# Galvanomagnetic manifestations of weak interactions that violate spatial parity

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The effect of a spatial-parity-violating weak interaction between neutral electron currents, on the one hand, and currents of nuclei, on the other, on the behavior of conduction electrons in magnets is considered. The possible galvanomagnetic effects are estimated.

The influence of weak interactions on the behavior of conduction electrons is obviously weak and is of interest only as a possible new independent way of observing weak interactions. The largest effect should be expected in conducting magnetized media, in which the conduction-electron spins are highly ordered in view of the large exchange forces. This question was previously touched upon in a paper by Labzovskiĭ,[1] but the results there are in error. According to Labzovskiĭ a weak spatial-parity violating interaction of neutral electrons and nuclei, of the form[2]

$$\mathscr{H}^w=\frac{1}{m}QZ\mathbf{s}(\mathbf{p}u+u\mathbf{p}) \tag{1}$$

(where $Q$ is a parameter proportional to the $\beta$-decay Fermi constant, $Z$ is the ratio of the charges of the nucleus and of the electron,

$$u=\sum_k\delta(\mathbf{r}-\mathbf{r}_k),\qquad \mathbf{p}=-i\hbar\nabla,$$

$\mathbf{r}_k$ are the coordinates of the nuclei, $\mathbf{s}$ is the electron spin, and $m$ is its mass) produces in a magnet a dc current proportional to $Q$ and to the length of the vector $[\langle\mathbf{s}\rangle\times\mathbf{B}]$, where $\mathbf{B}$ is the magnetic induction in the medium and $\langle\mathbf{s}\rangle$ is the average polarization of the conduction-electron spins.

It is obvious, however, that a constant magnetic field cannot produce a dc current in a medium of finite conductivity in the absence of a constant magnetic field (this was precisely the subject of Ref. 1), no matter whether the interactions conserve parity or not. The reason is simply that the energy conservation law would be violated, since a constant magnetic field does not transport energy—the Poynting vector is zero.

Analysis of the errors in the arguments of Ref. 1 makes it easy to correct the analysis there and estimate the galvanomagnetic manifestations (1) of weak interactions in magnets under nonequilibrium conditions. This is the subject of the present note. The gist of the reasoning of Ref. 1 is the following. The Hamiltonian of the conduction electron regarded as a free particle takes, when the interaction (1) is taken into account, the form (apart from small terms of order $Q^2$)

$$\mathscr{H}=\frac{1}{2m}\left(\mathbf{p}-\frac{e}{c}\mathbf{A}+2QZu\mathbf{s}\right)^2+e\varphi-\frac{e}{mc}\mathbf{B}\mathbf{s}, \tag{2}$$

where $\mathbf{R}=\text{curl}\mathbf{A}$, and $\mathbf{A}$ and $\varphi$ are the vector and scalar potentials of the magnetic and electric fields. The forces acting on a particle are given by

$$\mathbf{F}=m\dot{\mathbf{v}}=\frac{i}{\hbar}m(\mathscr{H}\mathbf{v}-\mathbf{v}\mathscr{H}),$$

where

$$\mathbf{v}=\dot{\mathbf{r}}=\frac{i}{\hbar}(\mathscr{H}\mathbf{r}-\mathbf{r}\mathscr{H}),$$

and contain in accordance with (2) the potential electric and magnetic forces $-e\nabla\varphi$ and $e\nabla(\mathbf{B}\cdot\mathbf{s})/mc$, the Lorentz force $e[\mathbf{v}\times\mathbf{B}]/c$ and also the term

$$\mathbf{F}=\frac{2eQZ}{mc}u[\mathbf{s}\mathbf{B}]. \tag{3}$$

The force (3) is of interest because in a constant field $\mathbf{B}$ it is analogous, averaged over the volume, to the action of an extraneous electric field $\mathbf{E}^w$ in the $[\langle\mathbf{s}\rangle\times\mathbf{B}]$ direction.[1)] This leads according to Ref. 1 to the possible existence of direct currents in magnets, since the polarization $\langle\mathbf{s}\rangle$ is manifest in them both by the magnetic field $\mathbf{b}$ and by a molecular field not of magnetic character, and $\langle\mathbf{s}\rangle$ can have an equilibrium orientation not collinear with $\mathbf{B}$.

The fallacy in this reasoning is that a molecular energy of nonmagnetic character is not accounted for directly in the model (2). The general structure of the corresponding molecular energy, since we are dealing with particles having spin 1/2, is

$$\mathscr{H}^{mol}=-e\mathbf{s}\mathbf{H}^{mol}/mc, \tag{4}$$

where $\mathbf{H}^{mol}$ is a molecular field independent of $\mathbf{s}$. Addition of the energy (4) to (2) modifies the potential magnetic force $e\nabla(\mathbf{B}\cdot\mathbf{s})/mc$ and the force (3). The vector $\mathbf{B}$ is replaced in these forces by the sum $\mathbf{H}^{mol}+\mathbf{B}$. Since the equilibrium orientation of $\langle\mathbf{s}\rangle$ is collinear with the resultant field $\mathbf{H}^{mol}+\mathbf{B}$, now the force $\mathbf{F}^w$ vanishes on the average and no electric current is produced, as should indeed be the case in accordance with the energy conservation law.

It is clear that not only the dc current of conduction electrons, but also constant flows of any particles with different values of the spin $s$ cannot be produced in a constant magnetic field via interactions that violate spatial parity, if the resistance to the particle motion in the medium is finite.

Let us estimate the temporal effects. We assume, as in Ref. 1, that the conduction-electron spin polarization is total and assume that $\langle u\mathbf{s}\rangle\approx\langle u\rangle\langle\mathbf{s}\rangle$. The electron density $\langle u\rangle$ at the nucleus increases with increase of $Z$. For outer atoms in atoms with large $Z$ we obtain, from known quasiclassical considerations and from estimates of the relativistic enhancement of the attraction near the nucleus (see, e.g., Ref. 2, p. 31), in order of magnitude,

$$\langle u\rangle=\varkappa q a_0^{-3}Z,$$

where $a_0$ is the Bohr radius, $q$ is the relativistic enhancement factor, and $\varkappa$ the weight of the $s$-waves in the Bloch functions of the conduction electron. For $Z\approx 80$ we have $q\approx 10$. For many metals $\varkappa\approx 1$. Note that the estimate of $\langle u\rangle$ in Ref. 1

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contains for some reason a small factor of the order of $10^{-3}$, which seems to be an error.

Taking into account the replacement of **B** in (3) by $\mathbf{H}^{mol} + \mathbf{B}$, we obtain for the extraneous field $\mathbf{E}^w$, which is equal to the mean value of $\mathbf{F}^w/e$ at the instant of time $t$, the estimate

$$\mathbf{E}^w = K[\mathbf{H}^{ef}\mathbf{M}]/M, \tag{5}$$

where **M** is the specific (per unit volume) magnetization of the conduction electrons at a given instant of time. Instead of $\mathbf{H}^{mol} + \mathbf{B}$ there was substituted in (5) a field that does not alter the results

$$\mathbf{H}^{ef} = \mathbf{H}^{mol} + \mathbf{B} - 4\pi\mathbf{M} = -\delta w/\delta\mathbf{M}$$

($w$ is the energy density in the magnet). The dimensionless (in the Gaussian system) parameter $K$ is equal to

$$K \approx QZ^2\varkappa q\hbar/mca_0^3 \approx 0{,}5\cdot 10^{-5}\varkappa q Z^2 ( \text{[illegible]} (e^2/\hbar c)^3 \approx 4\cdot 10^{-14}, \tag{6}$$

where $m_p$ is the proton mass, and the numerical estimate corresponds to $\varkappa q = 10$ and $Z = 80$. We shall use for $K$ the round number $10^{-11}$. At a nonequilibrium orientation of **M**, when the component $\mathbf{H}^{ef}$ normal to **M** is equal to $10^4$ Oe, it yields $E^w = 3\cdot 10^{-8}$ V/cm. Can such a field actually be observed in practice?

Since $\mathbf{E}^w$ appears under nonequilibrium conditions, it is obviously accompanied by a solenoidal electric field **E** induced by the changes of **B**. Let us compare $\mathbf{E}^w$ and **E**.

The changes of **M** are described by the equation

$$\dot{\mathbf{M}} = \gamma[\mathbf{M}\mathbf{H}^{ef}] + \mathbf{R}, \tag{7}$$

where $\gamma = e/mc$, **R** is a relaxation term that tends to align **M** with the field $\mathbf{H}^{ef}$. Neglecting **R** we have

$$\mathbf{E}^w = -K\dot{\mathbf{M}}/\gamma M. \tag{8}$$

Consider a one-domain sample in which homogeneous oscillations of $M$ take place in a constant external magnetic field. From the Maxwell equation $\mathrm{curl}\mathbf{E} = -(\partial\mathbf{B}/\partial t)/c$ we obtain, in order of magnitude,

$$E \approx 4\pi\dot{M}l/c, \tag{9}$$

where $l$ is the characteristic dimension of the sample. [For a sample in the form of a thin disk, when **M** precesses in the plane of the disk, $l$ is the disk thickness. The field **E**, just as the field (8), lies then in the plane of the disk, but is not homogeneous over its thickness: it is equal to zero in the midplane and is oppositely directed on opposite sides of this plane. For a sample in the form of a long cylinder, when $\mathbf{M}\perp\mathbf{n}$, where **n** is directed along the cylinder axis, $l$ is the cylinder radius, and the field $\mathbf{E}\|\mathbf{n}$ and is not uniform (alternates in sign) in the cross section.] Both fields vary synchronously and their ratio is

$$E^w/E \approx K\lambda_M/l, \tag{10}$$

where $\lambda_M = c/4\pi\gamma M$ and is equal to $\approx 1$ cm at $M \approx 10^2$ G. Recognizing that $K \approx 10^{-14}$ we see from (10) that even at $\lambda_M/l = 10^6$ the field $E^w$ is weaker than $E$ by 8 orders of magnitude.

It is difficult to hope to observe oscillations of $\mathbf{E}^w$ against the background of such predominant oscillations of **E**, notwithstanding the difference between the spatial distributions and polarization of the two fields.

The external field **H** was assumed constant in the estimates. In alternating fields, when $\dot{H} \ll 4\pi\dot{M}$, the same estimate (10) is obtained, while at $\dot{H} \gg 4\pi\dot{M}$ the ratio $E^w/E$ becomes even smaller, decreasing by a factor $\dot{H}/4\pi\dot{M}$. Optimal conditions would be if alternating fields were applied in the intermediate region to cancel out the demagnetization fields, so that $\mathbf{B} = 0$ in the medium. The cancellation required must be at least accurate to $10^{-5}$ and is not attainable in practice.

The relaxation term **R** was neglected in the foregoing estimates. Generally speaking, allowance for this term alters the situation. Thus, in a stationary (or quasistationary) **M**-precession regime, the presence of **R** can produce a constant (or slowly varying) component of $\mathbf{E}^w$, whereas the solenoidal field **E** does not contain such a component and the mean field $\langle\mathbf{E}^w\rangle$ or the mean current due to it can be filtered out. Assume that on the average $\langle\dot{\mathbf{M}}\rangle = 0$ in the stationary regime, it follows from (5) and (7) that

$$\langle\mathbf{E}^w\rangle = K\langle\mathbf{R}\rangle/\gamma M. \tag{11}$$

Interest can attach apparently to conditions when $R$ is relatively small, since heating of the system becomes appreciable with increase of $R$. With an aim at estimating the effects for like samples, we express **R** in the Gilbert form:

$$\mathbf{R} = \alpha[\mathbf{M}\dot{\mathbf{M}}]/M, \tag{12}$$

where $\alpha$ is a dimensionless relaxation parameter. High-grade samples have in the microwave region $\alpha \approx 10^{-3}$.

According to (12) and (11), if the vector **M** precesses in a plane, the vector $\langle\mathbf{E}^w\rangle$ is normal to the plane and in the case of precession at a frequency $\omega$ is given by

$$\langle E^w\rangle = \frac{\alpha\omega}{\gamma}\frac{M_\perp^2}{M^2}K, \tag{13}$$

where $M_\perp^2 = M_1M_2$, where $M_1$ and $M_2$ are the principal axes of the ellipse traced by the vector **M**. Expression (13) vanishes if the **M** oscillations are linearly polarized and is a maximum for circular polarization.

According to (13), the momentum of the force applied an the average to an electron during the time of free (in a constant external field) relaxation of **M** from the state $M_\perp = M$ to a state $M_\perp = 0$, is of the order of $eK/\gamma$. This is clear from the fact that the **M** relaxation time is $1/\alpha\omega_0$, where $\omega_0$ is the natural frequency of the homogeneous precession in the given field $\mathbf{H}^{ef}$. The momentum $eK/\gamma$ corresponds to an electron-velocity increment

$$\Delta v = eK/\gamma m = cK \approx 3\cdot 10^{-4}\ \text{cm/s}. \tag{14}$$

If the sample is a flat plate and the field $\mathbf{H}^{ef}$ is normal to it, then by attaching electrodes to both planes of the plate and closing the circuit, we obtain in the limit of zero resistance of the entire circuit, a "ballistic" current

$$I_b \approx necKS \approx 5\cdot 10^{-2}\ \text{A},$$

where $n$ is the density of the conduction electrons in the sample and $S$ is the area of the plate, and the values $n = 10^{21}$ cm$^{-3}$ and $S = 1$ cm$^2$ were used. Its maximum value, it the impedance $z(\omega)$ of the circuit changes little in the region $\omega \approx \alpha\omega_0$, is

$$I_a \approx \alpha\omega_0 dK/\gamma|z| \approx 10^{-7} \text{ A}, \quad (15)$$

where $d$ is the plate thickness; it was assumed in the estimates that $d = 1$ cm, $\omega_0/2\pi = 1$ GHz, and $|z| = 10^{-5}\ \Omega$. Note that it is possible to decrease both the imaginary part of the circuit impedance and the real one, by using active elements in the electric circuit. If a stationary precession with $M_\perp^2/M^2 = 10^{-6}$ is maintained in the plate at a frequency 1 GHz, the dc current flowing through the entire circuit will be

$$I_0 = \frac{\alpha\omega d}{\gamma r_0}\frac{M_\perp^2}{M^2}K \approx 10^{-13}\text{A}, \quad (16)$$

where $r_0$ is the circuit resistance at zero frequency, with the value $r_0 = 10^{-5}\ \Omega$ used in the estimate.

Is it really possible to increase substantially the values of $\alpha$ and $\omega$? The point is that increasing these parameters increases the dissipated power. Its specific power for the relaxation (12) is

$$P = \alpha\langle \dot{M}^2\rangle/M = \alpha\omega^2(M_1^2 + M_2^2)/2M. \quad (17)$$

It increase like $\omega^2$. At $\alpha = 10^{-3}$, $\omega/2\pi = 1$ GHz, $M = 10^2$ G, and $M_1 = M_2 = 10^{-3}M$ we obtain $P = 4\cdot 10^5$ W/cm$^2$. Magnetic films are preferable from the standpoint of heat transfer, but even if they are 1 $\mu$m thick the power obtained is 0.4 W/cm$^2$. This is a large thermal load for continuous operation. Besides the heating, the temperature gradients are of great significance. These gradients give rise to a thermoelectric power. The characteristic value of the thermoelectric coefficient is $10^{-6}$ V/deg, so that at a gradient $\nabla T = 1$ deg/cm we have a thermoelectric field intensity $E^T = 10^{-6}$ V/cm. It is clear therefore that in the general case the field $E^T$ is not small and experiment skill is needed to separate the field $\langle \mathbf{E}^w \rangle$ against the background $\mathbf{E}^T$, even though the direction of $\mathbf{E}^T$ is determined by the gradient $\nabla T$ and that of $\langle \mathbf{E}^w \rangle$ by the vector $\langle \mathbf{R} \rangle$.

[1)]Recognizing that the exchange forces orienting the spins in magnets are large enough, one can approximately assume in the phenomena considered that $\langle u\mathbf{s}\rangle = \langle u\rangle\langle \mathbf{s}\rangle$.


[1]L. N. Lazovskiĭ, Zh. Eksp. Teor. Fiz. **89**, 1921 (1985) [Sov. Phys. JETP **62**, 1108 (1985)].

[2]P. B. Khriplovich, Parity Nonconservation in Atomic Phenomena [in Russian], Nauka, 1981.


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