Nonuniform state in quasi-1D superconductors

A. I. Buzdin and S. V. Polonskiĭ

M. V. Lomonosov Moscow State University
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The region in which a nonuniform superconducting state exists in quasi-1D superconductors is considerably larger than that in quasi-2D or quasi-3D superconductors. The finite magnitude of the paramagnetic limit in this case results from scattering by impurities. Near the tricritical point, the nonuniform phase arises through a second-order transition. The phase diagram and the properties of the nonuniform superconducting state are examined. Possibilities for observing it experimentally in quasi-1D superconductors are discussed.

1. INTRODUCTION

Organic superconductors of the types $(TMTSF)_2X$ and $(BEDT\text{–}TTF)_2X$ which have recently been synthesized are characterized by substantially anisotropic electronic properties (see, for example, the reviews in Refs. 1-3). The compounds $(BEDT\text{–}TTF)_2X$ are actually quasi-2D superconductors, while the compounds $(TMTSF)_2X$ are closer to being quasi-1D superconductors. Near the superconducting transition temperature T_c, the upper critical field in the $(TMTSF)_2X$ family is highly anisotropic; it reaches its maximum when the field is directed along the stacks of TMTSF molecules, i.e., in the **a** direction (the conductivity is a maximum along this axis). This is because electrons experience difficulty in making transitions from one conducting chain to another. The slope of the $H_{c2}(T)$ curve becomes greater than that in an ordinary isotropic superconductor by a factor of $(t_\perp/\varepsilon_F)^{-2}$, where $t_\perp$ is the width of the electron band in the direction transverse to the chains. As a result, the condition $H_{c2} \gtrsim H_p$, where $H_p = 2^{-1/2}\Delta_0/\mu_B$ is the paramagnetic limit,[4] becomes satisfied as the temperature is lowered (in the BCS model we would have $\Delta_0 = 1.76T_c$). It is pertinent in this connection to note that Lebed'[5] has shown that the orbital effect is incapable in principle of suppressing the superconductivity completely in a description of the transverse motion of electrons in the strong-coupling approximation. Consequently, the paramagnetic effect may play an important role in the behavior of the critical field at low temperatures.

At low temperatures, $T<T^* = 0.56T_c$, the field $h=\mu_B H$, acting on the electron spins, gives rise through a second-order phase transition to superconductivity in the form of a nonuniform state with Cooper pairs that have finite momentum. This nonuniform superconducting state, the Larkin-Ovchinnikov-Fulde-Ferrell state[6,7] (LOFF state), exists in a 3D system at $T=0$ only in a narrow field interval $0.7\Delta_0 < h < 0.755\Delta_0$ (in fields below $0.7\Delta_0$, there is an ordinary BCS ground state.

The region in which the LOFF phase exists in the 3D case becomes even narrower when impurity scattering is present.[8] The net result is that in 3D systems it is essentially impossible to weaken the orbital effect in comparison with the paramagnetic effect and to set the stage for the onset of an LOFF phase. Exceptions to this rule may be magnetic superconductors[9] or superconductors with magnetic impurities. In such systems, the electron spins will also be acted upon by the exchange field of magnetic atoms polarized in the external field, and the large value of the exchange integral will result in a sort of amplification of the paramagnetic effect. Unfortunately, at this point we have no experimental evidence of any sort pointing to the appearance of an LOFF phase in these compounds.

In layered superconductors, the region in which the LOFF phase exists becomes slightly wider, and the critical field is $h_i = \Delta_0 = 2^{1/2}H_p\mu_B$ (Ref. 10). Even in this case, however, impurity scattering[11] suppresses the nonuniform state and severely complicates an experimental observation of this phase.

A special situation in regard to the paramagnetic limit occurs in quasi-1D superconductors. In the limit $T\to 0$, in the absence of impurities, there is no paramagnetic limit, and the region in which the nonuniform superconducting state exists becomes significantly broader for $T<T^*$. In this case it is possible to find a complete description of the LOFF phase, which is a soliton lattice.[12]

In the quasi-1D superconductivity model, only the presence of impurities makes the paramagnetic limit finite. This circumstance is responsible for the interest in the problem of the effect of impurity scattering on the (h, T) phase diagram of the LOFF state in a quasi-1D superconductor. This problem is taken up in Sec. 2 of this paper. Numerical calculations have made it possible to construct a family of phase diagrams for various impurity concentrations. Near the tricrital point T^* (at $T<T^*$) the phase transition to the LOFF state becomes a first-order transition. As the impurity concentration increases, the region in which the transition is first-order increases, and at a mean free path $l=0.42v_F/T_c$ the transition occurs as a first-order transition for all $T<T^*$.

In Sec. 3 we analyze the Ginzburg-Landau expansion for the superconducting order parameter, which can be constructed near the tricritical point T^*. In Sec. 4 we describe the nonuniform state in quasi-1D superconductors at low temperatures. In the Conclusion (Sec. 5) we discuss the question of the experimental observability of the LOFF phase in organic superconductors.

2. PHASE DIAGRAM OF A QUASI-1D SUPERCONDUCTOR WITH NONMAGNETIC IMPURITIES

Assuming that the magnetic field is directed parallel to the superconducting chains (the z axis), we can ignore the orbital effect. The Hamiltonian of this system can then be written as follows in the mean-field approximation:

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$$H=\int d\mathbf{r}\{\psi_\sigma^+(\mathbf{r})\hat{\xi}\psi_\sigma(\mathbf{r})+\sigma h\psi_\sigma^+(\mathbf{r})\psi_\sigma(\mathbf{r})+U(\mathbf{r})\psi_\sigma^+(\mathbf{r})\psi_\sigma(\mathbf{r})$$
$$+[\Delta(\mathbf{r})\psi_{+1}^+(\mathbf{r})\psi_{-1}^+(\mathbf{r})+\text{H. C.}]\}, \qquad (1)$$

where $\sigma=1$ (-1) if the electron spin is directed parallel (antiparallel) to the field $h=H\mu_B$; $U(\mathbf{r})$ is the potential of the impurities distributed at random along the chains; and the superconducting order parameter is $\Delta(\mathbf{r})=\lambda\langle\psi(\mathbf{r})\cdot\psi(\mathbf{r})\rangle$ (λ is the electron-phonon interaction constant).

The electron spectrum in a quasi-1D superconductor $(TMTSF)_2X$ is described well in the strong-coupling approximation for the transverse motion of the electrons[1-3] even in the momentum representation:

$$\hat{\xi}(\mathbf{p})=v_F(|p_z|-p_F)+t_x\cos p_x a+t_y\cos p_y b, \qquad (2)$$

where t_x and t_y are transition integrals for the corresponding directions, and a and b are the periods in these directions (for simplicity we are assuming a model with an orthorhombic structure).

The mean-field approximation used in writing (1), and thus the simplification of ignoring 1D fluctuations, is valid if the deviation from one-dimensionality is significantly large: $t_\perp \gg T_c$ (here $t_\perp$ means t_x or t_y). This condition holds in quasi-1D superconductors.[3] At the same time, all the specific features of the quasi-1D situation for the LOFF phase are manifested for $t_\perp \ll \varepsilon_F = p_F^2/2m$, i.e., for open Fermi surfaces. Our approach is thus valid for $T_c \ll t_\perp \ll \varepsilon_F$.

Our analysis of the behavior of the field at which the transition to the LOFF phase occurs and the nature of this transition is carried out on the basis of a solution to order $\Delta^3(\mathbf{r})$ of the Gor'kov equations for the normal Green's function $G(\mathbf{r}_1,\mathbf{r}_2)$ and the anomalous Green's function $F(\mathbf{r}_1,\mathbf{r}_2)$ (Ref. 13). In the temperature technique these equations take the following form after an average is taken over the impurities, which are assumed to interact as points:

$$(i\tilde{\omega}(\mathbf{r})-\hat{\xi}_+)G(\mathbf{r},\mathbf{r}')+\Delta_\omega(\mathbf{r})F(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}'),$$
$$(i\tilde{\omega}(\mathbf{r})+\hat{\xi}_-)F(\mathbf{r},\mathbf{r}')+\Delta_\omega^*(\mathbf{r})G(\mathbf{r},\mathbf{r}')=0, \quad \omega=(2n+1)\pi T, \qquad (3)$$

where

$$\tilde{\omega}(\mathbf{r})=\omega+\frac{i}{2\tau}G_\omega(\mathbf{r},\mathbf{r}), \quad \Delta_\omega(\mathbf{r})=\Delta(\mathbf{r})+\frac{1}{2\tau}F_\omega^*(\mathbf{r},\mathbf{r}),$$
$$\hat{\xi}_\pm(\mathbf{p})=\xi(\mathbf{p})\pm h, \qquad (4)$$

and τ is the mean free time of the electrons.

As usual, the system (3) is supplemented with the self-consistency equation

$$\Delta(\mathbf{r})=|\lambda|T\sum_\omega F_\omega(\mathbf{r},\mathbf{r}). \qquad (5)$$

Switching to integral equations for G and F, as in the procedure used to derive the Ginzburg-Landau equations,[13] and solving them by an iterative method to terms of third order in Δ, we find

$$F_\omega(\mathbf{r},\mathbf{r})=\int d\mathbf{r}'G_{-\omega}^-(\mathbf{r}-\mathbf{r}')G_\omega^+(\mathbf{r}'-\mathbf{r})\Delta_\omega^*(\mathbf{r}')$$
$$-\int d\mathbf{r}'\,d\mathbf{r}''\,d\mathbf{r}'''G_{-\omega}^-(\mathbf{r}-\mathbf{r}')$$
$$\cdot G_\omega^+(\mathbf{r}'-\mathbf{r}'')G_{-\omega}^-(\mathbf{r}''-\mathbf{r}''')G_\omega^+(\mathbf{r}'''-\mathbf{r})\Delta_\omega^*(\mathbf{r}')\Delta_\omega(\mathbf{r}'')\Delta_\omega^*(\mathbf{r}''')$$
$$-\frac{1}{2\tau}\int d\mathbf{r}'\,d\mathbf{r}''\,G_{-\omega}^-(\mathbf{r}-\mathbf{r}')\Delta_\omega^*(\mathbf{r}')G_\omega^+(\mathbf{r}'-\mathbf{r}'')$$
$$\times G_\omega^{(2)}(\mathbf{r}'',\mathbf{r}'')G_\omega^+(\mathbf{r}''-\mathbf{r})$$
$$+\frac{1}{2\tau}\int d\mathbf{r}'\,d\mathbf{r}''G_{-\omega}^-(\mathbf{r}-\mathbf{r}')G_\omega^{(2)}(\mathbf{r}',\mathbf{r}')$$
$$\times G_{-\omega}^-(\mathbf{r}'-\mathbf{r}'')\Delta_\omega^*(\mathbf{r}'')G_\omega^+(\mathbf{r}''-\mathbf{r}), \qquad (6)$$

where the functions $G^\pm$ solve the equations

$$[i\omega'\mp\hat{\xi}_\pm]G_\omega^\pm(\mathbf{r}-\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}'), \quad \omega'=\omega+\operatorname{sign}\omega/2\tau \qquad (7)$$

and in the momentum representation we have $G_\omega^\pm(\mathbf{p})=(i\omega'-\xi\pm h)^{-1}$. The function $G_\omega^{(2)}(\mathbf{r},\mathbf{r})$ determines the correction to $\tilde{\omega}(\mathbf{r})$ which is quadratic in $\Delta(\mathbf{r})$ and is given by

$$G_\omega^{(2)}(\mathbf{r},\mathbf{r})=-\int d\mathbf{r}'\,d\mathbf{r}''\,G_\omega^+(\mathbf{r}-\mathbf{r}')G_{-\omega}^-(\mathbf{r}'-\mathbf{r}'')$$
$$\times G_\omega^+(\mathbf{r}''-\mathbf{r})\Delta_\omega(\mathbf{r}')\Delta_\omega^*(\mathbf{r}''). \qquad (8)$$

We will be interested in solutions of Eq. (6) of two types:

$$\Delta^a(\mathbf{r})=\Delta_0^a e^{i\mathbf{qr}} \text{ with } F_\omega(\mathbf{r},\mathbf{r})=\Phi_\omega^a e^{i\mathbf{qr}} \qquad (9a)$$

and

$$\Delta^b(\mathbf{r})=\Delta_0^b\sin\mathbf{qr} \text{ with } F_\omega(\mathbf{r},\mathbf{r})=\Phi_\omega^b\sin\mathbf{qr}. \qquad (9b)$$

These two solutions obviously give the same field for the second-order transition, $h(T)$, but the nature of the transition may be completely different in the two cases.

Substituting (9) into (6), and omitting the superscripts "a" and "b" (to avoid overburdening the notation), we can write

$$\Phi_\omega=K_\omega^{(1)}\left(\Delta_0+\frac{1}{2\tau}\Phi_\omega\right)-K_\omega^{(3)}\left(\Delta_0+\frac{1}{2\tau}\Phi_\omega\right)^3, \qquad (10)$$

where the expressions for $K_\omega^{(1)}$ and $K_\omega^{(3)}$ follow directly from (6)–(8) and will be given below only for particular cases of interest. Solving (10) for Φ_ω, we find the self-consistency equation (5) in the form

$$\Delta_0=|\lambda|T\sum_\omega\left\{\frac{K_\omega^{(1)}(\mathbf{q})\Delta_0}{1-(2\tau)^{-1}K_\omega^{(1)}(\mathbf{q})}-\frac{K_\omega^{(3)}(\mathbf{q})\Delta_0^3}{[1-(2\tau)^{-1}K_\omega^{(1)}(\mathbf{q})]^4}\right\}. \qquad (11)$$

The field of the second-order transition can be found from the linearized version of (11) (in the case of a first-order transition, this field will be the boundary for an absolute instability of the supercooled normal state):

$$1=|\lambda|T\sum_\omega K_\omega^{(1)}(\mathbf{q},h)\left[1-\frac{1}{2\tau}K_\omega^{(1)}(\mathbf{q},h)\right]^{-1}, \qquad (12)$$

where $K_\omega^{(1)a}=K_\omega^{(1)b}=K_\omega^{(1)}$. Equation (12) implicitly specifies a relationship among the temperature, the critical field h, and the wave vector $\mathbf{q}$. At a fixed temperature, a transition occurs to the state which corresponds to the maximum value of h.

Equation (12) can easily be solved in the absence of impurities near the tricritical point ($T^*=0.56T_c$, $h^*=1.01T_c$). In this case the wave vector $\mathbf{q}$ is small, and we can restrict (12) to an expansion in powers of $\mathbf{q}$. As a result we find

$$A[h-h_0(T)]+\alpha_i Bq_i^2\left[\frac{h^*}{T^*}T-h_0(T)\right]+\beta_{ij}Cq_i^2q_j^2=0;\quad i,j=x,y,z, \tag{13}$$

where

$$A=\mathrm{Im}\,\Psi'(z)/4\pi T^*,\quad B=\frac{v_F^2}{2(4\pi T^*)^3}\mathrm{Im}\,\Psi'''(z),$$

$$C=\frac{v_F^4}{2\cdot 4!(4\pi T^*)^4}\mathrm{Re}\,\Psi^{IV}(z),\quad z={}^1\!/_2\left(1-\frac{ih^*}{\pi T^*}\right),$$

Ψ is the digamma function,

$$\alpha_z=1,\quad \alpha_x=(t_xa)^2/2v_F^2,\quad \alpha_y=(t_yb)^2/2v_F^2,$$
$$\beta_{zz}=1,\quad \beta_{xx}={}^3\!/_8(t_xa)^4/v_F^4,\quad \beta_{zx}=\beta_{xz}={}^3\!/_2(t_xa)^2/v_F^2,$$
$$\beta_{xy}=\beta_{yx}={}^3\!/_4(t_xt_yab)^2/v_F^4.$$

The field $h_0(T)$ is the well-known field of the second-order transition from a normal state to a uniform superconducting state.[4]

The reason why terms $\sim q^4$ must be considered in (13) is that the term $|\partial\Delta/\partial\mathbf{r}|^2$ enters with a minus sign for $T<T^*$ (it changes sign at the tricritical point), in contrast with the usual situation in the free-energy functional. The result is to favor the appearance of a nonuniform LOFF state.

As can be seen easily from (13), the strongest critical field is reached when the wave vector $\mathbf{q}$ is directly along the chains ($\mathbf{q}\|\mathbf{z}$). In this case we have $q=q_2=B/2C$, and the transition field is

$$h_i(T)=h_0(T)+[h^*T/T^*-h_0(T)]^2B^2/4AC.$$

The functional dependence $h_i(T)$ is given in Ref. 12 for arbitrary temperatures and for $q=q_z$; in the limit $T\to 0$ there is no paramagnetic limit.

In the case $q_z=0$, $q=q_x$, the kernel $K^{(1)}$ is the same as the corresponding kernel in a 2D superconductor, so the field at $T=0$ is $h_i=\sqrt{2}h_p=\Delta_0$ (Ref. 10); i.e., there is a paramagnetic limit. In a quasi-2D superconductor we would have $\varepsilon_F\gg t_\perp\gg T_c$, and at $T=0$ we would have $h_i=\sqrt{2}H_p\mu_B$. However, there is a degeneracy with respect to the direction of $\mathbf{q}$ both in the plane of the layers and for the direction perpendicular to the layers.

All these results are evidence that the most favorable direction for a modulation of $\Delta(r)$ in a quasi-1D superconductor is along the chains. We will accordingly assume below that the vector $\mathbf{q}$ is directed along the z axis.

For the kernel $K^{(1)}$ in this case we have the following representation:

$$K_\omega^{(1)}(q,h)=\frac{1}{2}\left[\frac{1}{\omega'-i(h+v_Fq/2)}+\frac{1}{\omega'-i(h-v_Fq/2)}\right]. \tag{14}$$

Using (14), we can write Eq. (12) as follows, making use of the digamma functions:

$$2\Psi\left(\frac{1-ih_0(T)/\pi T}{2}\right)-\Psi\left(\frac{1-w_1/\pi T}{2}\right)-\Psi\left(\frac{1-w_2/\pi T}{2}\right)+\frac{1/2\tau}{[(1/2\tau)^2-(v_Fq)^2]^{1/2}}\left(\Psi\left(\frac{1-w_2/\pi T}{2}\right)-\Psi\left(\frac{2-w_1/\pi T}{2}\right)\right)+\text{c.c.}=0, \tag{15}$$

where

$$w_{1,2}=\frac{1}{2}\left\{2ih-\frac{1}{2\tau}\pm\left[\left(\frac{1}{2\tau}\right)^2-(qv_F)^2\right]^{1/2}\right\}.$$

In the limit $T=0$ and at large values of τ ($\tau^{-1}\ll T_c$), Eq. (15) reduces to

$$\left(q\frac{v_F}{2}\right)^2=h^2-\left(\frac{1}{4\tau}\right)^2\pm\left[h_0^4-\left(\frac{h}{2\tau}\right)^2\right]^{1/2}, \tag{16}$$

where[4] $h_0=h_0(T=0)=\Delta_0/2\approx 0.88T_c$.

The expression in the radical in (16) vanishes when the maximum field is attained:

$$h_i(T=0)=2\tau h_0^2. \tag{17a}$$

Here the modulation wave vector is

$$q_i=\frac{2}{v_F}\left[(2h_0^2\tau)^2-\left(\frac{1}{4\tau}\right)^2\right]^{1/2}. \tag{17b}$$

The presence of impurities thus causes a qualitative change in the behavior of the critical field at low temperatures. It gives rise to a finite paramagnetic limit.

In the opposite case of a dirty superconductor, with $\tau^{-1}\gg T_c$, we find from (15)

$$\ln\frac{h_0}{h}-\left(\frac{\tau}{h}\right)^2\left[h^2-\left(q\frac{v_F}{2}\right)^2\right]+\tau^2\left(h^2-\frac{(qv_F)^2}{2}\right)-\tau^2(qv_F)^2\ln(2h\tau)=0, \tag{18}$$

from which we find in turn

$$h=h_0(1+\tau^2h_0^2\ln^2 4\tau^2h_0^2), \tag{19a}$$

$$(v_Fq)^2=-4h^2\ln 4h^2\tau^2. \tag{19b}$$

The phase transition in this case, however, is of first order, and the field (19a) is only the field corresponding to the supercooling of the normal phase. The actual field of the first-order transition is not known, but it might be assumed to be close to $h_p=\sqrt{2}h_0$, which is the field of the first-order transition from the normal state to the uniform superconducting state. In a dirty superconductor the LOFF phase apparently is possible only as a metastable phase.

Numerical methods have been used to study Eq. (15) for the general case of an arbitrary impurity concentration. The numerical calculations yielded curves of $h_i(T)$ and $q_i(T)$ over a broad range of τ; some of these curves are shown in Figs. 1 and 2. It can be seen from Fig. 1 that the impurity scattering may substantially reduce the field of the transition to the LOFF phase. It is important to note that the presence of impurities gives rise to a first-order transition on the $h(T)$ curve below the point (T^*, h^*). In this case, the coefficient of the gradient term $|\partial\Delta/\partial\mathbf{r}|^2$ in the Ginzburg-Landau functional is positive, while in the case $|\Delta_{q=0}|^4$ it is

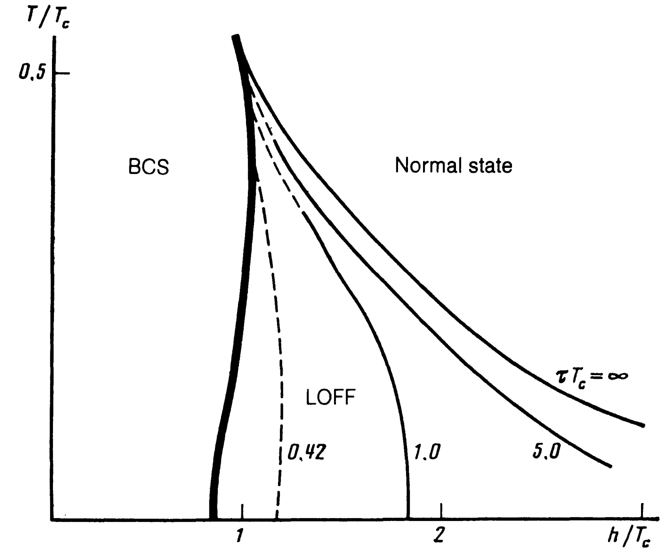

FIG. 1. The (h, T) phase diagram for various impurity concentrations. Dashed lines—regions of a first-order transition; heavy line—the field $h_0(T)$, at which the transition to a uniform superconducting state occurs.

negative. With increasing impurity concentation, the region of the first-order transition becomes broader, and at $\tau^{-1} = 2.4T_c$ it reaches the point $T = 0$.

Numerical methods are again mandatory for studying the nature of the phase transition over the entire temperature range. A first-order phase transition corresponds to a positive coefficient of Δ_0^3 in Eq. (11). Its magnitude depends strongly on the type of solution. Our calculations show that the solution $\Delta^b = \Delta_0^b \sin qr$ has a lower energy than the helicoidal solution $\Delta^a = \Delta_0^a e^{iqr}$ everywhere.

For the first type of functional dependence of the order parameter we have

$$K_\omega^{(3)b} = \frac{1}{4}\Big\{-i\frac{i\omega'+h}{(i\omega'+x)^2(i\omega'+y)^2} - \frac{i}{4}\Big[\frac{1}{(i\omega'+x)^3}+\frac{1}{(i\omega'+y)}\Big] - \frac{1}{\tau}\Big[\frac{1}{8}\Big(\frac{1}{(i\omega'+x)^2}+\frac{1}{(i\omega'+y)^2}\Big)^2 + \frac{1}{4}\frac{1}{(i\omega'+x)^2(i\omega'+y)^2}\Big]\Big\}, \tag{20}$$

$$x=h+v_Fq/2, \quad y=h-v_Fq/2.$$

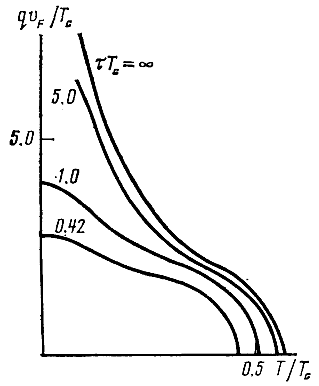

FIG. 2. Temperature dependence of the wave vector of the LOFF phase on the transition for various impurity concentrations.

Using expression (20) in (11), and using as $q(T)$ and $h(T)$ the values they have at the point of transition to the LOFF phase, we numerically determined the coefficient of Δ_0^3. As a result we also found the region of the first-order transition, which is shown by the dashed line in Fig. 1.

For the solution $\Delta_0^a e^{iqr}$, on the other hand, we have

$$K_\omega^{(3)a} = -\frac{1}{4}\Big\{i\Big(\frac{1}{(i\omega'+x)^3}+\frac{1}{(i\omega'+y)^3}\Big) - \frac{1}{4\tau}\Big(\frac{1}{(i\omega'+x)^2}+\frac{1}{(i\omega'+y)^2}\Big)^2\Big\}, \tag{21}$$

and the transition always occurs as a second-order transition. This result again emphasizes the fact that the solution $\Delta_0^b \sin qr$ is favored from the energy standpoint.

To conclude this section of the paper we note that in the 3D case the transition to the LOFF phase occurs as a first-order transition near the tricritical point, even in the absence of impurities. As the temperature is lowered, it becomes a second-order transition.[14] Our analysis shows that in the 2D case and also in the quasi-2D case with $t_\perp \gg T_c$ the transition to the LOFF phase always occurs as a second-order transition in the absence of impurities.[1)]

In addition to the scattering by nonmagnetic impurities we studied the effect of magnetic impurities on the phase diagram of the LOFF state in quasi-1D superconductors. According to our numerical calculations, the curves of $h_i(T)$ and $q_i(T)$ are similar to those in Figs. 1 and 2 (when we allow for the decrease in the critical temperature T_c with increasing concentration of the magnetic impurities). The transition to the LOFF phase, however, always occurs as a second-order transition, and the region in which the LOFF phase exists decreases with increasing impurity concentration much faster than it does in the case on nomagnetic impurities.

3. GINZBURG-LANDAU FUNCTIONAL FOR A QUASI-1D SUPERCONDUCTOR

Near the tricritical point, we can use the Ginzburg-Landau functional to describe the LOFF phase. In contrast with the ordinary situation, the coefficient of the gradient term is negative, and terms with second derivatives must also be retained in the functional.

A functional of this sort was studied in Refs. 15 and 16 for electronic phase transitions to a nonuniform state in the 3D case (corresponding to the LOFF transition in an isotropic superconductor); a functional of this type was also studied for quasi-2D superconductors in Ref. 17.

In the case at hand, that of a quasi-1D superconductor, we have

$$\frac{F}{N(0)} = A|\Delta|^2[h-h_0(T)] + \alpha_i B\left|\frac{\partial\Delta}{\partial x_i}\right|^2\left[\frac{h^*}{T^*}T-h_0(T)\right] + \beta_{ij}C\left|\frac{\partial^2\Delta}{\partial x_i\,\partial x_j}\right|^2 + \tilde{B}|\Delta|^4\left[\frac{h^*}{T^*}T-h_0(T)\right] + \alpha_i\Big\{8\tilde{C}|\Delta|^2\left|\frac{\partial\Delta}{\partial x_i}\right|^2 + \tilde{C}\Big[\Delta^2\Big(\frac{\partial\Delta^*}{\partial x_i}\Big)^2 + \text{c.c.}\Big]\Big\}, \tag{22}$$

where $\tilde{B} = B/v_F^2$ and $\tilde{C} = C/v_F^2$. We wish to call attention to the structure of the terms in braces (curly brackets) here. This structure arises because the superconducting order parameter $\Delta(\mathbf{r})$ is complex. This circumstance distinguishes the functional for the LOFF phase from the corresponding situation in the case of electronic transitions, e.g., the Peierls transition.

The equation for the superconducting order parameter found by varying function (22) is

$$A\Delta[h-h_0(T)]-\alpha_i B\frac{\partial^2\Delta}{\partial x_i}\left[\frac{h^*}{T^*}T-h_0(T)\right]+\beta_{ij}C\frac{\partial^4\Delta}{\partial x_i^2\,\partial x_j^2}$$
$$+2\tilde{B}|\Delta|^2\Delta\left[\frac{h^*}{T^*}T-h_0(T)\right]-\alpha_i\left\{8\tilde{C}\left(\Delta\left|\frac{\partial\Delta}{\partial x_i}\right|^2+|\Delta|^2\frac{\partial^2\Delta}{\partial x_i^2}\right)\right.$$
$$\left.+2\tilde{C}\left(\Delta^*\left(\frac{\partial\Delta}{\partial x_i}\right)^2-2\Delta\left|\frac{\partial\Delta}{\partial x_i}\right|^2-\Delta^2\frac{\partial^2\Delta^*}{\partial x_i^2}\right)\right\}=0. \tag{23}$$

Equation (23) is an extremely complicated nonlinear fourth-order equation. It is not difficult to see that Eq. (23) has a trivial complex solution for which the order parameter has a constant modulus, $\Delta^a=\Delta_0 e^{i\mathbf{qr}}$. In this case, it is also a simple matter to derive an exact solution of the Gor'kov equations. A helicoidal solution, however, does not correspond to the ground state of the functional (22). If we are interested in a real solution of (23), which depends only on one coordinate, we can immediately recognize the one nontrivial solution $\Delta(z)\sim\mathrm{sn}(z)$, which has an energy lower than $\Delta\sim e^{i\mathbf{qr}}$ and which in the limit $h\to h_i$ goes over to $\Delta^b=\Delta_0\sin\mathbf{qr}$ (Ref. 12). The reader is directed to Ref. 18 for a more detailed discussion of solution (23).

Functional (22) ignores the effect of the orbital field on the superconductivity. If the orbital effect is weak, it can be incorporated in (22) by the standard substitution $\partial/\partial x_i\to\partial/\partial x_i-(2ie/c)A_i$, where A_i is the vector potential of the orbital field [in principle, we would also need to incorporate a term $|\Delta|^2\mathbf{B}^2$, where $\mathbf{B}=\mathrm{curl}\,\mathbf{A}$, in (22)].

Interestingly, the expression which is found for the current from functional (22) is quite different from the usual expression in the Ginzburg-Landau theory. Writing the variational derivative $\delta F/\delta\mathbf{A}$, we find

$$j_k=-\frac{2ie}{c}\left\{\alpha_k B\Delta\left(\frac{\partial}{\partial x_k}+\frac{2ie}{c}A_k\right)\Delta^*\right.$$
$$+\beta_{ik}C\left[\frac{\partial(\Delta g_{ik})}{\partial x_i}-\left(\frac{\partial}{\partial x_i}-\frac{2ie}{c}A_i\right)\Delta(g_{ik}+g_{ki})\right]$$
$$-\alpha_k\tilde{C}\left[8|\Delta|^2\Delta\left(\frac{\partial}{\partial x_k}+\frac{2ie}{c}A_k\right)\Delta^*\right.$$
$$\left.\left.+2\Delta^{*2}\Delta\left(\frac{\partial}{\partial x_k}-\frac{2ie}{c}A_k\right)\Delta\right]\right\}+\text{c.c.}, \tag{24}$$

where

$$g_{ik}=\left(\frac{\partial}{\partial x_i}+\frac{2ie}{c}A_i\right)\left(\frac{\partial}{\partial x_k}+\frac{2ie}{c}A_k\right)\Delta,$$

[we are omitting from (24) a small paramagnetic contribution which comes from the dependence of the coefficients of the functional on the field h; generally speaking, this field is the field $\mu_B\mathbf{B}$ which is acting on the electron spins].

That expression (24) is specific in nature is emphasized by the fact that the solution $\Delta=\Delta_0 e^{i\mathbf{qr}}$ of Eq. (23), which describes the current state in an ordinary superconductor, corresponds in our case to $\mathbf{j}=0$. It is easy to see that the expression for $\mathbf{j}$ is the same as the derivative $\partial F/\partial q^2$.

The problem of calculating the orbital critical field for the functional (22) differs from its standard formulation for an ordinary superconductor.[17] This problem has not been finally resolved for a quasi-1D superconductor.

The specific nature of the LOFF state is also manifested in its thermodynamic properties, which are different from those of an ordinary superconductor. For example, the discontinuity in the specific heat at the second-order transition from the normal state to the LOFF phase is

$$\Delta c=\frac{N(0)}{2}\frac{T^{*4}A^2}{h-h^*}\left(\frac{dh_0}{dT}\right)^2\left(1+\frac{h^*}{T^*}\frac{dT}{dh_0}\right)^{-1}, \tag{25}$$

where the derivative dh_0/dT is taken at the point $T=T^*$.

The divergence of the discontinuity in the specific heat at the tricritical point results from the vanishing of the coefficient of $|\Delta|^4$ in the Ginzburg-Landau functional at this point.

Idlis and Kopaev[16] have called attention to the growth of fluctuations near the tricritical point. In an isotropic superconductor, as the boundary for an absolute instability of the normal phase with respect to a transition to an LOFF state is approached, the fluctuational increment in the specific heat becomes

$$\Delta c_{fl}^{3D}\sim\frac{\tau_0^{1/2}}{\tau^{3/2}}\left(\frac{T_c}{v_F}\right)^3,$$

where

$$\tau_0=\frac{h-h^*}{h^*},\qquad \tau=\left|\frac{T-T_c(h)}{T^*}\right|.$$

It is not difficult to show, through the use of expression (22) for the free energy, that in a quasi-1D suprconductor we would have

$$\Delta c_{fl}^{Q1D}\sim\frac{1}{\tau^{1/2}}\frac{1}{\tau_0^{3/2}}\left(\frac{T_c}{v_F}\right)^3\left(\frac{\varepsilon_F}{t_\perp}\right)^2.$$

The power-law dependence of τ is different from that in the 3D case because the degeneracy with respect to the direction of the wave vector $\mathbf{q}$ is lifted.

The requirement that Δc_{fl} be small in comparison with the discontinuity in the specific heat yields an estimate of the size of the fluctuation region: $\tau\gg(T_c/t_\perp)^4/\tau_0$ (this estimate is valid under the condition $\tau\ll\tau_0^2$).

4. THE LOFF PHASE IN QUASI-1D SUPERCONDUCTORS AT LOW TEMPERATURES

The nonuniform LOFF state can be determined for all temperatures in pure quasi-1D superconductors.[12] A similar approach was proposed by Machida and Nakanishi[19] for analyzing the LOFF phase in a 1D model of a ferromagnetic superconductor. The description is based on an exact solution of the Peierls continuum model at $T=0$, with an approximately half-filled band.[20]

In this section of the paper we examine the description of the properties of the LOFF phase at low temperatures, $T\ll T_c$, extending the approach of Ref. 12.

We start with the Bogolyubov-de Gennes equations for a superconductor in an exchange field[21]:

$$\begin{aligned}E_\uparrow u_\uparrow(\mathbf{r})&=(\hat{\xi}+h)u_\uparrow(\mathbf{r})+\Delta(\mathbf{r})v_\downarrow(\mathbf{r}),\\ -E_\uparrow v_\downarrow(\mathbf{r})&=(\hat{\xi}-h)v_\downarrow(\mathbf{r})-\Delta^*(\mathbf{r})u_\uparrow(\mathbf{r}).\end{aligned} \tag{26}$$

These equations are written for the case of a spin oriented parallel to the field; similar equations with $h\to -h$ hold for the opposite field direction. The functions u and v perform the transformation

$$\psi(\sigma,\mathbf{r})=\sum_n [u_n(\mathbf{r},\sigma)\gamma_{n,\sigma}+v_n^*(\mathbf{r},\sigma)\gamma^+_{n,-\sigma}],$$
$$\psi^+(\sigma,\mathbf{r})=\sum_n [u_n^*(\mathbf{r},\sigma)\gamma^+_{n,\sigma}+v_n(\mathbf{r},\sigma)\gamma_{n,-\sigma}], \tag{27}$$

which diagonalizes the Hamiltonian (1) in terms of the operators γ nd γ^+. We restrict the discussion below to the equations for $(u_\uparrow, v_\downarrow)\to(u,v)$, since the equations and the spectrum for $u_\downarrow, v_\uparrow$ are analogous, because of the symmetry under the substitution $h\to -h$ (this symmetry corresponds to the electron-hole symmetry in the problem of the Peierls transition).

The self-consistency equation for system (26) is

$$\Delta(\mathbf{r})=|\lambda|\sum_n uv^*[1-n(\varepsilon_n+h)-n(\varepsilon_n-h)], \tag{28}$$

where ε_n is an energy eigenvalue of (26) for $h=0$, and $n(\varepsilon)$ is the Fermi distribution function.

We seek solutions of (26) in the form

$$u=\tilde u^\pm(z)\exp\left[\pm i\left(p_F-\frac{\eta(p_\perp)}{v_F}\right)z+i\mathbf{p}_\perp\mathbf{r}_\perp\right],$$
$$v=\tilde v^\pm(z)\exp\left[\pm i\left(p_F-\frac{\eta(p_\perp)}{v_F}\right)z+i\mathbf{p}_\perp\mathbf{r}_\perp\right], \tag{29}$$

where $\mathbf{p}_\perp=(p_x,p_y)$, $\eta(\mathbf{p}_\perp)=t_x\cos p_x+t_y\cos p_y b$, and $\tilde u(z)$ and $\tilde v(z)$ are slowly varying functions of z. Substituting (29) into (26), and ignoring the second derivatives of $\tilde u$ and $\tilde v$, we find the following equation for $\tilde u^+$ and $\tilde v^+$ (the equations for $\tilde u^-$ and $\tilde v^-$ are similar and are found by making the substitution $v_F\to -v_F$; we will be omitting the indices $+$ and $-$):

$$(E-h)\begin{pmatrix}\tilde u\\ \tilde v\end{pmatrix}=\begin{pmatrix}-iv_F\dfrac{d}{dz} & \Delta\\ \Delta^* & iv_F\dfrac{d}{dz}\end{pmatrix}\begin{pmatrix}\tilde u\\ \tilde v\end{pmatrix}. \tag{30}$$

We might also note that in deriving (30) we assumed $\eta^2/\varepsilon_F\ll\Delta$ at $\eta\sim t\sim 50$–100 K, in accordance with real quasi-1D organic superconductors.[1-3] This condition holds; at the same time we have $t\gg\Delta$, T_c and 1D fluctuations are suppressed.

Squaring both sides of (30), and transforming to the new functions

$$U=(\tilde u+i\tilde v)/2,\qquad V=(\tilde u-i\tilde v)/2, \tag{31}$$

we write (30) as

$$\left[\varepsilon_n^2+v_F^2\frac{d^2}{dz^2}-\Delta^2-v_F\Delta'\right]U_n=0,$$
$$\left[\varepsilon_n^2+v_F^2\frac{d^2}{dz^2}-\Delta^2+v_F\Delta'\right]V_n=0, \tag{32}$$

where $\varepsilon_n=E_n-h$, and where we are assuming $\Delta(z)$ to be a real function. The self-consistency equation is

$$\Delta=2i\lambda\sum_n (VU^*+V^*U)[n(h-\varepsilon_n)-n(h+\varepsilon_n)]; \tag{33}$$

the factor of 2 arises because of the relations $\tilde v^-=\tilde u^+$ and $\tilde u^-=\tilde v^+$.

The system (32), (33) is analogous to the equations which describe a Peierls transition near a half-filled band. For it, the exact solution at $T=0$ is known,[20] as is the generalization of this solution to nonzero temperatures.[12,22] The behavior of the superconducting potential is described by a Jacobi elliptic function

$$\Delta(z)=\Delta\ \mathrm{sn}\,(\Delta z/v_F k_1,\ k_1); \tag{34}$$

i.e., $\Delta(z)$ is essentially a soliton lattice.

The thermodynamic potential of the system is conveniently written in the form

$$\Omega=\int\frac{\Delta^2(z)\,dz}{|\lambda|}-T\sum_n\ln\left(1+\exp\left[-\left(\frac{\varepsilon_n-h}{T}\right)\right]\right.$$
$$\left.\times\left(1+\exp\left[-\left(\frac{\varepsilon_n+h}{T}\right)\right]\right)\right). \tag{35}$$

Transforming to the new variable $\xi=z\Delta(1+k_1)/v_F 2k_1$ and from (34) to the function sn of the new modulus γ, where $\gamma'=(1-\gamma^2)^{1/2}$ and $k_1=(1-\gamma')/(1+\gamma')$, we find the following equation for V:

$$\left\{\tilde\varepsilon^2+\frac{1}{\gamma^2}\frac{d^2}{d\xi^2}-[2\,\mathrm{sn}^2(\xi,\gamma)-1]\right\}V=0, \tag{36}$$

where $\tilde\varepsilon_n^2=\varepsilon_n^2 k_1/\Delta^2$. An equation for U can be found from (36) by introducing the shift $\xi\to\xi+K(\gamma)$, i.e., $U=V(\xi+K(\gamma))$. Equation (36) is a Schrödinger equation with a potential sn^2, and its solution is

$$V=[\mathrm{sn}^2(\xi,\gamma)+b]^{1/2}\exp\left\{ic\int_0^\xi d\xi[\mathrm{sn}^2(\xi,\gamma)+b]^{-1}\right\},$$
$$b=\tilde\varepsilon^2-\gamma^2,\quad c^2=\tilde\varepsilon^2\gamma^2(1+b)b. \tag{37}$$

Making use of the cyclic boundary conditions, we find from (37) a relationship between the energy and the wave vector $q=2\pi n/L$ (L is the length of the chain, and $-\pi<q<\pi$):

$$q=\frac{c}{l}\int_0^{2K(\gamma)}\frac{d\xi}{\mathrm{sn}^2(\xi,\gamma)+b}, \tag{38}$$

where $l=2v_F(1-\gamma')K(\gamma)/\Delta$ is the period of the soliton lattice. The solution (37) satisfies the self-consistency condition (33); in this case we find equations for the soliton-lattice parameters $\Delta(h,T)$ and $\gamma(h,T)$. The same equation can be derived more conveniently, however, by varying the thermodynamic potential of the system (35).

The spectrum of Eq. (36) consists of three bands,[20] separated by two energy gaps with boundaries: $\tilde\varepsilon_+=1/\gamma$, $\tilde\varepsilon_-=(1/\gamma^2-1)^{1/2}$, and the forbidden energy region $\varepsilon_-^2<\varepsilon^2<\varepsilon_+^2$.

Evaluating the thermodynamic potential (35) at $T=0$, we find

$$\frac{\Omega-\Omega_n}{N(0)}=\frac{\tilde\Delta^2}{2}\left\{\ln\frac{\tilde\Delta}{\Delta_0}\left[-1+\frac{2}{\gamma^2}\left(1-\frac{E}{K}\right)\right]\right.$$
$$\left.+\frac{1}{2}+\frac{1}{\gamma^2}+\frac{2E}{\gamma^2K}\right\}-\frac{h\tilde\Delta\pi}{2\gamma K}, \tag{39}$$

where Ω_n is the value of the potential in the normal state

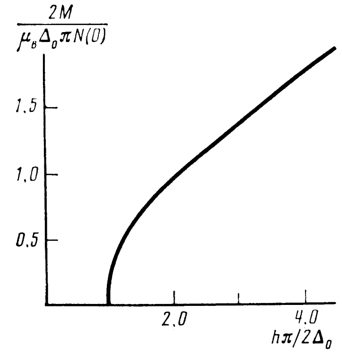

FIG. 3. Nonlinear dependence of the magnetic moment on the field for a superconducting soliton lattice.

with $h = 0$, and $\tilde{\Delta} = \Delta/k_1^{1/2}$. Varying (39), we find $\tilde{\Delta} = \Delta_0$ and

$$\frac{\Delta\Omega}{N(0)}=\frac{\Omega-\Omega_n}{N(0)}=\frac{\Delta_0{}^2}{2}\left(\frac{1}{2}-\frac{1}{\gamma^2}\right),\quad \frac{E(\gamma)}{\gamma}=\frac{h\pi}{2\Delta_0}. \tag{40}$$

In the case $h < h_c = 2\Delta_0/\pi$ there exists a uniform phase with $\gamma = 1$, $\Delta(z) = \text{const}$, and $\Delta = \Delta_0$. At $h = h_c$ we reach the threshold for the production of solitons, and at $h > hd$ the system goes into a soliton-lattice phase (the period of this phase diverges logarithmially as $h \to h_c + 0$).

As has already been mentioned, there is no paramagnetic limit for the case of the LOFF phase in quasi-1D superconductors; this phase constitutes a soliton lattice.

In the strong-field region we have

$$\gamma=(\Delta_0/h)[1-{}^1/_4(\Delta_0/h)^2],\quad \gamma\ll 1,$$

$$\Delta\Omega=[\Delta_0{}^2N(0)/2]\left[-\left(\frac{h}{\Delta_0}\right)^2-\frac{1}{32}\left(\frac{\Delta_0}{h}\right)^2\right],$$

and the paramagnetic susceptibility $\chi_p = \mu_B^2 \partial^2\Omega/\partial h^2$ is essentially the same as the susceptibility of a normal metal (while in an ordinary superconductor, with $\Delta = \text{const}$, we would have $\chi_p = 0$). A nonzero paramagnetic susceptibility arises only in the soliton-lattice phase, and near the threshold h_c the functional dependence $M(h)$ is highly nonlinear:

$$M=-\mu_B\frac{\partial\Omega}{\partial h}=\mu_B\frac{\Delta_0\pi N(0)}{2\gamma K(\gamma)}. \tag{41}$$

The function $M(h)$ is plotted in Fig. 3.

In general, numerical analysis would be required in order to determine how the temperature affects the soliton-lattice phase. For $T \ll \Delta$, however, we would need to consider only the excited states in the outer bands, $\varepsilon^2 \gtrsim \varepsilon_+^2$, since the chemical potential lies close to the edges of these bands in a field h. Furthermore, we can restrict the analysis to states at the band edges. The thermal increment in Ω is

$$\delta\Omega(T)=-2TL\int_{-\pi}^{\pi}\frac{dq}{2\pi}\exp\left[\frac{-\tilde{\varepsilon}(q)+h}{T}\right]. \tag{42}$$

Near a band edge, we find from (38)

$$\tilde{\varepsilon}^2=\tilde{\varepsilon}_+{}^2+v^{*2}(q-q_0)^2, \tag{43}$$

where $q = q_0 = \pi\tilde{\Delta}/2\gamma K(\gamma)v_F$ corresponds to the band edge, and

$$v^*=\frac{v_F}{\tilde{\Delta}}\left(\frac{K\gamma}{K-E}\right). \tag{44}$$

Calculating $\delta\Omega(T)$, we find

$$\Omega(T,\tilde{\Delta},\gamma)=\Omega(T=0,\tilde{\Delta},\gamma)$$
$$-N(0)\tilde{\Delta}^2\left(\frac{K-E}{K\gamma}\right)\left(\frac{T}{\tilde{\Delta}}\right)^{3/2}\left(\frac{\pi}{2}\right)^{1/2}\exp\left[\frac{h-\tilde{\Delta}/\gamma}{T}\right]. \tag{45}$$

Expression (45) can be used to determine the temperature dependence of the field of the second-order transition from the uniform superconducting phase to the soliton-lattice phase (see also Ref. 12):

$$h_s=\frac{2\Delta_0}{\pi}\left[1+\left(\frac{\pi}{2}\right)^{1/2}\left(\frac{T}{\Delta_0}\right)^{3/2}\exp\left[\left(\frac{2}{\pi}-1\right)\frac{\Delta_0}{T}\right]\right]. \tag{46}$$

It follows from (45) that the specific heat in the soliton-lattice phase depends exponentially on the temperature, in contrast with the predictions for the LOFF phase in the 3D model of Refs. 6, 7, and 23.

The onset of modulation of the superconducting order parameter is accompanied by the appearance of a spin density wave in the system[6]; i.e., the spin density has an oscillatory increment in addition to its constant component. Knowing the exact solution (37), we can easily calculate the distribution of the electron spin density $s(z)$. At $T = 0$ we find

$$s(\xi)=\mu_B\sum_{\tilde{\varepsilon}^2<\tilde{\varepsilon}^2_-}\left\{\frac{\text{sn}^2(\xi,\gamma)-\text{sn}^2(\xi+K,\gamma)-2\,\text{sn}^2(\xi,\gamma)}{2[b+\text{sn}^2(\xi,\gamma)]}+1\right\}. \tag{47}$$

Using the formula for transforming elliptic functions,

$$\text{sn}^2(\xi,\gamma)+\text{sn}^2(\xi+K,\gamma)=1+(1-\gamma')\,\text{sn}^2\left(\frac{\Delta z}{v_Fk_1},k_1\right),$$

along with the expression for the state density of the spectrum of the soliton lattice,[24]

$$\mathcal{N}(\varepsilon)=\frac{1}{\pi}\frac{dq}{d\varepsilon}=\frac{1}{\pi v_F}\left(\frac{1}{\gamma^2}\frac{E}{K}-\tilde{\varepsilon}^2\right)[(\tilde{\varepsilon}^2-\tilde{\varepsilon}_+{}^2)(\tilde{\varepsilon}^2-\tilde{\varepsilon}_-{}^2)]^{-1}, \tag{48}$$

we can put (47) in the form

$$s(z)=\mu_B\frac{\Delta_0}{v_F\gamma K(\gamma)}\left\{1-\gamma^2K(\gamma)K(\gamma')\left[1-\frac{2}{\gamma^2}\left(1-\frac{E(\gamma)}{K(\gamma)}\right.\right.\right.$$
$$\left.\left.\left.+(1-\gamma')\,\text{sn}^2\left(\frac{\Delta z}{v_Fk_1},k_1\right)\right)\right]\right\}. \tag{49}$$

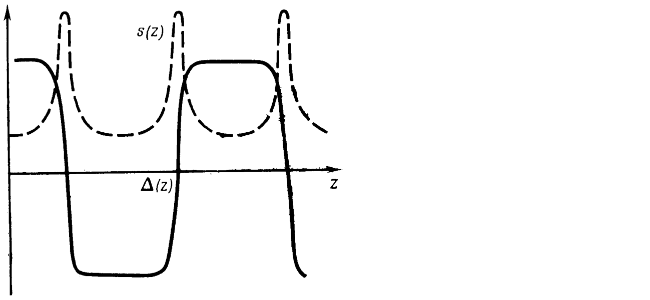

FIG. 4. Sketch of the electron spin density (dashed line) and of the order parameter in a soliton lattice.

It can be seen from this expression that the spin density reaches a maximum in regions where the quantity $\Delta^2(z)$ is small. Figure 4 is a sketch of the spin density in the soliton-lattice phase. In the region $\Delta(z) \approx 0$, the superconductivity is in a sense suppressed, and the spin angular momentum in the case $\gamma > 1$ (in the limit of a sparse soliton lattice) reaches nearly the same order of magnitude as in a normal metal. We should stress that the amplitude of the spin density wave in the LOFF phase is low, at the level of the paramagnetism of normal metals.

5. CONCLUSION

The conditions for the existence of a nonuniform LOFF superconducting phase depend strongly on the nature of the Fermi surface. These conditions are optimized in pure quasi-1D superconductors, where there is no paramagnetic limit because of the appearance of an LOFF phase. Impurity scattering leads to a finite paramagnetic limit, as was shown above. However, the mean free path of the electrons along the chains in the quasi-1D superconductors $(TMTSF)_2X$ reaches a magnitude of several hundred intermolecular distances,[1] so that the Chandrasekhar-Clogstone limit[4] can be exceeded by severalfold. A figure well above the paramagnetic limit has indeed been observed in the $(TMTSF)_2X$ family of superconductors.[1] The nature of the functional dependence $H_{c2}(T)$ at low temperatures is also unusual and reminiscent of the $H_i(T)$ curves in Fig. 1. At this point, however, we cannot assert with any confidence that the LOFF phase has been observed in quasi-1D superconductors. It is possible that a description of real quasi-1D superconductors will require going beyond the scope of the BCS theory and making use of the strong-coupling approximation. In the quasi-2D organic superconductor $(BEDT-TTF)_2AuI_2$, for example, there should be a very strong electron-phonon coupling according to tunneling measurements.[25]

It would be interesting to see a systematic study of the behavior $H_{c2}(T)$ in the quasi-1D superconductors $(TMTSF)_2X$ as the mean free path is varied in a controlled way, e.g., by means of irradiation. In this case the initial slope of the $H_{c2}(T)$ curve should increase (it is determined by the orbital effect), and the value of $H_{c2}(T)$ at low temperatures should decrease (it is determined by a paramagnetic effect).

The phase of a superconducting soliton lattice has several unusual properties (Sec. 4). In principle, the distinctive electron spectrum in this phase could be observed by IR spectroscopy, and the spin density wave could be detected from the broadening of NMR lines. The situation may be complicated by the circumstance that the LOFF state should exist in a mixed vortex state; this situation would also give rise to levels in the gap because of the cores of the vortices. It should also lead to a field modulation. The latter is not very important, since for $H \gg H_c$ this modulation would be weak, and the modulation of the field at the nuclei due to the spin density wave should play the major role.

In addition to the organic superconductors, superconductors in ultranarrow channels in strong fields might be suitable subjects for a study of the nonuniform state. Again in the case of these systems it has been found that the standard paramagnetic limit is exceed by severalfold.[26]

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[1)]Because of the degeneracy with respect to the direction of **q**, however, fluctuations may make the transition a first-order transition.[27]

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