Resonance self-focusing in the limit of strong nonlinearity saturation

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Analytic results are obtained for resonance self-focusing. They describe the evolution of the average beam radius under saturation conditions. The neck of the nonlinear focus is discussed in detail. It is shown that the transverse and longitudinal dimensions of the focal region have discontinuities as functions of the laser-beam parameters and density of the medium.

1. INTRODUCTION

The phenomenon of self-focusing[1] is quite general: after an intensity threshold has been reached, it accompanies the propagation of high-intensity radiation through any medium in which the polarizability is a nonlinear function of the electric field in the beam (cf., the reviews in Refs. 2–4). The self-focusing process is relatively simple to describe in general terms,[5] but turns out to be quite complicated when an attempt is made to carryout a detailed and comprehensive investigation of it. This applies particularly to experimental studies, in which the use of high intensities ensures that the phenomenon of self-focusing is accompanied by the stimulated generation of new waves[6,7] and by the ionization of the molecules.

Apart from factors such as the initial shape of the transverse beam profile, the time dependence of its intensity, and the ratio of the input to critical intensities, the saturation of the nonlinearity of the permittivity of the medium plays an important part in self-focusing.[8–12] This saturation leads to a weakening of the focusing properties of the medium, a reduction in the final focal diameter, the possibility of quasiwave propagation, and the evolution of a substructure in the beam intensity profile.

There is particular interest in the study of self-focusing under resonance conditions[13–25] because resonance processes play an important part in quantum electronics and nonlinear optics. The absence of thermal effects and the fact that ionization is of minor importance ensure that resonance gases are convenient for the investigation of self-focusing in its purest form. It is no accident that the agreement between theory and experiment on self-focusing (self-defocusing) of laser radiation has been achieved precisely for resonance gases.[17–26] We also not Ref. 18, which reports the observation of self-channeling and quasi-wave propagation of resonance radiation.

When self-focusing (self-defocusing) with allowance for permittivity saturation was investigated, the analytic resutls were obtained for paraxial rays. Analysis of the complete transverse distribution, on the other hand, is usually based on the numerical intergration of the parabolic equation describing the propagation of the beam in a passive medium. In the present paper, resonance self-focusing is examined by the method of moments,[27] which enables us, in the nonlinear saturation region, to obtain analytic results for the mean radius of the beam. We shall show that the dimensions of the nonlinear focal caustic exhibit discontinuities as functions of the density of the medium, the intensity, and the geometric beam factors at entrance to the medium.

2. SELF-FOCUSING AND SELF-DEFOCUSING WHEN THE OPTICAL TRANSITION IS STRONGLY SATURATED

For sufficiently large detuning from resonance, the interaction of high-intensity radiation with a resonance medium is adiabatic in character (precisely this case is considered below), and the permittivity of the medium can be represented by the expression[28,25]

$$\varepsilon=1+p(1+\xi)^{-1/2}\operatorname{sgn}\Delta. \tag{1}$$

where $p = 4\pi N|d|^2/\hbar|\Delta|$, N is the number density of resonance atoms, d is the transition matrix element, $\Delta=\omega_0-\omega$ is the detuning from resonance, ω_0 is the optical transition frequency, ω is the frequency of the high-intensity wave, $\xi=4|d^2|\,|E|^2/\hbar^2\Delta^2$ is a dimensionless intensity parameter of the light wave, and E is the electric field amplitu le in the wave. The propagation equation has the following form in the quasi-optical approximation[2,3]

$$2ik\partial E/\partial z=[\nabla_\perp{}^2+k^2\varepsilon_{nl}(|E|^2)]E, \tag{2}$$

where k is the wave vector, z the propagation coordinate, $\nabla_\perp^2$ the two-dimensional Laplace operator, written in terms of transverse coordinates, and

$$\varepsilon_{nl}(|E|^2)\equiv\varepsilon-\varepsilon(|E|^2=0)=p[(1+\xi)^{-1/2}-1]\operatorname{sgn}\Delta \tag{3}$$

is the nonlinear part of the permittivity (1).

For the chosen adiabatic interaction conditions, there is no absorption (the permittivity ε is real), so that we have the quantity[29]

$$\Pi_1=\int|E|^2\,d^2\rho, \tag{4}$$

is conserved; apart from the factor $c/2\pi$, this is the total energy flux flowing through the beam cross section. Whatever the particular form of ε, there is a further conserved quantity,[29] namely,

$$\Pi_2=\frac{1}{2k^2}\int[|\nabla_\perp E|^2-G(|E|^2)]d^2\rho, \tag{5}$$

where $G(|E|^2)$ is a solution of

$$\partial G(|E|^2)/\partial|E|^2=k^2\varepsilon_{nl}(|E|^2). \tag{6}$$

Using (2), (4), (5), and (6), we find that the mean square of the beam radius is given by[27]

$$R^2(z)=\frac{1}{\Pi_1}\int\rho^2|E|^2\,d^2\rho, \tag{7}$$

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and

$$\Pi_1 \frac{d^2R^2(z)}{dz^2} = 4\Pi_2 + \frac{2}{k^2}\int [2G(|E|^2) - k^2\varepsilon_{nl}(|E|^2)|E|^2]d^2\rho. \tag{8}$$

In the first nonlinear approximation, the integrand in (8) vanishes, and the resulting equation predicts the collapse of the beam for $\Pi_2 < 0$ (self-focusing) or the blowing up of the beam for $\Pi_2 > 0$ (self-defocusing).[30]

Substituting ε_{nl} and G from (3) and (6) into the right-hand side of (8), we can rewrite this equation in the form

$$\Pi_1 \frac{d^2R^2(z)}{dz^2} = 4\Pi_2 + 2\pi N\hbar\Delta \int \{2[2(1+\xi)^{1/2} - \xi - 2] - [(1+\xi)^{-1/2} - 1]\xi\}d^2\rho. \tag{9}$$

Now consider the case of high intensities:

$$\xi(z, \rho=0) \gg 1, \tag{10}$$

in which case, the permittivity of the medium (in the central part of the beam) is found to saturate to a considerable extent. We note that, for asymptotically high intensities ($\xi \to \infty$), the medium becomes completely trnasparent ($\varepsilon \to 1$) and the laser radiation propagates in the medium as if it were a vacuum, retaining its initial Gaussian profile. This suggests (see also Ref. 31) that we can use the Gaussian profile

$$\xi(z, \rho) = \xi_0 R^{-2}(z)\exp[-\rho^2/R^2(z)]$$

to evaluate the integral in (9), subject to condition (10). Retaining after integration all terms up to the first order in the small parameter $R^2(z)/\xi_0$, we obtain the following equation for the mean beam radius $\bar{r}(z) = [R^2(z)/R^2(0)]^{1/2}$:

$$2\bar{r}(z)\frac{d\bar{r}(z)}{dz} = \pm\left\{\left(\frac{1}{R_d^2} + \frac{4}{r^2}\right)\bar{r}^2(z) + \frac{\operatorname{sgn}\Delta}{R_f^2}[\bar{r}^3(z) - 1] - \frac{1}{R_d^2}\right\}^{1/2}, \tag{11}$$

where $R_d = kR^2(0)/2$ is the diffraction length, r is the radius of curvature of the wavefront at entry into the medium, and $R_f^2 = R(0)\xi_0^{1/2}/16p$ is a parameter determining the focusing properties of the medium. Note that the radiation intensity and density of the medium are represented by the single parameter R_f^2 in (11). An increase in intensity is completely analogous to a reduction in the medium density. This is natural for the saturation region because *an* increase in the intensity smooths out the transverse profile of the permittivity of the medium and thus reduces the focusing or defocusing properties of the medium. A reduction in the density of the medium also results in a weakening of the focusing or defocusing properties.

For $\Delta < 0$, Eq. (11) describes the self-focusing and, for $\Delta > 0$, the self-defocusing of high-intensity radiation. In the latter case, a beam that converges on entering the medium becomes irreversibly divergent after a single contraction. From now on, we shall mostly confine our attention to the self-focusing regime.

A relatively simple analysis of (1) for $\Delta < 0$ shows that a high-intensity wave propagates under quasi-wave conditions through the resonance medium. In general, the radius of the high–intensity beam that is focused on entering the medium will subsequently oscillate between the minimum

$$\bar{r}_1 = q_1[1 - 2\cos(\alpha/3 - \pi/3)] \tag{12}$$

and the maximum

$$\bar{r}_2 = q_2[1 + 2\cos(\alpha/3)] \tag{13}$$

values, where

$$\alpha = \arccos\frac{q_2}{2(q_1/3)^{3/2}}, \quad q_1 = \frac{1}{3}\left(\frac{4R_f^2}{r^2} + \frac{R_f^2}{R_d^2} + 1\right),$$
$$q_2 = \frac{2}{27}\left(\frac{4R_f^2}{r^2} + \frac{R_f^2}{R_d^2} + 1\right) - \frac{R_f^2}{R_d^2}.$$

The initial contraction of the beam from $\bar{r}(z=0) = 1$ to $\bar{r}(z) = \bar{r}_1$ is described by

$$2(\bar{r}_2+\bar{r}_3)^{-1/2}\{\bar{r}_3[F(\varphi(z), k) - F(\varphi(0), k)] - (\bar{r}_1+\bar{r}_3)[\Pi(\varphi(z), k^2, k) - \Pi(\varphi(0), k^2, k)]\} = z/2R_f, \tag{14}$$

and hence, for $\bar{r}(z) = \bar{r}_1$, we obtain the focal length, as well. The following designations are used in (14):

$$\varphi(z) = \arcsin\left\{\frac{(\bar{r}_2+\bar{r}_3)[\bar{r}(z) - \bar{r}_1]}{(\bar{r}_2-\bar{r}_1)[\bar{r}(z)+\bar{r}_3]}\right\}^{1/2}$$
$$k = \left(\frac{\bar{r}_2 - \bar{r}_1}{\bar{r}_2+\bar{r}_3}\right)^{1/2}, \quad \bar{r}_3 = q_1\left[1 - 2\cos\left(\frac{\alpha}{3} + \frac{\pi}{3}\right)\right]$$

and $F(\varphi,k)$, $\Pi(\varphi,k^2,k)$ are the elliptic integrals of the first and third kind, respectively. An expression analogous to (14) can readily be written down in terms of elliptic integrals for the beam expansion stage.

Let us now briefly consider the case where a parallel beam ($r = \infty$) is incident on the medium. When the intensities are not too high [condition (10) is, of course, assumed to be satisfied], so that $R_f^2 < 0.5R_d^2$, the beam entering the medium contracts and then expands to its initial size, and so on. However, when $R_f^2 > 0.5R_d^2$, the beam at first expands and then contracts to the initial size, and so on. When $R_f^2 = 0.5R_d^2$, diffraction is exactly balanced by nonlinear refraction, and self-channeling takes place. Substituting for R_f^2 and R_d^2, we can rewrite the last condition in the form

$$|E(z=0, \rho=0)|^2 = 16\pi^2N^2|d|^2[kR(0)]^4. \tag{15}$$

This essentially determines the new threshold for self-focusing, i.e., the maximum possible field above which focusing by the medium is weaker than diffraction, and a beam that is parallel at entry is found to expand. In contrast to the usual threshold,[2,3] which lies in the low-intensity region, in the present case, an expanding beam is brought together again because the beam intensity falls as the beam expands, there is weaker saturation and a corresponding increase in the focusing properties of the medium. At the same time, diffraction is also reduced, and focusing becomes stronger than diffraction. An analogous propagation picture obtains in the paraxial approximation. The latter has been examined in some detail[23] by numerical methods. The focal region plays a special part in different nonlinear interactions between intense laser radiation and a medium. Let us examine the dimensions of the neck of the nonlinear focus as functions of the

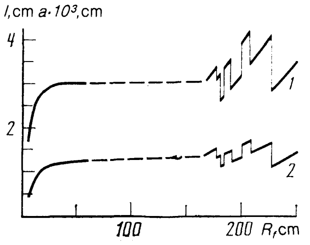

FIG. 1. Radius a (1) and length l (2) of the neck of the nonlinear focus as functions of R_f (radiation intensity and/or density of the medium). Here and in the subsequent figures, the broken lines are portions of the graphs on which the discontinuities are small. The calculations were performed for the $3s_{1/2}$–$3p_{3/2}$ transition in Na vapor. $R_d = 125$ cm, $R(0) = 0.05$ cm, $\bar r = 15$ cm.

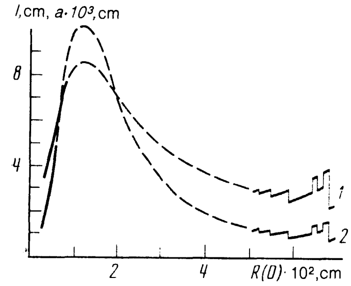

FIG. 3. Radius a (1) and length l (2) as functions of the initial beam radius $R(0)$ (for a constant energy flux in the beam: Π_i = const). The system parameters were chosen so that $R_f = 150$ cm, $R_d = 125$ cm for $R(0) = 0.05$ cm and $r = 15$ cm.

parameters of the interacting system. The neck radius a is given by $a = \bar r_1 R(0)$ and its length will be defined as the distance between the points at which two straight lines, drawn from the center of the caustic, touch the curve $\bar r(z)$. Since the lateral dimensions of the beam in the region of the neck remain sensibly constant, we find form (14) that

$$l \approx \frac{4\bar r_1^{1/2} R_\Phi}{[(\bar r_2-\bar r_1)(\bar r_1+\bar r_3)]^{1/2}}. \tag{16}$$

Substituting for R_f in (16), we find that the ratio of the neck length l to it radius a is

$$\frac{l}{a} = \left[\frac{\bar r_1}{(\bar r_2-\bar r_1)(\bar r_1+\bar r_2)}\right]^{1/2} \frac{[\xi_0/R^2(0)]^{1/4}}{p^{1/2}}. \tag{17}$$

Since the expression under the square root on the right-hand side is on the order of or less than unity, $\xi_0/R^2(0) \gg 1$ and $p \ll 1$ (for gas media), we find that $l \gg a$, i.e., the length of the neck is much greater than its width under fairly general conditions. Numerical calculations show that this result remains valid even under relatively hard focusing conditions ($r \lesssim 10$ cm) in the laser beam at entry to the medium.

Figure 1 shows the typical dependence of l and a on the parameter R_f. For small values of R_f, the size of the neck increases monotonically with R_f. However, after the latter reaches a certain value ($R_f \simeq 55$ cm in Fig. 1), the dependence becomes discontinuous. The first few jumps are small, but their size increases as F_f is increased further. For example, the relative size of a jump near $R_f = 100$ cm is about 2%, near $R_f = 150$ cm it increases to 8%, and at $R_f = 225$ cm it reaches 40%.

Analysis of the length of the neck in the paraxial approximation does not reveal this discontinuous behavior. The jumps must therefore be a consquence of nonlinear distortion of nonparaxial rays which, in turn, is due to the relatively complicated distribution of the focusing properties of the medium across the beam profile. In the central part of the beam, the medium is strongly saturated, so that, as shown above, its focusing properties deteriorate with increasing intensity. On the other hand, in peripheral regions, the field intensity is so low that there is no saturation and the focusing properties are enhanced with increasing intensity. These two regions are of course separated by an intermediate region in which the focusing properties of the medium are practically independent of the wave intensity. All this is taken into account in (9), in the integral on the right-hand side, since the expansion in terms of the parameter ξ^{-1} is performed after the integral has been evaluated.

The dependence of the neck size on r and $R(0)$ is also discontinuous. This is illustrated qualitatively by Figs. 2 and 3. The discontinuities in a and l appear in a wide range of values of the parameters of the interacting system, and can be seen in different experiments using powerful laser fields that saturate the resonance transition.

The lateral dimensions of the antinodes of the quasi-waveguide formed in the medium are continuous functions of the parameters of the system. As R_f increases, the antinode thickness monotonically increases, which is a consequence of the corresponding reduction in the focusing properties of the medium, whereas an increase in r and $R(0)$ is accompanied by a monotonic reduction in this thickness.

The discontinuous behavior of the dimensions of the neck of the nonlinear focus is also obtained for $\Delta > 0$, for which the medium becomes defocusing. All that is required is that the high-intensity radiation entering the medium be focused. Figure 4 shows some typical curves of a and l as

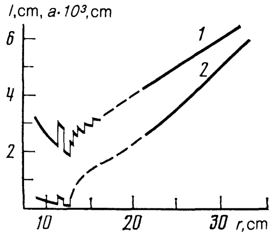

FIG. 2. Radius a (1) and length l (2) of the neck as functions of the radius of curvature r of the wavefront; $R_f = 150$ cm $R_d = 125$ cm, $R(0) = 0.05$ cm.

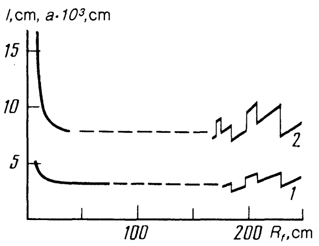

FIG. 4. Radius a (1) and length l (2) as functions of R_f during self-defocusing. $R_d = 125$ cm, $R(0) = 0.05$ cm, $r = 15$ cm.

functions of R_f. As expected, the relative size of the discontinuities for self-defocusing is smaller than for self-focusing, other things being equal.

We note in conclusion that the adiabatic resonance character of the interaction between high–intensity radiation and a medium manifests itself only in the fact that the permittivity ε assumes the form given by (1). The resonance condition is not used in subsequent calculations. The results presented above are therefore valid for any medium whose permittivity saturation can be approximately described by $\varepsilon = A + B(1 + I/I_{sat})^{-1/2}$, where A, B are constants, I is the radiation intensity, and I_{sat} is the saturated intensity.

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