Collective quantum effects in the transport for Boltzmann gases

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A simple phenomenological method is proposed for finding the kinematic virial corrections in the linearized transport equation for nondegenerate gases at arbitrary temperatures. These are precisely the corrections that determine the possible existence of collective modes even in the classical temperature region. The connection between the kinematic corrections and the virial expansion of the equilibrium distribution function is discussed. All the phenomenological quantities are expressed in the final results in terms of a microscopic quantity—the exact amplitude for scattering of two particles. A microscopic verification of the phenomenological equations is presented, and the nonlocal gradient corrections to the collision integral are calculated. The apparent contradictions between the macroscopic expressions and the results of the perturbation-theory calculations (the I_2-term problem and others) are resolved.

1. INTRODUCTION

It has become clear recently that collective phenomena connected in one way or another with particle interaction occur not only in degenerate or sufficiently dense systems, but can also manifest themselves in rarefied gases at high temperatures, when the molecule ensemble is described by classical Maxwell-Boltzmann statistics. Such phenomena can lead both to noticeable effects in the thermodynamics of the gas and to the appearance of specific collective modes. Probably the greatest accomplishment in this field was the prediction and experimental observation of collective spin waves in spin polarized gaseous H↑ and ^{3}He↑ (see Refs. 1 and 2). In certain cases, which are unfortunately quite difficult to realize, oscillations of zero-sound type can also propagate.[3] From the formal standpoint, spin-wave and zero-sound modes appear when account is taken of quantum virial corrections of the Fermi-liquid type in the kinematic (left-hand) part of the transport equation for Boltzmann gases. In contrast to the collision integral, these corrections, which are due to interaction between the particles, do not vanish when the macroscopic dynamics equations (the continuity equation, the Euler equation, and other) are obtained from the transport equation. In other words, these corrections lead to the appearance of additional terms in the relations for the macroscopic conservation laws. If the spectrum of the sound oscillations in a nonideal gas are defined with the aid of the transport equation, it is just (and only!) the kinematic virial corrections which lead to the known virial expansion of the speed of sound. Obviously, when acoustic oscillations are considered, the kinematic virial corrections can be retained in the kinetic equation for all temperatures, since all the remaining terms that depend on the interaction and enter in the collision integral vanish in the corresponding integrations as a result of the particle-number and momentum conservation laws. In the general case, the validity of retaining, in investigations of collective modes, terms that describe a self-consistent field of unusual form is verified by comparing these terms with nonlocal gradient additions to the collision integral.

It was noted already in the earliest studies[4,3,1] that at sufficiently low temperatures $T \ll \hbar^2/mr_0^2$, where m is the mass of the gas particle and r_0 is the atomic dimension, even the gradient self-consistent terms in the transport equation exceed significantly the nonlocal terms in the collision integral. Therefore the use of the transport equation with kinematic virial corrections and a local collision integral is fully justified at $T \ll \hbar^2/mr_0^2$. Lhouillier and Laloë,[5] using a T-matrix approach, obtained an expression for the local (i.e., independent of the distribution-function gradients) self-consistent field in the equation for the off-diagonal components of the polarization density matrix in a magnetized gas. The neglect of all the interaction-induced gradient terms in the transport equation turned in this case to be justified by the low gas density. It is precisely this circumstance which permits the results of Ref. 5 to be used for a description of transverse spin dynamics even at $T > \hbar^2/mr_0^2$. Allowance for the gradient virial terms in the transport equation would lead in this case to corrections, small in terms of the density, to the spectrum of the damped transverse spin modes. On the other hand, the absence of these gradient terms excludes completely collective solutions of zero-sound or spin-wave type in the argumentation of Ref. 5.

The frequency spectrum of homogeneous magnetic resonance in binary gases was obtained in Ref. 6 both in the framework of the macroscopic approach and with the aid of a transport equation with kinematic chiral corrections. The effective molecular field that leads to the chiral expansion of the magnetic-resonance spectrum can be taken into account in the transport equation at arbitrary temperatures. This approach is difficult to extend to a description of the dynamics of weakly inhomogeneous transverse-magnetization distributions.[1]

We propose here a phenomenological, in the sense of the Fermi-liquid theory, method of finding all the kinematic virial corrections to the transport equation at arbitrary temperatures. All the phenomenological quantities are expressed in the final results in terms of a microscopic property, viz., the exact scattering amplitude of two gas particles. Perturbation theory is used to verify and analyze the derived expressions and to calculate the gradient corrections in the collision integral. This explains automatically all the seeming contradictions between the phenomenological equations and the results of perturbation-theory calculations (e.g., the so-called I_2-term problem[7,8]).

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2. EQUILIBRIUM DISTRIBUTION FUNCTION

Having in mind ^{3}He gas, the superfluid ^{3}He–^{4}He solution, and nuclear-spin dynamics in atomic hydrogen, we consider for the sake of argument a system of spin-1/2 particles. Since the gas is assumed to be rarefied enough, $Nr_0^3 \ll 1$, where N is the number of atoms per unit volume, we can confine ourselves to pair collisions of the system particles. The contribution of the interaction to the total free energy of the gas is then given by a corresponding virial expansion that can be represented by a functional quadratic in the ideal-gas distribution function[1,4,6]:

$$F_{int} = \sum_{\mathbf{p},\mathbf{p}'} \Phi_{\alpha\beta,\mu\nu}(\mathbf{p},\mathbf{p}')\, n_{\beta\alpha}^{(0)}(\mathbf{p})\, n_{\nu\mu}^{(0)}(\mathbf{p}'). \qquad (2.1)$$

Here $n_{\alpha\beta}^{(0)}(\mathbf{p})$ is the polarization density matrix for an ideal gas:

$$n_{\alpha\beta} = \frac{n_+ + n_-}{2}\delta_{\alpha\beta} + \frac{n_+ - n_-}{2}\boldsymbol{\sigma}_{\alpha\beta}\mathfrak{M}, \qquad (2.2)$$

$\boldsymbol{\sigma}_{\alpha\beta}$ are Pauli matrices, $\mathfrak{M}$ is a unit vector in the spin-polarization direction, and n_+ and n_- are the occupation numbers of states oriented parallel and antiparallel to $\mathfrak{M}$. Summation over repeated spinor indices is implied in (2.1) and thereafter. We shall need later a microscopic expression for the interaction function $\Phi_{\alpha\beta,\mu\nu}$ in terms of the exact amplitudes of the triplet (f_+) and singlet (f_-) scattering of two particles[1,4,6]:

$$\Phi_{\alpha\beta,\mu\nu}(\mathbf{p},\mathbf{p}') = \psi(\mathbf{p},\mathbf{p}')\delta_{\alpha\beta}\delta_{\mu\nu} + \zeta(\mathbf{p},\mathbf{p}')\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\mu\nu}, \qquad (2.3)$$

$$\psi(\mathbf{p},\mathbf{p}') = \frac{3A_+(\mathbf{q}) + A_-(\mathbf{q})}{4}, \qquad 2\mathbf{q} = \mathbf{p} - \mathbf{p}',$$

$$\zeta(\mathbf{p},\mathbf{p}') = \frac{A_+(\mathbf{q}) - A_-(\mathbf{q})}{4}, \qquad (2.4)$$

$$A_\pm(\mathbf{q}) = -\frac{4\pi\hbar^2}{m}\Big\{\operatorname{Re} f_\pm(0,q) + \frac{mT}{\hbar}\Big[\operatorname{Re} f_\pm(\theta,q) \times \frac{\partial}{\partial q}\operatorname{Im} f_\pm(\theta,q) - \operatorname{Im} f_\pm(\theta,q)\frac{\partial}{\partial q}\operatorname{Re} f_\pm(\theta,q)\Big]\Big\}, \qquad (2.5)$$

where θ is the scattering angle in the c.m. system, and the values of $f_\pm$ can if necessary be expressed in terms of the phase shifts δ_l with the aid of the equation

$$f_\pm(\theta,q) = \frac{\hbar}{2iq}\sum_l (2l+1)(e^{2i\delta_l}-1)P_l(\cos\theta). \qquad (2.6)$$

Here $P_l(\cos\theta)$ are Legendre polynomials, and, since the particles are identical, the summation in (2.6) is carried out only over odd values of l when $f_+(\theta, q)$ is calculated and over even values for $f_-(\theta, q)$.

We obtain now the virial expansion of the distribution function. The quantum state of an individual particle in a rarefied gas is characterized by a definite momentum $\mathbf{p}$. This classification is valid, i.e., $\mathbf{p}$ is a good quantum number if the momentum uncertainty due to the finite mean free path is less than the momentum itself: $\Delta p \sim \hbar/d \ll p$, or

$$r_0/\Lambda_T \gg Nr_0^3, \qquad (2.7)$$

where $\Lambda_T = \hbar/(m/T)^{1/2}$ is the average de Broglie wavelength of the particle. Since $Nr_0^3 \ll 1$ always in a gas, the inequality (2.7) is certainly satisfied at high temperatures, when $\Lambda_T \ll r_0$, and the validity of (2.7) follows automatically from the condition $N\Lambda_T^3 \ll 1$ in the quantum region at $\Lambda_T \gg r_0$, i.e., when $T \ll \hbar^2/mr_0^2$.

Turning on the interaction, of course, alters the occupation numbers of the states in the gas. Since we assume that the classification properties of the energy levels of the gas are not changed by adiabatic switching-on of the interaction, the entropy of the interacting rarefied gas is determined by the usual combinatorial relation

$$S = -\sum_{\mathbf{p}} \{\hat{n}(\mathbf{p})\ln\hat{n}(\mathbf{p}) - [1-\hat{n}(\mathbf{p})]\ln[1-\hat{n}(\mathbf{p})]\}. \qquad (2.8)$$

To simplify the notation we have omitted here the spinor indices of the density matrix.

Varying (2.8) under the additional conditions that the total energy E and the total occupation numbers N_+ and N_- with different spin orientations be constant,

$$\delta S - b\sum_{\mathbf{p}} \varepsilon_{\alpha\beta}\delta n_{\beta\alpha} - d_{\alpha\beta}\sum_{\mathbf{p}} \delta n_{\beta\alpha} = 0, \qquad \varepsilon_{\alpha\beta} = \frac{\delta E}{\delta n_{\beta\alpha}}, \qquad (2.9)$$

we obtain the equilibrium distribution function, which takes the usual Fermi form

$$n_{\alpha\beta}(\mathbf{p}) = \frac{1}{2}\Big\{1 - \operatorname{th}\frac{b\varepsilon_{\alpha\beta}(\mathbf{p}) - d_{\alpha\beta}}{2}\Big\}. \qquad (2.10)$$

The indeterminate Lagrange multipliers are obtained, as usual, with the aid of the thermodynamic identity

$$dE = T\,dS + \mu_+\,dN_+ + \mu_-\,dN_- = T\,dS + \mu_{\alpha\beta}\,dN_{\beta\alpha}, \qquad (2.11)$$

where μ_+ and μ_- are the chemical potentials for subsystems with different spin orientations, and the following notation was introduced:

$$\mu_{\alpha\beta} = \frac{1}{2}(\mu_+ + \mu_-)\delta_{\alpha\beta} + \frac{1}{2}(\mu_+ - \mu_-)\boldsymbol{\sigma}_{\alpha\beta}\mathfrak{M},$$

$$N_{\alpha\beta} = \frac{N}{2}(\delta_{\alpha\beta} + \alpha\boldsymbol{\sigma}_{\alpha\beta}\mathfrak{M}), \qquad \alpha = \frac{N_+ - N_-}{N}. \qquad (2.12)$$

Comparing (2.9) and (2.11), we find that

$$b = T^{-1}, \qquad d_{\alpha\beta} = \mu_{\alpha\beta}T^{-1}. \qquad (2.13)$$

To avoid misunderstandings, we emphasize that we are considering now a spin-polarized gas, e.g., ^{3}He↑, in the absence of an external magnetic field, over times shorter than the relaxation time of the longitudinal magnetization. This allows us to regard the gas as a two-component system consisting of particles with different nuclear-spin orientations. The quantities N_+ and N_- are conserved in this case, and the values of μ_+ and μ_- are determined from the normalization conditions

$$\sum_{\mathbf{p}} n_\pm(\mathbf{p}) = N_\pm, \qquad N_+ + N_- = N. \qquad (2.14)$$

The generalization to the case of a gas polarized by an external magnetic field is self-evident.

According to the small-increment theorem, the total-energy change E_{int} due to the interaction of the gas molecule coincides with F_{int} from (2.1). We have therefore for the energy per particle, with allowance for the symmetry of the function $\Phi_{\alpha\beta,\mu\nu}$,

$$\varepsilon_{\alpha\beta}=\frac{p^2}{2m}\delta_{\alpha\beta}+2\sum_{\mathbf{p}'}\Phi_{\alpha\beta,\mu\nu}(\mathbf{p},\mathbf{p}')n_{\nu\mu}^{(0)}(\mathbf{p}'). \tag{2.15}$$

In the classical temperature region $T\gg\hbar^2N^{2/3}/m$ Eq. (2.2) takes the form

$$n_{\alpha\beta}^{(0)}(\mathbf{p})=(\delta_{\alpha\beta}+\alpha\sigma_{\alpha\beta}\mathfrak{M})n_0(\mathbf{p}),$$

$$n_0(\mathbf{p})=\frac{N}{2}\left(\frac{2\pi\hbar^2}{mT}\right)^{3/2}e^{-p^2/2mT}. \tag{2.16}$$

Substituting (2.3), (2.4), and (2.16) in (2.15), we get

$$\varepsilon_{\alpha\beta}(\mathbf{p})=\left[\frac{p^2}{2m}+4\sum_{\mathbf{p}'}\psi(\mathbf{p},\mathbf{p}')n_0(\mathbf{p}')\right]\delta_{\alpha\beta}$$

$$+4\alpha\sigma_{\alpha\beta}\mathfrak{M}\sum_{\mathbf{p}'}\zeta(\mathbf{p},\mathbf{p}')n_0(\mathbf{p}'). \tag{2.17}$$

Substituting next the virial expansion of (2.17) in the distribution function (2.10), taking (2.13) into account, and retaining for the Boltzmann region only the first two terms of the expansion of (2.10) in powers of the activities $\exp(\mu_\pm/T)\ll 1$, we get ultimately

$$n_{\alpha\beta}(\mathbf{p})=n_{\alpha\beta}^{(id)}(\mathbf{p})+\eta(\mathbf{p})\delta_{\alpha\beta}+\rho(\mathbf{p})\sigma_{\alpha\beta}\mathfrak{M},$$

$$\eta(\mathbf{p})=-\frac{4n_0(\mathbf{p})}{T}\sum_{\mathbf{p}'}[\psi(\mathbf{p},\mathbf{p}')+\alpha^2\zeta(\mathbf{p},\mathbf{p}')]n_0(\mathbf{p}'),$$

$$\rho(\mathbf{p})=-\alpha\frac{4n_0(\mathbf{p})}{T}\sum_{\mathbf{p}'}[\psi(\mathbf{p},\mathbf{p}')+\zeta(\mathbf{p},\mathbf{p}')]n_0(\mathbf{p}'), \tag{2.18}$$

where $n_{\alpha\beta}^{(id)}$ is given by Eq. (2.2) in which we must substitute the expressions

$$n_\pm=n_\mathbf{p}(\mu_\pm)-n_\mathbf{p}^2(\mu_\pm),\qquad n_\mathbf{p}(\mu_\pm)=\exp\left(\frac{\mu_\pm-\varepsilon_0}{T}\right),$$

$$\varepsilon_0=\frac{p^2}{2m}. \tag{2.19}$$

The equilibrium polarization of the density matrix $n_{\alpha\beta}(\mathbf{p})$ is diagonal, as it should be. The chemical potentials $\mu_\pm$, of course, also contain small virial corrections $\mu_\pm=\mu_\pm^{(0)}+\delta\mu_\pm$. With the aid of (2.18) and from the normalization conditions (2.14) we get

$$\delta\mu_\pm=\frac{T}{N_\pm}\sum_\mathbf{p}n_\mathbf{p}^2(\mu_\pm^{(0)})+\frac{1}{N_\pm}\sum_\mathbf{p}n_\mathbf{p}(\mu_\pm^{(0)})\delta\varepsilon_\pm(\mathbf{p}). \tag{2.20}$$

From (2.17), (2.19), and (2.20) we get

$$\delta\mu_\pm=\frac{\pi^{3/2}\hbar^3}{m^{3/2}T^{1/2}}N_\pm+\frac{8}{N}\sum_{\mathbf{p},\mathbf{p}'}[\psi(\mathbf{p},\mathbf{p}')\pm\alpha\zeta(\mathbf{p},\mathbf{p}')]n_0(\mathbf{p})n_0(\mathbf{p}'). \tag{2.21}$$

In spinor form we have ultimately

$$\mu_{\alpha\beta}=\left\{T\ln\left[\frac{N}{2}\left(\frac{2\pi\hbar^2}{mT}\right)^{3/2}(1-\alpha^2)^{1/2}\right]+2NX(T)\right\}\delta_{\alpha\beta}$$

$$+\left\{2\alpha NY(T)+\frac{1}{2}T\ln\frac{1+\alpha}{1-\alpha}\right\}\sigma_{\alpha\beta}\mathfrak{M}+\frac{\pi^{3/2}\hbar^3}{m^{3/2}T^{1/2}}N_{\alpha\beta},$$

$$X(T)=\frac{1}{(\pi mT)^{3/2}}\int\psi(\mathbf{q})e^{-q^2/mT}d^3q,$$

$$Y(T)=\frac{1}{(\pi mT)^{3/2}}\int\zeta(\mathbf{q})e^{-q^2/mT}d^3q. \tag{2.22}$$

Equation (2.22) for $\mu_{\alpha\beta}$ agrees fully with the purely thermodynamic calculations with the aid of the free energy (2.1) (see also Refs. 1 and 4). Thus, for example, the true chemical potential $\mu=\partial F/\partial N$ can be obtained from $\mu_{\alpha\beta}$ by using the obvious relation

$$\mu={}^1/_2\mu_+(1+\alpha)+{}^1/_2\mu_-(1-\alpha). \tag{2.23}$$

We have in fact obtained the equilibrium distribution function by using a Fermi-liquid scheme. There is another possible method, involving direct calculation of the mean value of the number of particles that are in a given quantum state. Calculations by such a method, performed for the distribution function in a gas of zero-spin particles, are given in the book by Akhiezer and Peletminskiĭ.[9] the use of this method for the present case leads, naturally, to the same results.

3. PHENOMENOLOGICAL EQUATIONS

To describe the properties of gases in the classical temperature region it is customary to use the traditional Boltzmann equation whose left-hand (kinematic) side corresponds to free motion of the gas particles, and all the changes in the particle states are described by the right-hand side of the kinetic equation (by the collision integral). As already shown, to allow for the interaction in the gas it is necessary to introduce not only the collision integral, but also specific additional terms that can be interpreted as virial corrections to the kinematic part of the Boltzmann equations and as describing in fact the interaction-induced deviation of the particle distribution function from Maxwellian in an ideal gas. A general microscopic derivation of the transport equation with the kinematic virial corrections at arbitrary temperatures is a rather complicated task. By a rather simple method, however, it is possible to obtain exact values of these corrections in the linearized Boltzmann equation.

The idea of the method is the following: the sought virial coefficient to the kinetic equation is expressed in the form of some functional that is linear in $n(\mathbf{p})$ and has an arbitrary kernel. The transport equation is then used to obtain the linearized hydrodynamic equations. All the hydrodynamic fluxes in the gases (the momentum-flux tensor and others) can be expressed in terms of purely thermodynamic quantities. Using the virial expansion of the thermodynamic functions (2.1) and (2.3)–(2.5) we can then obtain the actual form of the kernel of the linear functional for the kinematic virial correction in the Boltzmann equation. The use of linearized equations and linear functionals ensures uniqueness of the procedure. For simplicity and by way of illustration, we obtain here the kinematic virial corrections to the transport equation by transforming to hydrodynamic equations for an unpolarized ($\alpha=0$) nonideal gas.

It was shown[1,4] in the derivation of the transport equation at $T\ll\hbar^2/mr_0^2$ that the interaction between the gas molecules not only determines the collision integral, but yields also definite increments to the commutator $[\hat{p}^2/2m,\hat{n}]$ that describes the motion of the particles between the collisions. Accordingly, to describe the molecule motion in the effective self-consistent field within the framework of the linearized transport equation, we shall include the virial corrections to the commutator in the form of a linear functional of general form

$$\frac{\partial}{\partial t}\delta\hat{n}+\frac{i}{\hbar}\left[\frac{\hat{p}^2}{2m};\ \delta\hat{n}+\sum\hat{C}\,\delta\hat{n}\right]=\mathrm{St}\,\delta\hat{n}. \tag{3.1}$$

In the quasiclassical approximation, changing in (3.1) from the commutator to a Poisson bracket, we obtain

$$\frac{\partial}{\partial t}\delta n(\mathbf{p},\mathbf{r})+\mathbf{v}\nabla\left\{\delta n(\mathbf{p},\mathbf{r})+\int C(\mathbf{p},\mathbf{p}')\,\delta n(\mathbf{p}',\mathbf{r})\,d\Gamma'\right\}=\mathrm{St}\,\delta n,$$

$$d\Gamma=2\frac{d^3p}{(2\pi\hbar)^3}. \tag{3.2}$$

We do not specify the structure of the term St δn in Eqs. (3.1) and (3.2), but assume that this term satisfies the usual relations that correspond to the macroscopic conservation laws:

$$\int \mathrm{St}\,\delta n\,d\Gamma=0,\qquad \int \mathbf{p}\,\mathrm{St}\,\delta n\,d\Gamma=0. \tag{3.3}$$

The transport equation in the quantum temperature region $T\ll\hbar^2/mr_0^2$ (Refs. 1 and 4) corresponds at $\alpha=0$ to (3.2) with a functional kernel $C(\mathbf{p},\mathbf{p}')=2\pi a\Lambda_T n_0(\mathbf{p})$, where a is the s-scattering length. Introducing new symbols

$$C(\mathbf{p},\mathbf{p}')=-\frac{\partial n}{\partial\varepsilon}D(\mathbf{p},\mathbf{p}'), \tag{3.4}$$

we can rewrite (3.2) in a typical Fermi-liquid form:

$$\frac{\partial}{\partial t}\delta n(\mathbf{p},\mathbf{r})+\frac{\partial\varepsilon}{\partial\mathbf{p}}\nabla\delta n(\mathbf{p},\mathbf{r})-\frac{\partial n}{\partial\mathbf{p}}\nabla\varepsilon(\mathbf{p},\mathbf{r})=\mathrm{St}\,\delta n,$$

$$\varepsilon=\frac{p^2}{2m}+\int D(\mathbf{p},\mathbf{p}')\,\delta n(\mathbf{p}',\mathbf{r})\,d\Gamma'. \tag{3.5}$$

In (3.5) and hereafter we leave out of ε, i.e., in fact out of $\partial\varepsilon/\partial\mathbf{p}$ the equilibrium virial correction that contains $n_0(\mathbf{p})$. The equilibrium distribution function, with the virial corrections (2.18) taken into account, is normalized to the total number of particles:

$$\int n(\mathbf{p})\,d\Gamma=N. \tag{3.6}$$

Integration of (3.5), and of (3.5) multiplied by the momentum, over phase space allows us to change to the hydrodynamic formulation

$$\frac{\partial N}{\partial t}+\operatorname{div}\mathbf{j}=0,\qquad \frac{\partial}{\partial t}j_i+\frac{\partial}{\partial x_k}\Pi_{ik}=0,$$

$$\mathbf{j}=\left\langle\frac{\partial\varepsilon}{\partial\mathbf{p}}\right\rangle,\qquad \Pi_{ik}=\langle p_i v_k\rangle+\delta_{ik}(\langle\varepsilon\rangle-W),$$

$$\delta W=\int\varepsilon\delta n\,d\Gamma,\qquad \langle\ldots\rangle=\int\ldots d\Gamma,\qquad \mathbf{v}=\frac{\partial\varepsilon}{\partial\mathbf{p}}. \tag{3.7}$$

It is known from Fermi-liquid theory[10,11] that when conditions (3.3), (3.5), and (3.6) are met the momentum-flux tensor Π_{ik} accords with the hydrodynamic equations of an ideal (nondissipative) liquid if $W=E$, where E is the total energy of the system. Recognizing that $E_{\text{int}}=F_{\text{int}}$, we obtain with the aid of expression (2.1) for the free energy

$$\varepsilon=\frac{\delta W}{\delta n}=\frac{\delta E}{\delta n}=\frac{p^2}{2m}+2\int\psi(\mathbf{p},\mathbf{p}')\,n(\mathbf{p}')\,d\Gamma', \tag{3.8}$$

so that

$$D(\mathbf{p},\mathbf{p}')=2\psi(\mathbf{p},\mathbf{p}').$$

In the thermodynamic equilibrium state all the fluxes vanish. We express Π_{ik} in terms of the deviation $\delta n(\mathbf{p})$ of the distribution function from its equilibrium value $n(\mathbf{p})$. The equilibrium distribution function is expressed in terms of $n_0(\mathbf{p})$ by means of Eqs. (2.18) with $\alpha=0$:

$$n(\mathbf{p})=n^{(id)}(\mathbf{p})+2\frac{\partial n_0}{\partial\varepsilon}\int\psi(\mathbf{p},\mathbf{p}')\,n_0(\mathbf{p}')\,d\Gamma', \tag{3.9}$$

where $n^{(id)}(\mathbf{p})\approx n_{\mathbf{p}}(\mu)-n_{\mathbf{p}}^2(\mu)$ is the distribution function in an ideal Fermi gas. Within the accuracy limits we can replace $n_0(\mathbf{p})$ in the integral term of (3.9) by $n(\mathbf{p})$. To the same accuracy, the deviation of the distribution function $\delta\tilde{n}(\mathbf{p})$ from $n^{(id)}(\mathbf{p})$ can be written with the aid of (3.9) in the form

$$\delta\tilde{n}(\mathbf{p})=\delta n(\mathbf{p})-2\frac{\partial n_0}{\partial\varepsilon}\int\psi(\mathbf{p},\mathbf{p}')\,\delta n(\mathbf{p}')\,d\Gamma'. \tag{3.10}$$

Using (3.4), we readily see that

$$\delta n(\mathbf{p})+\int C(\mathbf{p},\mathbf{p}')\,\delta n(\mathbf{p}')\,d\Gamma'=\delta\tilde{n}(\mathbf{p}), \tag{3.11}$$

i.e., allowance for the virial corrections to the commutator $[p^2/2m,\hat{n}]$ in the right-hand side of the Boltzmann equation reduces to the fact that the deviation of the distribution function should be reckoned not from the true equilibrium distribution function $n(\mathbf{p})$ but from the distribution function $n^{(id)}(\mathbf{p})$ in an ideal gas, in view of the virial expansion of $n(\mathbf{p})$. Calculations using Eqs. (3.7) together with (3.10) also allow us to represent Π_{ik} in the form of a functional of $\delta\tilde{n}(\mathbf{p})$:

$$\Pi_{ik}=\int p_i v_k\,\delta\tilde{n}(\mathbf{p})\,d\Gamma. \tag{3.12}$$

Averaging (3.12) over the angles and recognizing that if the terms quadratic in $\mathbf{j}$ are neglected we should have $\Pi_{ik}=\delta P\,\delta_{ik}$, we get

$$\frac{2}{3}\int\frac{p^2}{2m}\,\delta\tilde{n}(\mathbf{p})\,d\Gamma=\delta P. \tag{3.13}$$

In an ideal gas in the absence of interaction, when the equation of state is $P=(2/3)E$, the identity (3.13) is obvious, i.e., in the principal approximation we have at any degree of gas degeneracy

$$\frac{2}{3}\int\frac{p^2}{2m}\,\delta n^{(id)}(\mathbf{p})\,d\Gamma=\delta P_{id}. \tag{3.14}$$

We now demonstrate the validity of (3.13) for a nonideal gas. We consider the simplest case, when the distribution-function deviation $\delta\tilde{n}(\mathbf{p})$ is due only to the pressure δP, which we expand in powers of $\delta\tilde{N}=\int\delta\tilde{n}(\mathbf{p})d\Gamma$. We represent $\delta\tilde{n}(\mathbf{p})$ also in form linear in δN:

$$\delta\tilde{n}(\mathbf{p})=-\frac{T}{N}\frac{\partial n}{\partial\varepsilon}\delta\tilde{N}\approx\frac{\delta\tilde{N}}{N}n_0(\mathbf{p}). \tag{3.15}$$

For the virial correction due to the particle interaction we obtain from (3.13) and (3.15) with the aid of (3.10)

$$\frac{2}{3}\int\frac{p^2}{2m}\,\delta\tilde{n}_{int}\,d\Gamma=2\int\psi(\mathbf{p},\mathbf{p}')\,n_0(\mathbf{p})\,\delta n_0(\mathbf{p}')\,d\Gamma\,d\Gamma'=\delta P_{int}, \tag{3.16}$$

where P_{int} is determined by expressions (2.1)–(2.5) at $\alpha=0$. The set (3.14) and (3.16) agrees fully with (3.13). Of course, this approach must not be regarded as a rigorous derivation of a transport equation with kinematic virial corrections. However, the obtained agreement with the linearized hydrodynamic equations confirms the validity of using, in the linear approximation, of a transport equation in the form (3.1), (3.2) (3.5) with a certain collision integral St δn that possesses the properties (3.3). The correspondence between the hydrodynamic equations and the collective self-consistent corrections to the transport equation was noted earlier by Miyake, Mullen, and Stamp.[12]

In the kinetic equation for a spin-polarized gas there appear also homogeneous (containing no gradients) kinematic virial corrections, due to the noncommutativity of the operators $\delta n_{\alpha\beta}$ and $\varepsilon_{\alpha\beta}$ in spin space. For weakly inhomogeneous distributions of the transverse magnetization it is therefore permissible, in the principal approximation in $Nr_0^3 \ll 1$, to retain in the transport equation only the local spin commutator $i[\hat{\varepsilon}, \hat{n}]_{\alpha\beta}/\hbar$ and the term $\mathbf{v}\cdot\nabla n_{\alpha\beta}$ that corresponds to free motion of the particles if the interaction is neglected.[4,1] This procedure leads to the equation

$$\frac{\partial}{\partial t}\boldsymbol{\lambda}_{\mathbf{p}}+(\mathbf{v}\nabla)\boldsymbol{\lambda}_{\mathbf{p}}+\frac{8\alpha}{\hbar}\sum_{\mathbf{p},\mathbf{p}'}\zeta(\mathbf{p},\mathbf{p}')\Big\{n_0(\mathbf{p})\,[\mathfrak{M}\boldsymbol{\lambda}_{\mathbf{p}'}]$$

$$-n_0(\mathbf{p}')\,[\mathfrak{M}\boldsymbol{\lambda}_{\mathbf{p}}]\Big\}=\mathrm{St}\,\boldsymbol{\lambda}_{\mathbf{p}},\qquad \delta n_{\alpha\beta}(\mathbf{p})=\boldsymbol{\lambda}_{\mathbf{p}}\boldsymbol{\sigma}_{\alpha\beta},\tag{3.17}$$

where St $\boldsymbol{\lambda}_{\mathbf{p}}$ is the local collision integral. Equation (3.17), just as the reasoning in Ref. 5, can be used to describe the transverse spin dynamics at arbitrary temperatures. Allowance for the gradient virial terms

$$-4\sum_{\mathbf{p},\mathbf{p}'}\zeta(\mathbf{p},\mathbf{p}')\left(\frac{\partial n_0}{\partial\mathbf{p}}\nabla\right)\boldsymbol{\lambda}_{\mathbf{p}}\tag{3.18}$$

in the left-hand side of (3.17) makes it possible to find the next term of the virial expansion of the spectrum of the transverse spin modes, and also to investigate the problem of collective solutions in the form of longitudinal spin waves of the zero-sound type, at least at sufficiently low temperatures, $T \ll \hbar^2/mr_0^2$. Application of an external magnetic field $\mathbf{H}\uparrow\uparrow\mathfrak{M}$ introduces into the kinetic equations a term $\Omega_H \mathfrak{M}\times\boldsymbol{\lambda}_{\mathbf{p}}$, where $\Omega_H = 2\beta H/\hbar$ is the Larmor frequency; this means that a gap $\omega(k=0)=\Omega_H$ appears in the spectrum of the transverse magnetization oscillations.

4. MICROSCOPIC ANALYSIS

To be specific, we return now to consideration of the transport equation in an unpolarized gas. Let the interaction between the articles be weak and let perturbation theory be applicable, so that a consistent quantum-theoretical derivation of the transport equation is possible (see, e.g., Refs. 13–15). In this section we carry out a direct microscopic perturbation-theory calculation of all the kinematic virial corrections in the transport equation, and compare the data obtained with the phenomenological results of the preceding part of the paper. To avoid excessivley complicated expressions, we confine ourselves here to sufficiently slow particles, $pr_0 \ll 1$ (for simplicity, we set the Planck constant $\hbar$ equal to unity), when the principal role is played by the matrix element of the interaction at zero momenta.

$$\langle\mathbf{p}_1',\mathbf{p}_2'|U|\mathbf{p}_1,\mathbf{p}_2\rangle=\int U(r)\exp[-i(\mathbf{p}_2'-\mathbf{p}_2)\mathbf{r}]\,d^3r$$

$$\approx\int U(r)\,d^3r\equiv U_0.\tag{4.1}$$

Here and elsewhere all the expressions are normalized to a unit volume. In second-order perturbation theory, the customarily employed transport equation[13,15] takes then the form (3.5), where ε now stands for

$$\varepsilon=\frac{p^2}{2m}+U_0\sum_{\mathbf{p}'}n(\mathbf{p}'),\tag{4.2}$$

and St n is a local collision integral in traditional form:

$$\mathrm{St}\,n_{\mathbf{p}}=2\pi U_0^2\int\frac{d^3p'\,d^3p_1'}{(2\pi)^6}\,d^3p_1\,\delta(\mathbf{p}+\mathbf{p}_1-\mathbf{p}'-\mathbf{p}_1')\,\delta(\varepsilon^{(0)}+\varepsilon_1^{(0)}$$

$$-\varepsilon^{(0)\prime}-\varepsilon_1^{(0)\prime})\,[n(\mathbf{p}')n(\mathbf{p}_1')-n(\mathbf{p})n(\mathbf{p}_1)],\qquad \varepsilon_i^{(0)}=\frac{p_i^2}{2m}.\tag{4.3}$$

Comparing the kinematic virial corrections in (3.5), obtained on the one hand by perturbation theory (4.2) and determined, on the other, independently with the aid of the thermodynamic virial expansions (2.4), (2.5), and (3.8), we easily verify that the traditional transport equation obtained by the perturbative approach and taking only (4.2) and (4.3) into account is not accurate. In fact, in this equation the kinematic virial corrections contain only terms linear in U_0, whereas all the terms quadratic in U_0 enter only in the collision integral (4.3). On the other hand, Eqs. (2.4), (2.5), and (3.8) show that the kinematic corrections must also contain those terms proportional to U_0^2 which enter in Re$f(0,q)$ [the terms containing Im$f(\theta,q)$ in $\psi(\mathbf{p},\mathbf{p}')$ and $\varepsilon(\mathbf{p})$ turn out to be of order U_0^3.] We shall show now that the term lost in the traditional scheme appears automatically in the calculation of the nonlocal corrections to the collision integral (4.3).

Indeed, once we retain the gradient kinematic corrections (linear in U_0) in the left-hand side of the transport equation, we must, at the same accuracy, take into account also the first term of the expansion in the gradients of the distribution function for the collision term (quadratic in U_0). The collision term, which determines the time variation of the single-particle density matrix $n_{1,2}=\langle\hat{a}_2+a_1\rangle$, can be represented in second-order perturbation theory in the form[9]

$$\left[\frac{\partial n_{1,2}}{\partial t}\right]_{coll}\equiv I_{12}=\pi U_0^2\sum_{1'2'3'4'}\sum_{1''2''3''4''}\delta_-(\varepsilon_{1''}^{(0)}+\varepsilon_{2''}^{(0)}-\varepsilon_{3''}^{(0)}-\varepsilon_{4''}^{(0)})$$

$$\times\,\delta_{4',2}\{n_{4'',2'}n_{3'',1'}\delta_{3',1''}\delta_{1,2''}-n_{3',1''}n_{1,2''}\delta_{1',3''}\delta_{4'',2'}\}+\text{h.c.}\tag{4.4}$$

The collision term (4.4) satisfies the principle of weaker correlations and regular correct boundary conditions as $t\to-\infty$ (Refs. 13 and 9), as manifested by the presence of the function $\delta_-(\varepsilon)$ defined by the relation

$$\int_{-\infty}^{0}e^{i\tau\varepsilon}\,d\tau=\pi\delta_-(\varepsilon)=\pi\delta(\varepsilon)-\frac{i}{\varepsilon}.\tag{4.5}$$

To construct a quasiclassical kinetic equation, it is convenient to change over from the density matrix $n_{1,2}\equiv n_{\mathbf{p}_1\mathbf{p}_2}$ to the Wigner distribution function $n_{\mathbf{p}}(\mathbf{r})$:

$$n_{\mathbf{p}}(\mathbf{r})=\frac{1}{(2\pi)^3}\int d^3k\,e^{-i\mathbf{k}\mathbf{r}}n_{\mathbf{p}-\mathbf{k}/2,\mathbf{p}+\mathbf{k}/2}.\tag{4.6}$$

In the Wigner representation (4.6) the collision term (4.4) can be written in the form

$$\left[\frac{\partial n_{\mathbf{p}}}{\partial t}\right]_{coll}\equiv I_{\mathbf{p}}=\frac{1}{(2\pi)^3}\int d^3k\,e^{-i\mathbf{k}\mathbf{r}}I_{\mathbf{p}-\mathbf{k}/2,\mathbf{p}+\mathbf{k}/2},\tag{4.7}$$

and for $n_{\mathbf{p}_1\mathbf{p}_2}$ we get with the aid of (4.6)

$$n_{\mathbf{p}_1\mathbf{p}_2}=\int d^3r'\,\exp[i(\mathbf{p}_2-\mathbf{p}_1)\mathbf{r}']\,n_{1/2(\mathbf{p}_1+\mathbf{p}_2)}(\mathbf{r}').\tag{4.8}$$

In the homogeneous case $n_{\mathbf{p},\mathbf{p}'} = n_{\mathbf{p}}\delta_{\mathbf{p},\mathbf{p}'}$, expression (4.4), naturally, coincides with the collision integral (4.3). We are interested in the following corrections to (4.3), which are proportional to $\nabla n_{\mathbf{p}}$

To calculate these corrections in the weakly inhomogeneous case it is necessary to substitute in the collision term (4.4) the representation (4.8) and expand all the distribution functions $n_q(\mathbf{r}')$ in a power series:

$$n_{\mathbf{q}}(\mathbf{r}')=n_{\mathbf{q}}(\mathbf{r})+\nabla n_{\mathbf{q}}(\mathbf{r}'-\mathbf{r}). \tag{4.9}$$

Following this procedure, the collision term (4.7) can be represented, in first order in the gradients, in the form

$$\delta I_{\mathbf{p}}=\sum_{i=1}^{3} X_i,$$

where

$$X_1=-\pi U_0^2\int d^3k\, d^3r e^{i\mathbf{kr}}$$

$$\times\sum_{\mathbf{q},\mathbf{q}'} n_{\mathbf{p}-\mathbf{k}/2}(\nabla n_{\mathbf{q}}\mathbf{r})\,\delta_-(\varepsilon^{(0)}_{\mathbf{q}+\mathbf{k}/2}+\varepsilon^{(0)}_{\mathbf{p}-\mathbf{k}/2}-\varepsilon^{(0)}_{\mathbf{p}'}-\varepsilon^{(0)}_{\mathbf{q}'})+\text{h.c.},$$

$$X_2=-\pi U_0^2\int d^3k\, d^3r e^{i\mathbf{kr}}$$

$$\times\sum_{\mathbf{q},\mathbf{q}'} n_{\mathbf{q}}(\nabla n_{\mathbf{p}}\mathbf{r})\,\delta_-(\varepsilon^{(0)}_{\mathbf{q}}+\varepsilon^{(0)}_{\mathbf{p}+\mathbf{k}/2}-\varepsilon^{(0)}_{\mathbf{p}'}-\varepsilon^{(0)}_{\mathbf{q}'})+\text{h.c.},$$

$$X_3=-2\pi U_0^2\int d^3k\, d^3r e^{i\mathbf{kr}}$$

$$\times\sum_{\mathbf{q},\mathbf{p}'} n_{\mathbf{p}'}(\nabla n_{\mathbf{q}}\mathbf{r})\,\delta_-(\varepsilon^{(0)}_{\mathbf{q}'}+\varepsilon^{(0)}_{\mathbf{p}-\mathbf{k}/2}-\varepsilon^{(0)}_{\mathbf{q}-\mathbf{k}/2}-\varepsilon^{(0)}_{\mathbf{p}'})+\text{h.c.} \tag{4.10}$$

When summing in (4.10) and hereafter we always assume momentum conservation, viz., $\mathbf{p}+\mathbf{q}=\mathbf{p}'+\mathbf{q}'$ for X_1 and X_2, and $\mathbf{p}+\mathbf{q}'=\mathbf{q}+\mathbf{p}'$ for X_3. The terms X_1 and X_2 stem from the term containing $n_{3',1''}\, n_{1,2''}$ in (4.4), while X_3 corresponds to the term proportional to $n_{4'',2'}\, n_{3'',1'}$. In the quasiclassical limit $p \gg k$, using the definition (4.5) and the simple integration

$$\int (\mathbf{ka})(\mathbf{rb})e^{i\mathbf{kr}}\, d^3k\, d^3r=i(\mathbf{ab}) \tag{4.11}$$

we obtain ultimately

$$\delta I_{\mathbf{p}}=\sum_{i=1}^{3} Y_i,$$

where

$$Y_1=U_0^2\left(\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}}\nabla-\nabla n_{\mathbf{p}}\frac{\partial}{\partial \mathbf{p}}\right)\sum_{\mathbf{q},\mathbf{q}'}\frac{n_q}{\varepsilon^{(0)}_{\mathbf{q}}+\varepsilon^{(0)}_{\mathbf{p}}-\varepsilon^{(0)}_{\mathbf{q}'}-\varepsilon^{(0)}_{\mathbf{p}'}},$$

$$Y_2=\frac{U_0^2}{m}\sum_{\mathbf{q},\mathbf{q}'} n_{\mathbf{p}}\frac{\nabla n_{\mathbf{q}}(\mathbf{q}-\mathbf{p})}{(\varepsilon^{(0)}_{\mathbf{q}}+\varepsilon^{(0)}_{\mathbf{p}}-\varepsilon^{(0)}_{\mathbf{p}'}-\varepsilon^{(0)}_{\mathbf{q}'})^2},$$

$$Y_3=2\frac{U_0^2}{m}\sum_{\mathbf{q},\mathbf{p}'} n_{\mathbf{p}'}\frac{\nabla n_{\mathbf{q}}(\mathbf{p}-\mathbf{q})}{(\varepsilon^{(0)}_{\mathbf{q}}+\varepsilon^{(0)}_{\mathbf{p}'}-\varepsilon^{(0)}_{\mathbf{p}}-\varepsilon^{(0)}_{\mathbf{q}})^2}. \tag{4.12}$$

The term Y_1 is the sought kinematic virial correction of second order in the interaction, which was lost in the transport equation with the self-consistent field (4.2) and the local collision integral (4.3). Indeed, transferring Y_1 to the left-hand side of the transport equation we easily verify with the aid of the expression for the scattering amplitude

$$f_-=-\frac{m}{4\pi\hbar^2}\left[U_0+U_0^2\sum_{\mathbf{p}_1'}\frac{1}{\varepsilon^{(0)}_{\mathbf{p}_1}+\varepsilon^{(0)}_{\mathbf{p}_2}-\varepsilon^{(0)}_{\mathbf{p}_1'}-\varepsilon^{(0)}_{\mathbf{p}_2'}}\right] \tag{4.3}$$

that the terms Y_1 correspond to the second-order perturbation-theory contributions to the energy of a particle in a self-consistent field, given by Eqs. (2.5), (2.17), and (3.8) and contained in the transport equation (3.5) as part of the combination

$$\frac{\partial \varepsilon}{\partial \mathbf{p}}\nabla\,\delta n-\frac{\partial n}{\partial \mathbf{p}}\nabla \varepsilon.$$

Thus, it is precisely the term Y_1 which ensures full agreement between the semiphenomenological derivation of the kinematic virial corrections, on the one hand, and the microscopic calculations in second-order perturbation theory, on the other.

The most important role in the derivation of the transport equation (3.5), (3.8) was played by satisfaction of the conditions (3.3). It is therefore necessary to verify that the remaining terms Y_2+Y_3 also satisfy these conditions. In this case, at $pr_0 \ll 1$ the requirement that (3.3) be valid for $\delta(\mathrm{St}\, n_{\mathbf{p}})=Y_2+Y_3$ becomes trivial, since it is readily seen that $Y_2+Y_3\equiv 0$ by virtue of the momentum conservation law. Indeed, the variable redefinition $\mathbf{p}\leftrightarrow\mathbf{p}'$ in Y_3 in (4.12) yields

$$\delta(\mathrm{St}\, n_{\mathbf{p}})=Y_2+Y_3$$

$$=\frac{U_0^2}{m}\sum_{\mathbf{p}',\mathbf{q}',\mathbf{q}}\frac{n_{\mathbf{p}}\nabla n_{\mathbf{q}}(2\mathbf{p}'-\mathbf{p}-\mathbf{q})}{(\varepsilon^{(0)}_{\mathbf{q}}+\varepsilon^{(0)}_{\mathbf{p}}-\varepsilon^{(0)}_{\mathbf{p}'}-\varepsilon^{(0)}_{\mathbf{q}'})^2}\delta_{\mathbf{p}+\mathbf{q},\mathbf{p}'+\mathbf{q}'}. \tag{4.14}$$

Taking into account the obvious relation

$$\sum_{\mathbf{p}',\mathbf{q}',\mathbf{q}} n_{\mathbf{p}}\frac{\nabla n_{\mathbf{q}}\mathbf{p}'\delta_{\mathbf{q}+\mathbf{p},\mathbf{q}'+\mathbf{p}'}}{(\varepsilon^{(0)}_{\mathbf{q}}+\varepsilon^{(0)}_{\mathbf{p}}-\varepsilon^{(0)}_{\mathbf{p}'}-\varepsilon^{(0)}_{\mathbf{q}'})^2}$$

$$=\sum_{\mathbf{p}',\mathbf{q}',\mathbf{q}} n_{\mathbf{p}}\frac{\nabla n_{\mathbf{q}}\mathbf{q}'\delta_{\mathbf{q}+\mathbf{p},\mathbf{q}'+\mathbf{p}'}}{(\varepsilon^{(0)}_{\mathbf{q}}+\varepsilon^{(0)}_{\mathbf{p}}-\varepsilon^{(0)}_{\mathbf{p}'}-\varepsilon^{(0)}_{\mathbf{q}'})^2}, \tag{4.15}$$

we get from (4.14)

$$Y_2+Y_3$$

$$=\frac{U_0^2}{m}\sum_{\mathbf{p}',\mathbf{q}'\mathbf{q}} n_{\mathbf{p}}\nabla n_{\mathbf{q}}\frac{(\mathbf{p}'+\mathbf{q}'-\mathbf{p}-\mathbf{q})}{(\varepsilon^{(0)}_{\mathbf{q}}+\varepsilon^{(0)}_{\mathbf{p}}-\varepsilon^{(0)}_{\mathbf{p}'}-\varepsilon^{(0)}_{\mathbf{q}'})^2}\delta_{\mathbf{q}+\mathbf{p},\mathbf{q}'+\mathbf{p}'}\equiv 0. \tag{4.16}$$

To prevent misunderstandings, we emphasize that although first-order perturbation theory in the pseudopotential was used to derive the transport equation subject to $T\ll\hbar^2/mr_0^2$ (Refs. 1, 3, 4), the final result was expressed, with the aid of the renormalization conditions, in terms of the true scattering length, which contains absolutely all the terms of the perturbation-theory expansion in the interaction between the particles. Therefore the calculations in Refs. 1, 3, and 4 for $T\ll\hbar^2/mr_0^2$ lead to correct kinematic virial corrections that correspond to (2.5), (2.17) and (3.8) in the limit $r_0/\Lambda_T\ll 1$.

5. THE PROBLEM OF THE I_2 TERM

Seeming discrepancies with the results of perturbation theory appear also in the formulation of the macroscopic equations of transverse spin dynamics. In this case, on the contrary, the perturbation scheme seems to acquire an extra term compared with the traditional macroscopic approach. This term appears already in the homogeneous case when the gradient corrections that depend on the interaction are neglected. The explicit form of such a term can be determined by using the results of Ref. 14:

$$I_2(\mathbf{p}_1)=\frac{U_0{}^2}{\hbar}\int\frac{d^3p_1{}'\,d^3p_2{}'}{(2\pi\hbar)^6}d^3p_2\,\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_1{}'-\mathbf{p}_2{}')$$

$$\times\frac{2\alpha}{\varepsilon^{(0)}_{\mathbf{p}_1}+\varepsilon^{(0)}_{\mathbf{p}_2}-\varepsilon^{(0)}_{\mathbf{p}_1{}'}-\varepsilon^{(0)}_{\mathbf{p}_2{}'}}\{n_0(\mathbf{p}_1)[\mathfrak{M}\lambda_{\mathbf{p}_2}]-n_2(\mathbf{p}_2)[\mathfrak{M}\lambda_{\mathbf{p}_1}]\}. \tag{5.1}$$

The existence of the additional term (5.1) cast doubts on the validity of the equations obtained in Ref. 5 at $\alpha\approx 1$, meaning also on the quantitative interpretation of the experiments on nuclear spin waves in gaseous H↑.[7]

It was proposed in Ref. 8 that in a consistent derivation of the transport equation the term I_2 is completely cancelled out and vanishes. An opinion was orally expressed that I_2 is only partly cancelled out, but in no way completely. F. Laloë was the first to suggest, and to advance strong corroborative arguments, that the I_2 term, while undoubtedly existing, is totally contained, without a remainder, in the kinematic virial corrections and is taken into account automatically if these corrections are expressed in terms of the exact scattering amplitude. The results of the present papers prove this statement to be correct. Indeed, expressions (4.13), (2.4), and (2.5) show that the term I_2 is exactly a second-order perturbation-theory correction to the kinematic virial term that contains the function $\zeta(\mathbf{p},\mathbf{p}')$ in Eq. (3.17).

There is thus no I_2-term problem in fact, and equations of type (3.17) are valid for all degrees of the polarization α. The quantitative analysis of the experimental data on transverse spin dynamics in gaseous H↑ (Refs. 16–18) and ^{3}He↑ (Refs. 19 and 20) on the basis of similar equations is perfectly correct. On the other hand, since the I_2 term is not related to the collision integral, one can see no reasons whatever for introducing two different relaxation times (with unequal temperature dependences) in the phenomenological description of the dissipative terms in the equations of motion for transverse magnetization.[21]

I am grateful to F. Laloë for interesting and useful discussions and for correspondence on the I_2-term problem.

APPENDIX

We present here a simple method of obtaining, for $T\ll\hbar^2/mr_0^2$, a collision integral in a spinor form suitable for specific analysis of various relaxation processes in a spin-polarized gas of spin-1/2 particles. The collision integral can be interpreted as the difference between the "arrival" and "departure" of particles in a given quantum state. In the temperature region $T\ll\hbar^2/mr_0^2$, when the interaction can be described by using perturbation theory followed by renormalization to the true scattering amplitude, the decrease and increase of the number of particles in the k th quantum state are given by the relations

$$W_k^{out}=\frac{2\pi}{\hbar}\sum_f|\langle f|\hat{H}_{int}|k\rangle|^2,\qquad W_k^{in}=\frac{2\pi}{\hbar}\sum_i|\langle k|\hat{H}_{int}|i\rangle|^2 \tag{A1}$$

so that

$$\mathrm{St}\,n_k=W_k^{in}-W_k^{out}. \tag{A2}$$

For states pure in spin, i.e., in the case when the polarization density matrix is diagonal, by substituting in (A1) and (A2) the Hamiltonian

$$\hat{H}_{int}=g\sum_{121'2'}\hat{a}^+_{\mathbf{p}_1{}'\alpha}\hat{a}^+_{\mathbf{p}_2{}'\beta}\hat{a}_{\mathbf{p}_2\beta}\hat{a}_{\mathbf{p}_1\alpha}=2g\sum_{121'2'}\hat{a}^+_{\mathbf{p}_1{}'\uparrow}\hat{a}^+_{\mathbf{p}_2{}'\downarrow}\hat{a}_{\mathbf{p}_2\downarrow}\hat{a}_{\mathbf{p}_1\uparrow}, \tag{A3}$$

where $g=2\pi a\hbar^2/m$ is the coupling constant and a is the s-scattering length, with allowance for the momentum and energy conservation in the elementary scattering act, leads, upon summation in (A1), to the known Fermi collision integral:

$$\mathrm{St}\,n_+(\mathbf{p}_1)=\left(\frac{2a}{m}\right)^2\int\frac{d\mathbf{p}_2}{(2\pi\hbar)^3}d\mathbf{p}_1{}'\,d\mathbf{p}_2{}'\,\delta$$

$$\times(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_1{}'-\mathbf{p}_2{}')\,\delta(\varepsilon^{(0)}_{\mathbf{p}_1}$$

$$+\varepsilon^{(0)}_{\mathbf{p}_2}-\varepsilon^{(0)}_{\mathbf{p}_1{}'}-\varepsilon^{(0)}_{\mathbf{p}_2{}'})\,[F_{\uparrow\uparrow}(\mathbf{p}_1{}',\mathbf{p}_2{}';\mathbf{p}_1,\mathbf{p}_2)-F_{\uparrow\uparrow}(\mathbf{p}_1,\mathbf{p}_2;\mathbf{p}_1{}',\mathbf{p}_2{}')],$$

$$F_{\uparrow\uparrow}(\mathbf{p}_1,\ \mathbf{p}_2;\ \mathbf{p}_1{}',\ \mathbf{p}_2{}')=n_+(\mathbf{p}_1)n_-(\mathbf{p}_2)[1-n_+(\mathbf{p}_1{}')][1-n_-(\mathbf{p}_2{}')]. \tag{A4}$$

It is implied in expression (A3) and (A4) that in the limit of slow collision, $|a|/\Lambda_T\ll 1$, only fermions with oppositely directed spins are scattered. In a Boltzmann gas, when the occupation numbers of the quantum states are small, we can neglect the functions $F_{\uparrow\uparrow}$ in (A4) the terms of third order in $n_\pm$, i.e.,

$$F_{\uparrow\uparrow}(\mathbf{p}_1,\ \mathbf{p}_2;\ \mathbf{p}_1{}',\ \mathbf{p}_2{}')\equiv F_{\uparrow\uparrow}(\mathbf{p}_1,\ \mathbf{p}_2)=n_+(\mathbf{p}_1)n_-(\mathbf{p}_2). \tag{A5}$$

For mixed spin states, transforming to an arbitrary polarization density matrix $n_{\alpha\beta}(\mathbf{p})=\langle a_\beta^+a_\alpha\rangle$, we obtain in place of (A4) and (A5)

$$\mathrm{St}\,n_{\alpha\beta}(\mathbf{p}_1)$$

$$=\left(\frac{2a}{m}\right)^2\int\frac{d\mathbf{p}_2}{(2\pi\hbar)^3}d\mathbf{p}_1{}'\,d\mathbf{p}_2{}'\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_1{}'-\mathbf{p}_2{}')\,\delta(\varepsilon^{(0)}_{\mathbf{p}_1}$$

$$+\varepsilon^{(0)}_{\mathbf{p}_2}-\varepsilon^{(0)}_{\mathbf{p}_1{}'}-\varepsilon^{(0)}_{\mathbf{p}_2{}'})\,[F_{\alpha\beta}(\mathbf{p}_1{}',\mathbf{p}_2{}')-F_{\alpha\beta}(\mathbf{p}_1,\mathbf{p}_2)],$$

$$F_{\alpha\beta}(\mathbf{p}_1,\mathbf{p}_2)=n_{\alpha\beta}(\mathbf{p}_1)\,n_{\mu\mu}(\mathbf{p}_2)$$

$$-\frac{1}{2}[n_{\alpha\mu}(\mathbf{p}_1)\,n_{\mu\beta}(\mathbf{p}_2)+n_{\alpha\mu}(\mathbf{p}_2)\,n_{\mu\beta}(\mathbf{p}_1)]. \tag{A6}$$

It is easy to verify that in the case of pure states, when $n_{\alpha\beta}=0$ for $\alpha\neq\beta$, expression (A6) goes over at $\alpha=\beta=\uparrow$, $n_{\uparrow\uparrow}=n_+$, $n_{\downarrow\downarrow}=n_-$ into (A4) with account taken of (A5). The vertex function $\Gamma_{\alpha\beta,\mu\nu}$ is defined by the expression

$$\Gamma_{\alpha\beta,\mu\nu}=\frac{4\pi a\hbar^2}{m}(\delta_{\alpha\beta}\delta_{\mu\nu}-\delta_{\alpha\mu}\delta_{\beta\nu}), \tag{A7}$$

which can be rewritten with the aid of the identity

$$\delta_{\alpha\beta}\delta_{\mu\nu}+\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\mu\nu}=2\delta_{\alpha\mu}\delta_{\beta\nu} \tag{A8}$$

in the traditional form:

$$\Gamma_{\alpha\beta,\mu\nu}=\frac{2\pi a\hbar^2}{m}(\delta_{\alpha\beta}\delta_{\mu\nu}-\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\mu\nu}). \tag{A9}$$

For density-matrix perturbations of the form $\delta n_{\alpha\beta}(\mathbf{p})=\boldsymbol{\lambda}(\mathbf{p})\boldsymbol{\sigma}_{\alpha\beta}$, $\boldsymbol{\lambda}\perp\mathfrak{M}$, the linearized collision integral (A6) takes the form

$$\mathrm{St}\,\boldsymbol{\lambda}(\mathbf{p}_1)$$

$$=\left(\frac{2a}{m}\right)^2\int\frac{d\mathbf{p}_2}{(2\pi\hbar)^3}d\mathbf{p}_1'\,d\mathbf{p}_2'\,\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_1'-\mathbf{p}_2')\,\delta(\varepsilon_{\mathbf{p}_1}^{(0)}+\varepsilon_{\mathbf{p}_2}^{(0)}$$

$$-\varepsilon_{\mathbf{p}_1'}^{(0)}-\varepsilon_{\mathbf{p}_2'}^{(0)})\,[\boldsymbol{\lambda}(\mathbf{p}_1')\,n_0(\mathbf{p}_2')-\boldsymbol{\lambda}(\mathbf{p}_1)\,n_0(\mathbf{p}_2)$$

$$+\boldsymbol{\lambda}(\mathbf{p}_2)\,n_0(\mathbf{p}_1)-\boldsymbol{\lambda}(\mathbf{p}_2')\,n_0(\mathbf{p}_1')\,]. \qquad \text{(A10)}$$

Expression (A10) for the collision term in the transport equation that describes the dynamics of the transverse magnetization, retains the same form also in the case of arbitrary deviations, $\delta n_{\alpha\beta}(\mathbf{p}) = \nu(\mathbf{p})\delta_{\alpha\beta} + \boldsymbol{\eta}(\mathbf{p})\sigma_{\alpha\beta}$, since the equilibrium polarization density matrix is diagonal, and the perturbations of the off-diagonal elements of the density matrix, i.e., $\boldsymbol{\eta}_\perp = (\eta_x, \eta_y) = \boldsymbol{\lambda}$, are not linked with the perturbations of the density $\nu(\mathbf{p})$, as in the case for the fluctuations of η_z. The collision integral St $n_{\alpha\beta}(\mathbf{p})$ of (A6) agrees in the corresponding limiting case with the analogous Boltzmann-equation term obtained in Ref. 14.

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