Rotation of a superfluid by container-wall vibrations (the vibrorotation effect)

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We propose a new method of rotating a superfluid while the vessel containing the liquid remains stationary. The rotation is induced by container-wall transverse vibrations in the form of a traveling wave propagating in the direction of rotation of the liquid. Effects connected with the existence, in the rotating superfluid, of a vortex structure—vortex pinning at the bottom of the vessel, flexure of the vortex lines, lag of the superfluid component behind the normal component—are investigated. The conditions are found under which these effects, which impair the rotation of the liquid "as a whole," are weak, and not stronger than the effect of viscous sticking of the normal component to the bottom of the vessel.

INTRODUCTION

In numerous experimental investigations of the properties of a rotating superfluid, the vessel containing the liquid was set in rotation. The aim of the present paper is to discuss another superfluid-spinning technique in which the vessel remains stationary. And precisely this can be achieved by generating a transverse wave running along the lateral surface of the vessel. In the process, as we shall see, in a definite range of values of the physical parameters of the problem, such as the vibration frequency, the viscosity of the liquid, the vessel dimensions, etc., the liquid will, on the average, execute "rigid-body" rotation.

In the case of a normal liquid such a spinning technique has been investigated theoretically and experimentally.[1,2]

In a superfluid the normal component is viscous and, like a normal liquid, experiences the twisting action of the walls. It is well known that, at normal-component angular velocities much higher than some critical velocity of the order of $\varkappa/R^2$ ($\varkappa$ is the vortex-circulation quantum and R is the vessel radius), there are formed in the interior of the liquid a large number $N = 2\pi\Omega R^2/\varkappa$ of quantized vortices simulating rigid-body rotation with angular velocity Ω. Using the proposed method, we can achieve angular velocities $\Omega \gtrsim 0.1$ rad/sec; the number of vortices is then high, and the vortex spacing $b \sim (\varkappa/\Omega)^{1/2}$ is small compared to the vessel dimensions, a circumstance which allows us to use for the description of the liquid motion continuum hydrodynamics[3] in which averaging over scales greater than the vortex spacing b has been carried out.

In our method of spinning a liquid the bottom of the vessel is stationary, and the vortices moving relative to it will interact with the irregularities of the solid surface, resulting in the appearance of a pinning force. This force, like the force responsible for the viscous sticking of vortices to the bottom of the vessel, will impede the rotation, impairing, generally speaking, its "rigid-body nature," i.e., the independence of the angular velocity of the distance from the axis of the vessel.

But, as will be shown below, by choosing the ratio of the vessel height to vessel radius to be sufficiently large, we can make the impairment of the rigid-body nature of the rotation insignificant.

Furthermore, we wish to emphasize that there exists an experimental setup in which it is generally possible to eliminate the influence of the bottom. For example, this can be achived by employing vessels in the form of a torus or the letter U.

In the first section we consider the effect whereby the vibrating walls drag the normal component of a liquid in a cylindrical vessel. The results of this section bear a direct analogy to the results obtained for a normal liquid in cylindrical geometry.[1] They also bear some analogy to the results obtained for a normal liquid in planar geometry.[4]

In the second section we analyze the motion of the superfluid component. We shall estimate the characteristic vessel height below which the pinning leads to appreciable impairment of the rigid-body character of the liquid motion. It turns out that, for a broad range of temperatures, this height does not exceed the vessel radius, so that the interaction with the bottom on account of pinning is not stronger than the effect of viscous friction on the bottom of the vessel.

1. NORMAL-COMPONENT ROTATION INDUCED BY WALL VIBRATIONS

Let the walls of a cylindrical vessel undergo periodic radial displacements with velocity

$$V(\varphi, t) = \mathrm{Re}\{V_k \exp i(k\varphi - \omega t)\}. \tag{1}$$

The operation Re will henceforth be implied in linear expressions.

As we shall see, in the first approximation in the vibration amplitude of the vessel walls, the wall motion produces a velocity field that automatically satisfies the incompressibility condition div $\mathbf{v} = 0$. In the second approximation we shall be interested only in the averaged azimuthal motion of the liquid. Therefore, we can use the Navier-Stokes equation for an incompressible liquid:

$$\frac{\partial \mathbf{v}_n}{\partial t} + (\mathbf{v}_n\nabla)\mathbf{v}_n = \frac{\nabla P}{\rho_n} + \nu_n \Delta \mathbf{v}_n, \tag{2}$$

where ρ_n is the density, ν_n is the kinematic viscosity, and P is the pressure. Following Ref. 2, we assume that the vessel height L is much greater than R, and ignore the effect of the bottom. In this approximation the problem is homogeneous in the coordinate z, which is oriented parallel to the axis of rotation.

In the cylindrical system of coordinates the components of the velocity can be given in terms of the stream function $\Psi(r,\varphi)$ as follows:

$$(v_n)_r=\frac{1}{r}\frac{\partial\Psi}{\partial\varphi},\quad (v_n)_\varphi=-\frac{\partial\Psi}{\partial r}. \tag{3}$$

Discarding the nonlinear term $(v_n\nabla)v_n$ in (2), we obtain

$$\frac{\partial}{\partial t}\nabla^2\Psi-\nu_n\nabla^4\Psi=0, \tag{4}$$

where

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}$$

is the Laplace operator. That solution to this equation which satisfies the linear boundary conditions

$$(v_n)_r|_{r=R}=V_k\exp[i(k\varphi-\omega t)],\quad (v_n)_\varphi|_{r=R}=0, \tag{5}$$

for the velocities has the form

$$\Psi_k=\frac{V_kR}{ik(1-J_k(\alpha R)k/J_k'(\alpha R)\alpha R)}\left[\left(\frac{r}{R}\right)^k-\frac{kJ_k(\alpha r)}{\alpha RJ_k'(\alpha R)}\right]\times\exp i(k\varphi-\omega t), \tag{6}$$

where $J_k(x)$ is a Bessel function of order k (k is assumed to be positive),

$$\alpha=\frac{1-i}{2^{1/2}}\left(\frac{\omega}{\nu_n}\right)^{1/2}. \tag{7}$$

The stationary rotation of the liquid is an effect of second order in the vibration amplitude V_k. To determine it, we must consider the Navier-Stokes equation averaged over the vibration period. The averaging leads to the vanishing of the radial velocity (this follows from the equation of continuity), and to the following equation for the azimuthal velocity $(\bar{v}_n)_\varphi$:

$$\nu_n\left[\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\frac{1}{r^2}\right](\bar{v}_n)_\varphi=U(r). \tag{8}$$

Here $U(r)$ is determined by averaging the nonlinear term $[(v_n\nabla)v_n]_\varphi$, where we use for v_n the first approximation to the velocity (Eq. (6)]:

$$U(r)=\frac{k}{2r}\operatorname{Im}[\Psi_k\nabla^2\Psi_k^*]$$
$$=-\frac{V_k^2R}{2r|1-J_k(\alpha R)k/J_k'(\alpha R)\alpha R|^2}\left(\frac{r}{R}\right)^k\operatorname{Im}\frac{J_k(\alpha r)}{J_k'(\alpha R)}. \tag{9}$$

To determine the boundary condition for Eq. (8), we must take account of the fact that the condition for the sticking of the viscous liquid is set at a vibrating wall, i.e.,

$$(v_n)_\varphi(R+\delta R)=0, \tag{10}$$

where

$$\delta R=-\frac{V_k}{i\omega}\exp i(k\varphi-\omega t).$$

Expanding (10) in a series, we find that, in second order,

$$(\bar{v}_n)_\varphi=-\overline{\delta R\frac{\partial(v_n)_\varphi}{\partial r}}\Big|_{r=R}$$
$$=\frac{V_k}{2\omega}I_m\left[\frac{\partial^2\Psi_k(R)}{\partial r^2}\exp\{-i(k\varphi-\omega t)\}\right]. \tag{11}$$

The solution to Eq. (8) with the boundary condition (11) has the form

$$(v_n)_\varphi(r)=v_0(r)+[(v_n)_\varphi(R)-v_0(R)]r/R, \tag{12}$$

where

$$v_0(r)=\frac{1}{\nu_nr}\int_0^r r'\,dr'\int_0^{r'}dr''\,U(r'') \tag{13}$$

(here and below the averaging sign is omitted).

These expressions are simpler in the case $|\alpha|R\gg1$, which is the case of greatest interest to us. Then

$$v_0(r)=-\frac{V_k^2}{2(2\nu_n\omega)^{1/2}}\exp[2^{-1/2}|\alpha|(r-R)]\times\cos\left[\frac{|\alpha|}{2^{1/2}}(r-R)+\frac{\pi}{4}\right], \tag{14}$$

$$(v_n)_\varphi(R)=\frac{V_k^2}{2^{3/2}(\nu_n\omega)^{1/2}}. \tag{15}$$

We see that the particular solution $v_0(r)$ decreases exponentially in the viscous interior, so that in the main volume the liquid rotates as a whole with angular velocity

$$\Omega=\frac{(v_n)_\varphi(R)-v_0(R)}{R}=\frac{V_k^2}{R(2\nu_n\omega)^{1/2}}. \tag{16}$$

For the iterative scheme used by us to solve the Navier-Stokes equation to be applicable, it is necessary that the wall vibration amplitude $r_k=V_k/\omega$, which guarantees an angular velocity of Ω according to the formula (16):

$$r_k=(\Omega R)^{1/2}(2\nu_n)^{1/4}\omega^{-3/4},$$

be much smaller than the viscous length $|\alpha|^{-1}$. This means that, for an angular velocity of $\Omega=0.1$ rad/sec to be achieved in a vessel of radius $R=1$ cm, the vibration frequency ω should be higher than 10^2 rad/sec in the case of He^4 ($\nu_n\sim10^{-4}$ cm^2/sec) and 0.1 rad/sec in the case of He^3 ($\nu_n\sim0.1$ cm^2/sec).

To discuss the applicability of the present rotation technique at extremely low temperatures ~1 mK, i.e., in the case of He^3, we must estimate the effect of the warmup of the liquid as a result of the viscous friction. The heat release occurs in a boundary layer of thickness $|\alpha|^{-1}$. Using (16), we obtain for its magnitude per unit vessel height the expression

$$\dot{Q}=\int\rho_n\nu_n\mathbf{v}_n\Delta\mathbf{v}_n\,dr=\rho_n\nu_n\omega\Omega R/2. \tag{17}$$

For $\rho_n\nu_n\sim0.04$ P (which corresponds to 2.4 mK for He^3), $\omega=1$ rad/sec, $\Omega=0.1$ rad/sec, and $\dot{Q}=4\times10^{-9}$ W/cm. It is known that the transfer of heat from the superfluid to the walls of the vessel is hindered (because of the Kapitza discontinuity). For the above-estimated heat output to be removable in the case of a temperature jump $\delta T\sim1$ mK, the thermal resistance of the He^3–solid state transition should not exceed

$$R_k=\delta T\cdot2\pi R/\dot{Q}=1\ \mathrm{K\cdot cm^2\cdot sec/erg}.$$

This value is, apparently, attainable for the wall materials used at present.

The applicability of our hydrodynamic theory is restricted also by the inequality $|\alpha|l\ll1$, where l is the mean free path of the normal excitations. As the temperature is lowered, the viscous depth $|\alpha|^{-1}$ becomes comparable to l, and the above-expounded quantitative theory becomes inapplicable, although, of course, qualitatively, the phenomenon can survive as long as the quantity l remains small compared to the radius of the vessel.

2. ROTATION OF A SUPERFLUID

We shall use the equations of the two-velocity hydrodynamics of a rotating incompressible superfluid

$$\frac{\partial \mathbf{v}_s}{\partial t}=[\mathbf{v}_L, \operatorname{rot} \mathbf{v}_s]-\nabla\left(\mu+\frac{v_s^2}{2}\right), \tag{18}$$

$$\frac{\partial \mathbf{v}_n}{\partial t}+(\mathbf{v}_n\nabla)\mathbf{v}_n=\nu_n\Delta\mathbf{v}_n+\frac{\rho_s}{\rho_n}[\mathbf{v}_L-\mathbf{v}_{sl}, \operatorname{rot}\mathbf{v}_s]-\frac{1}{\rho_n}\nabla\left[P-\rho_s\left(\mu+\frac{v_s^2}{2}\right)\right], \tag{19}$$

$$\mathbf{v}_L=\mathbf{v}_{sl}+\beta'(\mathbf{v}_n-\mathbf{v}_{sl})+\beta[\mathbf{s}, \mathbf{v}_n-\mathbf{v}_{sl}]. \tag{20}$$

Here the force of the friction between the normal and superfluid components is determined by the parameters $\beta=\rho_n B/2\rho$ and $\beta'=\rho_n B'/2\rho$, $\mathbf{s}=\operatorname{curl}\mathbf{v}_s/|\operatorname{curl}\mathbf{v}_s|$ is the unit vector tangential to the vortices,

$$\mathbf{v}_{sl}=\mathbf{v}_s+\nu_s \operatorname{rot}\mathbf{s} \tag{21}$$

is the local superfluid velocity in the neighborhood of a singular vortex line and differs from the mean superfluid velocity because of the bending of the vortices, and the parameter

$$\nu_s=\frac{\varkappa}{4\pi}\ln\frac{b}{r_0} \tag{22}$$

determines the tension in a vortex.

The above equations of the hydrodynamics of a rotating superfluid are suitable also for the description of the superfluid phases of ^{3}He if the symmetry of the vortex structures arising in them is sufficiently high.[5,6] In the case of a lower symmetry of the vortex structure Eqs. (19) and (20) are somewhat more complicated, but these complications are not a basic feature of the effect in question, and therefore the results obtained below can qualitatively apply also to superfluid ^{3}He. But in the case of the nonsingular vortex structure in ^{3}He-A the expression (22) relating ν_s to the circulation quantum no longer contains a logarithm, and v_{sl} cannot be interpreted as a local velocity.

Let us consider the motion of the superfluid component, assuming the normal-component velocity field to be given. We are interested in the fields $\mathbf{v}_S$ and $\mathbf{v}_L$ averaged over the period of the wall vibrations. We solve the problem for the case of small vortex flexures (i.e., the case in which s_r, $s_\varphi \ll 1$). Let us linearize Eqs. (20) and (21) with respect to s_r and s_φ, and average over z. Then taking account of the fact that $(\bar{v}_L)_r=(\bar{v}_s)_r=(\bar{v}_n)_r=0$ (a bar denotes averaging over z and t), we obtain for the velocity components the equations

$$0=\overline{(v_{sl})}_r(1-\beta')-\beta(\bar{v}_n-(\bar{v}_{sl})_\varphi), \tag{23}$$

$$\bar{v}_L=(1-\beta')(\bar{v}_{sl})_\varphi+\beta'\bar{v}_n-\beta(\bar{v}_{sl})_r, \tag{24}$$

$$\overline{(v_{sl})}_r=-\nu_s\overline{\frac{\partial s_\varphi}{\partial z}}=\frac{\nu_s}{L}s_\varphi(0), \tag{25}$$

$$(\bar{v}_{sl})_\varphi=\bar{v}_s+\nu_s\overline{\frac{\partial s_r}{\partial z}}=\bar{v}_s-\frac{\nu_s}{L}s_r(0). \tag{26}$$

In deriving the right-hand sides of (25) and (26), we used the boundary condition $s_{r,\varphi}(L)=0$ (where L is the height of the vessel) at the free surface of the liquid. Hence

$$v_L=v_n-\frac{\nu_s}{L}s_\varphi(0)\left[\frac{(1-\beta')^2}{\beta}+\beta\right], \tag{27}$$

$$v_s=v_n+\frac{\nu_s}{L}\left[s_r(0)+\frac{\beta'-1}{\beta}s_\varphi(0)\right] \tag{28}$$

(we drop the averaging sign—the superior bar).

Here $s_\varphi(0)$ and $s_r(0)$ are the components of the vector $\mathbf{s}$ at the bottom of the vessel ($z=0$). They are phenomenologically connected with the velocity $\mathbf{v}_L$ by the Bekarevich-Khalatnikov condition:

$$\mathbf{s}(0)=a\mathbf{v}_L+a'[\hat{z}, \mathbf{v}_L], \tag{29}$$

where $\hat{z}$ is the unit vector along the z axis.

With the aid of (29), we can eliminate $s_\varphi(0)$ and $s_r(0)$ from (27) and (28), and write the final expressions for v_L and v_s in terms of v_n:

$$v_L=\frac{v_n}{1+(a\nu_s/L)[(1-\beta')^2/\beta+\beta]}, \tag{30}$$

$$v_s=v_n\left[1+\frac{\nu_s}{L}\frac{a(\beta'-1)/\beta-a'}{1+(\nu_s/L)a[(1-\beta')^2/\beta+\beta]}\right]. \tag{31}$$

Let us determine the shape of the bent vortices in terms of the radial and azimuthal displacements $u_r(z)$ and $u_\varphi(z)$. Integrating the Euler equation (18) with respect to the time, we obtain for the azimuthal component of the velocity the equation

$$\frac{\partial v_s}{\partial z}=-2\Omega_s\frac{\partial u_r}{\partial z}, \tag{32}$$

where $2\Omega_s=|\operatorname{curl}\mathbf{v}_s|$. It follows from Eq. (20) that v_{sl} does not depend on z, since v_L and v_n do not depend on z. Hence, differentiating the linearized equation (21) with respect to z, we obtain

$$\frac{\partial^3 u_\varphi}{\partial z^3}=0, \qquad -\nu_s\frac{\partial^3 u_r}{\partial z^3}+\frac{\partial u_r}{\partial z}2\Omega_s=0. \tag{33}$$

Integrating (33), and using the boundary conditions for $s_\varphi=\partial u_\varphi/\partial z$ and $s_r=\partial u_r/\partial z$, we obtain

$$u_\varphi(z)=\frac{s_\varphi(0)}{L}\left(L-\frac{z}{2}\right)z, \tag{34}$$

$$u_r(z)=-s_r(0)\left(\frac{\nu_s}{2\Omega}\right)^{1/2}\exp\left[-\frac{z}{(\nu_s/2\Omega)^{1/2}}\right]. \tag{35}$$

We see that the radial displacement of the vortices decreases exponentially as we move away from the bottom of the vessel over a distance of $(\nu_s/2\Omega)^{1/2}$, which is sometimes called the thickness of the Ekman superfluid layer.[6] But the azimuthal displacements penetrate far along the vessel, causing the vortex lines to twist along the helical lines whose axis coincides with that of the vessel.

This is due to the fact that the vortex lattice is "incompressible:" its density is strictly prescribed by the angular velocity, and it admits of only shear deformations, permitting the "twisting" of the structure when the vortex lines, as they move along the axis of rotation, twist along helical lines.

The twisted structure can arise not only on account of the pinning, but also as a result of the creation of the appro-

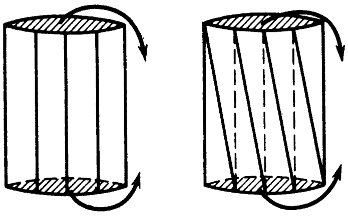

FIG. 1. Twisting of vortices along helical lines on a torus. The depicted cylinder is a torus that has been cut and straightened up. Depicted on the left and straight vortices corresponding to the normal structure; on the right, twisted vortices. When the ends of the cylinder are joined to form a torus, the end of each vortex at the top cross section joins onto the initial point of the neighboring vortex at the bottom cross section, as indicated by the dashed lines.

priate conditions for the onset of rotation of a liquid in a vessel with the shape of a torus. The vortices are then helical lines on toroidal surfaces. It is more convenient to depict these lines on a cylinder, which must then be mentally coiled into a torus (see Fig. 1). The figure depicts the case when, on going around the torus (in the figure, a cylinder), a vortex shifts sideways and gets to a point where the neighboring vortex begins. As a result all the vortices lying on the appropriate toroidal surface form a single closed vortex line. There exist a number of other possibilities. For example, the vortices, on going around the torus, can undergo a displacement of two intervortex distances, and, if the number of vortices is even, then the vortex system forms two closed lines that go around the torus several times, etc. No matter what topological structure arises, it will be stable. Here we have some analogy with a long solid rod twisted about is axis and then coiled into a ring. The resulting torsional deformation will exist indefinitely as long as the rod remains ideally elastic.

We have considered the problem in the linear approximation in the vortex flexures. If we use the pinning constant value $a \sim 10$ sec/cm obtained experimentally by Gamtsemlidze *et al.*,[7] then it turns out that, for a vortex velocity $v_L \sim 0.1$ cm/sec (which corresponds to $\Omega \sim 0.1$ rad/sec, $R \sim 1$ cm), the flexure is considerable, and we are at the limit of applicability of the theory. But it should be noted that the pinning force in superfluid ^{3}He, especially in the case of the nonsingular structures of the A phase, which do not have a small core scale, should, apparently, be much weaker than the corresponding force in ^{4}He. Therefore, there we can expect a regime with a small vortex slope right up to fairly high angular velocities. But to estimate the role of pinning in ^{4}He, it is expedient to turn directly to the formulas (27) and (28), which contain the flexures $s_\varphi(0)$ and $s_r(0)$, which should be considered to be quantities of the order of unity. It follows from these formulas that the lagging of the vortices and superfluid component behind the normal component is insignificant in the case when $L \sim 1$ cm, $\nu_s \sim 10^{-3}$ cm^2/sec, and $\beta, \beta' \sim 1$.

In the preceding discussions we assumed that the torque exerted by the normal component on the superfluid component and setting the latter in rotation in no way affects the motion of the normal component, which executes rigid-body rotation: $\mathbf{v}_n = \mathbf{\Omega} \times \mathbf{r}$. Let us now estimate the superfluid component's reaction to the normal component, a reaction which should lead to the slowing down of the normal component and the destruction of the rigid-body nature of the rotation.

The Navier-Stokes equation for the mean azimuthal motion has, after the elimination of $(v_{sl})_r$ with the aid of (25), the form [cf. Eq. (8)]

$$\frac{d^2}{dr^2}\bar{v}_n + \frac{1}{r}\frac{d}{dr}\bar{v}_n - \frac{v_n}{r^2} = \frac{U}{\nu_n} - \frac{\rho_s \nu_s}{\rho_n \nu_n}\,\mathrm{rot}_z\,\mathbf{v}_s\,\frac{s_\varphi(0)}{L}. \qquad (36)$$

As in (8), the first term on the right-hand side of (36) is a force acting in the vicinity of the vessel walls, and setting the normal component in rotation.

To find the upper bound for the force slowing down the normal component, and represented in Eq. (36) by the second term on the right hand side, we set curl $\mathbf{v}_S =$ curl $\mathbf{v}_n = 2\mathbf{\Omega}$ and $s_\varphi(0) = 1$. Then from (36) we can estimate the value of the ratio L/R at which the effect of the pinning force is certainly negligible:

$$L/R \gg \rho_s \nu_s / \rho_n \nu_n. \qquad (37)$$

It is especially easy to satisfy this inequality in ^{3}He because of its high viscosity.

In ^{4}He, in a broad range of temperatures, e.g., at $T > 1$ K,

$$\rho_s \nu_s / \rho_n \nu_n \leqslant 1$$

and the effect of the pinning is not stronger than the effect of the viscous friction between the normal component and the bottom of the vessel. In this case the superfluid rotates as a whole.

CONCLUSION

It follows from the foregoing that both superfluid ^{4}He and superfluid ^{3}He can be rotated in a stationary vessel through the generation of a sufficiently strong transverse wave running along the lateral walls of the vessel. This method opens up new possibilities in the investigation of the structures that arise in a rotating superfluid, including the diverse structures of the superfluid phases of ^{3}He. On the other hand, small deviations from the equilibrium rotation of the liquid can provide useful information about the interaction of the vortices with the surface, i.e., about the pinning effect.

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