Effect of plasma heating by laser radiation in stimulated Brillouin scattering

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It is shown that local violation of the mode locking condition in interaction of light waves with ion acoustic waves, which is due to plasma heating, can lead to an increase of the stimulated Brillouin scattering (SBS) threshold as well as to breakdown of the wavefront reversal (WFR). Mechanisms that lead to thermal suppression of SBS and WFR and are due to the increase of the ion-sound frequency and to the change, in space and in time, of the frequencies of the pump and of the Stokes wave with increase of plasma temperature are considered. The conditions under which there is no thermal suppression of SBS are determined.

Research into stimulated Brillouin scattering (SBS) has increased of late in view of its possible use for laser wavefront reversal (WFR).[1–4] This is particularly vital for CO_2 lasers, since SBS in the $\lambda = 10\ \mu$m band has been realized only in a plasma.[5–7] The typical situation of SBS in a plasma is additional heating of and ionization of the pre-conditioned plasma by the laser radiation. Plasma heating leads, on the one hand, to an increase of the ion-sound frequency $\Omega_s \sim T^{1/2}$, and on the other to a change, in space and in time, of the pump frequency ω_0 and of the Stokes-wave frequency, owing to the phase advance $\varphi_{0,1} = (\omega_{0,1}/c)\int\delta n dz$, due to the change $\delta n = (dn/dT)\delta T$ of the refractive index. Local violation of the condition of frequency synchronism of the light wave with the ion sound ($\omega_0 - \omega_1 - \Omega_s \neq 0$) can lead not only to an increase of the SBS threshold, but also to a shutoff of the WFR, for in the case of a multimode pump beam the opposing Stokes wave gain growth rate is decreased to a greater degree than the waves uncorrelated with the pump wave.

The influence of the violation of the frequency-synchronization conditions of the interacting waves on the stimulated scattering was discussed earlier in a number of papers. In Ref. 8, for example, it is indicated that phase modulation of the exciting radiation influences strongly the gain in stimulated temperature scattering in a liquid. The influence of the refractive-index modulation due to excitation of the molecule excitation via conversion of the excitation energy into heat are explained by the peculiarities of the stimulated Raman scattering.[9] Suppression of stimulated Raman scattering by the frequency shift of the optical phonon, due to the change of temperature, is considered in Ref. 10. It is also shown in Ref. 11 that absorption of the radiation lead to energy exchange, due to the onset of a local frequency difference, between opposing light waves in media with thermal nonlinearity.

It is shown in the present paper that nonstationary heating of a plasma by laser radiation influences strongly both the SBS excitation threshold and the WFR accompanying the SBS. This influence is due both to modulation of the phase of the interacting waves and to a change of the ion-sound frequency. We obtain conditions under which there is no SBS or WFR. We show that WFR realization in SBS in a plasma is possible only because of the high thermal conductivity of the plasma.

1. INITIAL EQUATIONS

Since thermal suppression of SBS can occur also in other media (solids, liquids, gases), we precede the actual estimates by a description of the influence of heating on SBS in general form. Let a pump wave

$$E_0 \exp[i(\omega_0 t - k_0 z)] + \text{c.c.}$$

be incident on the boundary of the medium in the $z = 0$ plane, and let a Stokes wave

$$E_1 \exp[i(\omega_1 t + k_1 z)] + \text{c.c.}$$

propagate counter to it. For a short nonlinear medium ($2L/c \ll t_p$, where t_p is the duration of the pump pulse), the backward SBS is described in the quasi-optical approximation, with account taken of the heating of the medium, by the set of coupled equations

$$\frac{\partial E_0}{\partial z} + \frac{i}{2k_0}\Delta_\perp E_0 + ik_0\frac{\delta n}{n}E_0 + \frac{\alpha_0}{2}E_0 = \frac{i}{2}g_1 E_1 P, \tag{1}$$

$$-\frac{\partial E_1}{\partial z} + \frac{i}{2k_1}\Delta_\perp E_1 + ik_1\frac{\delta n}{n}E_1 + \frac{\alpha_1}{2}E_1 = \frac{i}{2}g_2 E_0 P^*,$$

$$\partial P/\partial t + (i\delta\Omega + 1/\tau)P = ig_3 E_0 E_1^*,$$

where n is the refractive index of the medium, $\alpha_{0,1}$ are the pump and Stokes-wave absorption coefficients, τ is the relaxation time of the sound wave, $P\exp[i(\Omega_s t - qz)] + \text{c.c.}$ ($q = k_0 + k_1$), $\delta\Omega = \omega_0 - \omega_1 - \Omega_s$, and $g_{1,2,3}$ are constants that determine the growth rate $g_0 = g_2 g_3 \cdot 2\pi\tau/cn$ of the Stokes wave per unit length and per unit intensity. Since the frequency shift in SBS is small, we assume furthermore that $k_0 = k_1 = k$, $q = 2k$ and $\alpha_0 = \alpha_1 = \alpha$. The values of the refractive index and of the sound frequency depend on the temperature, and in the case of small temperature perturbations ($\delta T/T \ll 1$) their change is described by

$$\delta n = (dn/dT)\delta T, \qquad \delta\Omega_s = (d\Omega_s/dT)\delta T.$$

We recognize also that at the SBS threshold, in the given-pump-wave approximation, the heating of the medium is determined by the value of $|E_0|^2$. Neglecting the heat diffusion, we have for the temperature perturbation

$$\delta T(t, z, \mathbf{r}_\perp) = g_4 \int_0^t |E_0(t', z, \mathbf{r}_\perp)|^2 dt', \tag{4}$$

where $g_4 \sim \alpha$ is a constant of the medium.

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2. PLANE PUMP WAVE

We consider first the effect of thermal suppression of the SBS, using a plane pump wave as the example. Using Eq. (1) with zero right hand side, we readily find that

$$|E_0(t,z)|^2=|E_0(t,0)|^2\exp(-\alpha z).$$

Taking into account the connection between $\delta n(t,z)$ and $\delta T(t,z)$ the pump wave amplitude takes the form

$$E_0(t,z)=E_0(t,0)\exp\left[-\frac{\alpha}{2}z+i\Phi_0(t,z)\right], \tag{5}$$

where

$$\Phi_0(t,z)=-kg_4\left(\frac{1}{n}\frac{dn}{dT}\right)\frac{1-e^{-\alpha L}}{\alpha}\int_0^t |E_0(t',0)|^2\,dt'.$$

The phase advance $\Phi_0(t,z)$ due to the heating leads to a pump-frequency change that depends on the time and on the longitudinal coordinate, amounting to

$$\delta\omega_0(t,z)=\frac{d\Phi_0}{dt}=-\nu(1-e^{-\alpha z}),$$

$$\nu(t)=\frac{kg_4}{\alpha}\left(\frac{1}{n}\frac{dn}{dT}\right)|E_0(t,0)|^2.$$

The parameter $\nu(t)$, which characterizes the change of the pump frequency at a penetration depth α^{-1} into the absorbing medium, is determined by the properties of the nonlinear medium and by the pump-wave intensity. Similarly, the heating of the medium modulates the Stokes-wave frequency. Its additional phase advance is

$$\Phi_1(t,z)=-\int_0^t \nu(t')\,dt'\,(e^{-\alpha z}-e^{-\alpha L}).$$

The heat-induced change of the sound-wave frequency, due to the dependence of the sound velocity v_s on the temperature, can be written in the form

$$\delta\Omega_s=\mu e^{-\alpha z},\qquad \mu=g_4\frac{d\Omega_s}{dT}\int_0^t |E_0(t',0)|^2\,dt'.$$

In this case $\delta\Omega=\delta\Omega(0)-\delta\Omega_s$, where $\delta\Omega(0)=\omega_0-\omega_1-\Omega_s(0)$ is the initial detuning from resonance.

Under the conditions

$$|\delta\omega_{0,1}|\gg\left|\tau\frac{d\omega_{0,1}}{dt}\right|,\qquad \left|\frac{d\Omega_s}{dT}\right|\gg\left|\tau\frac{d}{dt}\left(\frac{d\Omega_s}{dT}\right)\right|$$

the SBS of a pump pulse of duration $t_p\gg\tau$ can be analyzed in a quasistationary approximation in which the time t enters the stationary solution as a parameter. Making the substitutions

$$E_1(t,z)=C_1(t,z)\exp[-{}^1\!/_2\alpha(L-z)+i\Phi_1(t,z)],$$
$$P(t,z)=\mathscr{P}(t,z)\exp\{i[\Phi_0(t,z)-\Phi_1(t,z)]\}, \tag{6}$$

we obtain from (2) and (3), with account taken of (5) and of the expressions for $\delta\omega_0$, $\delta\omega_1$, and $\delta\Omega_s$,

$$\frac{\partial C_1}{\partial z}=-\frac{g_0}{2}I_0\frac{C_1e^{-\alpha z}}{1+i\tau[\delta\Omega(0)-\nu(1+e^{-\alpha L}-2e^{-\alpha z})-\mu e^{-\alpha z}]}, \tag{7}$$

where $I_0=(cn/2\pi)|E_0(t,0)|^2$. Integrating (7), we obtain the total growth rate of the Stokes wave:

$$G_t=\ln\frac{|E_1(t,0)|^2}{|E_1(t,L)|^2}$$
$$=\frac{g_0I_0}{\alpha\tau(2\nu-\mu)}\{\mathrm{arctg}[\tau[\delta\Omega(0)+\nu(1-e^{-\alpha L})-\mu]]$$
$$-\mathrm{arctg}[\tau[\delta\Omega(0)-\nu(1-e^{-\alpha L})-\mu e^{-\alpha L}]]\}-\alpha L.$$

At the optimal value of the initial detuning $\delta\Omega_{\rm opt}(0)=1/2\mu(1+e^{-\alpha L})$ the value of G_t is a maximum

$$G_{t\,\max}=g_0I_0\frac{1-e^{-\alpha L}}{\alpha}\frac{\mathrm{arctg}\,\beta}{\beta}-\alpha L, \tag{8}$$

where $\beta=1/2\tau|2\nu-\mu|(1-e^{-\alpha L})$ is the Raman-scattering detuning corresponding to $\delta\Omega_{\rm opt}(0)$ and normalized to the sound bandwidth τ^{-1}. Neglecting the effect of heating on the SBS, i.e., letting $\nu\to 0$ and $\mu\to 0$, we obtain the known result of the stationary theory

$$\delta\Omega_{\rm opt}(0)=0,\qquad G_{t\,\max}=g_0I_0\frac{1-e^{-\alpha L}}{\alpha}-\alpha L,$$

which follows from (8) at $\beta\ll 1$. The growth rate decreases with increase of β like $\arctan\beta/\beta$, and this may be the reason why the SBS shuts off at $\beta\gtrsim 1$.

The effectiveness of the thermal suppression of the SBS by modulation of the pump frequency and of the Stokes-wave frequency is characterized by the heating rate dT/dt, and hence by the intensity I_0 of the pump wave. In crystals, the change of the refractive index by heating, meaning the onset of the suppression, take place without delay. In liquids, gases, and plasma the principal mechanism that alters n is thermal expansion of the substance, so that the suppression is turned on only after the time $t_a=a/v_s$ required for the sound to negotiate the characteristic scale of the pump beam (a is the beam radius). It is convenient to introduce the kinetic pump intensity I_0^* above which the thermal suppression of the SBS becomes substantial. The suppression threshold is determined by the quantity $\beta\approx 0.5$ which corresponds to a decrease of the total growth rate by 1–2 units relative to the threshold value ($M_{\rm thr}\approx 25$). Putting $\beta=0.5$ and $\mu=0$ we get

$$I_0^*\sim\alpha\left[\frac{4\pi}{cn}g_4\left(\frac{1}{n}\frac{dn}{dT}\right)k\tau(1-e^{-\alpha L})\right]^{-1}.$$

The SBS suppression due to the change of the sound frequency is characterized by the integral heating δT, i.e., by the pump-energy density

$$w_0=\int_0^t I_0(t')\,dt'.$$

Its influence is manifested after a certain time t^* in which the pump energy reaches a threshold value

$$w_0^*\sim\left[\frac{2\pi}{cn}g_4\tau\frac{d\Omega_s}{dT}(1-e^{-\alpha L})\right]^{-1},\qquad t^*\sim\frac{w_0^*}{I_0},$$

corresponding to $\beta\approx 0.5$ and $\nu=0$. Note that $\beta=0$ also at $\mu=2\nu$, i.e., there exists an instant of time

$$t_0\sim\frac{2k}{\alpha}-\left(\frac{1}{n}\frac{dn}{dT}\right)\left(\frac{d\Omega_s}{dT}\right)^{-1}$$

at which these effects cancel each other and there is no thermal suppression. Within the time interval

$$t_0-\Delta<t<t_0+\Delta, \quad \Delta\sim\left[\frac{4\pi}{cn}g_4 I_0\left|\frac{d\Omega_s}{dT}\right|\right]^{-1},$$

determined by the condition $|\beta|<0.5$ this makes it possible to obtain an SBS growth rate close to the unperturbed value.

3. MULTIMODE PUMP BEAM

To analyze the influence of heating on the WFR in the presence of SBS we assume that the pump beam $E_0(t,0,\mathbf{r}_\perp)=A(t)\Psi_0(0,\mathbf{r}_\perp)$ has a complicated spatial structure $\Psi_0(0,\mathbf{r}_\perp)$ with a characteristic transverse scale ρ much smaller than the beam radius a and an average intensity $\bar{I}_0$ that is constant over the cross section [$A(t)$ describes the pulse waveform]. We assume that the necessary WFR condition $g_0\bar{I}_0 z_c \ll 1$ (Ref. 12), where $z_c = k$ is the length of the longitudinal correlation of the radiation, is met. We assume also that

$$\left(k\frac{\langle\delta n\rangle}{n}L\right)\left(k\frac{\langle\delta n\rangle}{n}z_\text{к}\right)\ll 1,$$

where $\langle\delta n\rangle$ is the mean value of the perturbation of n. In this case the self-action of the pump wave does not distort its spatial structure and reduces only to an additional phase advance. The solution for the amplitude of the pump wave will be sought in the form

$$E_0(t,z,\mathbf{r}_\perp)=\bar{E}_0(t,z)\Psi_0(z,\mathbf{r}_\perp),$$

where $\Psi_0(z,\mathbf{r}_\perp)$ satisfies the equation

$$\hat{L}\Psi_0(z,\mathbf{r}_\perp)\equiv\frac{\partial\Psi_0}{\partial z}+\frac{i}{2k}\Delta_\perp\Psi_0=0.$$

After substituting $\delta T(t,z,\mathbf{r}_\perp)$ from (4) into the equation for the pump wave we multiply it by Ψ_0^* and integrate over the beam cross section. As a result we get

$$\frac{\partial\bar{E}_0}{\partial z}+i\frac{kg_4}{n}e^{-\alpha z}D_0\int_0^t|\bar{E}_0(t',0)|^2dt'\bar{E}_0+\frac{\alpha}{2}\bar{E}_0=0, \quad (10)$$

where

$$D_0=\left(\int_s|\Psi_0|^4d^2r_\perp\right)\Big/\left(\int_s|\Psi_0|^2d^2r_\perp\right).$$

The solution of (10) is

$$\bar{E}_0(t,z)=\bar{E}_0(t,0)\exp\left[-\frac{\alpha}{2}z+i\varphi_0(t,z)\right], \quad (11)$$

where

$$\varphi_0(t,z)=-D_0\int_0^t\nu'(t')dt'(1-e^{-\alpha z}),$$

$$\nu'=\frac{kg_4}{\alpha}\left(\frac{1}{n}\frac{dn}{dT}\right)|\bar{E}_0(t,0)|^2.$$

The phase advance $\varphi_0(t,z)$ changes the pump frequency by an amount

$$\delta\omega_0(t,z)=d\varphi_0/dt=-D_0\nu'(1-e^{-\alpha z}).$$

The parameter ν' has the same physical meaning as the parameter ν in the case of a plane pump wave. The quantity ν' characterizes the frequency shift, averaged over the cross section, of a spatially inhomogeneous pump beam at a penetration depth α^{-1}.

To calculate the growth rates of the Stokes waves that are correlated and uncorrelated with the pump, we represent the field E_1 in the form

$$E_1(t,z,\mathbf{r}_\perp)=\bar{E}_1(t,z)\Psi_1(z,\mathbf{r}_\perp), \quad (12)$$

where $\hat{L}^*\Psi_1(z,\mathbf{r}_\perp)=0$. After substituting (12) we arrive, with allowance for (11) in (2), at the equation

$$-\frac{\partial\bar{E}_1}{\partial z}+i\frac{kg_4}{n}e^{-\alpha z}D_{01}\int_0^t|\bar{E}_0(t',0)|^2dt'\bar{E}_1+\frac{\alpha}{2}\bar{E}_1=\frac{i}{2}g_2\bar{E}_0(t,0)\exp\left(-\frac{\alpha}{2}z+i\varphi_0\right)\left(\int_s\Psi_0\Psi_1P^*d^2r_\perp\right)\Big/\left(\int_s|\Psi_1|^2d^2r_\perp\right), \quad (13)$$

$$D_{01}=\left(\int_s|\Psi_0|^2|\Psi_1|^2d^2r_\perp\right)\Big/\left(\int_s|\Psi_1|^2d^2r_\perp\right).$$

The phase advance

$$\varphi_1(t,z)=-D_{01}\int_0^t\nu'(t')dt'(e^{-\alpha z}-e^{-\alpha L})$$

determines the quantity

$$\delta\omega_1(t,z)=d\varphi_1/dt=-D_{01}\nu'(e^{-\alpha z}-e^{-\alpha L}).$$

The change of the sound frequency as a result of the local dependence on the heating is in this case spatially inhomogeneous

$$\delta\Omega_s=\mu'|\Psi_0(z,\mathbf{r}_\perp)|^2e^{-\alpha z}, \quad \mu'=g_4\frac{d\Omega_s}{dT}\int_0^t|\bar{E}_0(t',0)|^2dt'.$$

Making in (3) and (13) a substitution similar to (6), we obtain an equation that describes the gain of the Stokes wave:

$$\frac{\partial c_1}{\partial z}=-\frac{g_0}{2}\bar{I}_0C_1e^{-\alpha z}\int_s|\Psi_0|^2|\Psi_1|^2\times\{1+i\tau[\delta\Omega(0)-\nu'D_0(1-e^{-\alpha z})+\nu'D_{01}(e^{-\alpha z}-e^{-\alpha L})-\mu'|\Psi_0|^2e^{-\alpha z}]\}^{-1}d^2r_\perp\Big/\left(\int_s|\Psi_1|^2d^2r_\perp\right), \quad (14)$$

where $\bar{I}_0=(cn/2\pi)|\bar{E}_0(t,0)|^2$. Integrating (14) we obtain, after reversing the substitution, the total growth rate of the Stokes wave

$$G=\ln\frac{|E_1(t,0)|^2}{|E_1(t,L)|^2}=g_0\bar{I}_0\int_0^L\int_s|\Psi_0|^2|\Psi_1|^2\times\{1+\tau^2[\delta\Omega(0)-\nu'D_0(1-e^{-\alpha z})$$

$$+\nu' D_{01}(e^{-\alpha z}-e^{-\alpha L})$$

$$-\mu'|\Psi_0|^2 e^{-\alpha z}]^2\}^{-1} d^2r_\perp dz \Big/ \Big(\int_s |\Psi_1|\, d^2r_\perp\Big) - \alpha L. \quad (15)$$

We replace the integration over the cross section in (15) by averaging the integrand over the ensemble of realizations of the pump field. If the pump field has Gaussian statistics, the statistical averaging can be carried out only with a distribution function $W(I_0) = (1/\bar{I}_0)e^{-I_0/\bar{I}_0}$. In this case $D_0 = 2$, the value of D_{01} for Stokes wave correlated with the pump ($\Psi_1 \sim \Psi_0^*$) coincides with D_0, and for the uncorrelated one ($\int_s \Psi_0 \Psi_1^* d^2r_\perp = 0$) we have $D_{01} = 1$. Integrating next (15) with respect to z, we obtain expressions for the total SBS growth rates for the Stokes radiation structure associated with the pump:

$$G_c = \frac{g_0 \bar{I}_0}{\alpha\tau} \int_0^\infty \frac{U^2 e^{-U}}{4\nu' - \mu' U} \{\mathrm{arctg}[\tau[\delta\Omega(0) + 2\nu'(1-e^{-\alpha L}) - \mu' U]]$$

$$-\mathrm{arctg}[\tau[\delta\Omega(0) - 2\nu'(1-e^{-\alpha L}) - \mu' U e^{-\alpha L}]]\} dU - \alpha L',$$

$$U = I_0/\bar{I}_0, \quad (16)$$

and for the components that are not correlated with the pump we get

$$G_u = \frac{g_0 \bar{I}_0}{\alpha\tau} \int_0^\infty \frac{U e^{-U}}{3\nu' - \mu' U} \{\mathrm{arctg}[\tau[\delta\Omega(0) + \nu'(1-e^{-\alpha L}) - \mu' U]]$$

$$-\mathrm{arctg}[\tau[\delta\Omega(0) - 2\nu'(1-e^{-\alpha L}) - \mu' U e^{-\alpha L}]]\} dU - \alpha L. \quad (17)$$

Recognizing that the relative contribution of each of the thermal-suppression mechanisms is different at different instants of time, we consider them independently of one another. Neglecting the change of the sound frequency ($\mu' = 0$), Eqs. (16) and (17) can be analytically integrated. As a result we get

$$G_c = \frac{M_c}{4\nu'\tau(1-e^{-\alpha L})} \{\mathrm{arctg}[\tau[\delta\Omega(0) + 2\nu'(1-e^{-\alpha L})]$$

$$-\mathrm{arctg}[\tau[\delta\Omega(0) - 2\nu'(1-e^{-\alpha L})]]\} - \alpha L, \quad (18)$$

$$G_u = \frac{M_u}{3\nu'\tau(1-e^{-\alpha L})} \{\mathrm{arctg}[\tau[\delta\Omega(0) + \nu'(1-e^{-\alpha L})]$$

$$-\mathrm{arctg}[\tau[\delta\Omega(0) - 2\nu'(1-e^{-\alpha L})]]\} - \alpha L,$$

where $M_c = g_0\bar{I}_0(1-e^{-\alpha L})/\alpha$ is the growth rate of the correlated wave in the absence of heating, and $M_c = 2M_u$. A zero initial detuning corresponds to a maximum value of the total growth rate of the correlated component (at $\alpha L \ll M_{\rm thr}$)

$$G_{c\,\max} = M_c\, \mathrm{arctg}\, \beta'/\beta', \quad \beta' = 2|\nu'|\tau(1-e^{-\alpha L}).$$

The total growth rate of the uncorrelated Stokes waves reaches a maximum at a nonzero detuning $(\delta\Omega_u)_{\rm opt} = \nu'(1-e^{-\alpha L})/2$ and amounts to

$$G_{c\,\max} = M_c\, \mathrm{arctg}(^3/_4\beta')/(^3/_4\beta').$$

When the change of the light-wave frequencies ($\sim\nu'$) is small compared with the sound bandwidth ($\sim 1/\tau$), i.e., $\beta' \ll 1$, we get $G_{c\,\max} \approx M_c$ and $G_{u\,\max} \approx M_u$, and their ratio is $\gamma = G_{c\,\max}/G_{u\,\max} \approx 2$. With increase of β', a decrease is observed in the growth rates of both the correlated and uncorrelated Stokes waves, and this leads to suppression of the SBS. In this case $G_{c\,\max}$ decreases with increase of β' more rapidly than $G_{u\,\max}$. This decreases the discrimination of the growth rates γ and leads to deterioration of the quality of the WFR. As $\beta' \to \infty$ the value of γ tends to $\gamma_\infty = 1.5$. Therefore, if a discrimination $\gamma = 1.5$ suffices to ensure good quality of the WFR, heating of the medium can lead only to suppression of the SBS.

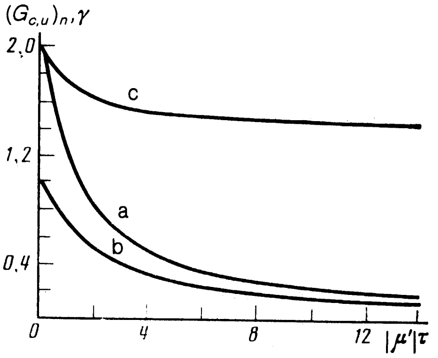

FIG. 1. Normalized total growth rate $(G_{c,u})_n = (G_{c,u})_{\max}/M_n$ of a correlated (a) and uncorrelated (b) Stokes wave, and discrimination γ (c) vs the thermal-suppression parameter.

We consider now the effect of thermal suppression of the WFR in SBS as a result of a change of the sound-wave velocity, assuming that $\nu' = 0$. In this case Eqs. (16) and (17) were integrated numerically. The results are shown in Fig. 1. In the case of a small change ($\sim\mu'$) of the sound-wave frequency within the sound band width, the influence of the heating on the SBS can be neglected ($|\mu'|\tau \ll 1$), therefore $G_{c\,\max} \approx M_c$, $G_{u\,\max} \approx M_u$, and $\gamma \approx 2$. With increase of $|\mu'|\tau$ the values of the growth rates decrease quite rapidly. Thus, $G_{c\,\max}$ decreases to one-half at $|\mu'|\tau \approx 1.5$, and $G_{u\,\max}$ at $|\mu'|\tau \approx 2$. Just as in the preceding case, $G_{c\,\max}$ decreases more rapidly with increase of $|\mu'|\tau$ than $G_{u\,\max}$, and this leads to a decrease of γ and to a deterioration of the quality of the WFR. Since γ changes much more slowly than $G_{c\,\max}$ and $G_{u\,\max}$ with increase of $|\mu'|\tau$ (the value of γ exceeds 1.4 even at $|\mu'|\tau = 20$), the SBS can shut off even before the onset of thermal suppression of the WFR. The frequency detuning $(\delta\Omega)_{\rm opt}$ corresponding to the maximum growth rate turns out to be different for correlated as well as uncorrelated Stokes waves, and increases monotonically with increase of heating (Fig. 2).

The joint analysis of the thermal-suppression effect was carried out also by numerical methods. Figure 3 shows the calculated maximum total growth rate of the correlated

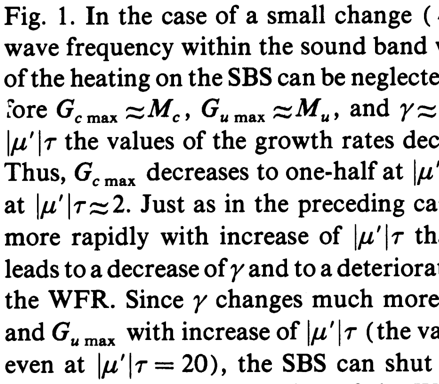

FIG. 2. Optimal initial detuning, normalized to the sound bandwidth, $\Omega_{\rm opt}(0)\tau$, for a correlated (a) and an uncorrelated (b) Stokes wave vs the thermal suppression parameter $|\mu'|\tau(\nu' = 0)$.

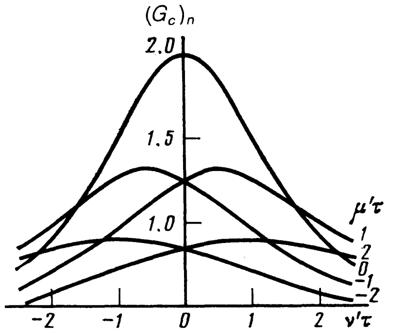

FIG. 3. Normalized total growth rate of a correlated Stokes wave vs the thermal suppression parameter $\nu'\tau$ at $\mu'\tau = -2;\ -1;\ 0;\ 1;\ 2$.

component for the region of the parameters $\nu'\tau$ and $\mu'\tau$, the boundaries of which correspond to a decrease of $G_{c\,\max}$ by more than 2 times. If $\nu'\tau$ and $\mu'\tau$ are of opposite sign, $G_{c\,\max}$ decreases monotonically in absolute value with increase of each of the parameters. If $\nu'\tau$ and $\mu'\tau$ are of the same sign, partial cancellation of the thermal-suppression effect takes place. The maximum growth rate corresponds therefore at $\mu'\tau \neq 0$ to a nonzero value of $\nu'\tau$. Since the modulation of the light-wave frequencies is determined by the average heating, and the change of the sound frequency by the local value, there is no complete cancellation and $G_{c\,\max}$ remains smaller than M_c.

4. ESTIMATES OF THE POSSIBILITY OF WFR FOR SBS IN A PLASMA

Before we consider the influence of heating on WFR for SBS in a plasma, we estimate the minimum threshold pump intensity $\bar{I}_{\min}$ without allowance for thermal effects. The maximum value of the growth rate is reached in a fully ionized plasma of hot electrons and cold ions ($T_e \gg T_i$). Such a plasma is practically always collisionless, i.e., Ω_s exceeds the effective frequency of the ion-ion collisions. In this case the ion-sound attenuation, which determines the sound relaxation time $\tau = 1/\alpha_s v_s$ is due principally to Landau damping[13]

$$\alpha_s = \frac{\omega_0}{c}\left(\frac{\pi Z}{2}\right)^{1/2}\left[\left(\frac{m}{M_i}\right)^{1/2} + \left(\frac{T_e}{T_i}\right)^{3/2} Z\exp\left(-\frac{ZT_e}{2T_i} - \frac{3}{2}\right)\right],$$

where m and M_i are respectively the electron and ion masses. Minimum attenuation α_s ($T_e/T_i \gtrsim 20$) corresponds to a maximum relaxation time

$$\tau = \frac{\lambda}{\pi^{3/2} v_s Z^{1/2}}\left(\frac{M_i}{2m}\right)^{1/2}.$$

Since $v_s = (Z\varkappa T_e/M_i)^{1/2}$, where z is the atomic number of the ions, and $\varkappa$ is the Boltzmann constant, the logal growth rate $g_0 \sim \lambda^3 N_e/T_e$ increases with increase of electron density N_e and with decrease of their temperature T_e. An increase of N_e/T_e, however, is accompanied by a damping decrement

$$\alpha = \nu_{ei} N_e / cN_{\rm cr} \sim N_e^2/T_e^{3/2}$$

(ν_{ei} is the effective frequency of the electron-ion collisions, $N_{\rm cr}$ is the critical density of the electrons). It follows hence that the minimum threshold intensity $\bar{I}_{\min}$ at which the unperturbed growth rate of the correlated Stokes wave

$$M_{c\,\rm tot} = 2g_0\bar{I}_0(1 - e^{-\alpha L})\alpha^{-1} - \alpha L$$

reaches the threshold value ($M_{\rm thr} \approx 20$ for a plasma) corresponds to an optimum optical thickness $(\alpha L)_{\rm opt} \approx 1.09$. On the basis of the last equation it is easy to obtain an expression for the optimal plasma density

$$N_{e\,\rm opt}\,[\mathrm{cm}^{-3}] \approx 3.5\cdot 10^{18}\frac{T_e^{3/2}[\mathrm{eV}]}{(Z\ln\Lambda\cdot L[\mathrm{cm}])^{1/2}\lambda[\mu\mathrm{m}]}$$

($\ln\Lambda \approx 5$–10 is the Coulomb logarithm) and the corresponding maximum nonlinearity constant

$$g_{0\,\rm opt}\left[\frac{\mathrm{cm}}{\mathrm{MW}}\right] \approx 6.3\cdot 10^{-4}\left(\frac{M_i/m}{Z\ln\Lambda\cdot L[\mathrm{cm}]}\right)^{1/2}\frac{\lambda^2[\mu\mathrm{m}]}{T_e^{1/4}[\mathrm{eV}]}.$$

Equating $M_{c\,\rm tot}$ to the threshold value, we get

$$\bar{I}_{\min}\left[\frac{\mathrm{MW}}{\mathrm{cm}^2}\right] \approx 5.6\cdot 10^{4}\left(\frac{Z\ln\Lambda}{L[\mathrm{cm}]}\right)^{1/2}\frac{T_e^{1/4}[\mathrm{eV}]}{\lambda^2[\mu\mathrm{m}]}.$$

For example, for a hydrogen plasma with length $L = 10$ cm and $T_e = 10$ eV we obtain in the $\lambda = 10\ \mu$m band the relatively low value $\bar{I}_{\min} \approx 0.6$ GW/cm^2. Let us see how these estimates change when electron heating by laser radiation is taken into account.

As already noted, suppression of WFR and SBS in a plasma by sweeping the frequencies of the pump and of the Stokes waves sets in after the time in which the sound travels over the characteristic scale of the pump wave. For a plane wave, this scale is the beam radius a ($t_a = a/v_s$). In the case of multimode pumping, the suppression is turned on after the sound has negotiated the inhomogeneity scale $t_\rho = \rho/v_s$. Since the discrimination γ falls off slowly with increase of $|\nu'|\tau$, the WFR shutoff is determined by the suppression of the SBS, the threshold of which $|\nu'|\tau \approx 0.5$ is introduced in analogy with Sec. 5. Recognizing that $g_4 = (cn/2\pi)\alpha/kN_e$ for a plasma in the adiabatic approximation ($dn/dT_e = -3(N_c/N_{\rm cr})T_e^{-1}/2$), we get $\nu' = -3kI_0/2kN_{\rm cr}T_e$. It is clear that the minimum of $|\nu'|$ when SBS is realized is reached for the minimum threshold pump intensity $JI_{\min}$. Following the substitution $JI_0 = JI_{\min}$ the condition $|\nu'|\tau \lesssim 0.5$ for the absence of suppression reduces to the following condition on the initial plasma temperature

$$T_{e0}\,[\mathrm{eV}] \gtrsim 4.2\cdot 10^3 (M_i/m)^{4/5}(\ln\Lambda/ZL[\mathrm{cm}])^{2/5},$$

the form of which is independent of λ. It follows from this inequality that the required plasma temperature must exceed 1 keV, which is difficult to obtain in experiment. Estimates show, however, that high thermal conductivity of the electrons can equalize the small-scale profile of the distribution of the temperature and limit the average heating. Over times exceeding the time τ_ρ ($t > \tau_\rho$) of heat transport over the inhomogeneity scale, heating of the medium does not influence the WFR in SBS. If $t > \tau_a$, on the other hand, where τ_a is the time of heat transport over the beam scale, the heating ceases to affect also the SBS. If the heat transport is determined by the free-passage mechanism (the electron mean free path l_f exceeds the considered transverse scale) and takes place at thermal velocities, the time of passage of the ion sound cannot in principle be shorter than the time of heat transport ($v_e/v_s \gg 1$, where v_e is the average electron velocity), thus excluding the thermal suppression effect. When the heat transport is by diffusion (l_f is shorter than the transverse scale), the ratio of the ion-sound travel time to the heat-transport time can vary and is determined by the specific parameters of the plasma. Estimates show that the free-passage mechanism can take place only for a small scale ρ. In this case the suppression of the WFR (the lowering of γ) does not take place, since $\tau_\rho \ll t_\rho$, but at $t_a < \tau_a$ suppression of the SBS (decrease of the growth rate) does take place in the time interval $t_a \lesssim t \lesssim \tau_a$. If $t_\rho < \tau_\rho$, suppression of both the WFR and the SBS can take place in the time interval $t_\rho \lesssim t \lesssim \tau_\rho$. The WFR is no longer suppressed at $t > \tau_\rho$, but the SBS suppression remains substantial all the way to $t \approx \tau_a$.

A high threshold pump intensity under the restricted dimensions of a laboratory plasma (L usually does not exceed several centimeters) is reached by sharp focusing of the radiation.[1)] In this case the pump-beam radius does not exceed several millimeters. It is easy to meet in such beams the conditions for the absence of suppression, in view of the modulation of the refractive index ($\tau_\rho < t_\rho$, $\tau_a < t_a$) because of the high rate of heat transport by the electrons. For example, at the optimal plasma parameters, corresponding to minimum threshold pump intensity in the $\lambda = 10\ \mu$m and $L = 1$–10 cm ranges, at an electron temperature $T_e \lesssim 40$ eV the heat spreads out by diffusion ($\tau_\rho \approx \rho^2/\pi^2 D$, $\tau_a \approx \tau_\rho a^2/\rho^2$), where $D = 5\cdot 10^{20} T_e^{5/2}[\mathrm{eV}]/N_e[\mathrm{cm}^{-3}]Z \ln \Lambda$ (Ref. 13) is the thermal conductivity), and takes place in the temperature range 10 eV $\lesssim T_e \lesssim 40$ eV before the ion sound can pass through. At higher temperature, the heat transport is even faster. At $T_e > 40$ eV, over a small scale of ρ, it is determined by the free-passage mechanism, thus excluding suppression of the WFR by modulation of the refractive index.

A distinctive feature of the thermal suppression of WFR and SBS as a result of a change $\delta\Omega_s$ of the ion-sound frequency is that this change is cumulative in time. The effect itself, however, becomes substantial only after a time t^*, during which $\delta\Omega_s \sim \mu'$ reaches a value comparable with the ion-sound bandwidth ($\sim 1/\tau$). A criterion for the shutoff of the SBS, and hence also of the WFR, is the value $|\mu'|\tau \approx 0.5$. Estimates show that even at minimum threshold intensity t^* does not exceed several nanoseconds. Even in this case, however, the SBS is not suppressed if the time τ_a of heat transport in the beam scale is shorter than the time t^*. The condition under which heating does not influence the discrimination γ of the growth rates ($\tau_\rho < t^*$) is met in this case automatically. Note that SBS suppression in the intermediate case $\tau_\rho < t^* \lesssim \tau_a$ is due only to the inhomogeneity of the average heating along z. It is practically impossible, however, to eliminate it by decreasing the optical thickness αL, since a change of αL decreases the total growth rate of the SBS, thus requiring an increase of $\bar{I}_0$ if M_{thr} is to be reached.

Calculations show that the suppression of the WFR in SBS, in a plasma of a focused beam ($a \sim 1$ mm) from a CO_2 laser, can take place ($t^* < \tau_\rho$) only at sufficiently low electron temperature, $T_e \lesssim 10$ eV. At $T_e > 10$ eV the principal mechanism of WFR suppression is the shutoff of the SBS itself on account of inhomogeneous average heating along the z, a heating that can also be limited by heat escape to the outside of the pump beam. At optimal hydrogen-plasma concentration corresponding to $\bar{I}_{\min}$, the required electron temperature is $T_e \gtrsim 50$ eV.

Thus, from the results of estimates in which account is taken of the possible effect of heating on WFR in SBS in a plasma, it follows that for its realization ($\tau_\rho < t_\rho$, $\tau_a < t_a$; $\tau_a < t^*$) it is necessary to have a plasma with a rather high initial temperature. Thus, for WFR of the radiation of CO_2 laser in SBS in a hydrogen plasma at $L = 5$ cm, $a = 1$ mm, and $a/\rho \sim 30$, the electron temperature must be not lower than $T_e \approx 75$ eV. Such a temperature corresponds to $N_{e\,\mathrm{opt}} \approx 1.4\cdot 10^{18}$ cm^{-3} and $\bar{I}_{\min} \approx 2$ GW/cm^2 ($\tau_a \approx 0.35$ ns, $t_a \approx 0.65$ ns, $t^* \approx 0.5$ ns).

Owing to the low threshold pump intensity compared with a plasma, and to the substantially higher (by 4–5 orders) heat capacity C_p, the change of the sound frequency in solids and liquids by heating is usually so small that this effect can be neglected. However, thermal suppression of SBS by modulation of the refractive index can be appreciable. For example, in pure Ge specimens ($dn/dT \approx 4\cdot 10^{-4}$ deg^{-1}, $\rho_0 \approx 5.3$ g/cm^3, $C_p \approx 0.32$ J/cm$^3\cdot$deg, $\alpha \approx 3\cdot 10^{-2}$ cm^{-1}, Ref. 14), at a CO_2-laser threshold intensity $\bar{I} \approx 50$ MW/cm^2 corresponding to an optical thickness $\alpha L \sim 1$, the parameter of the thermal suppression of WFR in SBS reaches a value $\nu'\tau \sim 1$ sufficient to stop the SBS. Although the low sound velocity ($V_s \sim 10^5$ cm/s) causes the suppression of WFR and SBS in solids and in liquids to set in with sufficient delay ($t_\rho > \lambda/v_s \sim 10$ ns), it is impossible to eliminate it with the aid of heat transport as in a plasma.

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[1)] To produce WFR in SBS of focused beams it is necessary that the length of the focal neck $L_f = 2ka\rho$ not exceed the length of the nonlinear medium ($L_f \lesssim L$), something likewise attainable by sharp focusing of the beam.

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