The $n \to n(\nu\bar{\nu})$ process in the field of a linearly polarized plane wave

V. V. Skobelev

(Submitted 4 March 1987)
Zh. Eksp. Teor. Fiz. **93**, 1168–1172 (October 1987)

A solution obtained for the generalized Dirac equation in the field of a plane wave is used for the first time to discuss the generation of neutrino pairs induced by the interaction of the neutron anomalous magnetic moment with the field of a plane wave. Expressions exact in the field strength are obtained for the probabilities in a wave of linear polarization and in a crossed field. Estimates are made of the possibility of observing the effect against the background of the competing process of beta decay.

A considerable number of studies have been devoted to interaction processes in the field of a plane wave. The calculation technique proposed by Ritus and Nikishov[1] permits taking into account in a relativistically invariant way the charged interaction with the wave field and obtaining a result which depends on the invariant field and energy parameters. Interest in problems of this type has been stimulated both by progress in laser technology and by the opening of new reaction channels which are forbidden in the free case. Among the latter, for example, are the electroweak processes $e^- \to e^-(\nu\bar{\nu})$,[2–4] $\nu \to \nu(e^+e^-)$,[5] $\nu \to \pi^+\mu^-$ and $e^\pm \to \pi^\pm \nu$[6], and others. In this connection there has previously been no consideration of reactions due to interaction of anomalous magnetic moments of neutral particles with the field of a plane wave. This may be due partly to the lack of a relativistically invariant calculation scheme similar to that of Ref. 1. In the present work the Ritus-Nikishov technique is extended to interactions of this type with specific application to calculating the probability of neutrino pair production by a neutron in the field of a linearly polarized plane wave.

1. The wave function of a neutral Fermi particle which has an anomalous magnetic moment μ satisfies the generalized Dirac equation (metric $(+---)$)

$$\left(i\hat{\partial}-m-\frac{i}{2}\mu\sigma_{\mu\nu}F^{\mu\nu}\right)\psi=0, \tag{1}$$

where F is the tensor of the external field. The potential of the plane-wave field has the form

$$A=A(\varphi),\quad \varphi=(kx),\quad k^2=(kA)=(kA')=0. \tag{2}$$

Squaring the initial equation (1), we shall seek the solution in the form

$$\psi=e^{-i(px)}\Phi(\varphi), \tag{3}$$

where $p^2=m^2$ holds and the function $\Phi(\varphi)$ obeys the equation

$$\Phi'=\frac{\mu}{2(kp)}(\hat{k}\hat{A}'\hat{p}+\hat{p}\hat{k}\hat{A}')\Phi. \tag{4}$$

From this we obtain

$$\Phi=\exp\left\{\frac{\mu}{2(kp)}(\hat{k}\hat{A}\hat{p}+\hat{p}\hat{k}\hat{A})\right\}\frac{u(p)}{(2p_0)^{1/2}}. \tag{5}$$

The expression for ψ determined by (3) and (5) is a solution of the initial equation (1) if the condition $(\hat{p}-m)u=0$ is satisfied, and where with normalization to a single particle per unit volume we have $\bar{u}u=2m$. Taking into account further that the expression in curly brackets in (5) satisfies the relation

$$\{\}^{2n}=(-1)^n z^{2n},\quad z=\mu(-A^2)^{1/2},$$

we eventually obtain

$$\Phi=\left[\cos z+\frac{\hat{k}\hat{A}\hat{p}+\hat{p}\hat{k}\hat{A}}{2(kp)(-A^2)^{1/2}}\sin z\right]\frac{u(p)}{(2p_0)^{1/2}}. \tag{6}$$

It is easy to verify that the current density is given by the expression

$$\bar{\psi}\gamma_\mu\psi=p_\mu/p_0,$$

which means that effective renormalization of the mass, in contrast to an interaction of the charged type,[1] does not occur in this case.

We note that the solutions of Eq. (1) previously obtained have an extremely cumbersome and noninvariant form, describing moreover particular cases of wave polarization.[7]

2. In the contact appoximation we shall write the effective Lagrangian $L_{(nn\nu\bar{\nu})}$ induced by the contribution of neutral currents in the form (the neutrino is massless)

$$L_{(nn\nu\bar{\nu})}=\frac{G}{2^{1/2}}[\bar{\psi}_n\Gamma_\alpha\psi_n][\bar{\psi}_\nu\gamma^\alpha(1+\gamma^5)\psi_\nu],$$
$$\Gamma_\alpha=\gamma_\alpha(C_V+C_A\gamma^5), \tag{7}$$

and the values of the vector and axial vector coupling constants C_V and C_A do not depend on the type of neutrino, since

$$g_{z\nu_e\nu_e}=g_{z\nu_\mu\nu_\mu}=g_{z\nu_\tau\nu_\tau}$$

We shall consider the process $n\to n(\nu\bar{\nu})$ in the field of a linearly polarized plane wave

$$A=a\sin\varphi. \tag{8}$$

The matrix element obtained with use of Eqs. (3), (6), and (7) contains integrals

$$I_{1,2,3}=\frac{1}{(2\pi)^4}\int d^4x\, e^{-i(p-p'-q)x}\{\cos^2 z,\ \sin^2 z,\ \sin z\cos z\}, \tag{9}$$

where p and p' are the momenta of the initial and final neutrons and q is the combined momentum of the neutrinos. Using an expansion in Fourier series, we obtain

 0038-5646/87/100659-03$04.00

$$I_1=-I_2=\frac{1}{2}\sum_{s=2}^{\infty} J_s(x)\delta(p-p'-q+sk), \tag{9a}$$

$$I_3=\frac{i}{2}\sum_{s=1}^{\infty} J_s(x)\delta(p-p'-q+sk), \tag{9b}$$

where the sum in (9a) is taken over even values of s and that in (9b) over odd values, and we retain only terms which make a nonzero contribution to the probability. In Eqs. (9) we use the notation $x=2\mu(-a^2)^{1/2}$ for the argument of the Bessel functions, which is similar in meaning to the parameter $e(-a^2)^{1/2}/m$ when the charged interaction with the wave field is included.[1]

The total probability of the process per unit time after integration over neutrino momentum is given by the expression

$$W=\sum_{s=1}^{\infty} W_s, \tag{10a}$$

where the partial contribution W_s corresponds to capture of s photons of the wave (when the radiation of neutrinos of all types is taken into account, the result must in addition be multiplied by 3):

$$W_s=\frac{G^2J_s^2}{12(2\pi)^4 p_0}\int\limits_{q^2>0}\frac{d^3p'}{2p_0'}(q^\mu q^\nu-q^2g^{\mu\nu})\left[\frac{1+(-1)^s}{2}\Gamma_{\mu\nu}^{(even)}+\frac{1-(-1)^s}{2}\Gamma_{\mu\nu}^{(odd)}\right],$$

$$\Gamma_{\mu\nu}={}^1/_4\,\mathrm{Sp}\,[(\hat p'+m)T_\mu(\hat p+m)\bar T_\nu],\quad q=p+sk-p',$$

$$T_\mu^{(even)}=\Gamma_\mu+\frac{M'\Gamma_\mu M}{(kp)(kp')a^2},$$

$$T_\mu^{(odd)}=\frac{M'\Gamma_\mu}{(kp')(-a^2)^{1/2}}+\frac{\Gamma_\mu M}{(kp)(-a^2)^{1/2}},$$

$$M=\hat k\hat a\hat p+(pk)\hat a-(pa)\hat k,\quad M'=\hat p'\hat a\hat k+(p'k)\hat a-(p'a)\hat k. \tag{10b}$$

The calculations show that

$$(q^\mu q^\nu-q^2g^{\mu\nu})\Gamma_{\mu\nu}^{(even)}=(q^\mu q^\nu-q^2g^{\mu\nu})\Gamma_{\mu\nu}^{(odd)}$$
$$=2(C_V^2+C_A^2)\Big\{2(pq)(p'q)+q^2(pp')+3q^2m^2+\frac{2m^2(kq)}{(kp)(kp')}[(kq)(pp')-(kp')(pq)-(kp)(p'q)]$$
$$-2m^2(kp)(kp')\frac{\alpha^2}{a^2}\Big\}+2(C_V^2-C_A^2)\Big\{-2(pq)(p'q)+q^2(pp')$$
$$-3m^2q^2+\frac{m^2}{(kp)(kp')}[-2m^2(kq)^2-q^2(kp)^2-q^2(kp')^2$$
$$+2(kp')(p'q)(kq)+2(kp)(pq)(kq)]+$$
$$+(kp)(kp')\frac{\alpha^2}{a^2}[4m^2+q^2-2(pp')-2(p-p')q]\Big\}.$$

where we use the notation

$$\alpha=(ap/kp-ap'/kp')$$

and have omitted the interference term proportional to $C_V C_A$, which does not contribute to the integrated probability.

As in Ref. 1, we introduce the invariant variables

$$y=2(kp)/m^2,\quad u=(kq)/(kp'),\quad \lambda=q^2/m^2$$

and the azimuthal angle φ_0 of the vector p' in the system $(\mathbf{k}\uparrow\downarrow\mathbf{p})$ so that, for example,

$$\left(\frac{m^2}{-a^2}\right)\alpha^2=\left\{\frac{2}{y}[(1+u)(\lambda_s-\lambda)]^{1/2}\cos\varphi_0\right\}^2,$$

$$\lambda_s=u(u_s-u)(1+u)^{-1},\quad u_s=sy.$$

In the new notation the partial probability is written in the form

$$W_s=w_0J_s^2\int_0^{u_s}\frac{du}{(1+u)^2}\int_0^{\lambda_s}d\lambda\int_0^{2\pi}\frac{d\varphi_0}{\pi}\left\{C_V^2\left[\frac{\lambda uu_s}{1+u}-\lambda^2\right]\right.$$
$$\left.+C_A^2\left[4(\lambda_s-\lambda)\cos^2\varphi_0+\frac{(u_s-2u)^2}{1+u}+\lambda\left(8+\frac{uu_s}{1+u}\right)-\lambda^2\right]\right\},$$
$$w_0=G^2m^6/48(2\pi)^3p_0. \tag{11}$$

3. The fairly elementary though awkward integration in (11) yields the result

$$W_s=w_0J_s^2(C_V^2F_V^{(s)}+C_A^2F_A^{(s)}), \tag{12}$$

where

$$F_V^{(s)}=\frac{u_s^3}{12}-u_s^2-\frac{107}{12}u_s-\frac{1}{6}+b_s\left(\frac{1}{6}-u_s\right)+\frac{u_sb_s^2}{12}-5(u_s+2)\ln b_s, \tag{12a}$$

$$F_A^{(s)}=\frac{5}{12}u_s^3-u_s^2-\frac{19}{12}u_s-\frac{15}{2}+b_s\left(u_s^2+7u_s+\frac{53}{6}\right)-b_s^2\left(\frac{u_s^2}{3}+\frac{5}{4}u_s+\frac{4}{3}\right)-(u_s+2)\ln b_s, \tag{12b}$$

$b_s=(1+u_s)^{-1}$, and the functions $F^{(s)}$, as one would expect, are positive definite in the range 0–∞ of values of u_s.

In the nonrelativistic case $u_s\ll 1$ to within terms of order U_s^7 we obtain

$$F_V^{(s)}=u_s^7/84, \tag{13a}$$

$$F_A^{(s)}=u_s^5\left(\frac{2}{5}-\frac{4}{5}u_s+\frac{101}{84}u_s^2\right), \tag{13b}$$

which means that the contribution of the axial vector structure constant dominates and the leading power of y is the same as in the charged interaction with a wave field.[4]

In a wave of low intensity $x\ll 1$ the main contribution in the sum over s in (10a) is from small values of s in accordance with the expansion[8]

$$J_s^2(x)=\left(\frac{x}{2}\right)^{2s}\left[\frac{1}{(s!)^2}-\frac{x^2}{2s!(s+1)!}+\ldots\right],$$

which with inclusion of (12) permits us to write out easily the contribution to W in any order in x^2. It is easy to see that the term linear in the wave energy density (proportional to x^2) does not depend on the polarization state of the wave, which is just like the situation in the charged interaction.[1]

Passage to the limit of a constant crossed field, $x\to\infty$, $y\to 0$, $\tilde\chi=xy=$ const is simpler to achieve if we repeat the calculations with a potential $A=a\varphi$ (for definiteness we shall assume that $k_0>0$). As can easily be seen, the integrals of the form (9) will now be equal to $(x=2|\mu|(-a^2)^{1/2})$

$$I_1=-I_2={}^1/_4\delta(p+xk-p'-q),$$

$$I_3=\frac{i}{4}\,\mathrm{sign}(\mu)\delta(p+xk-p'-q).$$

This means that the total probability of the process $n\to n(\nu\bar\nu)$ in a constant crossed field $W_\perp$ is obtained from

(10a) and (12) by removing the sum over s and making the substitutions $J_s^2 \to \frac{1}{4}$ and $u_s \to \tilde{\chi}$. In particular, for $\tilde{\chi} \ll 1$ we have

$$W_\perp = {}^1/_4 w_0 \left[C_V^2 \frac{\tilde{\chi}^7}{84} + C_A^2 \left(\frac{2}{5} \tilde{\chi}^5 - \frac{4}{5} \tilde{\chi}^6 + \ldots \right) \right], \quad (14a)$$

and for $\tilde{\chi} \gg 1$

$$W_\perp = {}^1/_{48} w_0 \tilde{\chi}^3 (C_V^2 + 5C_A^2). \quad (14b)$$

4. Present-day lasers can focus to an intensity of about 10^{18} W/cm^2,[9] which corresponds to a maximum wave field strength of about 10^7 to 10^8 gauss. Taking also the limiting value $(\omega/m_e) \sim 10^{-6}$, we obtain $x \sim 10^{-3}$, and the value of $y \sim 10^{-9}(p_0/m)$ will depend on the neutron energy. In this case it is possible to discuss just the contribution of the harmonic $s = 1$ to the probability, and with the additional condition $y \ll 1$ it is necessary to consider only the contribution of the axial vector part (13b). Although information on the value of the constant C_A in the Lagrangian (7) is not available to us, it is quite natural to assume $C_A \sim 1$. Then, using the standard expression for the probability of beta decay of a free neutron, transformed to an arbitrary system, we easily find that the probability of the process $n \to n(\nu\bar{\nu})$ is equal to the usual probability for extreme relativistic energy values, $p_0 \sim 10^{6-7} m$ (but with $y \ll 1$), which as yet are not accessible experimentally. We note that when the influence of the field on both processes is taken into account, it is possible in principle to separate them on the basis of the difference in the decay products. Therefore the hopes for detection of the process under discussion most likely rest on further increase of the power and frequency of laser radiation.

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Translated by Clark S. Robinson