Magnetic transition from an incommensurate phase into a commensurate intermediate one, and low-temperature spin correlations

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(Submitted 24 October 1986; resubmitted 31 December 1986)
Zh. Eksp. Teor. Fiz. **93**, 255–265 (July 1987)

The transition of an incommensurate structure into a commensurate intermediate state, induced in Heisenberg magnets by temperature and by an external magnetic field, is investigated. With rise of temperature, both the phase and the amplitude fluctuations decrease the wave vector of the modes (in crystals with inversion centers). When the field is increased, the thermal fluctuations of the phases (in contrast to the amplitude fluctuations) increase, so that the temperature interval in which a modulated structure is produced becomes narrower. In easy-plane magnets, quantum fluctuations lead to an additional decrease of the wave vector of the modes, and in an external field this effect becomes stronger. The spatiotemporal spin correlations are calculated in the region of low temperatures and are used to investigate the stability of the incommensurate states on different lattices.

1. INTRODUCTION

There are now many compounds in which a modulated magnetic structure is realized. The phase-modulation period usually varies with temperature (various experimental temperature dependences are contained in the review.[1] In systems with linear Lifshitz invariants, the change of the structure period is due to the presence of anisotropy. The magnetic transition from an incommensurate to a commensurate phase in such systems was investigated also by Dzyaloshinskiĭ[2]: with decrease of temperature, the purely sinusoidal (helical) structure changes into a soliton structure whose period increases continuously to its limiting commensurate value (see also Ref. 3). In other systems, where instability of conical points is produced by dipole interaction,[4,5] the change of the period of the modes takes place in one of two intermediate states, when a longitudinal and transverse modulation wave are simultaneously produced. The interaction of these waves also leads to an increase of their period with decrease of temperature. The resultant configurations feature both a spatial change of the phases and a spatial change of the amplitudes of the nonlinear wave.[6]

Heisenberg magnets (which include halides of transition metals[7,8]) are at present the subject of intense experimental study. Since their neutron-diffraction patterns reveal, on the contrary, an increase of the helix period with rise of temperature, and transition to an intermediate antiferromagnetic state. The point of transition between these two states can be varied in a wide temperature range either by changing the pressure (the helical structure can even vanish at a certain critical pressure[9]) and by partially replacing some nonmagnetic ions by others.[10] Resonance measurements[11] of these compounds have shown that a modulated structure is realized in crystals having an inversion center as a result of competition between exchange interactions of opposite sign.

The task of the present paper is to study the change of an incommensurate phase in a Heisenberg magnet into a commensurate intermediate state under the influence of pressure and of an external magnetic field. We confine ourselves here to the field **H** oriented along the c axis of a crystal containing an inversion center. It was already indicated earlier by Villain,[12] with classical two-dimensional (XY) spins as an example, that it is important to take into account, in temperature measurements of the wave vector of the structure, the thermal renormalizations, by the interaction of spin waves, of the exchange constants. These renormalizations are achieved even in the framework of the self-consistent harmonic approximation with a temperature dependent effective Hamiltonian. For planar spins, this approximation is equivalent to the variational procedure[13] used in the adiabatic problem of the nonlinear response in a two-dimensional isotropic nematic; on the other hand, the results obtained in Ref. 13 agree with (or are close to) the more rigorous results of Berezinskiĭ[14] for the same physical problem. We use below the Villain approach for our case of three-dimensional (Heisenberg) spins and consider the field dependence of the wave vector both at temperatures corresponding to the classical limit and in the region where quantum effects are significant. We consider also the transition point from an incommensurate phase into a commensurate one as a function of the change of the exchange interaction J' between spins located in different (neighboring) layers of the crystal lattice (on which a finite modulation period is in fact realized).

We investigate the spatiotemporal behavior of spin correlations in the limit of low temperatures. The stability of the incommensurate states depends strongly not only on the dimensionality of space, but also on the type of lattice (triangular or quadratic) the spins are located in the crystal layers, since the spectrum of the long-wave fluctuations differs substantially for states made up of these different lattices.

2. WAVE VECTOR OF STRUCTURE. EXTERNAL FIELD

Consider a Heisenberg magnet with one-ion anisotropy of the easy-plane type, placed in an external field. The Hamiltonian of such a system is given in the form

$$\mathscr{H}=-\sum_{i,j} J_{ij}\mathbf{S}_i\mathbf{S}_j+D\sum_i (S_i^z)^2-H\sum_i S_i^z, \qquad (1)$$

where the anisotropy constant is $D>0$, and the field **H** is applied along the crystal c axis which coincides with the z axis. We assume that exchange interactions of opposite signs exist in such a magnet. Let these interactions result in a ground state having a modulated structure. The wave vector **q** of such a structure at nonzero temperatures, as well as the

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dc component of the magnetization m along the field **H**, should be obtainable from the condition that the free energy F be a minimum.

We investigate first the variation of **q** and m at temperatures corresponding to the classical approximation. It is convenient next to consider the Hamiltonian (1) in terms of cononical variables, viz., the azimuthal angle $\varphi_i = \arctan(S_i^y/S_i^x)$ and the spin projection S_i^z, The variables φ_i and S_i^z are generalized coordinates and momenta, and satisfy the commutation relations[15,16]

$$\{\varphi_i, S_j^z\} = i\delta_{ij}, \tag{2}$$

where the curly brackets correspond to Poisson brackets. We express the fluctuating quantities φ_i and S_i^z in the form

$$\varphi_i = \mathbf{q}\mathbf{r}_i + \theta_i, \quad S_i^z = m + s_i^z. \tag{3}$$

Here θ_i and s_i^z are small deviations from the equilibrium values of the helix $\mathbf{q}\cdot\mathbf{r}_i$ at site i and of the dc component of the spin m along the field.

At sufficiently low temperatures (when the system is far from the transition to the paramagnetic state), the probability distributions of θ_i and s_i^z are Gaussians. The effective Hamiltonian $\tilde{\mathscr{H}}$ corresponding to this distribution can be obtained by approximating $\mathscr{H}$ in (1) by a temperature-dependent harmonic Hamiltonian[12,13]

$$\tilde{\mathscr{H}} = \sum_{\mathbf{k}} (a_\mathbf{k}|\theta_\mathbf{k}|^2 + b_\mathbf{k}|s_\mathbf{k}^z|^2), \tag{4}$$

where $a_\mathbf{k}$ and $b_\mathbf{k}$ are variational parameters determined from the conditions that F be independent of $a_\mathbf{k}$ and $b_\mathbf{k}$: $\partial F/\partial a_\mathbf{k} = 0$, $\partial F/\partial b_\mathbf{k} = 0$. The varied free energy is of the form[17]

$$F = -T\ln Z + \langle \mathscr{H} - \tilde{\mathscr{H}} \rangle, \quad Z = \int D\theta Ds^z \exp(-\beta\tilde{\mathscr{H}}). \tag{5}$$

The angle brackets $\langle \ldots \rangle$ denote here thermodynamic averaging with a distribution function determined by $\tilde{\mathscr{H}}$: $\rho = \exp(-\beta\tilde{\mathscr{H}})/\tilde{Z}$. Averaging in (5) with the distribution function ρ and assuming that $m \ll S$ and $(S_i^z)^2 \ll S^2$ we get

$$F = \Big[(m^2 - S^2)N + \sum_{\mathbf{k}} \langle |s_\mathbf{k}^z| \rangle \Big] \sum_{i-j} \tilde{J}_{ij} \cos \mathbf{q}\mathbf{r}_{ij} + m^2(D - J_0)N$$
$$+ \sum_{\mathbf{k}} (D - J_\mathbf{k})\langle |s_\mathbf{k}^z|^2 \rangle - HmN + \frac{T}{2} \sum_{\mathbf{k}} \ln(a_\mathbf{k} b_\mathbf{k}), \tag{6}$$

where N is the number of spins, $\tilde{J}_{ij} = J\langle \cos(\theta_i - \theta_j) \rangle$ are the exchange interaction constants renormalized by the thermal fluctuations, and

$$J_\mathbf{k} = \sum_{i-j} J_{ij} \exp(i\mathbf{k}\mathbf{r}_{ij}).$$

For Gaussian fluctuations we have

$$\langle \cos(\theta_i - \theta_j) \rangle = \exp[-{}^1/_2 \langle (\theta_i - \theta_j)^2 \rangle],$$

so that

$$\tilde{J}_{ij} = J_{ij} \exp\Big[-\frac{1}{N} \sum_{\mathbf{k}} \langle |\theta_\mathbf{k}|^2 \rangle (1 - e^{i\mathbf{k}\mathbf{r}_{ij}}) \Big]. \tag{7}$$

In the classical limit $T \gg \hbar\omega_\mathbf{k}$, where $\omega_\mathbf{k} = 2(a_\mathbf{k} b_\mathbf{k})^{1/2}$ are the frequencies of the Hamilton equations, we have

$$\dot{\theta}_\mathbf{k} = i\{\tilde{\mathscr{H}}, \theta_\mathbf{k}\}, \quad \dot{s}_\mathbf{k}^z = i\{\tilde{\mathscr{H}}, s_\mathbf{k}^z\}.$$

The correlators of θ and s^z in (6) and (7) are determined by the corresponding variational parameters $a_\mathbf{k}$ and $b_\mathbf{k}$:

$$\langle |\theta_\mathbf{k}|^2 \rangle = T/2a_\mathbf{k}, \quad \langle |s_\mathbf{k}^z|^2 \rangle = T/2b_\mathbf{k}. \tag{8}$$

The wave vector of the helix **q**, the dc component of the spin along the field m, and the parameters $a_\mathbf{k}$ and $b_\mathbf{k}$ are found from a system of equations obtained by minimizing (6) with respect to these variables:

$$\sum_{i-j} \tilde{J}_{ij}\mathbf{r}_{ij} \sin(\mathbf{q}\mathbf{r}_{ij}) = 0, \quad m = \frac{1}{2} \frac{H}{D + \tilde{J}_0^s - J_0}; \tag{9}$$

$$a_\mathbf{k} = (\tilde{J}_0^s - \tilde{J}_\mathbf{k}^s) \Big(S^2 - m^2 - \frac{1}{N} \sum_{\mathbf{k}} \langle |s_\mathbf{k}^z|^2 \rangle \Big), \tag{10}$$

$$b_\mathbf{k} = \tilde{J}_0^s - J_\mathbf{k} + D,$$

$$\tilde{J}_\mathbf{k}^s \equiv \sum_{i-j} \tilde{J}_{ij} \cos(\mathbf{q}\mathbf{r}_{ij}) e^{i\mathbf{k}\mathbf{r}_{ij}} = \frac{1}{2} (\tilde{J}_{\mathbf{q}+\mathbf{k}} + \tilde{J}_{\mathbf{q}-\mathbf{k}}).$$

It follows from (9) that the temperature-induced changes of m, just as those of the wave vector **q**, depend on the thermal renormalizations $\tilde{J}_{ij}$. In the classical approximation, and if $T \ll DS^2$, the contribution of the fluctuations of s^z to these renormalizations becomes small compared with the fluctuations of the phases θ, i.e., at these temperatures the Heisenberg spins are effectively planar.

We consider below systems in which change interactions of opposite sign are realized only in the basal plane, and the spins are located in this plane in sites of a triangular (or quadratic) lattice. Since furthermore the basal plane is simultaneously also the easy-magnetization plane, the wave vector of the helix in such system will lie in the spin-polarization plane (a state of the type of a plane spiral). The energy spectra of the $a_\mathbf{k}$ phase oscillations and also the spectra of the spin waves $\omega_\mathbf{k} = 2(a_\mathbf{k} b_\mathbf{k})^{1/2}$ are substantially different for structures produced on layers made up of triangular and quadratic lattices. Thus, at $T = 0$ the spectra of the phase oscillations of helical structures with wave vector $\mathbf{q}_0$ are given in the limit of small **k** by the following expressions (the x axis is directed along the elementary translation vector $(\mathbf{a} = a(1,0), k_z = 0)$:

$$a_\mathbf{k} = 3S_\perp^2 \varepsilon k_x^2, \quad \mathbf{q}_0 = (8\varepsilon/3J_1)^{1/2}(1,0) \tag{11a}$$

for triangular lattices and

$$a_\mathbf{k} = 2S_\perp^2 \varepsilon (k_x^2 + k_y^2), \quad \mathbf{q}_0 = (2\varepsilon/J_1)^{1/2}(1,1) \tag{11b}$$

for square ones. Here $S_\perp^2 = S^2 - m^2$ and $\varepsilon = 4|J_2| - J_1 > 0$, where J_1 and J_2 are the exchange integrals in the basal plane, with $J_1 > 0$ for the nearest spins and $J_2 < 0$ for spins with a common neighbor about which they are symmetric. It is seen from (11) that for structures made up of triangular lattices the long-wave spectrum in the (k_x, k_y) plane is anisotropic, whereas for square ones it is isotropic.

Far from the point of transition to the paramagnetic state, $\tilde{J}_{ij}$ takes the form

$$\tilde{J}_{ij} = J_{ij} \Big[1 - \frac{T}{2N} \sum_{\mathbf{k}} \frac{1 - \exp(i\mathbf{k}\mathbf{r}_{ij})}{a_\mathbf{k}} \Big], \tag{12}$$

where $a_\mathbf{k}$ is defined in (10). With increase of temperature

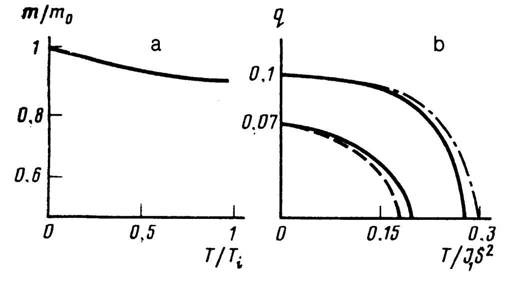

FIG. 1. a) Temperature dependence of the magnetization m along the c axis in a rhombohedral lattice (weak fields: $J_2/J_1 = -0.255$, $J'/J_1 = -0.1$, $D/J_1 = 0.05$; b) temperature dependence of structure wave vector q on triangular lattices ($D/J_1 = 0.05$): Solid lines—in field $H = 0$, $J'/J_1 = -0.1$; dashed—at $H/J_1S = 0.01$, $J'/J_1 = -0.1$; dash-dot—at $H = 0$, $J'/J_1 = -0.15$.

(and field), $\tilde{J}_{ij}$ in (12) decreases but the Fourier component

$$\mathcal{J}_0^s = \sum_{i-j} \mathcal{J}_{ij} \cos(\mathbf{q}\mathbf{r}_{ij})$$

increases, on the contrary, at $\mathbf{q} \neq 0$. This leads in turn to a decrease of the magnetization in (9) with increase of this growth of T. The dependence of m on T for rhombohedral lattices is shown in Fig. 1a, and is similar to the dependence observed in experiment.[7] It is easy to obtain for the same lattice an equation for q in the limit $T \ll DS_\perp^2$, here $a_\mathbf{k} \approx (S^2 - m^2)(\tilde{J}_0^S - J_\mathbf{k}^s)$ in (10):

$$q^2 = q_0^2 - \frac{4T}{(S^2 - m^2)J_1} F(q^2),$$
$$F(q^2) = \int \frac{(3k_x^2 + k_y^2)\, d^3k}{q^2 k_x^2 + {}^1/_4 (k_x^2 + k_y^2)^2 + {}^8/_{27} (|J'|/J_1) k_z^2}. \quad (13)$$

Here $q_0 \ll 1$ and J' is the exchange interaction constant between the nearest spins of the neighboring layers (triangular lattices). The external-field-induced decrease of the spin projection in the basal plane, $S_\perp = (S^2 - m^2)^{1/2}$ leads to an increase of the phase fluctuations characterized by the last term of Eq. (13). As a result, the wave vector of the helix decreases with rise of temperature more strongly than in the case when no field is applied to the system. In a second-order phase transition, q vanishes at the point $T_i = (S^2 - m^2) J_1 q_0^2 / 4F(0)$. The contribution due to allowance for the nonlinear terms in the Hamiltonian (4) is small if $TF^2(q^2)/J_1 \ll 1$. Thus, at the transition point T_i the influence of the fluctuation is also small when the inequality $q_0^2(F(0) \ll 1$ is satisfied. At small values of J', the principal term of the integral F at $q = 0$ is proportional to $\ln(J_1/|J'|)$, so that near the point of transition from the incommensurate to the commensurate phase the approximation of the varied Hamiltonian in harmonic form is no longer valid if the condition $\varepsilon \ln(J_1/|J'|) \ll J_1$ is not met. The fluctuations can in this case be small, as before, only in the region of sufficiently low T, where $T \ll T_i$.

We consider now the change of q in the temperature interval $k_B T \lesssim \hbar\omega_\mathbf{k}$, where the quantum spin fluctuations are significant. In this case the canonical variables φ_i and s_i^z are Hermitian operators whose commutation relations are analogous to the commutation relations (2). The effective Hamiltonian $\tilde{\mathcal{H}}$ is easily diagonalized after changing from $\theta_\mathbf{k}$ and $s_\mathbf{k}^z$ in (4) to the boson operators

$$c_\mathbf{k} = \frac{1}{\sqrt{2}} \left[\left(\frac{a_\mathbf{k}}{b_\mathbf{k}} \right)^{1/4} s_\mathbf{k}^z + i \left(\frac{b_\mathbf{k}}{a_\mathbf{k}} \right)^{1/4} \theta_\mathbf{k} \right],$$
$$c_\mathbf{k}^+ = \frac{1}{\sqrt{2}} \left[\left(\frac{a_\mathbf{k}}{b_\mathbf{k}} \right)^{1/4} s_{-\mathbf{k}}^z - i \left(\frac{b_\mathbf{k}}{a_\mathbf{k}} \right)^{1/4} \theta_{-\mathbf{k}} \right]. \quad (14)$$

Calculating the free energy with the density matrix

$$\rho = \exp(-\beta\tilde{\mathcal{H}}) / \mathrm{Sp} \exp(-\beta\tilde{\mathcal{H}})$$

and again minimizing with respect to the wave vector $\mathbf{q}$, the dc spin component m, and the parameters $a_\mathbf{k}$ and $b_\mathbf{k}$, we obtain for these quantities the same system of equations (9) and (10) with renormalized exchange integrals in which, however, the correlators of the fluctuating quantities are now specified with allowance for the zero-point oscillations ($\hbar = 1$):

$$\langle |\theta_\mathbf{k}|^2 \rangle = \frac{\omega_\mathbf{k}}{4a_\mathbf{k}} \operatorname{cth} \frac{\omega_\mathbf{k}}{2T}, \qquad \langle |s_\mathbf{k}^z|^2 \rangle = \frac{\omega_\mathbf{k}}{4b_\mathbf{k}} \operatorname{cth} \frac{\omega_\mathbf{k}}{2T}. \quad (15)$$

In the quantum region, in contrast to (8), these relations are expressed in terms of the spectrum of the spin waves $\omega_\mathbf{k}$ and do not depend individually on $a_\mathbf{k}$ and $b_\mathbf{k}$. As a result, the wave vector $\mathbf{q}$ becomes explicitly dependent on the anisotropy constant D. Figure 1b shows a plot of q and T for q_0 equal to 0.07 and 0.1. The curves obtained take into account the thermal and quantum fluctuations of both θ and s^z. With increase of H, the temperature region of the helical structure decreases, as can be seen from a comparison of the lower curves of the figure for $q_0 = 0.07$. With increase of interplanar interaction $|J'|$, however, the region of existence of an incommensurate phase, conversely, increases and this can be seen in turn from a comparison of the upper curves for $q_0 = 0.1$.

At $T = 0$ the renormalization of the exchange integrals in (7) is due only to the zero-order approximations ($\langle |\theta_\mathbf{k}|^2 \rangle = (a_\mathbf{k}/b_\mathbf{k})^{1/2}/2$), so that for the case of triangular lattices the value of q will be determined from the equation

$$q^2 = q_0^2 - \left[\frac{6D}{(S^2 - m^2)J_1} \right]^{1/2} \Phi(q^2),$$
$$\Phi(q^2) = \int \frac{(3k_x^2 + k_y^2)\, d^3k}{[q^2 k_x^2 + {}^1/_4 (k_x^2 + k_y^2)^2 + {}^8/_{27} (|J'|/J_1) k_z^2]^{1/2}}. \quad (16)$$

This leads to the following: the quantum fluctuations decrease the value of $q = q_0 \equiv (8\varepsilon/3J_1)^{1/2}$, and this value decreases both with increase of D and with increase of the field H. If the wave vector of the helix is small enough, the sinusoidal state vanishes even in weak magnetic fields.

3. SPATIOTEMPORAL CORRELATIONS

We consider in this section the behavior of the pair correlation function in the investigated system. We confine ourselves to calculation of the most important transverse part connected with fluctuations without a helix in the easy-magnetization plane

$$G_{ij}(t) = \langle \cos(\varphi_i - \varphi_j(t)) \rangle \propto \langle S_i^x S_j^x(t) \rangle + \langle S_i^y S_j^y(t) \rangle. \quad (17)$$

Substituting in (17) the values from (3) for the phases at the sites and using again the properties of Gaussian fluctuations, we get

$$G_{ij}(t) = \exp\left\{ -\frac{1}{N} \sum_\mathbf{k} [\langle |\theta_\mathbf{k}|^2 \rangle - \langle \theta_\mathbf{k}^* \theta_\mathbf{k}(t) \rangle \cos \mathbf{k}\mathbf{r}_{ij}] \right\} \cos \mathbf{q}\mathbf{r}_{ij}, \quad (18)$$

where the expression for the unequal-time correlator $\langle \theta_{\mathbf{k}}^{*}\theta_{\mathbf{k}}(t)\rangle$ is obtained from the equations of motion with the Hamiltonian (4):

$$\langle \theta_{\mathbf{k}}^{*}\theta_{\mathbf{k}}(t)\rangle=\left(\frac{b_{\mathbf{k}}}{a_{\mathbf{k}}}\right)^{1/2}\left[\frac{1}{2}\exp(i\omega_{\mathbf{k}}t)+\frac{\cos\omega_{\mathbf{k}}t}{\exp(\beta\omega_{\mathbf{k}})-1}\right], \quad (19)$$

so that with allowance for the quantum fluctuations we have for $G_{ij}(t)$

$$G_{ij}(t)=g_{ij}(t)\gamma_{ij}(t)\cos\mathbf{qr}_{ij}, \quad (20)$$

$$g_{ij}(t)=\exp\left\{-\frac{1}{2N}\sum_{\mathbf{k}}\left(\frac{b_{\mathbf{k}}}{a_{\mathbf{k}}}\right)^{1/2}[1-\exp(i\omega_{\mathbf{k}}t)\cos\mathbf{kr}_{ij}]\right\}, \quad (21a)$$

$$\gamma_{ij}(t)=\exp\left[-\frac{1}{N}\sum_{\mathbf{k}}\left(\frac{b_{\mathbf{k}}}{a_{\mathbf{k}}}\right)^{1/2}\frac{1-\cos\omega_{\mathbf{k}}t\cos\mathbf{kr}_{ij}}{\exp(\beta\omega_{\mathbf{k}})-1}\right]. \quad (21b)$$

We neglect hereafter in the temperature dependences of $a_{\mathbf{k}}$ and $b_{\mathbf{k}}$; this is valid, naturally, only in the region of very low $T(T\ll\varepsilon)$. We consider first the properties of $G_{ij}(t)$ for the simplest case of one-dimensional systems with competing interaction. Then, retaining in (10) the terms quadratic in k, we get $a_k\approx 2S^2\varepsilon k^2$, $b_k\approx D$ $(H=0)$. In the ground state $\gamma_{ij}(t)=1$, and the behavior of $G_{ij}(t)$ in (20) is set by the factor $g_{ij}(t)$. Over small scales of r_{ij} and t the mean squared deformations of the phases, determined by double the exponent of the reciprocal function $g_{ij}^{-1}(t)$, increase quadratically with distance and with time:

$$\langle[\theta_i-\theta_j(t)]^2\rangle=(r_{ij}/l_0)^2+(t/\tau_0)^2, \quad (22)$$

where

$$l_0=S^{1/2}(J_1/D)^{1/4},\quad \tau_0=(3/S)^{1/2}D^{-3/4}J^{-1/4}.$$

On the other hand, over large spatiotemporal scales the deformations due to the quantum fluctuations diverge logarithmically:

$$\langle(\theta_i-\theta_j(t))^2\rangle\approx\alpha\begin{cases}\ln r_{ij}, & r_{ij}\geqslant ct\\ \ln ct, & r_{ij}\leqslant ct\end{cases}. \quad (23)$$

Here $\alpha=(2\pi S)^{-1}(D/2\varepsilon)^{1/2}$ and $c=2S(2\varepsilon D)^{1/2}$ is the spin-wave velocity. As a result we have for the static correlation function

$$G_{ij}=r_{ij}^{-\alpha}\cos(qr_{ij}),\quad q=(2\varepsilon/J_1)^{1/2}, \quad (24)$$

i.e., G_{ij} takes a form typical of states of the floating-phase[18] type.

At nonzero temperatures we have $\gamma_{ij}(t)<1$. Over small scales r_{ij} and t, the thermal fluctuations, just as the quantum ones, increase quadratically, so that $\gamma_{ij}(t)$ takes on at these scales a Gaussian form:

$$\gamma_{ij}(t)=\exp[-(r_{ij}/l_1)^2-(t/\tau_1)^2] \quad (25)$$

with characteristic lengths at times

$$l_1=4\pi^{1/2}S^{3/2}(2D)^{1/4}\varepsilon^{3/4}T^{-1},\quad \tau_1=2\pi^{1/2}S^{-1/2}\varepsilon^{1/4}(2D)^{-1/4}T^{-1}. \quad (26)$$

Over large spatiotemporal scales, the transverse correlations fall off exponentially. Neglecting in the long-wave limit the final quantum corrections, we obtain from (21b)

$$\gamma_{ij}(t)\approx\begin{cases}\exp(-r_{ij}/l), & r_{ij}\geqslant ct\\ \exp(-t/\tau), & r_{ij}\leqslant ct\end{cases}, \quad (27)$$

where $l=8S^2\varepsilon/T$ and $\tau=l/c=2S(2\varepsilon/D)^{1/2}T^{-1}$. The fluctuations that increase linearly with distance destroy completely the long-range order at $r_{ij}\gtrsim l$. Nonetheless, at sufficiently low temperatures, when

$$T\ll\frac{4}{\pi}S^2\varepsilon\left(\frac{2\varepsilon}{J_1}\right)^{1/2},$$

a helical short-range order is possible at times $t<r_{ij}/c$, since its length l is much longer than the wave period $2\pi/q=\pi(2J_1/\varepsilon)^{1/2}$ of the structure.

For exponentially decreasing correlations (27), the Fourier component of the function $G_{ij}(t)$ takes in the classical limit the form

$$G(k,\omega)=f(k+q,\omega)+f(k-q,\omega), \quad (28)$$

where

$$f(k,\omega)=\frac{4c\Omega^2}{(\omega^2+\omega_k^2+\Omega^2)^2-4\omega_k^2\omega^2},\quad \omega_k=ck$$

$(\Omega=\tau^{-1}=c\varkappa,\ \varkappa=l^{-1}=T/8S^2\varepsilon)$. At low values of $|k\pm q|(<\varkappa)$ the maximum of the function $G(k,\omega)$, which is even in ω, has zero frequency. At $q>\varkappa$, however, i.e., when a helical short-range order exists, the maximum of the spectrum $G(k,\omega)$ is located at frequencies that differ from zero. Thus, at $k=0$ there is one maximum at the frequency $\omega=(\omega_q^2-\Omega^2)^{1/2}$ (curve 1 of Fig. 2). With increase of k, this maximum splits into two peaks, one of which shifts towards the wings of the $G(k,\omega)$ spectrum, and the other to the region of small ω (curve 2). At $q-|k|\leqslant\varkappa$ there remains one peak (curves 3 and 4) with nonzero frequency $(\omega=(\omega_{q+|k|}^2-\Omega^2)^{1/2})$. Further increase of k leads again to the appearance of a second peak (curve 5) at the frequency $\omega=(\omega_{|k|-q}^2-\Omega^2)^{1/2}$, if the condition $|\varkappa|-q>\kappa$ is met. Thus, in structures with helical short-range order, the appearance in the spectrum of spin waves propagating counter to the direction of the wave vector of q of the helix is possible (in contrast to waves propagating along q) only at $|q-k|>\varkappa$. At $q<\varkappa$ (there is no helical short-wave order) there exists in the spectrum not more than one peak; in the limit $q\to 0$ only spin waves with $|k|>\varkappa$ can be observed.[19]

In two-dimensional spin systems, the spatiotemporal correlations depend substantially on the type of the plane

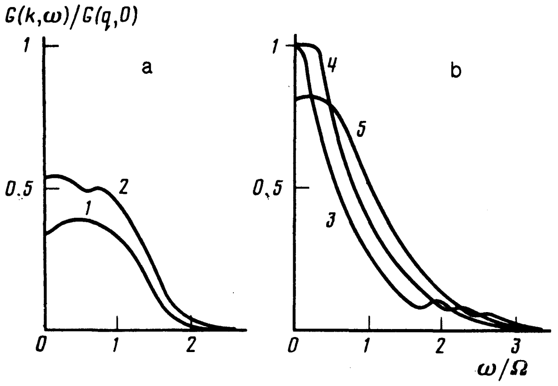

FIG 2. Fourier component of correlation function $G_{ij}(t)$ for structures with helical short-range order $(q>\varkappa)$; a) for wave vectors $|k|<q$; curves 1 and 2 correspond to $k=0$ and $k=0.1$; b) for $|k|>q$. Curves 3, 4, and 5 were constructed respectively for $k=q$, $k=q+\varkappa$, and $k=q+1.2\varkappa$; the last two curves are drawn in a different scale ($G(q+\varkappa,0)$ is taken to be unity).

lattices. We consider here the correlation properties of such systems using as examples triangular and square lattices, for which $a_{\mathbf{k}}$ (in the approximation quadratic in $\mathbf{k}$) and $\mathbf{q} = \mathbf{q}_0$ are given by (11) (in this case, just as for one-dimensional systems, $b_{\mathbf{k}} \approx D$). For the case of triangular lattices $\gamma_{ij}(t)$ in large spatiotemporal scales is independent of t and of one of the coordinates y_{ij}:

$$\gamma_{ij}(t) = \exp(-x_{ij}/L), \quad L = 6\pi^2 S^2 \varepsilon / T. \tag{29}$$

Thus, mean square phase deformations that are constant in time (and are characterized by the argument of the exponential in (29) will increase linearly along a direction corresponding to the orientation of the wave vector $\mathbf{q} = q(1,0)$: at $x_{ij} > L$ the helical long-range order is completely destroyed as before. The Fourier component is $G(\mathbf{k}) = \widetilde{G}(k_x)\delta(k_y)$, in which the function $\widetilde{G}(k_x)$ is described by two Lorentzian lines whose maxima are located at $k_x = \pm q$.

At the same time, the calculation of $\gamma_{ij}(t)$ for the case of square lattices using Eq. (21b) yields a power-law dependence of the correlation function at large values of r_{ij} and t (Ref. 20):

$$\gamma_{ij}(t) = \begin{cases} (k_c r_{ij})^{-\alpha}, & r_{ij} \geqslant ct \\ [ck_c t + ((ck_c t)^2 - (k_c r_{ij})^2)^{1/2}]^{-\alpha}, & r_{ij} \leqslant ct \end{cases}, \tag{30}$$

where the cutoff parameter k_c and the exponent α are of the form

$$k_c = \min(T/c, \pi), \quad \alpha = T/8\pi S^2 \varepsilon$$

(the velocity c of the spin wave on a square lattice is equal to the value of c determined in (23) for one-dimensional spin waves). The static function $G(\mathbf{k}, t=0)$ diverges at the points $\mathbf{k} = \pm \mathbf{q}$ if $\alpha < 2$: $G(\mathbf{k}, t=0) \sim |\mathbf{k} \mp \mathbf{q}|^{\alpha - 2}$. For t different from zero, $\gamma_{ij}(t)$ in (30) increases with increase of r_{ij} until $r_{ij} < ct$. At $r_{in} = ct$ expression (30) has a square-root singularity, which leads at values $\alpha \leqslant 1$ to divergence of the Fourier components of $G(\mathbf{k}, \omega)$ at frequencies $\omega = \omega_{\mathbf{k} \pm \mathbf{q}}$ and $\omega = -\omega_{\mathbf{k} \pm \mathbf{q}}$: at $\alpha < 1$ these frequencies are branch points of the function $G(\mathbf{k}, \omega) \sim |\omega \pm \omega_{\mathbf{k} \pm \mathbf{q}}|^{\alpha - 1}$, and $\alpha = 1$ the function $G(\mathbf{k}, \omega)$ diverges logarithmically.

We note here that besides the linear excitations described by the harmonic Hamiltonian, it is necessary, generaly speaking, to take into account also the contribution of the topological excitations. These can exist even in Heisenberg two-dimensional systems characterized in the ground state by a wave vector $\mathbf{q}$ of the structure. In this case, just as in a three-sublattice Heisenberg magnet,[21] point defects (dislocations[18]) are produced, denoted only by the vortex numbers zero or unity (Z_2 structures). At low temperatures, however, the density of the Z_2 vortical configurations is low. Thus, for example, for a two-dimensional three-sublattice magnet with Heisenberg spins, Monte Caro calculations[21] show that the spin density increases abruptly only near the point of transition into the paraphase. For the XY model with competing interaction, as in the ANNI model,[22] linear defects (domain walls) can be formed in addition to point defects. Their presence in a planar system is due to the appearance of discrete symmetry together with the continuous one, viz., two degenerate spiral configurations with opposite orientations of the field vector q can no longer be continuously transformed into each other. Nontheless, in contrast to the ANNI model, where the ground state is known to be strongly degenerate,[23,24] the nonlinear excitations in the planar system cannot lower strongly the temperature of the transition into the paraphase. In the region of sufficiently low T it is therefore possible to confine oneself, as before, to the linear approximation.

Exchange interaction between spins located in different layers of square lattices leads to the onset of long-range order. On the other hand, in systems made up of layers of triangular lattices, a long-range order is realized only in definite directions, to which there correspond in three-dimensional space the planes $C_1 x_{ij} + C_2 y_{ij} = 1$ and $C_3 y_{ij} + C_4 z_{ij} = 1$ (C_i are arbitrary constants). Actually, at finite values (of any sign) of the exchange integral J' along the z axis, the spatial correlations γ_{ij} are described, neglecting quantum corrections in the following manner (the integrals in Eq. (21b) are calculated in the long-wave limit with $a_{\mathbf{k}} = S^2(3\varepsilon k_x^2 + |J'| k_z^2)$):

$$\gamma_{ij} = \begin{cases} x_{ij}^{-\delta}, & x_{ij} \leqslant z_{ij}/l_{\parallel} \\ (z_{ij}/l_{\parallel})^{-\delta}, & x_{ij} \geqslant z_{ij}/l_{\parallel} \end{cases}$$

where we have for the dimensionless parameters δ and $l_{\parallel}$

$$\delta = \frac{T}{4\pi S^2 (3\varepsilon |J'|)^{1/2}}, \quad l_{\parallel} = \left(\frac{|J'|}{3\varepsilon} \right)^{1/2}.$$

It is assumed in Eqs. (31) that $z_{ij} \gtrsim 1$, and that $x_{ij} \gtrsim l_{\parallel}^{-1}$ (if $x_{ij} < l_{\parallel}^{-1}$, then γ_{ij} is given as before by the exponential expression (29)). It follows from (31) that the spatial correlations between spins that do not lie in the planes indicated above fall off in power-law fashion. In the particular case $z_{ij} = \text{const}$ the correlations are finite, so that a long-range helical order exists between the basal planes located at the indicated distance (although, of course, this order becomes weaker with increase of the fixed values of z_{ij}).

CONCLUSION

Thus, a temperature- and field-induced transformation of an incommensurate structure (such as a flat spiral) into a commensurate intermediate state was considered for easy-plane Heisenberg magnets whose crystal lattice contains an inversion center. The spatiotemporal dependences of the spin correlations were investigated in the low-temperature limit.

It was shown that the thermal fluctuations of the helix phases increase in an external field applied along the c axis, and therefore the wave vector of the modes decreases additionally (in comparison with $H = 0$), while the transition point between the ordered states shifts towards lower T. At the same time, the increase of the exchange interaction J' between spins located in different layers of the lattice expands the temperature range in which the incommensurate phase can exist. Next, owing to thermal renormalizations of the exchange constants, the susceptibility along the c axis decreases smoothly with increase of temperature, something that coincides with its analogous behavior observed in experiment.[7] On the other hand, at temperatures close to absolute zero, the zero-point oscillations of the phases become substantial and decrease, together with the thermal oscillations, the wave vector $\mathbf{q}$ of the structure. In an external field this effect (the quantum shortening of $\mathbf{q}$) becomes stronger and can even destroy the helical structure at $T = 0$ if the wave vector $\mathbf{q}$ is small enough.

The stability of structures with helical order depends strongly not only on the dimensionality of space, but also on the type of lattices on which interactions of opposite signs are realized. In the one-dimensional case, at sufficiently low temperature, a helical short-range order is possible at times $t < r_{ij}/c$ is possible; for structures with such an order, spin waves can appear in the spectrum of the correlation function (in ω, **k** space) for wave vectors **k** that are not too close to the wave vector **q** of the helix.

Substantial differences between the correlation properties appear already in the two-dimensional case: on triangular lattices the spin correlations are independent of time and fall off exponentially with distance, whereas on quadratic ones they decrease at a power-law rate—at times $t < r_{ij}/c$ only a spatial growth of the mean squared phase deformations takes place, while at $t > r_{ij}/c$ the deformations increase with time. In the three-dimensional case the spatial correlations are finite in spin systems made up of layers of quadratic lattices. If, however, the layers are triangular lattices, long-range order exists only in certain directions. While there is no long-range order in the other directions, the spatial correlations fall of nevertheless slowly—in power-law fashion.

The author is grateful to V. a. Ignatchenko, V. V. Val'kov, V. I. Ponomarev, A. F. Sadreev, and I. S. Sandalov for helpful discussions and critical remarks.

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Translated by J. G. Adashko