}\tau$, falling at high values of V proportionally to $V^{-1/2}$. This fall is due to suppression of the order parameter of the a normal metal at high contact voltages. The current-voltage characteristic is strongly nonlinear for

$$D \gg 2\tau^3/u. \qquad (17)$$

It should be pointed out that this condition is not related to the approximations used to write down the system (15), i.e., the current-voltage characteristic is significantly nonlinear when the condition (17) is obeyed irrespective of the validity of the condition (14). However, in this case it is no longer possible to obtain the expression for the current-voltage characteristic from the simple system of equations (15).

We can find the current-voltage characteristic of an illuminated contact by assuming in the system (15) that V is a function of time:

$$V = V_0 + A\cos\omega t. \qquad (18)$$

Then, the order parameter is described by

$$\psi(x,t) = \exp\left(i\frac{A}{\omega}\sin\omega t\right)\sum_{-\infty}^{+\infty} J_n\left(\frac{A}{\omega}\right)\exp(-in\omega t)\,\psi_{V_0+n\omega}(x), \qquad (19)$$

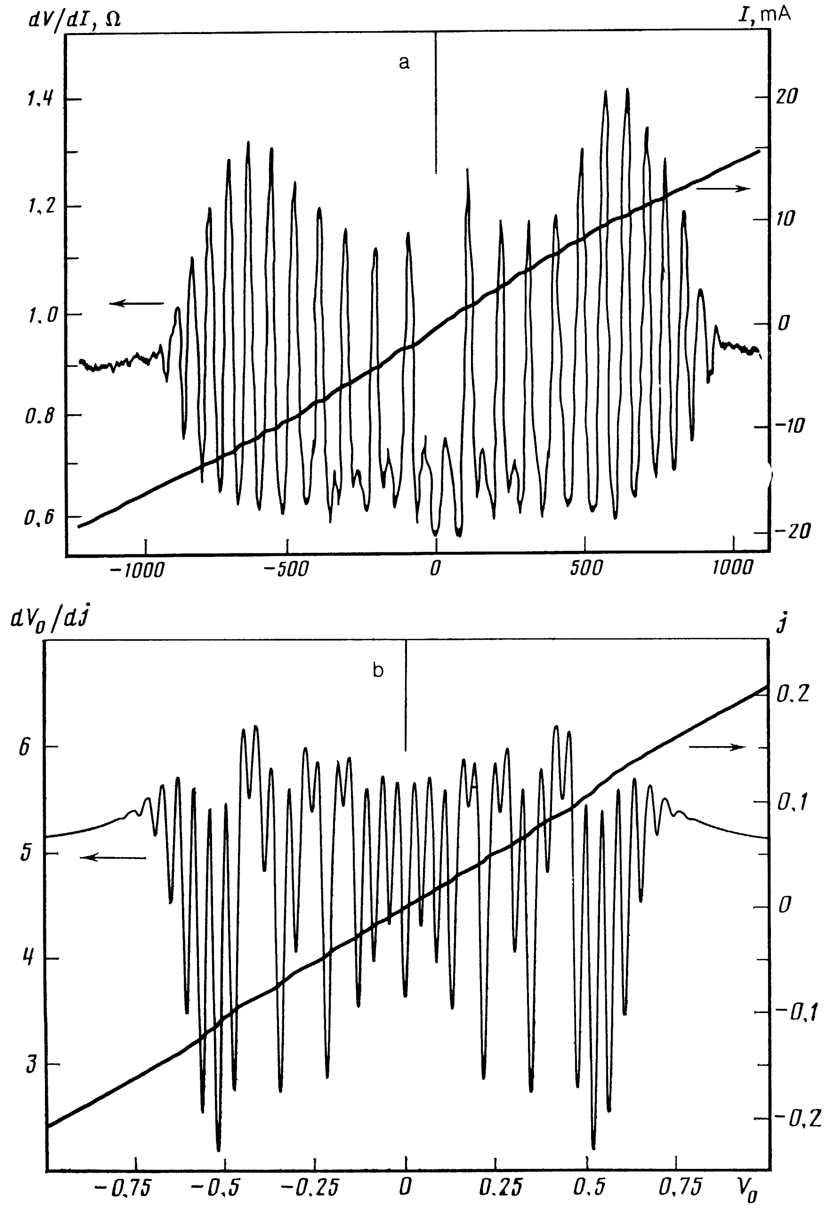

FIG. 3. Current-voltage characteristics: a) experimental results (Fig. 1a in Ref. 1), UBe_{13}/Nb, $\nu = 25$ GHz, $T = 4.17$ K; b) theory, $D = 0.2$, $\tau = 0.4$, $A/\omega = 14$.

where J_n is the Bessel function and $\psi_{V_0+n\omega}(x)$ is the order parameter for a constant voltage across the contact amounting to $V_0+n\omega$. Separating the constant component of the current, we obtain

$$j(V_0)=\sum_{-\infty}^{+\infty} J_n^2\left(\frac{A}{\omega}\right) j^0(V_0+n\omega) \tag{20}$$

(exactly the same relationship between the current-voltage characteristic in the presence of illumination with the characteristic in the absence of illumination was obtained earlier in Ref. 9).

If the condition (17) is satisfied, then near $V=0$ the dc current-voltage characteristic is nonlinear. It is clear from Eq. (20) that in this case the current-voltage characteristic in the presence of illumination remains nonlinear at $V\approx n\omega$, by analogy with the Shapiro steps. The current-voltage characteristics obtained from Eq. (16) and (20) are compared with the experimental results in Fig. 3. We can see that the proposed model describes the main features of the current-voltage characteristic of a real contact.

For the sake of completeness, we also carried out a numerical integration of the system (2) subject to the boundary conditions (13) in the case of significant transparency of the barrier, when the condition (14) is not satisfied. It was found that in this case the condition is steady-state and the current-voltage characteristic is again close to the experimental results.

5. CONCLUSIONS

We shall now summarize briefly the main results.

In this model there are no time-dependent solutions near the boundary between a superconductor and a normal metal and the current-voltage characteristic is nonlinear.

In the presence of illumination there are kinks in the current-voltage characteristics only in the case of an *SIN* contact (the current-voltage characteristic of an *SIN* contact is understood to be the dependence of the current j across the contact on the difference between the potentials at a barrier V). The distance between the inflections satisfies the usual Josephson relationship $V=\hbar\omega/2e$. In the experiments reported in Refs. 1 and 2 the critical current was 10–100 times less than estimated for such an Nb–Nb contact. Hence, the order parameter induced in UBe_{13} is small. In our model the ratio of the critical currents is of the same order of D, which gives $D\sim 0.1$–0.01. Therefore, our assumption that D is small is justified.

We shall now consider the role of the three-dimensional nature of a contact. In the experiments of Refs. 1 and 2 the potential difference was measured not at the contact itself but at macroscopically large distances. Since the contact was of the point type, the potential in the normal metal rapidly reached a constant value. Clearly, an allowance for the three-dimensional nature of the real contact changes all the dependences only slightly, but the results remain qualitatively the same as before. The experimentally determined voltage is related to the chemical potential in the one-dimensional problem at a distance of the order of the contact size a.

In the case of an *SN* junction the chemical potential near the contact changes significantly at distances of order ξ. Therefore, in the presence of several short circuits the scatter of their dimensions undoubtedly would flatten out the inflections even if they were to appear in the current-voltage characteristic of a one-dimensional *SN* contact. An estimate obtained in Ref. 1 suggests that $D\ll 1$, i.e., that the junction should be regarded as an *SIN* contact. In this case the chemical potential changes insignificantly in a distance of the order of ξ and the experimentally determined voltage corresponds to a discontinuity of the chemical potential at a barrier in the one-dimensional problem.

It therefore follows that the experimental results[1)] reported in Refs. 1 and 2 on the existence of kinks of the current-voltage characteristic of a point contact subjected to illumination can be explained also without assuming the existence of time-dependent solutions at a contact. This conclusion on the absence of time-dependent solutions near an interface in the one-dimensional case seems to us to apply also in the case of a three-dimensional contact.

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[1)] After writing the present paper we learnt of the experimental results reported in Ref. 10, in which an analogous effect was observed for Nb–Ta contacts.

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