Global structure of inflationary universe

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The global structure of the universe is analyzed in the chaotic inflation scenario. It is shown that under certain conditions the expansion of the universe in this scenario does not have an end and may not have a beginning. The greater part of the physical volume of the universe must always be in the inflation stage at a density of the order of the Planck density. During the time of inflation, the universe separates into exponentially large regions, within which all possible types of metastable vacuum states and all possible types of compactification compatible with the presence of the inflation regime are realized. The investigation is made by means of the diffusion equation for a fluctuating scalar field φ in the inflationary universe.

1. INTRODUCTION

In recent years, the main hopes for the creation of a consistent cosmological theory have been based on the development of the inflationary universe scenario. By means of this scenario, it has been possible to solve numerous problems relating to the earliest stages in the evolution of the universe. Much less well known (but no less important) are the consequences of this scenario relating to the structure of the universe in the large. The essence of these new insights is that the inflationary universe appears locally as part of a homogeneous and almost flat Friedmann universe, but the global structure of the universe is very different from the Friedmann geometry.[1–4] The corresponding differences do not affect the structure of the observable part of the universe, and in this sense they are of no interest from a narrowly pragmatic point of view. However, the differences between the global structure of the universe and the structure of the Friedmann universe are of the greatest importance when one is considering the problem of the origin of the universe (the singularity problem), the future of the universe, the application of the so-called anthropic principle in cosmology, etc.[1–4]

Historically, many different variants of the inflationary universe scenario have existed.[5–9] From our point of view, the simplest and most natural of them is the so-called chaotic inflation scenario[9] (for a review of the present status of the inflationary universe scenario see Ref. 1). This scenario can be realized in a large class of theories, including theories of scalar fields with polynomial effective potentials, $V(\varphi) \sim \varphi^n$ (Ref. 9), in an extended variant of Starobinskiĭ's model,[10] in grand unification theories,[11] in $N = 1$ supergravity,[12] in Kaluza–Klein theories,[13] and in superstring theory.[14 1)]

Recent investigations have shown that in this scenario inflation of the universe never ends in the greater part of the physical volume of the universe.[2–4] The universe is broken up into a large number of locally Friedmann regions, in each of which the properties of space-time and elementary particles can be different. The aim of this paper is to make a more detailed study of this question by analyzing solutions of the diffusion equation for a fluctuating scalar field φ in an inflationary universe.[15,16]

In Sec. 2, we recall the general scheme of the scenario of chaotic inflation, and in Sec. 3 the elementary theory of quantum fluctuations of the scalar field during the time of inflation and the influence of these fluctuations on the global structure of the universe. Section 4 contains an analysis of solutions of the diffusion equation for the scalar field φ, and in Sec. 5 we discuss the significance of our results for the development of our ideas about the global structure of the universe. The Appendix contains a brief derivation[17] of the diffusion equation that we employ.[15,16]

2. CHAOTIC INFLATION

To illustrate the basic idea of chaotic inflation, we consider the very simple theory of a scalar field φ with Lagrangian

$$L = -\frac{M_P^2}{16\pi} R + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{\lambda}{4}\varphi^4. \tag{2.1}$$

Here, $M_P^{-2} = G$ is the gravitational constant, $M_P \sim 10^{19}$ GeV is the Planck mass, R is the curvature scalar, and $\lambda \ll 1$. We consider a region of the universe with scale greater than $2H^{-1}$ (H^{-1} is the radius of the event horizon in the inflationary universe, $H = \dot a/a$ is the Hubble parameter, and $a(t)$ is the scale factor of the universe). If the classical field φ is sufficiently homogeneous and varies sufficiently slowly in this region ($\partial_\mu\varphi\partial^\mu\varphi \leqslant V(\varphi)$), its behavior in this region does not depend on the physical processes outside the horizon[18,19] and is determined by the Klein-Gordon equation for the homogeneous field φ,

$$\ddot\varphi + 3H\dot\varphi = -dV/d\varphi, \tag{2.2}$$

where $V(\varphi) = \lambda\varphi^4/4$ is the effective potential of the scalar field in the theory (2.1), and also by the Einstein equation

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3M_P^2}\left[\frac{1}{2}\dot\varphi^2 + V(\varphi)\right], \tag{2.3}$$

where $k = \pm 1, 0$, respectively, for a (locally) closed, open, or flat universe. For $\varphi \gtrsim M_P$, the solution of Eqs. (2.2), (2.3) reaches rapidly (within a time of order H^{-1}) the asymptotic regime:

$$\varphi(t) = \varphi_0\left\{\exp\left[-\frac{\lambda^{1/2}M_P}{(6\pi)^{1/2}}t\right]\right\}, \tag{2.4}$$

$$a(t) = a_0\left\{\frac{\pi}{M_P^2}\left[\varphi_0^2 - \varphi^2(t)\right]\right\}. \tag{2.5}$$

In this regime $\dot\varphi^2 \ll V(\varphi)$, $\ddot\varphi \ll 3H\dot\varphi$, and $\dot H \ll H^2$. The last inequality means that during the time $\Delta t \lesssim H^{-1}$

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$(\Delta t \lesssim (6\pi)^{1/2}/\lambda^{1/2} M_P)$ the value of $H(\varphi)$ hardly changes and the universe expands quasiexponentially (it inflates):

$$a(t+\Delta t) \approx a(t)\, e^{H\Delta t}. \tag{2.6}$$

The inflation regime is realized for $\varphi \gtrsim M_P/3$. For $\varphi \lesssim M_P/3$, the field φ oscillates rapidly and its potential energy $V(\varphi \sim M_P/3) \sim \lambda M_P^4$ goes over into thermal energy $\sim T^4$. The temperature of the universe T_R after its heating may be of order $\lambda^{1/4} M_P$ or less, depending on the strength of the interaction of the field φ with the other fields. It is important that T_R does not depend on the initial value φ_0 of the field φ for $\varphi_0 \gg M_p$. The only parameter that depends on φ_0 is the scale factor $a(t)$, which increases by $\exp(\pi\varphi_0^2/M_P^2)$ times during the inflation time.

If, as is usually assumed, description of the universe in terms of a classical space-time becomes possible when the matter energy-momentum tensor becomes less than M_P^4, then at this instant $\partial_\mu \varphi \partial^\mu \varphi \lesssim M_P^4$ and $V(\varphi) \lesssim M_P^4$. Therefore, the only restriction on the initial amplitude $\varphi = \varphi_0$ of the field in the considered theory is the condition $l\varphi^4/4 \lesssim M_P^4$, and the typical initial value of the field φ is of the order

$$\varphi_0 \sim \lambda^{-1/4} M_P. \tag{2.7}$$

To be definite, we consider a closed universe of typical initial scale $O(M_P^{-1})$. One can show that if primordially $\partial_\mu \varphi \partial^\mu \varphi \lesssim V(\varphi) \sim M_P^4$ in this universe, then after a time of order M_P^{-1} the value of $\partial_\mu \varphi \partial^\mu \varphi$ becomes much less than $V(\varphi)$, and the subsequent evolution of the universe is described by Eqs. (2.2)–(2.5). After inflation, the total scale of the universe is in accordance with (2.5)

$$l \sim M_P^{-1} \exp(\pi\varphi_0^2/M_P^2) \sim M_P^{-1} \exp(\pi/\lambda^{1/2}). \tag{2.8}$$

For $\lambda \sim 10^{-12}$ (see below) this gives $l \sim 10^{10^6}$ cm, many orders of magnitude greater than the scale ($\sim 10^{28}$ cm) of the observable part of the universe.

As a result of the inflation, the term k/a^2 in (2.3) becomes negligibly small compared with H^2, and this means that the universe becomes flat and its geometry locally Euclidean. For similar reasons, the universe becomes locally homogeneous and isotropic. The density of all "undesirable" objects (monopoles, domain walls, etc.) created before or during the time of inflation becomes exponentially small, and they are never recreated if the temperature T_R is not too high.

We wish to emphasize that for the realization of this scenario it is sufficient if the condition $\partial_\mu \varphi \partial^\mu \varphi \lesssim V(\varphi) \sim M_P^4$ is satisfied in the region of the smallest possible scale $O(M_P^{-1})$. Since the inequality $\partial_\mu \varphi \partial^\mu \varphi \lesssim M_P^4$ is satisfied in any region of classical space-time, this condition seems completely natural.[20] (Moreover, the condition $(\partial_0 \varphi)^2 \lesssim V(\varphi)$ can in reality be significantly relaxed.[10,21,20])

We have discussed above the general scheme of the chaotic inflation scenario without allowance for quantum effects. These effects are indeed unimportant for the study of the local structure of the inflationary universe. It is however remarkable that it is precisely the quantum effects that determine the structure of the universe on the very greatest scales.

3. QUANTUM FLUCTUATIONS IN THE INFLATIONARY UNIVERSE

As was shown in Ref. 22, inflation leads to the generation of very specific quantum fluctuations of the scalar field φ. The spectrum of these fluctuations with wavelength $l < H^{-1}(\varphi)$ has the same form as the usual spectrum of quantum fluctuations in Minkowski space. At the same time, the spectrum of the fluctuations with momentum $k = l^{-1} < H$ has the form of the spectrum of particles in quantum statistics with anomalously large occupation numbers, $n_k \sim (H/k)^3 \gg 1$. For this reason, the quantum fluctuations with wavelength $l \gg H^{-1}$ can be interpreted as inhomogeneities $\delta\varphi(x)$ in the distribution of the *classical* scalar field φ.[22,23] The description of this phenomenon is completely analogous to the description of Bose condensation in samples of finite size.[24] This effect is the basis of the theory of the formation of density inhomogeneities $\delta\rho(x)$ during the time of inflation[25–27] (in the considered case, these inhomogeneities are proportional to the inhomogeneities $\delta\varphi(x)$ of the scalar field[26,27]). Referring the reader to the original studies[22,23,26,27] for a detailed discussion of this effect (see also Refs. 15–17 and the Appendix), we recall here the basic phenomenological features of the process of formation of the inhomogeneities $\delta\varphi(x)$ of the classical field φ during the time of inflation.

The field $\varphi(x)$ in the inflationary universe can be represented as a sum of a relatively homogeneous (over scales much greater than H^{-1}) classical field φ_c that satisfies Eq. (2.2) and quantum fluctuations $\delta\varphi(x)$ near φ_c. As in Minkowski space, the quantum fluctuations can be represented in the form of a set of waves with different momenta **k**. However, in the inflationary universe the momentum **k** corresponding to each given wave is exponentially decreased by the inflation. When the corresponding wavelength $\sim k^{-1}$ becomes greater than H^{-1} the fluctuations of the field in this wave cease and there develops in space a field distribution $\delta\varphi(x)$ whose characteristic wavelength continues to increase as e^{Ht}, while the amplitude hardly depends on the time ($\delta\varphi(x)$ decreases slowly as $\dot\varphi$). The reason for the "freezing" of the long-wave fluctuations of the field φ is to be sought in the presence of the term $3H\dot\varphi$ in (2.2), which has the meaning of a friction force. In the subsequent expansion of the universe, more and more new fluctuations $\delta\varphi(x)$ of the scalar field "freeze" in amplitude, having at the time of freezing the wavelength $l \sim H^{-1}$. As a result of this, there are formed on the background of the originally homogeneous classical field φ inhomogeneities $\delta\varphi(x)$ with characteristic wavelength H^{-1}. These inhomogeneities are stretched (without their amplitude changing), the distribution of the field φ again becomes homogeneous, inhomogeneities of wavelength H^{-1} again arise on this background, etc. The rms amplitude of the inhomogeneities generated during the time t is

$$\Delta = [\langle \delta\varphi^2(x) \rangle]^{1/2} = \frac{H}{2\pi}(Ht)^{1/2} = \left[\frac{\lambda^{3/2}}{3(6\pi)^{1/2}} \frac{\varphi^6}{M_P^3} t\right]^{1/2}. \tag{3.1}$$

In particular, the rms amplitude of the inhomogeneities $\delta\varphi(x)$ with wavelength $\sim H^{-1}$ generated during the characteristic time $\Delta t = H^{-1}$ is

$$|\delta\varphi(x)| \sim \frac{H}{2\pi} \sim \frac{1}{M_P}\left[\frac{2V(\varphi)}{3\pi}\right]^{1/2} \sim \frac{\lambda^{1/2}}{(6\pi)^{1/2}} \frac{\varphi^2}{M_P}. \tag{3.2}$$

During this same time $\Delta t = H^{-1}$ the classical field φ_c is decreased in accordance with (2.4) by

$$\Delta\varphi(x) = M_P^2/2\pi\varphi. \tag{3.3}$$

Comparison of (3.2) and (3.3) shows that for $\varphi \ll \varphi^*$, where

$$\varphi^* = \lambda^{1/6} M_P, \tag{3.4}$$

we have $|\delta\varphi(x)| \ll \Delta\varphi(x)$, so that the fluctuations of the field φ hardly influence the rolling down of the field φ to the minimum of $V(\varphi)$.

Much more complicated and interesting is the behavior of the regions of space with $\varphi \gg \varphi^*$. In such regions, $|\delta\varphi(x)| \gg \Delta\varphi$. As a result, during the time $\Delta t = H^{-1}$ any region of initial size $O(H^{-1})$ expands by e times linearly and e^3 times in volume, so that it can be represented in the form of a collection of $O(e^3)$ regions each having the dimension H^{-1}. In almost half of these regions the field φ does not decrease during this time but increases. It is important that, in accordance with the "no hair" theorem, for de Sitter space the events taking place within each of the regions of dimension $l \gtrsim O(H^{-1})$ are practically independent of what happens in the other regions of the universe.[18,19] This means that during the time $\Delta t = H^{-1}$ any region of scale $l \gtrsim O(H^{-1}(\varphi))$ containing a field $\varphi \gg \varphi^*$ increases and breaks up into $O(e^3)$ independent regions (mini-universes) of scale $O(H^{-1})$, and in almost half of these regions the field φ increases. As a result, during the inflation time the total volume of space filled by constantly increasing fields $\varphi \gg \varphi^*$ increases approximately as $\exp[(3 - \ln 2)H(\varphi)t]$.[2-4 2)] If it is now borne in mind that $H(\varphi)$ increases with increasing φ (3.2), then it may be concluded that the greater part of the physical volume of the inflating universe must be in the state with maximally large value of φ, i.e., with $\varphi \sim \varphi_P \sim \lambda^{-1/4} M_P$, above which the classical description of the evolution of the universe becomes impossible.[2-4] It is interesting to note that as φ approaches $\varphi_P \sim \lambda^{-1/4} M_P$ the process of creation of inflationary mini-universes with increasing field φ becomes suppressed, since for $V(\varphi) \gtrsim M_P^4$ the energy density associated with the field inhomogeneity $\delta\varphi(x)$ and proportional to $\partial_\mu(\delta\varphi)\partial^\mu(\delta\varphi) \sim H^4$ becomes greater than $V(\varphi)$ and the corresponding regions of the universe cease to inflate.[4]

Note that these results by no means require the entire universe to be eternally in a state with $\varphi \sim \lambda^{-1/4} M_P$ and with Planck (or almost Planck) energy density $V(\varphi) \sim M_P^4$. It is necessary to distinguish between two possible formulations of the problem. In the first case, we need to know what fraction of the initial volume of the universe is in the state with given field φ. To solve this problem, we must find the distribution function $P_c(\varphi)$ of the field φ in the comoving frame or calculate the mean value of the field φ and the standard deviation Δ of the distribution of this field in unit coordinate volume (i.e., without allowance for the increase in the volume due to the expansion of the universe). As will be shown in the following section, the value of Δ during the time of inflation is always less than the mean field φ_m (for $\varphi_m < \lambda^{-1/4} M_P$). Therefore, the mean field φ_m behaves like the classical field φ (2.4). This means that the field φ increases in the greater part of the initial (coordinate) volume of the universe in accordance with the results of Sec. 2.

In the other formulation of the problem, one must find what fraction of the *physical* volume of the universe, i.e., the volume with allowance for its increase during the time of inflation) is at the present time in the state with given field φ. It is this question that we investigated above, and the result is that the inflation leads to a growth in the volume of the regions of the universe filled with the largest possible fields φ. A more detailed discussion of this question is contained in the following section.

4. DIFFUSION OF THE SCALAR FIELD DURING INFLATION

As we have already said, the value of the inhomogeneities of the field φ with wavelength $l \gtrsim H^{-1}$ changes during a time $\Delta t = H^{-1}$ on the average by $|\delta\varphi(x)| \sim (2\pi)^{-1}$ and during this same time the entire distribution of the field φ is displaced toward the minimum of $V(\varphi)$ by the amount $(\Delta t/3H)(dV/d\varphi)$. The evolution of the field with wavelength $l \gtrsim H^{-1}$ looks like Brownian motion (diffusion) in the field of an external force $-\partial V/\partial\varphi$. We first attempt to understand qualitatively the behavior of the distribution function $P_c(\varphi, t)$ of the diffusing scalar field,[2-4] and we then study $P_c(\varphi, t)$ in more detail by solving the corresponding diffusion equation.

We recall first of all that inhomogeneities of the classical scalar field $\varphi(x)$ occur in the inflationary universe only on scales exceeding H^{-1}.[22] This makes possible the following formulation of the problem: We assume that the field φ in a region of scale $O(H^{-1})$ was homogeneous, $\varphi = \varphi_0$, and we then study the dynamics of the development of inhomogeneities in this region in the course of its inflation. Thus, $P_c(\varphi, t=0) \propto \delta(\varphi - \varphi_0)$ in the considered region. The further evolution of the distribution $P_c(\varphi, t)$ in the theory with $V(\varphi) = \lambda\varphi^4/4$ is divided into two basic stages.

The first stage has duration $\Delta t \sim (\lambda^{1/2} M_P)^{-1}$. In this stage, in accordance with (2.4), the classical field φ hardly changes. But on the background of the field $\varphi \approx \varphi_0$ there appear inhomogeneities with wavelength $l \gtrsim H^{-1}(\varphi_0)$ and standard deviation $\Delta \sim (H/2\pi)(Ht)^{1/2}$ (3.1), this increasing by the end of the period to

$$\Delta \sim \frac{H}{2\pi}\left(\frac{H}{\lambda^{1/2} M_P}\right)^{1/2} \sim \frac{\lambda^{1/2}\varphi_0^3}{M_P^2}. \tag{4.1}$$

It follows from (4.1) that for $V(\varphi_0) = \lambda\varphi_0^4/4 \ll M_P^4$ the standard deviation of the inhomogeneities $\delta\varphi(x)$ of the field φ is negligibly small compared with the mean value of this field, i.e., $\varphi_m \approx \varphi \approx \varphi_0$.

In the following stage of the expansion of the universe ($t \gtrsim (\lambda^{1/2} M_P)^{-1}$) fluctuations $\delta\varphi(x)$ are also produced. However, their amplitude, which is proportional to $H(\varphi)$, decreases with decreasing field φ as $\varphi^2(t)/\varphi_0^2$, and their standard deviation decreases as $\varphi^3(t)/\varphi_0^3$. At the same time, as is shown in Ref. 26, the amplitude of the previously created inhomogeneities decreases only as $\dot\varphi \sim \varphi(t)$ (2.4). The behavior of the standard deviation $\Delta(t)$ of the inhomogeneities formed in the first stage is the same:

$$\Delta(t) \sim \lambda^{1/2}\varphi(t)\varphi_0^2/M_P^2. \tag{4.2}$$

Therefore, the form of the distribution $P_c(\varphi, t)$ in the second stage is almost completely determined by the fluctuations formed in the first stage of the process:

$$P_c(\varphi, t) \sim \exp\{-[\varphi-\varphi_m(t)]^2/2\Delta^2(t)\}$$
$$\sim \exp\{-M_P^4[\varphi-\varphi_m(t)]^2/2\lambda\varphi^2(t)\varphi_0^4\}, \qquad (4.3)$$

where the mean field $\varphi_m(t)$ is determined by Eq. (2.4). (In accordance with (4.2), the standard deviation $\Delta(t)$ of the distribution of the field φ is always less than its mean value $\varphi_m(t)$ for $V(\varphi) \sim \lambda\varphi_0^4/4 \ll M_P^4$. This justifies the determination of the mean field $\bar{\varphi}_m$ by means of Eq. (2.4) for the classical field $\varphi(t)$ in all stages of the considered process. For potentials $V(\varphi)$ of a more complicated form, the condition $\Delta \ll \varphi$ may be violated in the late stages in the evolution of the field $\varphi(t)$, and then the determination of the mean field $\varphi_m(t)$ may become a somewhat more complicated problem.)

We can go over from the distribution of the field φ in the comoving coordinate system (4.3) to the distribution $P_p(\varphi)$, which takes into account the relative increase in the volume of the space occupied by the large field φ. In the general case, the transition from $P_m(\varphi)$ to $P_p(\varphi)$ is fairly complicated. However, for us it will be sufficient to have the relationship between $P_p(\varphi)$ and $P_m(\varphi)$ that is approximately satisfied when $\varphi - \varphi_0 \gg \Delta$ after a time $\Delta t \sim (\lambda^{1/2}M_P)^{-1}$ from the beginning of the diffusion process:

$$P_p(\varphi, t) \approx e^{3H(\varphi)\Delta t}P_c(\varphi, t), \qquad (4.4)$$

whence

$$P_p(\varphi, t) \approx \exp\left\{-\frac{(\varphi-\varphi_0)^2 3(6\pi)^{1/2}M_P^3}{2\lambda\lambda^{1/2}\varphi_0^6\Delta t} + \frac{(6\pi\lambda)^{1/2}\varphi^2\Delta t}{M_P}\right\}$$
$$\sim \exp\left[\varphi^2\left(\frac{A}{M_P^2} - \frac{BM_P^4}{\lambda\varphi_0^6}\right)\right], \qquad (4.5)$$

where $A, B = O(1)$. It follows from (4.5) that in regions with $\varphi_0 \gtrsim \lambda^{-1/6}M_P$ during $\Delta t \sim (\lambda^{1/2}M_P)^{-1}$ the delta-function distribution $P_p(\varphi, 0) = P_c(\varphi, 0) \propto \delta(\varphi - \varphi_0)$ goes over into a distribution that *increases* with increasing φ. This conclusion agrees completely with the results obtained in the previous section.

For a more detailed analysis of the behavior of the distribution, we can use the diffusion equation (Fokker–Planck equation) for the long-wave component of the field φ:

$$\frac{\partial}{\partial t}P_c(\varphi, t) = \frac{\partial}{\partial\varphi}\left[\frac{\partial}{\partial\varphi}(DP_c) + \frac{1}{3H}\frac{\partial V}{\partial\varphi}P_c\right], \qquad (4.6)$$

where $D = H^3(\varphi)/8\pi^2$ is the diffusion coefficient and $(3H)^{-1}$ is the mobility. This equation was first obtained by Starobinskiĭ.[15,16] However, in Ref. 15 the derivation of this equation was merely sketched, a more detailed derivation is contained in the rather inaccessible publication of Ref. 16. For completeness of the exposition, we derive this equation in the Appendix, following our earlier paper.[17]

To study the solutions of Eq. (4.6), we shall find it convenient to introduce the dimensionless variables

$$\tau = M_P t/4\pi, \quad x = \varphi/M_P, \quad h = H/M_P. \qquad (4.7)$$

In these variables, Eq. (4.6) takes the form (we omit the subscript c of $P_c(x, \tau)$)

$$\frac{\partial}{\partial\tau}P = \frac{\partial}{\partial x}(-A(x)P) + \frac{1}{2}\frac{\partial^2}{\partial x^2}(B(x)P), \qquad (4.8)$$

where $A(x) = -\partial h/\partial x$, $B(x) = h^3/\pi$. The relative importance of the first ("convective") and second ("diffusion") terms in (4.8) can be estimated by means of the ratio

$$|A(x)| \Big/ \frac{\partial}{\partial x}B(x) \approx [h(x)]^{-2}.$$

Thus, the "diffusion" effects are most important in the region of maximally large fields φ such that

$$H(\varphi) \sim M_P \quad (V(\varphi) \sim M_P^4).$$

As before, we consider the evolution of a distribution $P(\varphi)$ concentrated at the initial time at $\varphi = \varphi_0$:

$$P(\varphi, 0) = \delta(\varphi - \varphi_0). \qquad (4.9)$$

We consider below two possibilities: 1) $V(\varphi_0) \ll M_P^4$ and 2) $V(\varphi_0) \sim M_P^4$.

In the first case, we can, in accordance with the asserted smallness of the "diffusion" effects, immediately say that in the distribution $P(\varphi, t)$ there will be a sharp peak displaced in the direction of decrease of the potential $V(\varphi)$. The width of the peak will depend on the position of the maximum of $P(\varphi)$. To study the behavior of $P(\varphi, t) = P(x, \tau)$, we introduce the function $\bar{x}(t)$, which satisfies the differential equation $\dot{\bar{x}} = A(\bar{x})$ and the initial condition $\bar{x}(0) = x_0 \equiv \varphi_0/M_P$. In Eq. (4.8), we make the change of variables $(x, \tau) \to (y, s)$:

$$s = t, \quad y = x - \bar{x}(t).$$

At the same time

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial s} - \dot{\bar{x}}\frac{\partial}{\partial y} = \frac{\partial}{\partial s} - A(\bar{x})\frac{\partial}{\partial y}, \qquad \frac{\partial}{\partial x} = \frac{\partial}{\partial y},$$

and therefore Eq. (4.8) takes the form

$$\frac{\partial}{\partial s}P = -\frac{\partial}{\partial y}\{[A(\bar{x}+y) - A(\bar{x})]P\} + \frac{1}{2}\frac{\partial^2}{\partial y^2}[B(\bar{x}+y)P]. \qquad (4.10)$$

The function $\bar{x}(t)$ actually characterizes the position of the maximum of $P(x)$, while y characterizes the deviation from the maximum. Because the peak in the distribution $P(x)$ is narrow, we can restrict ourselves to the leading terms in the expansions of the functions $A(\bar{x} + y)$ and $B(\bar{x} + y)$ in powers of the small parameter y, so that Eq. (4.10) takes the form

$$\frac{\partial}{\partial s}P = -A'(\bar{x})\frac{\partial}{\partial y}(yP) + \frac{1}{2}B(\bar{x})\frac{\partial^2}{\partial y^2}P. \qquad (4.11)$$

The solution of Eq. (4.11) with initial condition $P(y, 0) \propto \delta(y)$ is given by a Gaussian distribution with $\langle y(s)\rangle = 0$ and time-dependent standard deviation. The variance

$$\Delta^2(s) \equiv \langle y^2(s)\rangle = \int dy\, y^2 P(s, y)$$

can be found without explicit solution of Eq. (4.11). From Eq. (4.11) there follows an equation for the variance:

$$\frac{d}{ds}\Delta^2(s) = 2A'(\bar{x}(s))\Delta^2(s) + B(\bar{x}). \qquad (4.12)$$

Since we are not in fact interested in the dependence of the variance on the time but on the position $\bar{x}$ of the maximum of the distribution P, we make in Eq. (4.12) the change of variables $s \to \bar{x}$:

$$\frac{d}{d\bar{x}}\Delta^2=\frac{1}{\dot{\bar{x}}}\frac{d}{ds}\Delta^2=2\frac{A'(\bar{x})}{A(\bar{x})}\Delta^2+\frac{B(\bar{x})}{A(\bar{x})}. \tag{4.13}$$
$$\Delta^2(x_0)=0.$$

The solution of the Cauchy problem (4.13) has the form

$$\Delta^2(\bar{x})=A^2(\bar{x})\int_{x_0}^{\bar{x}}\frac{B(z)}{A^3(z)}dz. \tag{4.14}$$

Suppose the Hubble constant depends as a power on the field, $h(x)=\beta x^n/n$, this corresponding to a potential $V(\varphi)=g\varphi^{2n}/2n$ and $\beta=(4\pi ng/3)^{1/2}M_P^{n-2}$. In such a case, the variance is

$$\Delta^2(\bar{x})=\frac{\beta^2\bar{x}^{2n-2}}{4\pi n^3}(x_0^4-\bar{x}^4). \tag{4.15}$$

It can be seen from this formula that with decreasing $\bar{x}$ the variance of the distribution initially increases from zero and then begins to decrease if $n>1$ or only increases if $n\leqslant 1$. When the peak of the distribution function has moved sufficiently far from the initial position, $\bar{x}\ll x_0$, the distribution function in the neighborhood of the maximum is proportional to

$$\exp\left[-\frac{(x-\bar{x})^2}{2\Delta^2(\bar{x})}\right]\sim\exp\left[-\frac{3}{2}n^2g^{2/n-1}\frac{M_P^4}{(2nV_0)^{2/n}}\frac{(\varphi-\bar{\varphi})^2}{\bar{\varphi}^{2n-2}}\right]. \tag{4.16}$$

Our discussion is valid if we consider deviations from the maximum of the distribution $P(\varphi)$ sufficiently small for restriction to the leading terms in the Taylor expansions of the coefficients of Eq. (4.10) to be valid. In the case of the potential $V(\varphi)=\lambda\varphi^4/4$, this means that we must have

$$B'(\bar{x})y\leqslant B(\bar{x}), \tag{4.17}$$

i.e.,

$$y\leqslant {}^1/_6\bar{x}\quad(\varphi-\bar{\varphi}\leqslant {}^1/_6\bar{\varphi}).$$

In particular, the conclusions about the Gaussian form of the distribution $P(\varphi)$ are valid if $\Delta(\bar{x})\lesssim\bar{x}/6$.

For $V(\varphi)=\lambda\varphi^4/4$ it follows from (4.15) that

$$\frac{\Delta^2(\bar{x})}{\bar{x}^2}=\frac{\lambda}{12}(x_0^4-\bar{x}^4)=\frac{V_0-V(\bar{\varphi})}{3M_P^4}. \tag{4.18}$$

It is important that in this case $V_0\equiv V(\varphi_0)\ll M_P^4$

$$\Delta^2(\bar{x})/\bar{x}^2<V_0/3M_P^4\ll 1,$$

and therefore the peak of the $P(\varphi)$ distribution can be assumed to be narrow and one can speak of a Gaussian $P(\varphi)$ distribution in the neighborhood of the maximum. It is easy to show that our result (4.16) for the $\lambda\varphi^4/4$ theory is in agreement with our previous estimates (4.3). In the case $V_0\sim M_P^4$, the peak of the distribution $P(\varphi)$ is narrow only for a small "departure" of $\bar{\varphi}$ from φ_0: $V(\varphi_0)-V(\bar{\varphi})\ll M_P^4$. With further decrease of $\bar{\varphi}$, the width of the $P(\varphi)$ distribution increases sharply and becomes of order $\bar{\varphi}$.

We assume that the representation of the distribution function in the semiclassical form $P\sim e^{-S}$ with $S\gg 1$ is valid. In this case, the leading term on the right-hand side of Eq. (4.8) is the second term, and the coefficient $B(x)$ can be taken outside the differentiation sign. We then write the equation for $P(x)$ in the form

$$\frac{\partial}{\partial\tau}P=\left[\frac{B(x)}{2}\right]^{1/2}\frac{\partial}{\partial x}\left\{\left[\frac{B(x)}{2}\right]^{1/2}\frac{\partial}{\partial x}P\right\}. \tag{4.19}$$

Going over here from x to the variable

$$\xi=\int^x\frac{dx'}{[B(x')/2]^{1/2}},$$

we obtain the usual diffusion equation

$$\frac{\partial}{\partial\tau}P=\frac{\partial^2}{\partial\xi^2}P. \tag{4.20}$$

The solution of Eq. (4.20) that is nonzero at $\tau=0$ only at the point $x=x_0$ is proportional to

$$\exp\{-[\xi-\xi(x_0)]^2/4\tau\}. \tag{4.21}$$

For the potential $V(\varphi)=g\varphi^{2n}2n$, we have

$$\xi=-\frac{1}{3n/2-1}\left(\frac{2\pi n^3}{\beta^3}\right)^{1/2}\frac{1}{x^{3n/2-1}},$$

so that the distribution function is proportional to

$$P(x)\sim\exp\left\{-\frac{\pi n^3}{2\beta^3\tau}\frac{1}{(3n/2-1)^2}\left[\frac{1}{x^{3n/2-1}}-\frac{1}{x_0^{3n/2-1}}\right]^2\right\}. \tag{4.22}$$

It can be seen that for $x\ll x_0$ the solution does not depend at all on x_0:

$$P(x)\sim\exp\left\{-\frac{\pi n^3}{2\beta^3\tau}\frac{1}{(3n/2-1)^2}\frac{1}{x^{3n-2}}\right\}. \tag{4.23}$$

The result (4.23) is actually obtained under the assumption that the "convective" term in Eq. (4.8) is small compared with the "diffusion" term. This is justified for times τ so short that the "convective" classical motion is not capable of significantly shifting the value of x. (This interval of time τ may be different for each value of x.) We estimate the corresponding interval τ during which Eq. (4.23) can be used. The "convective" motion of the point x is determined by the equation

$$\dot{x}=A(x)=-\partial h/\partial x=-\beta x^{n-1}, \tag{4.24}$$

so that $\Delta x\lesssim x$ for $\tau\lesssim\beta^{-1}x^{2-n}$, whereas at large τ the diffusion regime is replaced by the regime of classical "rolling down" of the field φ to the minimum of $V(\varphi)$. Substituting in (4.23) the value $\tau=O(\beta^{-1}x^{2-n})$, we obtain for the distribution $P(x)$ at the end of the "quantum" stage the expression

$$P_c(x)\sim\exp\left\{-O(1)\frac{1}{\beta^2x^{2n}}\right\}, \tag{4.25}$$

i.e.,

$$P_c(\varphi)\sim\exp\left\{-O(1)\frac{M_P^4}{V(\varphi)}\right\}. \tag{4.26}$$

The physical meaning of this result is very interesting: $P(\varphi)$ determines the probability of occurrence of an inflationary mini-universe of scale $l\gtrsim O(H^{-1}(\varphi))$ due to quantum diffusion (tunneling) from a locally de Sitter region of scale $l\gtrsim O(M_P^{-1})$ with $V(\varphi_0)\sim M_P^4$. The probability of existence of such regions in a space-time foam for $V(\varphi_0)\sim M_P^4$ should not be exponentially suppressed.[31] Therefore, the expression (4.26) could be interpreted as the probability of quantum creation of an inflationary universe in accordance with the previous estimates of the probability of such a process.[31-34] However, we are not considering here the quantum creation of the entire universe "from nothing," but the creation of an inflationary mini-universe with $V(\varphi) \ll M_P^4$ from a "ground state" with $V(\varphi) \sim M_P^4$ filling the greater part of the physical volume of the universe.

5. DISCUSSION

The results above (see also Refs. 2–4) lead to somewhat unexpected ideas about the global structure of the universe. Prior to creation of the inflationary universe scenario, the main guide for cosmologists was the Friedmann model. According to this, the universe in the large was represented as a hot expanding sphere created from "nothing" (from a singularity, before which there was no space-time at all). In the distant future, the universe would either have to cool to zero temperature (if open or flat) or again disappear in a singularity (if closed). This picture appeared to be an almost inescapable consequence of the general theory of relativity and the high degree of homogeneity of the universe on scales accessible to observation. At the present time, this picture has been replaced by that of a self-reproducing inflationary universe that only locally resembles the Friedmann universe. The existence of such a universe never ends even if the universe is closed, i.e., in the universe there is no finite global spacelike singular hypersurface whose existence would amount to the "end of time." Moreover, in this scenario there are no grounds for fearing the existence of an initial global spacelike singular hypersurface, i.e., a common "beginning of time," before which "nothing existed."[3)]

An important feature of the considered picture, which arises only in the framework of the scenario of chaotic inflation, is that the greater part of the physical volume of the universe must be at a density close to the Planck density, $V(\varphi) \sim M_P^4$ (although the actual regime of creation of inflationary regions of the universe with increasing field φ also takes place at densities many orders of magnitude less than the Planck density). The process of diffusion to smaller values of the field φ is essentially a process of creation of inflationary mini-universes. This process takes place independently in different causally unconnected regions of the universe. As a result of this, there arise inflationary regions (mini-universes) of *all* possible types corresponding to all possible types of symmetry breaking and all possible types of compactification (in Kaluza-Klein theories) compatible with inflation of the universe. This makes it possible to justify the anthropic principle in cosmology.[4] It seems to us that the change in the ideas about our position in the universe and its global structure is one of the most important consequences of the creation of the inflationary universe scenario.

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APPENDIX: FOKKER–PLANCK EQUATION FOR SMOOTHED FIELD IN AN INFLATIONARY UNIVERSE

In the main text of the paper, we discussed on several occasions the behavior of the variance Δ^2 of the field $\varphi(\mathbf{x})$ in the process of inflation of the universe. As in flat space-time, the magnitude of the quadratic fluctuation of the field $\varphi(\mathbf{x})$ in quantum theory is infinite even in the vacuum state as a result of the addition of the zero-point vibrations of infinitely many modes. However, averaged over a finite volume of space, the field operator has finite quadratic fluctuation.

Writing down the Fourier expansion of the operator $\varphi(\mathbf{x})$ with respect to the spatial variables (for simplicity, we assume that space is flat)

$$\varphi(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} [a_\mathbf{k}\varphi_\mathbf{k} e^{i\mathbf{kx}} + \text{c.c.}], \qquad (\text{A.1})$$

we introduce a field operator φ_b, averaged over the volume b^3, as follows:

$$\varphi_b = \int \frac{d^3k}{(2\pi)^{3/2}} \theta(-k+b^{-1}) [a_\mathbf{k}\varphi_\mathbf{k} + a_\mathbf{k}^+\varphi_\mathbf{k}^*]. \qquad (\text{A.2})$$

Here, $a_\mathbf{k}$ and $a_\mathbf{k}^+$ are ordinary annihilation and creation operators with standard commutation relations.

We consider the evolution of the field $\varphi(\mathbf{x})$ in an inflationary universe with $H = \text{const}$ (in de Sitter space) with metric

$$ds^2 = -dt^2 + R^2(t) d\bar{x}^2, \quad R(t) = e^{Ht}.$$

Being interested in the field φ averaged over a physical volume $\gtrsim H^{-3}$, we set in (A.2) $b = (\varepsilon RH)^{-1}$, $\varepsilon \ll 1$. Denoting the field operator averaged in this manner by the symbol

$$\Phi = \int \frac{d^3k}{(2\pi)^{3/2}} \theta(-k+\varepsilon RH) [a_\mathbf{k}\varphi_\mathbf{k} + a_\mathbf{k}^+\varphi_\mathbf{k}^*], \qquad (\text{A.2}')$$

we obtain for its rate of change the formula

$$\dot{\Phi} = \int \frac{d^3k}{(2\pi)^{3/2}} \theta(-k+\varepsilon RH) [a_\mathbf{k}\dot{\varphi}_\mathbf{k} + a_\mathbf{k}^+\dot{\varphi}_\mathbf{k}^*] + f(t), \qquad (\text{A.3})$$

where

$$f(t) \equiv \varepsilon RH^2 \int \frac{d^3k}{(2\pi)^{3/2}} \delta(k-\varepsilon RH) [a_\mathbf{k}\varphi_\mathbf{k} + a_\mathbf{k}^+\varphi_\mathbf{k}^*]. \qquad (\text{A.4})$$

The equation of motion of the field $\varphi(x)$ in the de Sitter universe has the form

$$\ddot{\varphi} + 3H\dot{\varphi} - R^{-2}\nabla^2\varphi + \frac{dV(\varphi)}{d\varphi} = 0. \qquad (\text{A.5})$$

Ignoring the interaction between the short-wave modes with $k \gtrsim \varepsilon RH$, we obtain a linear equation determining the dynamics of such modes:

$$\ddot{\varphi}_\mathbf{k} + 3H\dot{\varphi}_\mathbf{k} + k^2R^{-2}\varphi_\mathbf{k} + V''(\Phi)\varphi_\mathbf{k} = 0. \qquad (\text{A.6})$$

The last term in Eq. (A.6) can be ignored if

$$V''(\Phi) \lesssim k^2/R^2 \quad (k \geqslant \varepsilon RH). \qquad (\text{A.7})$$

In the special case of the potential $V(\varphi) = \lambda\varphi^4/4$, the condition (A.7) takes the form

$$3\lambda\Phi^2 \ll \varepsilon^2 H^2 = \varepsilon^2 \frac{2\pi}{3M_P^2} \lambda\Phi^4,$$

i.e.,

$$\varepsilon^2\Phi^2 \gg 9M_P^2/2\pi. \qquad (\text{A.7}')$$

The requirement (A.7) can be satisfied for $\varepsilon \ll 1$ if $\Phi \gg M_P$, but it is precisely this region of Φ values that is investigated in the paper.

Thus, in the region $\Phi \gg M_P$ the short-wave modes ($k \geqslant \varepsilon RH$) satisfy the equation

$$\ddot{\varphi}_k + 3H\dot{\varphi}_k + k^2 R^{-2}\varphi_k = 0, \tag{A.6'}$$

whose solutions possessing the correct normalization are the functions

$$\varphi_k = \frac{H}{(2k)^{1/2}}\left(\eta + \frac{1}{ik}\right) e^{-ik\eta}, \quad \eta = -(HR)^{-1}.$$

For $k = \varepsilon RH$, we can set

$$\varphi_k = -iH/2^{1/2}k^{3/2} \quad (k = \varepsilon RH). \tag{A.8}$$

By means of Eq. (A.8) we can readily show that on averaging over a state $|\rangle$ in which there are no quanta of the short-wave field, $a_k|\rangle = 0$ ($k \geqslant \varepsilon RH$), the different-time moment functions of the term $f(t)$ on the right-hand side of (A.3) have a form corresponding to a Gaussian random process:

$$\langle f(t_1)\ldots f(t_{2n-1})\rangle = 0,$$
$$\langle f(t_1)\ldots f(t_{2n})\rangle = \sum_P \langle f(t_{i_1}) f(t_{i_2})\rangle \ldots \langle f(t_{i_{2n-1}}) f(t_{i_{2n}})\rangle, \tag{A.9}$$

where the summation is over all possible partitionings of the indices 1, 2,..., $2n$ into pairs. At the same time,

$$\langle f(t_1) f(t_2)\rangle = \varepsilon^2 H^4 R_1 R_2 \int \frac{d^3k\, d^3q}{(2\pi)^3} \delta(k - \varepsilon R_1 H)$$
$$\times \delta(q - \varepsilon R_2 H)\langle a_k a_q^+\rangle \varphi_k(t_1)\varphi_q^*(t_2)$$
$$= \varepsilon^2 H^4 R_1 R_2 \int \frac{d^3k}{(2\pi)^3} \delta(k - \varepsilon R_1 H)$$
$$\times \delta(k - \varepsilon R_2 H)\varphi_k(t_1)\varphi_k^*(t_2) = \frac{\varepsilon H^5 R_2}{(2\pi)^2}\delta(\varepsilon R_1 H - \varepsilon R_2 H)$$
$$= \frac{H^3}{4\pi^2}\delta(t_1 - t_2) \equiv 2D\delta(t_1 - t_2). \tag{A.10}$$

The first term on the right-hand side of Eq. (A.3) can be reduced to a convenient form by substituting in the equation of motion (A.5) the Fourier expansion (A.1) of the field $\varphi(\mathbf{x})$ after first dividing the momentum space into two regions with $k \lessgtr \varepsilon RH$. Then by virtue of Eq. (A.6') there is no contribution to the first three terms of Eq. (A.5) from the region $k > \varepsilon RH$. For the integration over the region $k < \varepsilon RH$, the most important of the first three terms of Eq. (A.5) is the second, so that we can write

$$3H\int \frac{d^3k}{(2\pi)^{3/2}}\theta(-k + \varepsilon RH)[a_k\dot{\varphi}_k e^{i\mathbf{kx}} + \text{c.c.}] + \frac{dV(\varphi(\mathbf{x}))}{d\varphi} = 0. \tag{A.11}$$

Averaging Eq. (A.11) over the volume $b^3 = (\varepsilon RH)^{-3}$ and assuming that $\overline{[\varphi(\mathbf{x}) - \Phi]^2} \ll \Phi^2$, we obtain for the first term on the right-hand side of (A.3) the expression

$$\int \frac{d^3k}{(2\pi)^{3/2}}\theta(-k + \varepsilon RH)[a_k\dot{\varphi}_k + a_k^+\dot{\varphi}_k^*] = -\frac{1}{3H}\frac{dV(\Phi)}{d\Phi}.$$

As a result, Eq. (A.3) takes the form

$$\dot{\Phi} = -\frac{1}{3H}\frac{\partial V(\Phi)}{\partial\Phi} + f(t). \tag{A.12}$$

This is an operator equation written down in the Heisenberg picture We shall be interested in the expectation values of functions of the operator Φ in a quantum state $|t_0\rangle$, that possesses the property

$$a_k|t_0\rangle = 0, \quad k \geqslant \varepsilon R(t_0)H. \tag{A.13}$$

We note that the condition (A.13) does not completely describe the state vector $|t_0\rangle$ since the population of the modes with $k \leqslant \varepsilon R(t_0)H$ is not determined by this condition. For a state vector satisfying the condition (A.13) there is obviously satisfied the requirement $a_k|t_0\rangle = 0$, $k \geqslant \varepsilon R(t)H$, used for calculation of the statistical properties of $f(t)$, if $t \geqslant t_0$. Therefore, the time t_0 in (A.13) must be shorter than the times over which we consider the dynamics of the system.

Equation (A.12) has the same form as the Langevin equation used in the theory of random processes. However, a difference from the standard situation is that the two terms on the right-hand side of Eq. (A.12) do not commute. However, this circumstance does not affect the asymptotic behavior of the moments

$$m_s(t;\tau) \equiv \langle t_0|[\Phi(t+\tau) - \Phi(t)]^s|t_0\rangle$$

as $\tau \to 0$. Therefore, with allowance for the properties (A.9) and (A.10) of the "random force" $f(t)$ we can obtain in the usual manner from the operator Langevin equation (A.12) the Fokker-Planck equation for the distribution function $P(\Phi)$ of the random variable Φ:

$$\frac{\partial}{\partial t}P(\Phi) = \frac{\partial}{\partial\Phi}\left[\frac{1}{3H}\frac{\partial V(\Phi)}{\partial\Phi}P(\Phi)\right] + \frac{\partial^2}{\partial\Phi^2}[DP(\Phi)], \tag{A.14}$$

where

$$D = H^3/8\pi^2.$$

This equation for the case H = const was obtained for the first time by Starobinskiĭ[15] (see also Ref. 17). It is also valid for the case of slowly varying H ($\dot{H} \ll H^2$), i.e., for the quasiexponential expansion that takes place during the time of inflation. Equation (A.14) for the regime of slow variation of H was obtained in a somewhat different but equivalent form in Ref. 16.

So as not to complicate the system of notation, in the main text of the paper we denote by the symbol φ the long-wave part Φ of the scalar field in (A.14).

[1)] In the last two cases, however, the realization of the inflationary universe scenario is still far from completion.[3]

[2)] A similar phenomenon can occur in either the "old" or the "new" inflationary scenario.[6,28-30]

[3)] Belief in the existence of a "beginning of time" was based on the singularity theorems in conjunction with the assumption that the global geometry of the universe is close to the geometry of the Friedmann model at least in the sense of homogeneity of the universe. In the considered scenario, the global geometry of the universe has nothing at all in common with the geometry of a homogeneous universe. If the universe is noncompact, the existence of a global initial spacelike singular hypersurface would contradict the causality principle.

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