Low-frequency spin dynamics of superfluid ^{3}He-B in a magnetic field

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The Leggett equations for ^{3}He-B are expanded in small frequencies and gradients about the periodic spatially uniform solutions found previously. Equations are obtained which describe the low-frequency spin dynamics superposed on the Larmor precession—in particular, the spin-current flow and other phenomena observed recently by Borovik-Romanov *et al.* superposed on this precession. Equations describing the low-frequency dynamics of the textures are also obtained, and the low-frequency precession of the axis of the order parameter anisotropy in a magnetic field is considered. The dispersion law is found for spin waves propagating on the background or precession with a frequency exceeding the Larmor frequency.

1. INTRODUCTION

In the superfluid B phase of ^{3}He a class of phenomena in which the decisive role is played by superfluid spin currents has recently been discovered and investigated.[1–4] These phenomena can be interpreted theoretically[5,6] by starting directly from the complete system of equations for the spin dynamics of the superfluid phases of ^{3}He—the Leggett equations.[7] It is possible, however, to simplify the description substantially by using the fact that all the observable phenomena are long-wavelength phenomena, i.e., the scale l_∇ characterizing the spatial nonuniformity of the structures arising in the experiments is large in comparison with the characteristic lengths appearing in the Leggett equations. There are two such lengths—the dipole length $l_D \sim c/\Omega$, and the magnetic length $l_H \sim c/\omega L$, where c is the spin-wave velocity, Ω is the longitudinal-resonance frequency, and ω_L is the Larmor frequency corresponding to a constant magnetic field H_0. The dipole length $l_D \sim 10^{-3}$ cm, and the magnetic length l_H depends on the magnitude of the field. In experiments, usually, $\omega_L \gtrsim \Omega$, and therefore, $l_H \lesssim l_D$ and it is sufficient to require that the inequality $l_\nabla \gg l_D$ be fulfilled. The experiments of Refs. 1–4 were performed by a pulsed NMR method, i.e., all the phenomena were played out against the background of spin precession with frequency close to the Larmor frequency. Of greatest interest here are those motions of the spin and order parameter for which the deviation from precession is characterized by frequencies ω_∇ that are small both in comparison with ω_L and in comparison with Ω. Thus, in the problem there appear small parameters l_D/l_∇ and ω_∇/Ω in terms of which the Leggett equations can be expanded. In Sec. 2 of the present article we describe a procedure for this expansion, and by means of this procedure obtain two equations in place of the original six; these two equations describe, in the leading approximation in l_D/l_∇ and ω_∇/Ω, the motions of the spin and order parameter in ^{3}He-B that are superposed on the Larmor precession. These equations make it possible to describe in a unified manner the larger part of the observed phenomena and to compare them with analogous phenomena that can be observed in other superfluid systems. A brief account of the application of the equations thus obtained to the description of the stationary flow of spin current along a long channel has already been published.[8]

It was shown previously[9] that the Leggett equations have four types of stable stationary solutions describing the joint precession of the spin and order parameter with a specified angular frequency ω_p; these solutions were denoted as Ia), Ib), IIa), and IIb). In particular, the above-mentioned Larmor precession is described by the solution Ib). The expansion procedure described in Sec. 2 is, with slight changes, also applicable to the other three types of stationary solution. Here we must bear in mind that for solutions of type IIa) and IIb) fixing the frequency ω_p determines the solution to within the initial phase of the precession, i.e., there is degeneracy with respect to one parameter, whereas for a unique determination of the stationary solutions of types Ia) and Ib) for a fixed ω_p one must specify two parameters. Thus, the degeneracy space of the IIa) and IIb) solutions is a one-parameter space, and that of the Ia) and Ib) solutions is a two-parameter space. As a result of expanding to leading order in l_D/l_∇ and ω_∇/Ω for each stationary solution, equations describing the motion of the order parameter through its degeneracy space are obtained. For the Ia) and Ib) solutions these are equations for two independent variables, while for the IIa) and IIb) solutions they are equations for only one variable.

The Ia) solutions describe spatially uniform static textures of the anisotropy axis $\mathbf{n}$ of the order parameter of ^{3}He-B. The expansion of the Leggett equations about these solutions is performed in Sec. 3 of the present paper. As a result, two equations describing the low-frequency dynamics of the textures are obtained. As an example of the application of the equations obtained we consider the slow precession of the vector $\mathbf{n}$ in a magnetic field.

In Sec. 4 we obtain the equations of the long-wavelength dynamics against the background of the IIb) solution describing the stationary precession of the spin in a magnetic field at tilt angles $\beta > \theta_0 = \arccos(-1/4)$. Using the equations obtained we find the spectrum of the small long-wavelength oscillations on the background of this precession.

Since the main aim of the paper is to derive the equations, and not to apply them, we have considered only the simplest examples that do not require long calculations. All the equations have been obtained without dissipative terms, although the expansion procedure makes it possible to take direct account of dissipation as well, if the latter is small.

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2. THE EXPANSION PROCEDURE

The order parameter in ^{3}He-B is proportional to the rotation matrix $\hat{R}(\mathbf{n}, \theta)$, which depends on three parameters, e.g., the rotation-axis direction $\mathbf{n}$ and rotation angle θ. On the scale of energies of the order of the characteristic Cooper-pairing energy Δ in ^{3}He there is degeneracy in $\mathbf{n}$ and θ. The motion of the order parameter through this three-parameter degeneracy space is described by the spin-dynamics equations—the Leggett equations.[7] An important role in the spin dynamics of the superfluid phases of ^{3}He is played by the spin–orbit or dipolar interaction U_D, which lifts the degeneracy of the order parameter referred to above. A B-phase feature of importance in what follows is the fact that in this phase the dipole energy

$$U_D \sim (\cos\theta + {}^1/_4)^2$$

depends only on θ, and the degeneracy with respect to the direction $\mathbf{n}$ is not lifted. Because of this property, in ^{3}He-B there exist those degeneratefamilies of stationary solutions that were mentioned in the Introduction.

In the following it will be convenient to describe the motion of the order parameter by means of Euler angles, which are defined in the usual way:

$$\hat{R}(\mathbf{n}, \theta) = \hat{R}(\alpha, \beta, \gamma) = \hat{R}_z(\alpha)\hat{R}_y(\beta)\hat{R}_z(\gamma), \tag{1}$$

with the z axis oriented in the direction opposite to $\mathbf{H}_0$. The Leggett equations can be written as Hamilton equations, by choosing as the variables the angles $\alpha, \beta, \Phi = \alpha + \gamma$ and the following combinations (canonically conjugate to these angles) of spin projections: $P = S_z - S_\zeta$, S_β, S_ζ, where S_z is the projection of the spin onto the z axis, S_ζ is the projection onto the axis $\zeta = \hat{R}\hat{z}$, and S_β is the projection onto the line of nodes. As before,[9] we choose the units so that $\chi = g = 1$, where χ is the magnetic susceptibility of a unit volume of ^{3}He, and g is the gyromagnetic ratio for the ^{3}He nuclei; then the spin will have the dimensions of frequency, and the energy will have the dimensions of frequency squared. In these variables and units the Leggett Hamiltonian has the form

$$\mathscr{H} = \frac{1}{1+\cos\beta}\left[S_\zeta^2 + PS_\zeta + \frac{P^2}{2(1-\cos\beta)}\right] - \omega_L(P+S_\zeta) + \frac{1}{2}S_\beta^2 + F_\nabla + U_D(\beta, \Phi). \tag{2}$$

The form of the gradient energy F_∇ is established from symmetry considerations[10,11]:

$$F_\nabla = {}^1/_2[c_\parallel^2\delta_{ik}\delta_{\xi\eta} + (c_\parallel^2 - c_\perp^2)(\delta_{i\xi}\delta_{k\eta} + \delta_{i\eta}\delta_{k\xi})]\omega_{i\xi}\omega_{k\eta}, \tag{3}$$

where

$$\omega_{1\xi} = -\alpha_{,\xi}\sin\beta\cos\gamma + \beta_{,\xi}\sin\gamma,$$

$$\omega_{2\xi} = \alpha_{,\xi}\sin\beta\sin\gamma + \beta_{,\xi}\cos\gamma, \quad \omega_{3\xi} = \alpha_{,\xi}\cos\beta + \gamma_{,\xi}, \tag{4}$$

$\alpha_{,\xi} = \partial\alpha/\partial x_\xi$, etc., and $c_\parallel^2$ and $c_\perp^2$ are the squares of the velocities of the two types of spin waves (see Ref. 5). The dipolar potential in the variables α, β, Φ has the form

$$U_D(\beta, \Phi) = {}^2/_{15}\Omega^2[\cos\beta - {}^1/_2 + (1+\cos\beta)\cos\Phi]^2; \tag{5}$$

it does not depend on α, and reaches a minimum $U_D = 0$ on the line

$$\cos\beta + \cos\Phi + \cos\beta\cos\Phi = {}^1/_2. \tag{6}$$

Both these properties are an expression, in the variables α, β, Φ, of the lack of dependence of U_D on the direction $\mathbf{n}$. The equations of motion generated by the Hamiltonian (2) have the form

$$\frac{\partial\alpha}{\partial t} = \frac{\partial\mathscr{H}}{\partial P}, \quad \frac{\partial P}{\partial t} = -\frac{\delta\mathscr{H}}{\delta\alpha},$$
$$\frac{\partial\beta}{\partial t} = \frac{\partial\mathscr{H}}{\partial S_\beta}, \quad \frac{\partial S_\beta}{\partial t} = -\frac{\delta\mathscr{H}}{\delta\beta},$$
$$\frac{\partial\Phi}{\partial t} = \frac{\partial\mathscr{H}}{\partial S_\zeta}, \quad \frac{\partial S_\zeta}{\partial t} = -\frac{\delta\mathscr{H}}{\delta\Phi}, \tag{7}$$

where the symbol δ denotes the variational derivative; e.g.,

$$\frac{\delta\mathscr{H}}{\delta\alpha} = \frac{\partial\mathscr{H}}{\partial\alpha} - \frac{\partial}{\partial x_\xi}\left(\frac{\partial\mathscr{H}}{\partial\alpha_{,\xi}}\right) = \frac{\partial\mathscr{H}}{\partial\alpha} - \frac{\partial}{\partial x_\xi}\left(\frac{\partial F_\nabla}{\partial\alpha_{,\xi}}\right).$$

In the spatially uniform case ($F_\nabla = 0$) the Hamiltonian (2) does not contain the variable α, and as a result the angular momentum P conjugate to α is conserved. This leads to the existence of stationary solutions of the system (7) (Ref. 9):

$$\frac{\partial\beta}{\partial t} = \frac{\partial\Phi}{\partial t} = \frac{\partial P}{\partial t} = \frac{\partial S_\beta}{\partial t} = \frac{\partial S_\zeta}{\partial t} = 0, \tag{8}$$

$$\partial\alpha/\partial t = -\omega_p. \tag{9}$$

If in place of α we introduce the variable $\psi = \alpha + \omega_p t$, which transforms the Hamiltonian into

$$\tilde{\mathscr{H}} = \mathscr{H} + \omega_p P, \tag{10}$$

the solutions described by (8) and (9) will also be stationary in ψ: $\partial\psi/\partial t = 0$. By virtue of the system (7), in the spatially uniform case these solutions are extrema of the new Hamiltonian $\tilde{\mathscr{H}}$ with respect to all the arguments.

In order to expand the system (7) in l_D/l_∇ and ω_∇/Ω, we must separate the Hamiltonian (10) into two parts: $\tilde{\mathscr{H}} = \tilde{\mathscr{H}}^{(0)} + V$, where $\tilde{\mathscr{H}}^{(0)}$ is that part of $\tilde{\mathscr{H}}$ that is independent of the coordinates and spatial derivatives. Included in the perturbation V are the gradient energy F_∇ and also all small terms that give rise to spatial gradients of the solutions or to a slow dependence of the solutions on the time. Thus, if the magnetic field is nonuniform, then

$$V = F_\nabla + (\omega_L^{(0)} - \omega_L)(P + S_\zeta), \tag{11}$$

where $\omega_L^{(0)}$ is the value of the Larmor frequency at any particular point of the volume under consideration. Here $\tilde{\mathscr{H}}^{(0)}$ is taken at $\omega_L = \omega_L^{(0)}$. In the perturbation V we can also include an oscillatory pumping field, a term proportional to $(\mathbf{n}\cdot\mathbf{H}_0)^2$ that lifts the degeneracy in $\mathbf{n}$, and other small corrections. Henceforth, for definiteness, we shall consider a perturbation of the form (11). We now rewrite the system (7) in such a way that terms containing the perturbation V and time derivatives are in the right-hand sides of the equations:

$$0 = \frac{\partial P}{\partial t} + \frac{\delta V}{\delta\psi}, \tag{12}$$

$$\frac{\partial\tilde{\mathscr{H}}^{(0)}}{\partial\beta} = -\frac{\partial S_\beta}{\partial t} - \frac{\delta V}{\delta\beta}, \tag{13}$$

$$\frac{\partial \tilde{\mathscr{H}}^{(0)}}{\partial \Phi} = -\frac{\partial S_\beta}{\partial t} - \frac{\delta V}{\delta \Phi}, \tag{14}$$

$$\frac{\partial \tilde{\mathscr{H}}^{(0)}}{\partial P} = \frac{\partial \psi}{\partial t} - \frac{\partial V}{\partial P}, \tag{15}$$

$$\frac{\partial \tilde{\mathscr{H}}^{(0)}}{\partial S_\beta} = \frac{\partial \beta}{\partial t}, \tag{16}$$

$$\frac{\partial \tilde{\mathscr{H}}^{(0)}}{\partial S_\zeta} = \frac{\partial \Phi}{\partial t} - \frac{\partial V}{\partial S_\zeta}. \tag{17}$$

In the zeroth approximation we must replace all the right-hand sides by zero, and we return to the system of equations for the stationary solutions. As the initial form of the solution we choose the stationary solution Ib), which is of greatest interest in connection with the experiments of Refs. 1–4. For this solution, $\omega_P = \omega_L$, and, in addition,

$$P=\omega_p(\cos\beta-1), \quad S_\beta=0, \quad S_\zeta=\omega_p,$$
$$\cos\Phi=(^1/_2-\cos\beta)/(1+\cos\beta). \tag{18}$$

The formulas (18) describe the stationary precession of a spin tilted through angle β from the equilibrium direction. The angle β varies in the interval $0 \leqslant \beta < \theta_0 = \arccos(-1/4)$. Direct substitution of the solution (18) into the formulas (10) and (2) gives

$$\tilde{\mathscr{H}}^{(0)} = -\omega_p^2/2, \tag{19}$$

i.e., the Hamiltonian $\tilde{\mathscr{H}}^{(0)}$ is degenerate in β. In a uniform magnetic field the solution Ib) is realized only for one value $\omega_p = \omega_L$. If, however, the field is nonuniform, the choice of ω_p becomes nonunique. Depending on the actual conditions in the experiments, either the precession frequency ω_p is specified directly, or $\int P dV \equiv \mathscr{P}$ is specified, where the integration is performed over the volume occupied by the ^{3}He. The change from the Hamiltonian $\tilde{\mathscr{H}}$ to $\mathscr{H}$ in accordance with Eq. (10) is a change variable from $\mathscr{P}$ to ω_p and is analogous to the change from a description with a fixed number of particles to a description with a fixed chemical potential in thermodynamics. In the following we shall assume that the frequency ω_p is specified. The determination of the value of ω_p corresponding to a given $\mathscr{P}$ does not give rise to difficulties; see Ref. 5.

To derive the first-order equations we must substitute the zeroth-order solutions (18) into the right-hand sides of Eqs. (12)–(17); as a result, the latter will be expressed in terms of the two variables ψ and β. Equation (12), without further transformations, gives one of the equations for these variables. To obtain the second equation we must represent all the variables except ψ in the form of expansions:

$$\beta=\beta^{(0)}+\beta^{(1)}+\dots$$
$$\Phi=\Phi^{(0)}+\Phi^{(1)}+\dots \tag{20}$$

where $\Phi^{(0)}$, $P^{(0)}$, etc. are the zeroth-approximation solutions, and substitute these expansions into the left-hand sides of Eqs. (13)–(17). There will be no zeroth-order terms, and the first-order terms will give a system of linear equations for $\beta^{(1)}$, $\Phi^{(0)}$, ... To simplify the notation we denote the variables $\beta, \Phi, P, S_\beta, S_\zeta$ by $q_1, q_2, \dots, q_5$, respectively; then the system for the first-order corrections will have the form

$$\sum_k \left(\frac{\partial^2 \tilde{\mathscr{H}}^{(0)}}{\partial q_i \partial q_k}\right)_{q_i=q_i^{(0)}} q_k^{(1)} = A_i(\beta,\psi), \tag{21}$$

where A_i are the right-hand sides of Eqs. (13)–(17). The determinant of the system (21) is equal to zero. This is easily seen by differentiating the relation (19) twice with respect to $u = \cos\beta$. As a result we obtain

$$\sum_{i,k} \frac{\partial^2 \tilde{\mathscr{H}}^{(0)}}{\partial q_i \partial q_k} \frac{dq_i^{(0)}}{du} \frac{dq_k^{(0)}}{du} = 0. \tag{22}$$

Earlier it was shown[9] that the solution Ib) is stable against small spatially uniform perturbations. This means that among the eigenvalues of the matrix $\partial^2 \mathscr{H}^{(0)}/\partial q_i \partial q_k$ there are no negative ones. Therefore, it follows from the equality (22) that there is at least one zero eigenvalue. On the other hand, it is known that after the separation of the angle α (or ψ) degeneracy remains in only one variable; therefore, the zero eigenvalue is the only one, and $dq_i^{(0)}/du$ is the eigenvector corresponding to the zero eigenvalue. The condition for solvability of the system (21),

$$\sum_k A_k \frac{dq_k^{(0)}}{du} = 0 \tag{23}$$

is the second equation connecting the variables ψ and β. For the solution of (17),

$$dq_i^{(0)}/du = (d\beta/du,\ d\Phi/du,\ \omega_p, 0, 0), \tag{24}$$

which leads to the equation

$$-\frac{d\beta}{du}\frac{\delta V}{\delta\beta} - \frac{d\Phi}{du}\frac{\delta V}{\delta\Phi} + \omega_p\left(\frac{\partial\psi}{\partial t} - \frac{\partial V}{\partial P}\right) = 0. \tag{25}$$

In this equation the arguments q_i of the perturbation V are expressed in terms of u by the formulas (18), i.e.,

$$V=V(u,\psi)=V(q_1^{(0)}(u)\dots q_5^{(0)}(u),\ \psi),$$

and hence

$$\frac{\delta V}{\delta u} = \sum_i \frac{dq_i^{(0)}}{du}\frac{\delta V}{\delta q_i}, \tag{26}$$

which makes it possible to write Eq. (25) in the form

$$\omega_p \frac{\partial\psi}{\partial t} = \frac{\delta V(u)}{\delta u}. \tag{27}$$

Analogously, Eq. (12) can be rewritten in the form

$$\omega_p \frac{\partial u}{\partial t} = -\frac{\delta V}{\delta\psi}. \tag{28}$$

The equations (27) and (28) form a closed system describing the slow variation of the variables ψ and u (or α and β) against the background of the stationary precession of the spin with angular frequency ω_p. The equations have the Hamiltonian $V(u, \psi)$ with respect to the canonically conjugate variables ψ and $\omega_p \cos\beta$.

We can give Eq. (28) the form of the law of conservation of the longitudinal spin component $S_z = \omega_p u$. For this it is necessary to write out the expression for V explicitly. The gradient energy appearing in this expression has different forms, depending on whether ψ and u are varying in the direction of the magnetic field (the z axis) or in the direction perpendicular to the field. Suppose first that the variables depend only on the longitudinal coordinate z; then, substituting Eqs. (4) into (3), after simple transformations we have

$$F_{\nabla\parallel}=(1-u)c^2(u)\left(\frac{\partial\psi}{\partial z}\right)^2+\frac{1}{2}\left[\frac{3c^2(-1)}{(1+4u)(1+u)^2}+\frac{c^2(1)}{1-u^2}\right]\left(\frac{\partial u}{\partial z}\right)^2\mp(1-u)\frac{c^2(-1)}{1+u}\left(\frac{3}{1+4u}\right)^{1/2}\frac{\partial\psi}{\partial z}\frac{\partial u}{\partial z}. \quad (29)$$

In this expression the angle ψ does not appear explicitly, and in this case Eq. (28) has the form

$$\frac{\partial(\omega_p u)}{\partial t}+\frac{\partial j_{zz}}{\partial z}=0, \quad (30)$$

where

$$j_{zz}=-\frac{\partial F_{\nabla\parallel}}{\partial\psi_z}=(u-1)\left[2c^2(u)\frac{\partial\psi}{\partial z}\mp\frac{c^2(-1)}{1+u}\left(\frac{3}{1+4u}\right)^{1/2}\frac{\partial u}{\partial z}\right] \quad (31)$$

is the flux of S_z in the direction of the z axis, induced by the nonuniformity of the order parameter; it is customarily called the zz-component of the superfluid spin current.[12,13]

In the case when ψ and u depend on the transverse coordinates x and y, the expression (3) for the gradient energy contains sines and cosines of the angle α. Because of this, in Eq. (28), besides the term with the divergence $\partial(\partial F_\nabla/\partial\psi_k)/\partial x_k$, there remains the derivative $\partial F_\nabla/\partial\psi$. This derivative, however, oscillates with frequency ω_p. In accordance with the van der Pol procedure, Eq. (28) can be averaged over a period of the oscillations, causing the oscillating term to vanish. This averaging only affects terms of the next orders in ω_∇/ω_p, and so does not reduce the accuracy of Eq. (28). After the averaging, that part of the gradient energy which depends on the transverse derivatives has the following form:

$$F_{\nabla\perp}=\frac{1}{2}(1-u)[(1-u)c_\parallel^2+(1+u)c_\perp^2]\psi_{,k}\psi_{,k}\mp c_\parallel^2\frac{1-u}{1+u}\left(\frac{3}{1+4u}\right)^{1/2}\psi_{,k}u_{,k}+\frac{1}{2}\left[\frac{c_\perp^2}{1-u^2}+\frac{3c_\parallel^2}{(1+u)^2(1+4u)}\right]u_{,k}u_{,k}, \quad (32)$$

Here, summation over $k=1, 2$ is implied. In this case too, Eq. (28) expresses the law of conservation of S_z:

$$\frac{\partial(\omega_p u)}{\partial t}+\frac{\partial j_{zk}}{\partial x_k}=0. \quad (33)$$

For the transverse components of the spin-current tensor we obtain the following expression:

$$j_{zk}=(u-1)[(1-u)c_\parallel^2+(1+u)c_\perp^2]\frac{\partial\psi}{\partial x_k}\pm c_\parallel^2\frac{1-u}{1+u}\left(\frac{3}{1+4u}\right)^{1/2}\frac{\partial u}{\partial x_k}. \quad (34)$$

In the case when u and ψ depend on all three coordinates the divergence in Eq. (33) will contain derivatives of the three current components (31) and (34). We note that both expressions for the spin-current components, like all the equations in the present paper, are written with the assumption that the ^3He is stationary.[1)]

The system (27), (28) is analogous in many respects to the equations of "time-dependent Ginzburg–Landau theory" for a neutral superfluid liquid. The role of the square of the modulus of the order parameter here is played by the conserved quantity

$$-P=\omega_p(1-u),$$

while that of the phase is played by the angle ψ. It should be noted, however, that the energy density V, unlike the Ginzburg–Landau functional, does not contain terms proportional to the fourth and higher powers of the order parameter. Because of this, in the spatially uniform case equilibrium can be reached only on the boundaries of the range of variation of u, i.e., for $u=1$ or $u=-1/4$, depending on the sign of the difference $\omega_p-\omega_L$. In particular, this circumstance leads to the appearance, in a nonuniform magnetic field, of a two-domain precessing structure[1,2,5] that has no analog in superconductors. The equations describing the stationary two-domain structure are obtained by equating the time derivatives in Eqs. (27) and (28) to zero, which leads to equations coinclding with those considered previously.[5] The system (27), (28) with time derivatives makes it possible to describe the dynamics of the two-domain structure as well.

3. THE LOW-FREQUENCY DYNAMICS OF THE TEXTURES

We now consider, as the initial stationary solution Ia),

$$S_z=\omega_L,\quad S_\beta=0,\quad S_\zeta=\omega_L\cos\beta,\quad \cos\Phi=\frac{(1/2-\cos\beta)}{(1+\cos\beta)}. \quad (35)$$

This solution is realized for $\omega_p=0$ and is therefore static. It describes static textures—spatial distributions of the rotation axis **n** that are determin by the competition of boundary energies with volume energies. In the statics of the textures an important role is played by the volume energy term $\chi\tilde{a}(\mathbf{n},\mathbf{H})^2$ that orients **n** along the magnetic field. This term is small in comparison with the dipolar energy, by the factor $(\omega_L/\Delta)^2$. In the dynamical case, from symmetry considerations we can also add to the Hamiltonian terms of the form $[\mathbf{n}\cdot(\mathbf{S}-\mathbf{H})]^2$ and $(\mathbf{n}\cdot\mathbf{H})[\mathbf{n}\cdot(\mathbf{S}-\mathbf{H})]$. For motions with frequencies that are small in comparison with ω_L these terms are small in comparison with $(\mathbf{n}\cdot\mathbf{H})^2$, and we shall omit them. When the above is taken into account the perturbation V in the dynamics of the textures has the form

$$V=F_\nabla-\tilde{a}\omega_L^2n_z^2. \quad (36)$$

When V is substituted into the equations of motion, n_z^2 must be expressed in terms of the Euler angles. For the solutions Ia) and Ib), $\cos\theta=1/4$, and

$$n_z^2=4(\cos\beta+1/4)/5. \quad (37)$$

From a comparison of formulas (35) and (18) it can be seen that the solution Ia) is obtained from Ib) by interchanging the spin projections S_z and S_ζ. It is natural to expect that in this case the angles α and γ conjugate to these projections will also exchange roles. Therefore, as the initial variables we choose the following pairs: γ and $Q=S_\zeta-S_z$; β and S_β; Φ

and S_z. The Hamiltonian $\mathscr{H}^{(0)}$ with $\omega_p = 0$ coincides with $\mathscr{H}^{(0)}$ expressed in the new variables. Thus, the starting system of equations has the following form:.

$$0=\frac{\partial Q}{\partial t}+\frac{\delta V}{\delta\gamma}, \tag{38}$$

$$\frac{\partial \mathscr{H}^{(0)}}{\partial\beta}=-\frac{\partial S_\beta}{\partial t}-\frac{\delta V}{\delta\beta}, \tag{39}$$

$$\frac{\partial \mathscr{H}^{(0)}}{\partial\Phi}=-\frac{\partial S_z}{\partial t}-\frac{\delta V}{\delta\Phi}, \tag{40}$$

$$\frac{\partial \mathscr{H}^{(0)}}{\partial Q}=\frac{\partial\gamma}{\partial t}-\frac{\partial V}{\partial Q}, \tag{41}$$

$$\frac{\partial \mathscr{H}^{(0)}}{\partial S_\beta}=\frac{\partial\beta}{\partial t}-\frac{\partial V}{\partial S_\beta}, \tag{42}$$

$$\frac{\partial \mathscr{H}^{(0)}}{\partial S_z}=\frac{\partial\Phi}{\partial t}-\frac{\partial V}{\partial S_z}. \tag{43}$$

As in the preceding section, to obtain the equations of the first approximation in l_D/l_∇ and ω_∇/Ω we must express all the variables in the right-hand sides of Eqs. (38)–(43) in terms of γ and u by means of the formulas (35). Equation (38) will then be one of the equations for γ and u, and to obtain the second equation we must substitute the variables β, Φ, ... in the form of the expansions (19) into the left-hand sides of Eqs. (39)–(43) and write out the system of equations for the first-order corrections $\beta^{(1)}$, $\Phi^{(1)}$, ... The solution of the corresponding homogeneous system in this case will be

$$dq_i^{(0)}/du=(\omega_L, 0, 0, d\beta/du,\ d\Phi/du). \tag{44}$$

The condition that the right-hand sides of Eqs. (39)–(43) be orthogonal to this solution gives the second equation for γ and u. After transformations analogous to those carried out in the preceding section, the system of equations for γ and u is brought to the form

$$\omega_L\frac{\partial\gamma}{\partial t}-\frac{\delta V(\gamma,u)}{\delta u}=0, \tag{45}$$

$$\omega_L\frac{\partial u}{\partial t}+\frac{\delta V(\gamma,u)}{\delta\gamma}=0. \tag{46}$$

This is the system describing the low-frequency dynamics of the textures. The projections of the axis **n** are expressed in terms of γ and $u = \cos\beta$, n_z is determined from the formula (37), and the transverse projections are found from

$$n_x\pm in_y=\pm4(15)^{-1/2}\sin\beta\cos\Phi\exp\{\mp i(\gamma-\Phi/2)\}. \tag{47}$$

The cartesian projections of the spin are expressed in terms of the time derivatives of the angles γ and β:

$$S_x\pm iS_y=(\dot\gamma\sin\beta\pm i\dot\beta)e^{\pm i(\Phi-\gamma)}, \tag{48}$$

$$S_z=\frac{d\Phi}{du}\dot u+\dot\gamma(u-1). \tag{49}$$

It is easy to convince oneself that in the case when γ and u depend only on the longitudinal coordinate z the gradient energy F_∇ does not depend on γ. Equation (46) in this case takes the form of the law of conservation of Q or $S_\zeta = \omega_L u$:

$$\omega_L\frac{\partial u}{\partial t}+\frac{\partial j_{\zeta z}}{\partial z}=0, \tag{50}$$

where

$$j_{\zeta z}=-\frac{\partial F_\nabla}{\partial\gamma'}=(1-u)\left\{-2c^2(u)\frac{\partial\gamma}{\partial z}+[(1+2u)c_\parallel^2-2uc_\perp^2]\frac{d\Phi}{du}\frac{\partial u}{\partial z}\right\} \tag{51}$$

is the current of the ζ-component of the spin in the z direction. If there is also dependence on the transverse coordinates x and y, the angle γ appears explicitly in the expression for the gradient energy. In contrast to the case considered in Sec. 2, this angle cannot be eliminated by averaging, and Eq. (46) now does not have the form of a conservation law.

As an example of the application of the system (45), (46) we shall consider a spatially uniform stationary solution of this system, of the form $\partial u/\partial t = 0$, $\partial\gamma/\partial t = -\tilde\omega_p$. From Eq. (45) we have $\tilde\omega_p = 4\tilde a\omega_L/5$, i.e.,

$$\gamma=\gamma_0-4\tilde a\omega_L t/5. \tag{52}$$

According to the formula (47), this solution describes precession of the vector **n** with angular frequency $\tilde\omega_p$ about the direction of the magnetic field. According to the experiments of Ref. 14, near the melting curve we have $\tilde a\sim5\cdot10^{-6}$, so the precession frequency is small; for a field ~ 1 kOe, $\tilde\omega_p \sim 100$ rad/sec. The precession of **n** leads to the appearance of transverse components of the spin, and this can be used to excite such precession by a transversely polarized oscillating magnetic field. In the continuous NMR spectrum we should observe a weak line at $\omega = \tilde\omega_p$. The limiting angle $\beta = \theta_0$ corresponds, according to formula (37), to tilt of **n** through 90° from the magnetic-field direction. Upon further increase of the angle β the system goes over to the solution IIa) (see Ref. 9) and the precession frequency will be determined by the dipolar energy. Thus, the situation is analogous in many respects to that which obtains in spin precession. In a nonuniform field an initially uniform precession of **n** should also go over into two-domain precession. In one of the domains **n** is parallel to $\mathbf{H}_0$, while in the other domain **n** is perpendicular to $\mathbf{H}_0$. In this case the domain-wall thickness $\lambda\sim(c^2/\tilde a\omega_L\nabla\omega_L)^{1/3}$, which, for $H_0 = 100$ Oe and $\nabla H_0\sim1$ Oe/cm, gives $\lambda\sim1$ cm. It should also be borne in mind that the influence of the walls on the orientation of **n** penetrates over large distances into the liquid, and in vessels of realistic size it is practically always necessary to take this influence into account.[15] For $\partial\psi/\partial t = 0$ and $\partial u/\partial t = 0$ Eqs. (45) and (46) go over into the equilibrium equations, from which, by specifying the boundary conditions, one can find the static textures.

4. DYNAMICS SUPERPOSED ON PRECESSION WITH FREQUENCY EXCEEDING THE LARMOR FREQUENCY

Uniform precession with frequency $\omega_p > \omega_L$ is realized for $\beta > \theta_0$ and is described in the notation of Ref. 9 by the solution IIb), i.e.,

$$\omega_p=\tfrac{1}{2}\{\omega_L+[\omega_L^2-\tfrac{16}{15}\Omega^2(1+4\cos\beta)]^{1/2}\}, \tag{53}$$

$$P=(\omega_L-2\omega_p)(1-\cos\beta), \tag{54}$$

$$S_\zeta=\omega_L\cos\beta+\omega_p(1-\cos\beta), \tag{55}$$

$$\Phi=0. \tag{56}$$

It can be seen from formula (53) that specifying the precession frequency determines $\cos\beta\equiv u=u(\omega_p)$ uniquely. Therefore, the low-frequency dynamics on the background of the solution IIb) should be described by an equation in only one free variable—the angle ψ. Substitution into Eq. (12) of P, u, and Φ, expressed in terms of ω_p by means of the formulas (53)–(56), leads an equation describing the stationary flow of the conserved combination $P=S_z-S_\xi$ of spin projections:

$$\partial j_{pk}/\partial x_k=0, \tag{57}$$

where $j_{pk}=-\partial F_\nabla/\partial\psi_k$ is the flux of the quantity P. In the expression for F_∇ in the leading approximation there remain only terms containing of the angle ψ. As a result, for the components of j we obtain the following expressions:

$$j_{px}=-\mu_\perp\partial\psi/\partial x,\ j_{py}=-\mu_\perp\partial\psi/\partial y,\ j_{pz}=-\mu_\parallel\partial\psi/\partial z, \tag{58}$$

where

$$\mu_\perp=(1-u)[(1-u)c_\parallel^2+(1+u)c_\perp^2],$$

$$\mu_\parallel=2(1-u)[uc_\parallel^2+(1-u)c_\perp^2].$$

Substitution of (58) into (57) leads to an equation for the angle ψ:

$$\mu_\perp\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)+\mu_\parallel\frac{\partial^2\psi}{\partial z^2}=0, \tag{59}$$

which can be reduced to the Laplace equation by a change of scales. Thus, the stationary flow of the spin current for $\beta>\theta_0$ differs from the stationary flow of an ideal incompressible liquid only in the anisotropy of the "densities" μ. The formulation of the correct boundary conditions on Eq. (59) requires separate analysis. These conditions should depend both on the properties of the surface and on the magnitude and direction of the magnetic field. It is evident that a satisfactory condition for most surfaces is the requirement that the spin-current component normal to the surface vanish. The simplest solutions of Eq. (59) are

$$\nabla\psi=\mathbf{h}=\text{const}, \tag{60}$$

and describe stationary one-dimensional flow of the spin. The anisotropy μ leads to the result that the direction of the current coincides with the direction of $\mathbf{h}$ only if $\mathbf{h}\parallel\hat{\mathbf{z}}$ or $\mathbf{h}\perp\hat{\mathbf{z}}$.

In order to obtain equations which for $\omega_p>\omega_L$ also describe the time variation of the spin currents, we must write out for this case the system of equations (21) for the first-order corrections $P^{(1)}$, $u^{(1)}$, $S_\zeta^{(1)}$, $S_\beta^{(1)}$, and $\Phi^{(1)}$. In the right-hand sides of these equations we must retain only the terms depending on derivatives of the angle ψ:

$$P^{(1)}+(1-u)S_\zeta^{(1)}-[\omega_p+u(\omega_p-\omega_L)]u^{(1)}=\frac{\partial\psi}{\partial t}(1-u^2), \tag{61}$$

$$P^{(1)}+2S_\zeta^{(1)}-\omega_L u^{(1)}=0, \tag{62}$$

$$[\omega_p+u(\omega_p-\omega_L)]P^{(1)}+(1-u)\omega_L S_\zeta^{(1)}-\mathscr{H}_{\beta\beta}u^{(1)}=\frac{\partial F_\nabla}{\partial u}(1-u^2), \tag{63}$$

$$\frac{\partial^2 U}{\partial\Phi^2}\Phi^{(1)}=\frac{\partial}{\partial x_\xi}\left(\frac{\partial F_\nabla}{\partial\Phi_{,\xi}}\right), \tag{64}$$

$$S_\beta^{(1)}=0, \tag{65}$$

where

$$\mathscr{H}_{\beta\beta}=(1-u^2)\frac{\partial^2 U}{\partial u^2}+\omega_L^2+2(1+u)\omega_p(\omega_p-\omega_L).$$

The part of $\partial F_\nabla/\partial u$ that depends on the derivatives of ψ has the form

$$\frac{\partial F_\nabla^\psi}{\partial u}=\frac{1}{2}\left\{\frac{d\mu_\perp}{du}\left[\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2\right]+\frac{d\mu_\parallel}{du}\left(\frac{\partial\psi}{\partial z}\right)^2\right\}. \tag{66}$$

We now express $P^{(1)}$ and $u^{(1)}$ in terms of the space and time derivatives of ψ by means of Eqs. (61)–(63):

$$P^{(1)}=\frac{\mu_\perp}{s_\perp^2}\frac{\partial\psi}{\partial t}-B\frac{\partial F_\nabla^\psi}{\partial u}, \tag{67}$$

$$u^{(1)}=B\frac{\partial\psi}{\partial t}-\frac{15}{16\Omega^2}\frac{\partial F_\nabla^\psi}{\partial u}, \tag{68}$$

where for brevity we have introduced the notation

$$s_\perp^2=16\Omega^2\mu_\perp/[(1-u)32\Omega^2+60\omega_p(\omega_p-\omega_L)+15\omega_L^2],$$

$$B=15(2\omega_p-\omega_L)/16\Omega^2.$$

Substituting the expressions (67) and (68) into Eq. (12) and keeping the terms with the leading derivatives, we obtain the following equation for ψ:

$$\frac{\mu_\perp}{s_\perp^2}\frac{\partial^2\psi}{\partial t^2}-\mu_\perp\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)-\mu_\parallel\frac{\partial^2\psi}{\partial z^2}$$
$$-2B\left[\frac{d\mu_\perp}{du}\left(\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial x\,\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\,\partial t}\right)+\frac{d\mu_\parallel}{du}\frac{\partial\psi}{\partial z}\frac{\partial^2\psi}{\partial z\,\partial t}\right]=0. \tag{69}$$

In the leading approximation in ω_∇/Ω and l_D/l_∇, Eq. (69) coincides with the wave equation with an anisotropic velocity of propagation. The accuracy of the procedure makes it possible, however, to retain also the fourth, nonlinear term in Eq. (69). Below it will be shown that allowance for this term leads to spin-wave drag by the spin current.

As an example of the application of Eq. (69) we shall consider the propagation of small oscillations of ψ superposed on a state with $\nabla\psi=\mathbf{h}=\text{const}$. Setting $\psi=\mathbf{h}\cdot\mathbf{r}+\tilde\psi$, linearizing Eq. (69) in $\tilde\psi$, and substituting $\psi\sim\exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)]$, we obtain the following dispersion law for such oscillations:

$$\omega=\frac{Bs_\perp^2}{\mu_\perp}\left[\frac{d\mu_\perp}{du}(h_xk_x+h_yk_y)+\frac{d\mu_\parallel}{du}h_zk_z\right]$$
$$\pm s_\perp\left[k_x^2+k_y^2+\frac{\mu_\parallel}{\mu_\perp}k_z^2\right]^{1/2}. \tag{70}$$

It can be seen from the formula obtained that for $\mathbf{h}=0$ the spin waves on the background of precession with $\omega_p>\omega_L$ have a linear dispersion law; $s_\perp$ is the velocity of propagation of such waves if $\mathbf{k}$ lies in the plane perpendicular to the magnetic field. But if $\mathbf{k}$ is parallel to the field, for the wave-propagation velocity we have

$$s_{\|}^2=\frac{\mu_\|}{\mu_\perp}s^2=(1-u)\frac{32\Omega^2[c_\|^2u+(1-u)c_\perp^2]}{32\Omega^2(1-u)+60\omega_p(\omega_p-\omega_L)+15\omega_L^2}. \quad (71)$$

For $\Omega\ll\omega_L$ this velocity goes over into the velocity obtained previously,[16] while for $u\to-1/4$ it goes over into the velocity of propagation of "torsional" oscillations of the two-domain structure.[3,6] The terms proportional to the components of $\mathbf{h}$ in Eq. (70) describe the entrainment of spin waves by the spin current; the expression $(d\mu_\|/du)(Bs_\perp^2h_z/\mu_\perp)$ plays the role of the velocity of the spin current for $\mathbf{h}\|\tilde{\mathbf{z}}$, while the expression $(d\mu_\perp/du)(Bs_\perp^2h/\mu_\perp)$ plays the analogous role for $\mathbf{h}\perp\mathbf{z}$. In the region of applicability of the approach used here, this correction to the velocity of the waves is small in proportion to sh/Ω.

Equation (69) cannot be applied in the immediate vicinity of the angle $\beta=\theta_0$ or for ω_p close to ω_L. For applicability of Eq. (69) it is necessary, in any case, that the correction $u^{(1)}$ be small in comparison with $|u+1/4|$, i.e., according to (68),

$$B\frac{\partial\psi}{\partial t}-\frac{15}{16\Omega^2}\frac{\partial F_\nabla{}^\psi}{\partial u}\ll\left|u+\frac{1}{4}\right|. \quad (72)$$

Using the estimate $\partial\psi/\partial t\sim\omega_\nabla$ we obtain from this

$$\left|\frac{1}{4}+u\right|\gg\frac{\omega_L\omega_\nabla}{\Omega^2}. \quad (73)$$

Using the formula (53) we can formulate this condition as a restriction on the frequency:

$$\omega_\nabla\ll\omega_p-\omega_L. \quad (73')$$

Analogously, estimating $\nabla\psi$ as ψ/l_∇ we arrive at a condition on u:

$$|u+{}^1/_4|\gg c^2/\Omega^2l_\nabla^2 \quad (74)$$

or on $\omega_P-\omega_L$:

$$\omega_p-\omega_L\gg c^2/l_\nabla^2\omega_L. \quad (74')$$

The right-hand sides of both criteria (73) and (74) contain small quantities, and, therefore, there exists a region

$$1\gg|u+{}^1/_4|\gg c^2/\Omega^2l_\nabla^2,\quad \omega_L\omega_\nabla/\Omega^2,$$

and this makes it possible to go over to the limit $u\to-1/4$ in expressions that do not have a singularity at $u=-1/4$—in particular, in Eq. (71).

The stationary solution IIa) is obtained from IIb) by the replacement $S_z\rightleftarrows S_\zeta$, and the angles α and γ then exchange roles. One can go over from the low-frequency dynamics superposed on the solution IIb) to the dynamics superposed on the solution IIa) in the same way as was done in Sec. 3 of the present paper for solutions of type I. It should be noted, however, that the spatially uniform precession of the vector $\mathbf{n}$, described by the solution IIa), has still not been observed experimentally, and therefore a detailed discussion of the dynamics on the background of this solution is premature.

5. CONCLUSION

In the present paper we have obtained the equations of motion of the spin only in the leading approximation in l_D/l_∇ and ω_∇/Ω. For the interpretation of certain phenomena, e.g., oscillations of the two-domain structure,[3] we require the equations of the next approximation. The scheme developed in Sec. 2 makes it possible to obtain such equations in a natural manner.

An important qualitative feature of the equations obtained is their similarity to the hydrodynamic equations of a superfluid. This makes it possible to use them to interpret and predict the analogs for the ^{3}He-B spin system of the phenomena observed in other superfluid systems. It should be noted, however, that the form of the equations of the low-frequency dynamics depends on the degeneracy space of the system under consideration. In this sense, only for solutions of the type II (Sec. 4) is there an analogy with ordinary superfluid systems, since in both cases the degeneracy space is a circle. For the solutions Ia) and Ib) the degeneracy space is equivalent to a hemisphere, and this leads to differences from the hydrodynamics of an ordinary superfluid liquid and to the appearance of phenomena that are specific for this case.

To describe real phenomena we must take into account dissipative terms in the equations of motion. The inclusion of dissipative terms in the scheme developed in Sec. 2 does not give rise to difficulties in the technical sense, but the equations of motion obtained as a result have an important qualitative difference. In the absence of dissipation, in almost all the cases considered, one of the equations has the form of a conservation law for a certain combination of spin projections. This combination thereby appears in the equations in the same way as the density of the liquid. When dissipation is taken into account the conservation law fails to hold, by an amount determined by the magnitude of the dissipative terms. The result is a form of fluid dynamics with a nonconserved mass. The situation is closer to normal for the solutions Ia) and Ib), where, in the uniform case, dissipation is absent and arises only in proportion to the deviation of the motion of the order parameter from the original stationary motion.

We note, finally, that irrespective of whether or not there exists an analogy with ordinary superfluid systems, the equations obtained are useful for describing the situations realized in experiments. The dynamics superposed on the solution Ib) has essentially already been used.[5,8,16] The dynamics of the textures is also important, since in experiments with ^{3}He-B one practically always has to take into account the influence of the specific texture on the result. In this connection, e.g., the question of the time in which the equilibrium texture is established is of interest. This question can be analyzed by adding dissipative terms to Eqs. (45) and (46).

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[1)]The two signs in front of $(1+4u)^{-1/2}$ in the formulas (29)–(34) correspond to the two different branches of the function $\Phi(\beta)$ determined by Eq. (6).

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