The theory of phase conjugation during stimulated scattering in a self-intersecting light beam

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A theory is developed for stimulated scattering in intersecting light beams with arbitrary wave fronts in a nonlinear medium. It is shown that, in the optical ring cavity configuration, the stimulated scattering process exhibits absolute instability with a very low threshold that corresponds to a convective gain on the order of unity. The wave front structure is determined for the scattered radiation produced during emission. Conditions are found for which the scattered and pump beams are nearly phase conjugate.

1. Stimulated scattering in the field of two light waves is attracting considerable attention mostly because, under these conditions, stimulated scattering can give rise to absolute instability (generation) with a very low threshold.[1–6] Processes of this kind can be used, when powerful laser radiation interacts with plasmas, in order to interpret the low stimulated scattering threshold and to explain the observed angular and spectral parameters of the scattered radiation.[7,8] In nonlinear optics, such processes are promising for the excitation of stimulated scattering in media with a low breakdown threshold, the development of efficient laser frequency converters, and the dynamic correction of the wave fronts of powerful light beams.

Phase conjugation by stimulated scattering (PCSS) has been examined in detail, both theoretically and experimentally, in the convective case.[9] Highly energy-efficient low-threshold phase conjugation systems (the optical ring cavity) that rely on generation by stimulated scattering have recently been proposed.[10,11] Despite the difference between the physical mechanisms employed, the common feature of Refs. 10 and 11 is that stimulated scattering occurs in crossed light beams (Fig. 1), one of which is the incident beam and the other is the initial beam returned by mirrors after passing through a nonlinear medium. Emission by stimulated scattering in the region in which the two light beams cross one another is essentially a two-dimensional problem because the longitudinal and lateral dimensions of the interaction region in the plane of intersection are generally comparable, and the interacting waves propagate in different directions.

The last point was not previously taken into account, so that existing theory[1,6] cannot predict the lateral structure of the scattered field, and cannot therefore deal with the quality of phase conjugation in emission by stimulated scattering.

In this paper, we develop a theory of emission by stimulated scattering in the region of interaction between two crossed light beams, assuming that the size of this region is small in comparison with the Fresnel length of each of the beams (this is the experimental situation[11]).

We shall use the example of Mandel'shtam-Brillouin stimulated scattering (MBSS) to show that Stokes radiation propagating opposite to the pump beam is generated in the optical ring cavity (Fig. 1) in which stimulated scattering takes place on refractive index perturbations whose wave vector is equal to the sum of the wave vectors of the incident waves. We shall show that the generation threshold is not very dependent on the structure of the pump wave front, and will determine the spatial structure of the scattered radiation near the generation threshold.

2. Consider stimulated scattering in a nonlinear medium in the region in which two light beams cross. We shall assume that the two pump beams have the same frequency ω_0 and that they cross at an angle 2θ. The electric field of two light beams in the medium will be represented by

$$E_z(\mathbf{r},t)=\frac{1}{2}\sum_{\sigma=\pm1}E_{0\sigma}(\mathbf{r})\times\exp(i\sigma k_0x'\sin\theta+ik_0y'\cos\theta-i\omega_0t)+\text{c.c.}\qquad(1)$$

where $k_0=\omega_0 n/c$, n is the linear refractive index of the medium, and the amplitudes $E_{0\pm1}(\mathbf{r})$ describe the field distributions across the direction of propagation. We shall confine our attention to processes occurring in the interaction region, and will neglect nonlinear effects along the propagation path in the medium outside this region. This assumption is justified when stimulated scattering in the field of the two waves exhibits absolute instability, and its threshold lies below the stimulated scattering threshold in a single beam.

In the beam overlap region, we can have MBSS corresponding to the stimulated scattering of both pump waves by a single sound wave (Fig. 2) with wave vector $\mathbf{k}_s=\mathbf{k}_{01}$

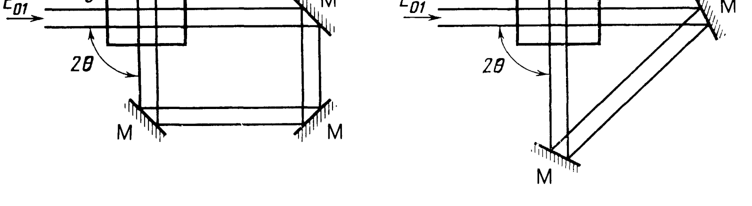

FIG. 1. Optical ring cavity with three (a) and two (b) mirrors M; C is the nonlinear medium.

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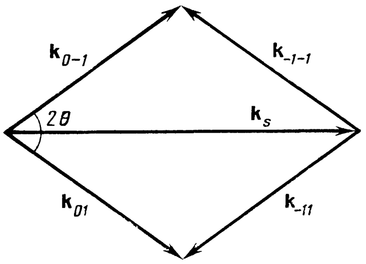

FIG. 2. Disposition of the wave vectors of the pump ($k_{0\pm1}$) and scattered Stokes ($k_{-1\pm1}$) waves; k_s is the wave vector of the sound waves.

$+\mathbf{k}_{0-1}$, directed along the bisector of the angle between the incident waves, and frequency $\omega \simeq \Omega = 2k_0 v_s \cos\theta$, where v_s is the velocity of sound (cf. Refs. 4 and 5). Stimulated scattering of the wave E_{01} in the medium produces the Stokes wave E_{-11} propagating against the wave E_{0-1} whereas stimulated scattering of the wave E_{0-1} produces the wave E_{-1-1} propagating against E_{01}. The electric field of the scattered waves will be written in the form

$$E_{sz} = \frac{1}{2}\sum_{\sigma=\pm1} E_{-1\sigma}(\mathbf{r})\exp[i\sigma k_0 x'\sin\theta - ik_0 y'\cos\theta - i(\omega_0-\omega)t] + \text{c.c.}, \quad (2)$$

where $\omega \simeq \Omega$ is the frequency shift of the scattered waves. The set of truncated equations for the scattered-wave amplitudes, deduced from the hydrodynamic equations and Maxwell's equations, is (cf., Ref. 5)

$$\left(\sigma\sin\theta\frac{\partial}{\partial x'} - \cos\theta\frac{\partial}{\partial y'}\right)E_{-1\sigma} = \frac{k_0 Y^2\Omega E_{0\sigma}}{64\pi n^2\rho v_s^2\gamma_s(1-i\Delta)}(E_{01}^* E_{-11} + E_{0-1}^* E_{-1-1}), \quad (3)$$

where ρ is the density, $Y = \rho(\partial\varepsilon/\partial\rho)$ is the nonlinear coupling parameter γ_s is the sound damping constant, and $\Delta = (\Omega-\omega)/\gamma_s$ is the detuning of the scattered-wave frequency from the maximum of the amplification band.

In deriving (3), we have discarded time derivatives of the amplitudes, assuming that the time taken by light to traverse an interaction region with linear dimensions $\sim D/\sin 2\theta$ is small in comparison with the sound damping constant, i.e., $\gamma_s D/c \sin 2\theta \ll 1$ (D is the pump beam diameter). We have also neglected the derivatives at right-angles to the direction of propagation of the beam, assuming that the typical scale of the lateral structure of the Stokes beams, a, is not too small ($l_F = k_0 a^2 \gg D/\sin 2\theta$). Moreover, it is assumed in (3) that $\theta > (\gamma_s/\Omega)^{1/2}$, so that we can ignore backward MBSS, which is a nonresonant process at the chosen frequency Ω.

3. Equations (3) describe two coupled processes. The terms $\sim |E_{0\sigma}|^2 E_{-1\pm1}$ determine the convective amplification of scattered waves, and the interference terms $\sim E_{0\sigma} E^*_{0-\sigma} E_{-1\pm1}$ describe the distributed coupling between scattered waves propagating in different directions.

In the special case of scattered waves propagating in opposite directions, equations similar to (3) describe the absolute instability of stimulated scattering, i.e., when the pump wave amplitudes are large enough, the scattered waves grow exponentially if the amplitudes vanish at entry into the interaction region (see Refs. 1–6 and 12).

On the other hand, when the scattered waves propagate at an angle to one another, the distributed feedback is insufficient for the onset of absolute instability because perturbations are then transported out of the interaction region in the direction of the y' axis. This means that stimulated scattering in crossed beams, with zero scattered-wave amplitudes at entry to the interaction region, exhibits convective amplification. To produce generation, a proportion of the energy transported by the waves out of the interaction region must be returned to it along a different direction. This can be done with a system of mirrors which, together with the nonlinear medium, form an optical ring cavity. The configurations illustrated in Fig. 1 were implemented in Ref. 10 to produce low-threshold generation and phase conjugation (Fig. 1a), using MBSS in an optical fiber; photorefractive crystals were used in Ref. 11 (Fig. 1b).

Let us substitute

$$x = x'\cos\theta + y'\sin\theta, \quad y = -x'\cos\theta + y'\sin\theta.$$

If we cut the interaction region with a plane parallel to the beam axes, the resulting cross section is rectangular, with sides parallel to the coordinate axes and center lying at the origin, which is also the point of intersection of the beam axes (Fig. 3). We can then rewrite (3) in the form

$$\frac{\partial}{\partial y}E_{-11}(x,y,z) = -\frac{K}{1-i\Delta}[|E_{01}(y,z)|^2 E_{-11} + E_{01}(y,z)E^*_{0-1}(x,z)E_{-1-1}], \quad (4)$$

$$\frac{\partial}{\partial x}E_{-1-1}(x,y,z) = -\frac{K}{1-i\Delta}[|E_{0-1}(x,z)|^2 E_{-1-1} + E^*_{01}(y,z) \times E_{0-1}(x,z)E_{-11}], \quad (5)$$

where $K = k_0 Y^2\Omega/64\pi n^2\rho v_s^2\gamma_s \sin 2\theta$. The pump wave E_{01} propagates along the x-axis, the function $E_{01}(y,z)$ describes the shape of its wave front, and $E_{0-1}(x,z)$ describes the wave front of the second pump wave, propagating along the y-axis. The z-coordinate is measured along the axis perpendicular to the plane of intersection of the two beams, and appears in (4) and (5) as a parameter.

The field E_{-11} of the scattered Stokes wave propagating in the negative y direction will be sought in the form

$$E_{-11}(x,y,z) = E^*_{0-1}(x,z)f(x,y,z)\exp\left[-\frac{K}{1-i\Delta}\int^{y} dy'|E_{01}(y',z)|^2 - \frac{K}{1-i\Delta}\int^{x} dx'|E_{0-1}(x',z)|^2\right]. \quad (6)$$

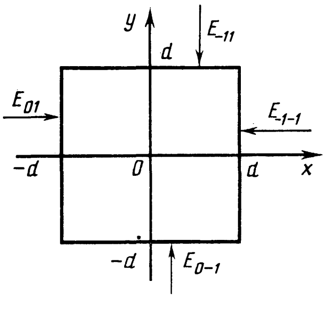

FIG. 3. Section through the interaction region by a plane parallel to the pump-beam axes.

The field of the second scattered wave propagating in the negative x direction can then be found from (4):

$$E_{-1-1}(x,y,z)=-\frac{1-i\Delta}{KE_{01}(y,z)}\frac{\partial f}{\partial y}\exp\left[-\frac{K}{1-i\Delta}\int^{y}dy'\,|E_{01}|^2-\frac{K}{1-i\Delta}\int^{x}dx'\,|E_{0-1}|^2\right], \quad (7)$$

whereas (5) gives us the equation for the function $f(x,y,z)$:

$$\frac{\partial^2 f}{\partial x\,\partial y}-\left(\frac{K}{1-i\Delta}\right)^2|E_{01}(y,z)|^2|E_{0-1}(x,z)|^2 f(x,y,z)=0. \quad (8)$$

The solution of this equation (cf., Ref. 13) determines the values of the function f in the interaction region in terms of its values and the values of its first derivatives on the boundary of the region:[14]

$$f(x,y,z)=f(x,d,z)+\int_x^d dx'\,\vartheta(x,y;x',d;z)f(x',d,z)-\int_y^d dy'\,\vartheta(x,y;d,y';z)\frac{\partial}{\partial y'}f(d,y'\,z). \quad (9)$$

We now impose the boundary conditions outside the beam interaction region, near its boundary. Without loss of generality, we can assume that the boundary conditions are defined on planes parallel to the z axis, i.e., we assume that d is independent of z. The function ϑ in (9) is given by

$$\vartheta(x,y;x',y';z)=I_0\left(\frac{2K}{1-i\Delta}\left[\int_y^{y'}dy''\,|E_{01}(y'',z)|^2\times\int_x^{x'}dx''\,|E_{0-1}(x'',z)|^2\right]^{1/2}\right), \quad (10)$$

where I_0 is the modified Bessel function.

We note at once that the absence of absolute instability mentioned above follows from (9) if the amplitudes of the two scattered waves at entry to the interaction region are zero. Actually, (9) has a nontrivial solution if $f(x,d,z)\neq 0$ or $\partial f(d,y,z)/\partial y\neq 0$. However, the function $f(x,d,z)$ is shown by (6) to be proportional to the wave amplitude E_{-11} at entry to the interaction region ($y=d$), and $\partial f(d,y,z)/\partial y$ is proportional to the amplitude of the second scattered wave on the boundary $x=d$. The condition for generation is that at least one of these two functions is not zero. Next, since we are interested in the optical ring cavity (Fig. 1), we shall assume that $E_{-11}(x,d,z)=0$, i.e., $f(x,d,z)=0$.

4. The wave amplitudes E_{-1-1} and E_{0-1} at entry to the nonlinear interaction region are determined by the propagation conditions in the optical channel. Consider the case where the channel length L is small in comparison with the Fresnel length of the pump beam: $L\ll l_F=k_0a^2$.

If the rays in the ring resonator are confined to a plane, as is the case, for example, for the three-mirror scheme of Fig. 1a, the boundary conditions become

$$E_{0-1}(x,z)=rE_{01}(x,z),\quad E_{-1-1}(d,y,z)=re^{i\varphi}E_{-11}(y,-d,z), \quad (11)$$

where φ is the relative phase gain of the Stokes wave along the cavity length and r is the reflectivity of the mirror system. From (11), we then have the following boundary condition:

$$\frac{\partial}{\partial y}f(d,y,z)=-\frac{K}{1-i\Delta}|r|^2e^{i\varphi}|E_{01}(y,z)|^2f(y,-d,z)\times\exp\left[\frac{K(1-|r|^2)}{1-i\Delta}\int_{-d}^{y}dy'\,|E_{01}|^2+\frac{\varkappa|r|^2}{1-i\Delta}\right],$$

where

$$\varkappa(z)=K\int_{-d}^{d}dy'\,|E_{01}(y',z)|^2$$

is the mean MBSS convective gain over the interaction length, averaged over the beam cross section. This relation and $f(x,d,z)=0$ together define the function $f(x,y,z)$ in (9) throughout the interaction region. When $y=-d$, we obtain the following equation for $f(x,-d,z)$:

$$f(x,-d,z)=\frac{K|r|^2}{1-i\Delta}e^{i\varphi}\times\int_{-d}^{d}dy'\vartheta(x,-d;d,y';z)|E_{01}(y',z)|^2f(y',d,z)\times\exp\left[\frac{\varkappa|r|^2}{1-i\Delta}+\frac{K(1-|r|^2)}{1-i\Delta}\int_{-d}^{y'}dy''\,|E_{01}(y'',z)|^2\right]. \quad (12)$$

This is a linear Fredholm equation of the second kind, which has a solution only for a discrete set of eigenvalues $\varkappa$, Δ, defining the absolute instability thresholds and pump-wave frequencies.

Let us first consider the solution of (12) in the two-dimensional model,[13,15] neglecting the dependence on z. The first step is to introduce the function $F(s)=f(x,-d)$, where

$$s=K\int_x^d dy'\,|E_{01}(y')|^2,$$

which, according to (6), defines the shape of the wave front of the wave E_{-11} leaving the interaction region. The equation for $F(s)$ does not depend on the lateral structure of the pump field, and is determined by the total intensity alone:

$$\varkappa=K\int_{-d}^{d}dy\,|E_{01}(y)|^2:$$

$$F(s)=\frac{|r|^2}{1-i\Delta}\exp\left(i\varphi+\frac{\varkappa|r|^2}{1-i\Delta}\right)\int_0^{\varkappa}ds'\,I_0\left(\frac{2|r|}{1-i\Delta}(ss')^{1/2}\right)\times F(\varkappa-s')\exp\left(\frac{1-|r|^2}{1-i\Delta}s'\right). \quad (13)$$

It has a denumerable set of distinct eigenvalues $F^{(m)}(s)$.

Let us demonstrate this in the case of low reflectivities $|r|\ll 1$, so that, in the limit as $\varkappa|r|\ll 1$, we can expand the Bessel function in the kernel of (13) into a power series. Since the dependence on s in (13) involves only the Bessel function, the function $F(s)$ is also a power series in this limit:

$$F(s)=\sum_{k=0}^{\infty}F_k[s|r|/(1-i\Delta)]^k/(k!).$$

The coefficients F_k can be found from (13), and are given by the following set of linear algebraic equations:

$$F_i=\sum_{j=0}^{\infty} a_{ij}F_j,$$

$$a_{ij}=\frac{|r|}{i!j!}\exp\left(i\varphi+\frac{\varkappa|r|^2}{1-i\Delta}\right)\left(\frac{\varkappa|r|}{1-i\Delta}\right)^{i+j+1}$$

$$\times\int_0^1 d\zeta\,\zeta^i(1-\zeta)^j\exp\left(\frac{1-|r|^2}{1-i\Delta}\varkappa\zeta\right). \qquad (14)$$

The dispersion relation $\det(a_{ik}-\delta_{ik})=0$ has a denumerable set of solutions. The first three are:

$$\begin{aligned}&|r|^2\exp[i\varphi+\varkappa_0/(1-i\Delta_0)]=1,\\ &|r|^4\exp[i\varphi+\varkappa_1/(1-i\Delta_1)]=-1,\\ &|r|^6\exp[i\varphi+\varkappa_2/(1-i\Delta_2)]=1.\end{aligned} \qquad (15)$$

The corresponding eigenfunctions are

$$F^{(0)}(s)\approx 1+\frac{\varkappa_0|r|^2}{1-i\Delta_0}s+o\left(\frac{\varkappa_0|r|^2}{1-i\Delta_0}s\right)^2\approx 1,$$

$$F^{(1)}(s)\approx 1-\frac{s}{1-i\Delta_1}\left[1+\frac{\varkappa_1|r|^2}{2(1-i\Delta_1)}s+o\left(\frac{\varkappa_1|r|^2}{1-i\Delta_1}s\right)^2\right]\sim s, \qquad (16)$$

$$F^{(2)}(s)\approx 1-\frac{2s}{1-i\Delta_2}+\frac{s^2}{2(1-i\Delta_2)^2}\left[1+o\left(\frac{\varkappa_2|r|^2}{1-i\Delta_2}s\right)\right]\sim s^2.$$

The equations in (15) are transcendental. Each has a denumerable set of solutions, i.e., to each eigenfunction $F^{(m)}(s)$ there corresponds a whole set of wavelengths with different frequencies Δ_{mn} and pump thresholds $\varkappa_{mn}$. The index m will be referred to as transverse because it labels solutions with different functions $F(s)$, i.e., solutions with different transverse structure of the excited wave field. Waves with different n (but the same m) have similar transverse structures, but different frequencies, i.e., different wave vectors in the direction of propagation. We shall therefore refer to n as the longitudinal index. From (15), we then have

$$\begin{aligned}&\varkappa_{0n}=(1+\Delta_{0n}^2)\ln(1/|r|^2), \quad \Delta_{0n}=(2\pi n-\varphi)\ln^{-1}(1/|r|^2),\\ &\varkappa_{1n}=2(1+\Delta_{1n}^2)\ln(1/|r|^2),\\ &\Delta_{1n}={}^1/_2[(2n+1)\pi-\varphi]\ln^{-1}(1/|r|^2).\end{aligned} \qquad (17)$$

The minimum threshold $\varkappa_{00}=\ln(1/|r|^2)$ is achieved for $\varphi=2\pi n_0$. Since $F^{(0)}(s)\simeq$const by (16), the scattered Stokes wave produced in this case is phase conjugate with respect to the pump wave, i.e., it has a reversed wave front. The pump threshold for the (1,0) mode, for which the quality of phase conjugation is less good [$F^{(1)}(s)\sim s$], is higher by a factor of approximately two ($\varkappa_{10}\simeq 2\varkappa_{00}$).

The generation threshold can be reduced, and the relative separation between the pump thresholds for different modes can be increased, by reducing $|r|$. When $|r|=1$, $\varphi=0$, we have

$$\varkappa_{00}=0.54, \quad F^{(0)}(s)\approx 1+0.4s+0.08s^2.$$

A qualitatively similar situation obtains for the two-mirror arrangement (see Fig. 1b), for which the boundary conditions can be written in the form [see (11)]

$$E_{0-1}(x)=rE_{01}(-x), \quad E_{-1-1}(d,y)=re^{i\varphi}E_{-11}(-y,-d).$$

When $|r|\ll 1$, the dispersion relation for the principal lateral mode ($m=0$) is again given by (15), whereas for $|r|=1$, $\varphi=0$, the minimum threshold corresponds to $\varkappa_{00}=0.52$.[13,15]

We must emphasize at this point that the phase conjugation of the pump beam in the field of the scattered radiation, deduced above, applies only to the coordinate y in the plane of intersection of the beams because (12) does not enable us to determine f as a function of z.

Nonlocal coupling in z between the waves entering and leaving the interaction region must be produced for the structure of the Stokes beam in both directions to be determined by the pump field. In other words, the beam path in the cavity must not be confined to a plane. This can be achieved, for example, by rotating the beam cross section through an angle differing from zero or 180°, or by changing the scale of the beam cross section in the feedback channel. It will be shown below that this gives rise to a discrete set of nondegenerate modes in z, i.e., we have the possibility of field structure selection.

5. As an example, consider the case where the cross-section of the light beam leaving the nonlinear interaction region is turned through 90° by four mirrors arranged in pairs into two two-sided corner reflectors with crossed edges (see Ref. 16, Section 3.6). Neglecting diffraction in the feedback channel, we find that the conditions on the boundary of the nonlinear interaction region are [cf., (11)]

$$E_{0-1}(x,z)=rE_{01}(z,x), \quad E_{-1-1}(a,y,z)=re^{i\varphi}E_{-11}(z,-a,y).$$

Using (6) and (7), this immediately yields the boundary condition for the function f:

$$\frac{\partial}{\partial y}f(d,y,z)=-\frac{K}{1-i\Delta}|r|^2e^{i\varphi}|E_{01}(y,z)|^2f(z,-d,y)$$

$$\times\exp\left[\frac{K}{1-i\Delta}\int^{y}dy'|E_{01}(y',z)|^2-\frac{K}{1-i\Delta}\int^{-d}dy'|E_{01}(y',y)|^2\right.$$

$$\left.+\frac{K|r|^2}{1-i\Delta}\int^{d}dy'|E_{01}(z,y')|^2-\frac{K|r|^2}{1-i\Delta}\int^{z}dy'|E_{01}(y,y')|^2\right].$$

Substitution of this in (9), together with $f(x,d,z)=0$, determines the function $f(x,y,z)$ throughout the interaction region. Next, assuming that $y=-d$, we obtain the integral equation [cf., (12)]

$$f(x,-d,z)=\frac{K|r|^2}{1-i\Delta}e^{i\varphi}\int_{-d}^{d}dy\,\vartheta(x,-d;d,y;z)$$

$$\times|E_{01}(y,z)|^2f(z,-d,y)\exp\left[\frac{K}{1-i\Delta}\int_{-d}^{y}dy'|E_{01}(y',z)|^2\right.$$

$$\left.+\frac{K|r|^2}{1-i\Delta}\int_{-d}^{d}dz'|E_{01}(z,z')|^2-\frac{K|r|^2}{1-i\Delta}\int_{-d}^{z}dz'|E_{01}(y,z')|^2\right]. \qquad (18)$$

Its eigenfunctions determine the lateral structure of the scattered beams, and the corresponding eigenvalues determine the generation thresholds and pump wave frequencies. In particular, the structure of the beam $E_{-1-1}(-d,y,z)$ leaving the interaction region is determined by the form factor $F(y,z)\sim E_{-1-1}(-d,y,z)/E^*_{01}(y,z)$, which is related to the function $f(x,-d,z)$ by

$$F(y,z)=f(z,-d,y)\exp\Big[\frac{K|r|^2}{1-i\Delta}\int_{-d}^{d}dz'\,|E_{01}(z,z')|^2$$

$$-\frac{K|r|^2}{1-i\Delta}\int_{-d}^{z}dz'\,|E_{01}(y,z')|^2\Big]-|E_{01}(y,z)|^{-2}$$

$$\times\int_{-d}^{d}dy'\Big[\frac{\partial}{\partial y}\vartheta(-d,y;d,y';z)\Big]\,|E_{01}(y',z)|^2 f(z,-d,y')$$

$$\times\exp\Big[\frac{K}{1-i\Delta}\int_{y}^{y'}dy''|E_{01}(y'',z)|^2+\frac{K|r|^2}{1-i\Delta}\int_{-d}^{d}dz'\,|E_{01}(z,z')|^2$$

$$-\frac{K|r|^2}{1-i\Delta}\int_{-d}^{z}dz'\,|E_{01}(y',z')|^2\Big].\qquad(19)$$

We first note that both (18) and the form factor (19) depend only on the intensity distribution $|E_{01}(y,z)|^2$ over the cross-section of the pump beam, and do not depend on the phase distribution over the beam cross section. Hence, it follows that the backscattered field has the complex conjugate phase, but the intensity distribution over the cross section of the scattered beam may, in general, be different from the corresponding distribution over the cross section of the pump beam.

To illustrate the last point, let us consider a pump beam of square cross-section ($2d\times 2d$) with uniform intensity distribution (phase-modulated pump). Equation (18) then assumes the form

$$f(x,-d,z)=\frac{\varkappa|r|^2}{2d(1-i\Delta)}\exp\Big[i\varphi+\frac{1}{2}\frac{\varkappa|r|^2}{1-i\Delta}\Big(1-\frac{z}{d}\Big)\Big]$$

$$\times\int_{-d}^{d}dy\,I_0\Big\{\frac{\varkappa|r|}{1-i\Delta}\Big[\Big(1+\frac{y}{d}\Big)\Big(1-\frac{x}{d}\Big)\Big]^{1/2}\Big\}f(z,-d,y)$$

$$\times\exp\Big[\frac{\varkappa}{2(1-i\Delta)}\Big(1+\frac{y}{d}\Big)\Big],\qquad(20)$$

where $\varkappa=2K|E_0|^2d$ is the convective amplification of the scattered waves within the beam thickness.

Equation (20) is similar to (13) in having a set of different eigenfunctions $f(x,-d,z)$ (lateral modes). To determine them, let us expand the Bessel function in the kernel of (20) into a power series. The solution of (20) then assumes the form

$$f(x,-d,z)=\exp\Big[\frac{\varkappa|r|^2}{2(1-i\Delta)}\Big(1-\frac{z}{d}\Big)\Big]\sum_{i,k=0}^{\infty}\frac{f_{ik}}{i!k!}$$

$$\times\Big(\frac{\varkappa|r|}{2(1-i\Delta)}\Big)^{i+k}\Big(1-\frac{x}{d}\Big)^{i}\Big(1-\frac{z}{d}\Big)^{k},$$

where the coefficient f_{ik} are given by the set of linear equations

$$f_{ik}=\sum_{j=0}^{\infty}a_{ij}f_{kj},\qquad(21)$$

whose elements a_{ij} are given by (14). However, in contrast to (14), the eigenvalues of (21) and, accordingly, the eigenvalues of (20) are now labeled by two indices, namely, m,m'.

Only the first few elements a_{ij} need be taken into account in the limit as $|r|\ll 1$. The following dispersion relations are then obtained for the first four eigenstates in (21):

$$|r|^2\exp\Big(i\varphi+\frac{\varkappa_{00}}{1-i\Delta_{00}}\Big)=1,$$

$$|r|^3\exp\Big(i\varphi+\frac{\varkappa_{01}}{1-i\Delta_{01}}\Big)=i,\quad |r|^3\exp\Big(i\varphi+\frac{\varkappa_{10}}{1-i\Delta_{10}}\Big)=-i,$$

$$|r|^4\exp\Big(i\varphi+\frac{\varkappa_{11}}{1-i\Delta_{11}}\Big)=-1.\qquad(22)$$

As in the case of (15), each of the equations in (22) has a denumerable set of solutions differing by the pump-wave frequency. In particular, (17) is again valid for $\varkappa_{00,n}$, Δ_{00} $\varkappa_{11,n}$, and $\Delta_{11,n}$. Similarly, the generation thresholds are reduced as $|r|$ is increased. The minimum generation threshold $\varkappa_{00,0}=0.54$ occurs for $|r|=1$, $\varphi=0$.

According to (19), in the case of the modes (22), we obtain the following expressions for the form factors $F^{(m,m')}(y,z)\simeq f^{(m,m')}(z,-d,y)$, which define the difference between the wave front of the escaping beam and the complex conjugate pump beam:

$$F^{(0,0)}(y,z)\approx 1+\frac{\varkappa_{00}|r|^2}{2(1-i\Delta_{00})}\Big[\Big(1-\frac{y}{d}\Big)+\Big(1-\frac{z}{d}\Big)\Big]$$

$$+o(\varkappa|r|)^4,$$

$$F^{(0,1)}(y,z)\approx-\frac{\varkappa_{01}}{2(1-i\Delta_{01})}\Big(1-\frac{y}{d}\Big)$$

$$+1+\frac{i\varkappa_{01}|r|}{2(1-i\Delta_{01})}\Big(1-\frac{z}{d}\Big)+o(\varkappa^4|r|),$$

$$F^{(1,0)}(y,z)\approx-\frac{\varkappa_{10}}{2(1-i\Delta_{10})}\Big(1-\frac{y}{d}\Big)$$

$$+1-\frac{i\varkappa_{10}|r|}{2(1-i\Delta_{10})}\Big(1-\frac{z}{d}\Big)+o(\varkappa^4|r|),$$

$$F^{(1,1)}(y,z)\approx-\frac{\varkappa_{11}^2}{4(1-i\Delta_{11})^2}\Big(1-\frac{y}{d}\Big)\Big(1-\frac{z}{d}\Big)-\frac{\varkappa_{11}}{2(1-i\Delta_{11})}$$

$$\times\Big[\Big(1-\frac{y}{d}\Big)+\Big(1-\frac{z}{d}\Big)\Big]+1+o(\varkappa^2|r|^2).$$

The principal-mode form factor is practically constant over the beam cross-section, i.e., it corresponds to phase conjugation. Since the principal-mode thresholds lie below the thresholds for the other modes by a factor of about 1.5–2 [cf., (17)], the Stokes radiation should be nearly phase conjugate for a small excess above the generation threshold.

The above analysis can be extended to the case of a pump with an arbitrary intensity distribution over the beam cross-section. However, it is then important to note that the above example is "ideal." For a beam with an arbitrary intensity distribution over the cross-section, the form factor of the radiation leaving the cavity is no longer constant even for the principal mode: it is determined by the specific form of the intensity distribution and by losses in the cavity.

6. Let us now use our theory to examine phase conjugation in an optical ring cavity without rotation of the beam cross section, but assuming that the beam is compressed in the optical channel in the direction perpendicular to the plane in which self-crossing occurs. We shall now use the configuration of Fig. 1a, for which, if there are lenses in the channel, the boundary conditions can be written in the form

$$E_{0-1}(y,z)=(r/(\alpha\beta)^{1/2})E_{01}(y/\beta,z/\alpha),\qquad(23)$$

$$E_{-1-1}(d,y,z)=re^{i\varphi}(\alpha\beta)^{1/2}E_{-11}(\beta y,-d,\alpha z).$$

The coefficients α and β then determine the variation in the scale of the beam cross-section in z and y, respectively. Using

(6) and (7), we obtain the following boundary conditions for f:

$$\frac{\partial}{\partial y}f(d,y,z)=-\frac{K|r|^2}{1-i\Delta}e^{i\varphi}|E_{01}(y,z)|^2f(\beta y,-d,\alpha z)$$

$$\times\exp\left[\frac{K}{1-i\Delta}\left(1-\frac{|r|^2}{\alpha}\right)\int_{-d}^{y}dy'|E_{01}(y',z)|^2+\frac{1}{1-i\Delta}\frac{|r|^2}{\alpha}\varkappa(z/\alpha)\right],$$

$$\varkappa(z)=K\int_{-d}^{d}dy|E_{01}(y,z)|^2.$$

Substituting this and $f(x,d,z)=0$ in (9), we obtain the relation between the function f in the interaction region and its values on the boundary $f(\beta y,-d,\alpha z)=\Psi(y,z)$. Next, substituting $y=-d$ in (9), we obtain the integral equation for Ψ:

$$\Psi(y,z)=\frac{K|r|^2}{1-i\Delta}e^{i\varphi}\int_{-d}^{d}dy'\vartheta(\beta y,-d;d,y';\alpha z)\times|E_{01}(y',\alpha z)|^2\Psi(y',\alpha z)$$

$$\times\exp\left[\frac{K}{1-i\Delta}\left(1-\frac{|r|^2}{\alpha}\right)\int_{-d}^{y'}dy''|E_{01}(y'',\alpha z)|^2+\frac{1}{\alpha}\frac{|r|^2}{1-i\Delta}\varkappa(z)\right]. \quad (24)$$

Although the parameter β is formally present in the kernel ϑ, the use of its explicit form (10) and of the boundary conditions (23) shows that ϑ does not depend on β. It follows that the variation in the scale of the beam in the plane of intersection has no influence on the generation conditions or the structure of the scattered beam. Equation (24) becomes identical with (12) when $\alpha=1$.

In contrast to (12), when $\alpha<1$, Eq. (24) determines Ψ as a function of both coordinates as a consequence of the scaling transformation with respect to z. Substituting $z=0$ in (24), we obtain a closed equation for

$$\Psi(y,0)=F(s),\quad s=K\int_{y}^{d}dy'\ |E_{01}(y',0)|^2,$$

which is analogous to (13), but differs from it by the fact that the exponentials and the argument of the Bessel function contain $|r|/\alpha^{1/2}$ instead of $|r|$ and, moreover, $\varkappa=\varkappa(0)$. Consequently, the generation threshold is determined by the gain in the beam layer in which the ray paths are planar. The analysis of the equation for $\Psi(y,0)$ can be performed by analogy with the corresponding analysis of (13). When the radiation is strongly attenuated in the optical system ($|r|^2\ll\alpha$), the generation conditions for the mode with lateral index $m=0$ are determined by the first formula in (17). For the $m=1$ threshold, we have

$$\varkappa_{1n}=(1+\Delta_{1n}^2)\ln(\alpha/|r|^4),\quad \Delta_{1n}=[(2n+1)\pi-\varphi]\ln^{-1}(\alpha/|r|^4),$$

where $F^{(0)}(s)\simeq1$, $F^{(1)}(s)\simeq s$. Consequently, in this case, the threshold for the principal lateral mode does not depend on the compression of the beam, whereas the threshold for the first mode decreases logarithmically with decreasing α.

In the other limiting case, i.e., $\alpha\ll|r|^2$, the principal-mode generation threshold no longer depends on α:

$$\varkappa_{0n}=(1+\Delta_{0n}^2)(\alpha/|r|^2)\ln(1/\alpha),\quad \Delta_{0n}=(2\pi n-\varphi)\ln^{-1}(1/\alpha),$$

$$\varkappa_{1n}=(1+\Delta_{1n}^2)(\alpha/|r|^2)\ln(|r|^2/\alpha^2),$$

$$\Delta_{1n}=[(2n+1)\pi-\varphi]\ln^{-1}(|r|^2/\alpha^2),$$

where

$$F^{(0)}(s)=1+\alpha^{1/2}s/|r|(1-i\Delta)\approx1,\quad F^{(1)}(s)\approx1-\alpha^{1/2}s/\varkappa|r|\sim s.$$

Since the thresholds for modes with $m\neq0$ are much higher than the principal-mode threshold, we turn to the analysis of Ψ as a function of z for the mode $m=0$. We shall use the fact that the eigenfunction is then practically independent of y for any $|r|$ and α. Moreover, we shall allow for the fact that, in (24), we can then neglect the departure of the kernel ϑ from unity. From (24), the equation for the function $\Psi^{(0)}(y,z)\equiv\Psi^{(0)}(z)$ then assumes the form

$$\Psi^{(0)}(z)=\frac{|r|^2e^{i\varphi}}{(1-|r|^2/\alpha)}\Psi^{(0)}(\alpha z)\exp\left[\frac{\varkappa(z)|r|^2}{\alpha(1-i\Delta)}\right]\times\left\{\exp\left[\frac{\varkappa(\alpha z)}{1-i\Delta}\left(1-\frac{|r|^2}{\alpha}\right)\right]-1\right\}\equiv D(z)\Psi^{(0)}(\alpha z).$$

Its solution under the normalizing condition $\Psi^{(0)}(0)=1$ is

$$\Psi^{(0)}(z)=\prod_{n=0}^{\infty}D(\alpha^nz), \quad (25)$$

and determines the lateral structure of the scattered beam both in the ring cavity at entry to the interaction region, $E_{-1-1}(d,y,z)$, and after leaving the cavity, $E_{-1-1}(-d,y,z)$:

$$E_{-1-1}(d,y,z)=|r|^2e^{i\varphi}E_{01}^*(y,z)\Psi^{(0)}(z)\times\exp\left[-\frac{K}{1-i\Delta}\frac{|r|^2}{\alpha}\int_{-d}^{y}dy'|E_{01}(y',z)|^2\right], \quad (26)$$

$$E_{-1-1}(-d,y,z)=E_{-1-1}(d,y,z)\exp\left[\frac{|r|^2}{\alpha(1-i\Delta)}\varkappa(z/\alpha)\right]$$

$$\times\left\{1+\frac{|r|^2}{\alpha(1-|r|^2/\alpha)}\varkappa(z/\alpha)\times\exp\left[\frac{K}{1-i\Delta}\left(1-\frac{|r|^2}{\alpha}\right)\int_{y}^{d}dy'|E_{01}(y',z)|^2\right]\right\}. \quad (27)$$

The expression for $\Psi^{(0)}(z)$ is particularly simple when $\alpha\ll1$. If we use the approximation $\varkappa(^nz)=\varkappa(0)+\alpha^nz\varkappa'(0)$, we then have

$$\Psi^{(0)}(z)\approx\exp\left[\frac{|r|^2}{\alpha(1-i\Delta)}(\varkappa(z)-\varkappa(0))+\frac{\alpha z\varkappa'(0)}{(1-i\Delta)(1-\alpha)}\right]. \quad (28)$$

It follows from (25)–(28) that the structure of the wave fronts of the scattered radiation (26)–(27) depends on the degree of compression of the beam. When $\alpha\lesssim|r|^2$, the function $\Psi^{(0)}(z)$ given by (28) and, together with it, the ratio $E_{-1-1}(\pm d,y,z)/E_{01}^*(y,z)$, are very dependent on z over the cross-section of the beam. The quality of phase conjugation is then poor. If, on the other hand,

$$|r|^2\ln(1/|r|^2)<\alpha<\ln^{-1}(1/|r|^2), \quad (29)$$

the function $\Psi^{(0)}(z)$ is a slowly-varying function of z, and $E_{-1-1}(d,y,z)\sim E_{01}^*(y,z)$, i.e., the scattered radiation in the ring cavity is phase conjugate to the pump. The scattered field $E_{-1-1}(-d, y,z)$ at exit from the nonlinear interaction region does not then reproduce the complex conjugate wave front of the pump, $E^*_{01}(y,z)$, because of the distortions that arise during the second crossing of the interaction region, which are described by the second term in braces in (27). These distortions are localized in a narrow band $\Delta z \sim \alpha d$ near the right-hand edge of the beam $y \simeq d$, where they can reach the substantial figure $\sim \alpha^{-1} \ln(1/|r|^2) \gg 1$. Despite the fact that the localization region is narrow, the distorted part of the backscattered beam leaving the interaction region contains practically all the energy. This means that, when the coefficients of the scaling transformation and the reflectivities are low and satisfy (29), a beam with high-grade phase conjugation cannot be extracted from the nonlinear interaction region in the backward direction, but it *can* be extracted from the interior of the optical channel through one of the semitransparent mirrors.

When attenuation in the system is low ($r \sim 1$), formulas (25)–(27) show that the formfactor Ψ will vary appreciably with z over the cross section of the beam. In general, the explicit form of $\Psi(z)$ does not then depend on the intensity distribution over the beam cross section, or on the magnitude of α. For example, when $\alpha = 1 - \delta$, where $\delta \ll 1$, the function $\Psi(z)$ varies from unity at $z = 0$ to $\exp(\Delta\varkappa/\varkappa(0)\delta)$ for $|z| \sim d$, where $\Delta\varkappa = \varkappa(d) - \varkappa(0)$. When $\alpha \ll 1$, the function $\Psi(z)$ varies from $\Psi(0) = 1$ to $\exp(\Delta\varkappa|r|^2/\alpha) \sim 1/\alpha$ for $|z| \sim d$. On the other hand, when $|r|^2 = \alpha$, and the beam has the parabolic gain distribution $\varkappa(z) = \varkappa(0)(1 - z^2/d^2)$, the smallest distortions occur for $\alpha \simeq 0.33$ and $\Psi(z) \simeq \exp(-1.3z^2/d^2)$. Accordingly, we reach the more general conclusion that distortions $\Delta\Psi/\Psi \sim 1$ occur when $r \sim 1$ for $\alpha \sim 0.3$–0.5.

7. The above theory of generation by stimulated scattering in a self-intersecting beam enables us to examine the structure of the amplitude and phase distributions in the scattered field, and indicates the conditions under which this field structure can be fully determined. This enables us to consider the quality of phase conjugation in different possible systems exploiting this phenomenon for generation, using self-crossing light beams.

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