

# Some characteristics of the superlattice conductivity in a quantizing magnetic field

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Variation of a quantizing magnetic field  $H$  applied along the axis of a one-dimensional superlattice induces metal-insulator phase transitions due to the dependence of the density of states in the minibands on  $H$ . An analysis is made of the existence of the intervals of  $H$  in which Fermi electrons moving along the magnetic field become localized. The characteristics of the photoconductivity of a superlattice in a quantizing magnetic field are determined.

## 1. ENERGY SPECTRUM

The interest in the electronic properties of superlattices is due to the unique ease of controlling their band structure and of imparting a number of properties not attainable in natural crystals.<sup>1,2</sup> They include, in particular, a strong anisotropy. In the case of one-dimensional superlattices (to which our treatment will be confined) the electrons which move across the layers have to overcome relatively high potential barriers distributed with a period  $d$  and their energy spectrum in this direction can be described in the tight-binding approximation. However, the electrons traveling in the planes of the layers are free and they retain the dispersion law of the original crystal  $\varepsilon_0(\mathbf{k})$ . In a wide range of carrier densities typical of semiconductors this dispersion law can be regarded as quadratic (for simplicity, we shall assume that it is isotropic). Consequently, the energy spectrum of a superlattice is

$$\varepsilon_i(\mathbf{k}) = \hbar^2(k_x^2 + k_y^2)/2m_{\perp} + \varepsilon_i + \frac{1}{2}\Delta_i(1 - \cos k_z d), \quad (1)$$

where  $\varepsilon_i$  is the position of the bottom of the  $i$ th miniband and  $\Delta_i$  is the width of this miniband. The low probability of tunneling across a barrier, which justifies the use of the tight-binding approximation, means that  $\Delta_i \ll \varepsilon_{i+1} - \varepsilon_i$ . This in turn indicates that longitudinal effective masses near the miniband extrema (which represent the motion along the superlattice axis) are very large for fairly low values of  $\Delta_i$ :

$$m_{\parallel}^{(i)} = 2\hbar^2/\Delta_i d^2 \gg m_{\perp}. \quad (2)$$

If  $\Delta_i \gtrsim 5 \times 10^{-3}$  eV and  $d \approx 50$  Å, the longitudinal effective mass  $m_{\parallel}^{(i)}$  is larger than the mass of a free electron.

If the electron gas in a superlattice is degenerate, its principal properties are known to be determined by the shape of the Fermi surface.<sup>3</sup> It readily follows from Eq. (1) that in the majority of cases the Fermi surface of a superlattice is a system of corrugated cylinders. Only when the Fermi energy  $\zeta$  lies within a narrow interval of width  $\Delta_i$ , defined by

$$\varepsilon_i < \zeta < \varepsilon_i + \Delta_i, \quad (3)$$

is one of the sheets in the form of a strongly elongated ovaloid and at the Fermi energy close to the left-hand limit of this interval the Fermi surface is a strongly elongated ellipsoid with the ratio of the axes equal to  $(m_{\parallel}^{(i)}/m_{\perp})^{1/2} \gg 1$ .

It therefore follows that variation of the carrier density  $n$  or of the superlattice parameters can alter the Fermi surface topology (as first pointed out in Ref. 4). Then, at the

points where  $\zeta = \varepsilon_i$  we can expect phase transitions of order two-and-half (see Ref. 3) due to the appearance of a new Fermi surface sheet, whereas at the points where  $\zeta = \varepsilon_i + \Delta_i$  these phase transitions are due to conversion of a closed Fermi surface into an open one. This should result in disappearance of saturation of one component of the magnetoresistance in a classically strong magnetic field applied to the superlattice axis.<sup>1)</sup>

We shall now consider the properties of a superlattice in a strong magnetic field  $\mathbf{H}$  parallel to the superlattice axis. Such a field quantizes motion in the plane of the layers and lifts the spin degeneracy, giving rise to an energy spectrum of the type

$$\varepsilon_{i\sigma}(k_z) = \hbar\omega_c(q + \frac{1}{2}) + \sigma\mu_0 H + \varepsilon_i + \frac{1}{2}\Delta_i(1 - \cos k_z d). \quad (4)$$

Here,  $\omega_c = eH/m_{\perp}c$ ;  $\mu_0$  is the paramagnetic moment of an electrons which, in the absence of the spin-orbit interaction, is identical with the Bohr magneton;  $q = 0, 1, 2, \dots$ ;  $\sigma = \pm 1$ . The energy spectrum of Eq. (4) corresponds to the density of states

$$g(\varepsilon) = \frac{eH}{2\pi^2 \hbar c d} \sum_{i,q,\sigma} \left\{ \frac{\Delta_i^2}{4} - \left[ \varepsilon - \hbar\omega_c \left( q + \frac{1}{2} \right) - \sigma\mu_0 H - \varepsilon_i - \frac{\Delta_i}{2} \right]^2 \right\}^{-1/2}, \quad (5)$$

which at the miniband edges has square-root singularities typical of quasideimensional motion.

In relatively weak magnetic fields ( $\hbar\omega_c \ll \Delta_i$ ) the Landau quantization has the usual consequences: at low temperatures all the electronic characteristics of a superlattice oscillate as a function of  $1/H$  with periods  $\Delta(1/H) = 2\pi\hbar e/cS_{\text{extr}}$ , where  $S_{\text{extr}}$  is an extremal section of the Fermi surface (de Haas-van Alphen and Shubnikov-de Haas effects). Each corrugated cylinder, with extremal section differing little from that of another cylinder (i.e., when  $\zeta$  is not too close to  $\varepsilon_i + \Delta_i$ ), should give rise to beats in the form of amplitude oscillations with a large period<sup>2)</sup> independent of  $n$ :

$$\Delta^{(b)}(1/H) = e\hbar/cm_{\perp}\Delta_i. \quad (6)$$

## 2. METAL-INSULATOR TRANSITIONS

A further increase of the magnetic field alters considerably the properties of a superlattice associated with the appearance of energy gaps in the density of states described by Eq. (5). The appearance of such gaps is mentioned in Refs. 6

and 7. The gaps occur when the separations between the energy levels exceed the miniband width, i.e., if

$$\hbar\omega_c, 2\mu_0 H > \Delta_i. \quad (7)$$

In this case the spectrum splits into separate allowed minibands, each of which has

$$N = eH/2\pi\hbar cd \quad (8)$$

states per unit volume. In the limit  $\Delta_i \rightarrow 0$  these minibands are transformed into  $\delta$ -like peaks of the density of states, which are typical of two-dimensional systems in a magnetic field.<sup>8</sup>

Since the capacity of  $N$  of each miniband depends on the magnetic field, the change in the field in the case of a constant total carrier density  $n$  alters the band populations in a superlattice. Then, at  $T = 0$  the minibands with the period

$$\Delta(1/H) = e/2\pi\hbar cd \quad (9)$$

are completely filled and emptied, i.e., metal-insulator transitions take place.

It follows that every time when the field has the value

$$H = H_s = 2\pi\hbar cd/es, \quad s = 1, 2, 3, \dots, \quad (9a)$$

i.e., every time that the carrier density is an integral multiple of the miniband capacity  $N$ , the chemical potential level is "switched" from one miniband to another.

It should be stressed that the periodicity of these metal-insulator transitions is independent of which of the minibands are neighbors. In moderate fields these minibands are separated by a gap  $\sim \hbar\omega_c$ , whereas in extremely high fields the gaps are of the order of  $\varepsilon_{i+1} - \varepsilon_i$ . It should be stressed that in this case we have an unusual situation: oscillations are not due to a change in the population of the Landau levels, but due to the filling or emptying of minibands with different serial numbers  $i$  [see Eqs. (1) and (4)].

In an ideal sample at  $T = 0$  this switching of the chemical potential from one miniband to another is abrupt, but at  $T \neq 0$  the dependence  $\zeta(H)$  has no singularities (see Figs. 1 and 2). We shall now determine the nature of the dependence  $\zeta(H)$  near  $H = H_s$ . We shall consider two minibands between which the Fermi level is located in a field  $H = H_s$  and we shall assume that energies are measured from the bottom of the lower miniband (Fig. 1). If  $\hbar\omega_c \gg T$ , the other minibands play no role. We can find  $\zeta$  from the equation

$$\int_0^\Delta \left\{ 1 / \left[ \exp\left(\frac{\varepsilon - \zeta}{T}\right) + 1 \right] + 1 / \left[ \exp\left(\frac{\hbar\omega_c + \varepsilon - \zeta}{T}\right) + 1 \right] \right\} \times \frac{d\varepsilon}{[\varepsilon(\Delta - \varepsilon)]^{1/2}} = \pi \frac{H_s}{H}. \quad (10)$$

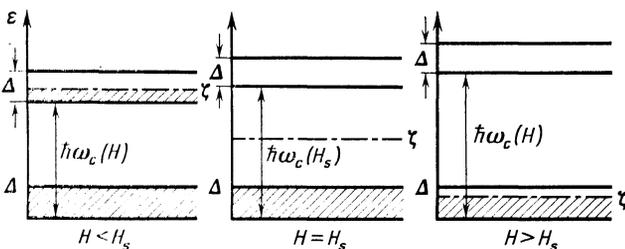


FIG. 1. Electron occupancy of minibands.

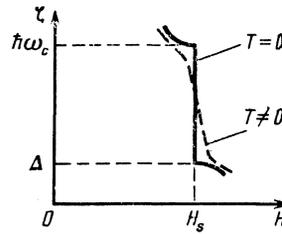


FIG. 2. Dependence of the chemical potential on the magnetic field.

We are assuming here that both minibands correspond to neighboring Landau levels with the same quantum number  $i$  (in the case when  $\hbar\omega_c < \varepsilon_{i+1} - \varepsilon_i$  this is the most probable situation). The width of the two minibands is the same (we shall omit the index  $i$ ).

The solution of Eq. (10) for  $T \ll \Delta$  and  $H \approx H_s$  is

$$\zeta = \begin{cases} \Delta - \frac{1}{4}\pi^2\Delta(H_s/H-1)^2, & H - H_s \gg H_s(T/\Delta)^{1/2}, \\ \frac{1}{2}(\hbar\omega_c + \Delta) + T \ln[(1 + \delta^2)^{1/2} + \delta], & |H - H_s| \ll H_s(T/\Delta)^{1/2}, \\ \hbar\omega_c + \frac{1}{4}\pi^2\Delta(H_s/H-1)^2, & H_s - H \gg H_s(T/\Delta)^{1/2}, \end{cases} \quad (11)$$

where

$$\delta = \frac{1}{2} \left( \frac{\pi\Delta}{T} \right)^{1/2} \left( \frac{H_s}{H} - 1 \right) \exp\left(\frac{\hbar\omega_c - \Delta}{2T}\right).$$

In the case when the deviation of  $H$  from  $H_s$  is exponentially small, we have

$$\zeta \approx \frac{\hbar\omega_c + \Delta}{2} + \frac{1}{2}(\pi T \Delta)^{1/2} \exp\left(\frac{\hbar\omega_c - \Delta}{T}\right) \left( \frac{H_s}{H} - 1 \right), \\ \left| \frac{H_s}{H} - 1 \right| \ll \frac{\hbar\omega_c}{(T\Delta)^{1/2}} \exp\left(-\frac{\hbar\omega_c}{T}\right). \quad (11a)$$

If a gap between minibands contains localized states, the shift of the Fermi energy  $\zeta$  from a miniband to another is smeared out and depends on the density of states in the "forbidden" band.

Since a metal-semiconductor transition described here results in a redistribution of electrons between minibands which are relatively far apart on the energy scale (when  $\hbar\omega_c \gg T$ ), we have to consider the relaxation time  $\tau_R$  of such a transition. At first sight it might appear that the attainment of an equilibrium requires an exponentially long time:  $\tau_R \propto \exp(\hbar\omega_c/T)$ . In reality this is not true.<sup>3)</sup> Figure 1 illustrates the energies of elementary excitations: in a field  $H = H_s$  the excitation of the system requires overcoming of a gap amounting to  $\hbar\omega_c - \Delta$ , whereas in a field  $H \neq H_s$  there is no need to overcome this gap. We must remember that when  $H$  is varied (in particular, when the value  $H = H_s$  is passed), the energy of the ground state of the whole system changes continuously. In fact, it readily follows from Eqs. (5) and (11) that the contribution of the miniband being filled to the energy of the system tends to zero in the limit  $H \rightarrow H_s$ . Therefore, the change in the ground state due to an adiabatic change in the magnetic field may occur instantaneously. This is not in conflict with the fact that in a field  $H \lesssim H_s$  some of the electrons are in a miniband with the energy equal to or even exceeding  $\hbar\omega_c$ . The region for the transition is the need to satisfy the requirements of the Pauli principle. In other words, the whole electron system should be modified (collective effect) and the value of  $\hbar\omega_c$  (one-electron gap) should be divided by the total number of elec-

trons. Consequently, in the thermodynamic limit a single electron need not acquire a finite energy for the transition to the upper miniband: the true gap is zero. As long as the magnetic field is varying, the system does not achieve a steady state and electrons do not have a definite energy; however, at the end of the process some of the electrons (governed by the difference between  $H$  and  $H_s$ ) are in the upper miniband.

We shall conclude this section by considering the case of extremely strong fields:

$$H > 2\pi\hbar cnd/e, \quad (12)$$

when only the lowest miniband of width  $\Delta_0$  is partly filled, i.e., when a metallic state is obtained. The Fermi energy measured from the bottom of the miniband decreases on increase in the field:

$$\zeta = \Delta_0 \sin^2(p_z^F d/2\hbar), \quad p_z^F = (2\pi\hbar)^2 nc/eH. \quad (13)$$

If  $p_z^F d/\hbar \ll 1$ , we have

$$\zeta = \frac{1}{m_{\parallel}^{(0)}} \frac{(2\pi\hbar)^4 n^2 c^2}{e^2 H^2} = \frac{16}{9} \frac{m_{\perp}}{m_{\parallel}^{(0)}} \zeta_{\perp} \left( \frac{\zeta_{\perp}}{\hbar\omega_c} \right)^2, \quad (14)$$

$$\zeta_{\perp} = \frac{(2\pi\hbar)^2}{2m_{\perp}} \left( \frac{3n}{8\pi} \right)^{1/2}, \quad \hbar\omega_c \gg \zeta_{\perp}.$$

We can see that in this quantum limit of very high fields the anisotropy induces strongly (in accordance with the mass ratio) the Fermi energy. This means that in the case of a superlattice we can observe lifting of the degeneracy of an electron gas: an increase in  $H$  may give rise to the case when  $\zeta \ll T$ , even when  $T \ll \Delta_0$ . Similar behavior can be expected also at  $H \approx H_s$ , when the Fermi energy measured from the edge of the relevant miniband is less than the temperature  $T$  of a sample and the temperature is less than the width of the corresponding miniband.

### 3. LONGITUDINAL CONDUCTIVITY IN A QUANTIZING MAGNETIC FIELD

We shall consider galvanomagnetic properties of a superlattice in the range of strong fields defined by Eq. (7) when there are energy gaps in the density of states. As pointed out already, in this case a superlattice is in many respects similar to a purely two-dimensional system in a quantizing field. In a plane  $xy$  perpendicular to the field the diagonal components of the conductivity  $\sigma_{xx}$  and  $\sigma_{yy}$  should vanish periodically and, because of the localization effects, this should occur in finite intervals of  $H$ . In the same intervals the Hall conductivity  $N$  should exhibit a plateau typical of the quantum Hall effect, but because of the parallel connection of  $N$  superlattice layers, this should be observed for  $\sigma_{xy} = jNe^2/2\pi\hbar(j, 1, 2, 3, \dots)$ . Such a plateau (after allowance for the ineffectiveness of the outer layers in real superlattices) has indeed been observed.<sup>9</sup>

However, there is an effect which distinguishes a superlattice in a basic manner from two-dimensional systems: this is the conductivity  $\sigma_{zz}$  along the applied magnetic field. We shall consider this quantity in more detail. The periodic metal-insulator transitions mentioned above should give rise to giant oscillations of the value of  $\sigma_{zz}$ . A fairly unusual situation appears: variation of the magnetic field can alter qualitatively the nature of the conductivity not only across, but also along the field.

Some general expression for the longitudinal conductivity of a superlattice in a magnetic field in the case of scattering by phonons are deduced by Polyanovskii<sup>7,10</sup> and experimental evidence of the vanishing of  $\sigma_{zz}$  at  $H = H_2$  is given in Ref. 9 [our notation of the fields is used: see Eq. (9a)]. We shall make clear the physics of these phenomena and consider the main features of the metal-insulator transitions described above by estimating the conductivity  $\sigma_{zz}$  using the relaxation time ( $\tau$ ) approximation.

In the degenerate case the contribution to the conductivity comes only from a partly filled band and we have

$$\sigma_{zz} = (e^2 H \tau / 2\pi^2 \hbar^2 c) |v_z^F|, \quad (15)$$

where the Fermi velocity is  $|v_z^F| = d [\zeta(\Delta_i - \zeta)]^{1/2}/\hbar$ , if the Fermi energy  $\zeta$  is measured from the bottom of the corresponding miniband.

The transitions described above are manifested as vanishing of  $v_z^F$  at the points  $H = H_s$ . The actual nature of the dependence  $\sigma_{zz}(H)$  is governed not only by  $v_z^F$ , but also by the dependence of  $\tau$  on  $H$ , which itself may be singular. The special nature of a superlattice lies in that, because of the law of conservation of energy, a particle with a wave vector  $k_z$  can be scattered elastically only to a state  $-k_z$ . A similar situation occurs in the case of the longitudinal magnetoconductance of three-dimensional systems in the extreme quantum limit,<sup>11,12</sup> except that in our case the dispersion law  $\varepsilon(k_z)$  is not quadratic. We then have

$$\tau^{-1} = \frac{g(\zeta)c}{eH} \int dq_x dq_y \exp \left[ -\frac{c\hbar}{2eH} (q_x^2 + q_y^2) \right] \times |V_{q_x, q_y, -2k_z, F}|^2, \quad (16)$$

where  $k_z^F = d^{-1} \cos^{-1}(1 - 2\zeta/\Delta)$ ;  $V_q$  is the Fourier component of the scattering potential. We can show (see Table II in Ref. 11) that for the majority of the elastic scattering mechanisms the dependence of the integral in Eq. (16) on  $\zeta$  is of little importance. Then,  $\tau$  vanishes at  $H = H_s$  entirely due to the singularity of  $g(\zeta)$  and on the whole near the transition points we have

$$\sigma_{zz} \propto \sigma_{zz}^{(0)} [(H - H_s)/H_s]^2, \quad (17)$$

where

$$\sigma_{zz}^{(0)} \approx \frac{e^3 H \tau}{\pi^2 \hbar^2 c} \frac{d}{\hbar} \Delta \quad (18)$$

is the value of the conductivity  $\sigma_{zz}$  in fields  $H \neq H_s$  and  $\tau^{-1}$  is understood to be the probability of elastic backscattering of a particle moving at a velocity  $\sim d\Delta/\hbar$ . The above formula may be useful in estimating the value of  $\tau$  by comparison with experimental results. We recall that vanishing of the conductivity described by Eq. (17) should be repeated periodically and the period is given by Eq. (9)

In fact,  $\sigma_{zz}$  should vanish not only at individual points  $H = H_s$ , but in whole intervals  $\Delta H$  of magnetic field because of the presence of localized states. If we assume that the localization criterion is the Ioffe-Regel condition  $k_z^F l \sim 1$  ( $l$  is the mean free path), it then follows from  $l \propto (H - H_s)^2$ , and  $k_z^F \propto |H - H_s|$  that these intervals  $\Delta H$  can be estimated from

$$\Delta H \propto H_s (d/l)^{1/2}, \quad l \gg d, \quad (19)$$

where  $\tau$  is the mean free path in fields  $H$  far from the critical

values  $H_s$ . We note that the intervals in which the component of the transverse conductivity  $\sigma_{xx}$  vanishes should be different from those in which  $\sigma_{yy}$  vanishes because there are no grounds for assuming identical localization conditions in the planes of the layers and along the magnetic field (in particular, this is true because of the difference between the structures of the wave functions along different directions).

The three-dimensional nature of a superlattice should be manifested particularly clearly in the possibility of propagation of an undamped helicoidal electromagnetic wave (helicon) at  $H \approx H_s$ . The absence of damping is due to vanishing of the dissipative components of the conductivity.<sup>13</sup>

In the derivation of the formulas in the present section we are assuming that  $T = 0$ . Obviously, at finite temperatures and in fields  $H \rightarrow H_s$ , when the level of the chemical potential is close to the middle of the gap [see Eq. (11a)], we have

$$\sigma_{zz} \propto \exp(-\hbar\omega_c/2T).$$

We shall conclude by considering the behavior of  $\sigma_{zz}$  in very high magnetic fields when all the carriers are near the bottom of the lowest miniband. If we also have  $T \ll \zeta$  [see Eq. (14)], then denoting by  $\sigma_{zz}^\infty$  the values of  $\sigma_{zz}$  in extremely high fields, we find that

$$\sigma_{zz}^\infty = (ne^2/m_\parallel^{(0)})\tau_\infty(H), \quad H \gg H_1. \quad (20)$$

The extreme simplicity of the above formula allows us to use the value of  $\sigma_{zz}^\infty$  to determine the scattering mechanism. We shall compare  $\sigma_{zz}$  with  $(\sigma_{zz}^\infty)_{is}$ , which is the conductivity of a three-dimensional isotropic semiconductor with an effective mass  $m_1$  is the same limit  $H \rightarrow \infty$  [the formulas for  $(\sigma_{zz}^\infty)_{is}$  are given in Ref. 12]. It is clear from Eq. (16) that the ratio of the corresponding relaxation times is governed by the ratio of the density of states at the Fermi level, which for same density is  $m_1/m_\parallel$ . The ratio of the values of  $|v_z^F|$  is then equal to the same quantity and on the whole we have

$$\sigma_{zz}^\infty / (\sigma_{zz}^\infty)_{is} \approx (m_1/m_\parallel)^2, \quad (21)$$

i.e., the superlattice conductivity in extremely strong magnetic fields is higher than the conductivity of a "normal" semiconductor under the same conditions. We recall however that both conductivities decrease on increase in the magnetic field and for some finite value of  $H$  we can expect localization. In a superlattice such localization occurs in a field

$$H \sim H_1 = \hbar(I/d)^{1/2}, \quad I \gg d, \quad (22)$$

where  $\hbar I$  is the field in which the lower miniband is approximately half-filled and  $\hbar I$  is the corresponding mean free path.

#### 4. PHOTOCONDUCTIVITY EFFECTS

The problem of the photoconductivity of purely two-dimensional systems and superlattices in quantizing magnetic fields is very interesting. It is known that the photoconductivity  $\Delta\sigma$  of materials with a very high carrier density is usually extremely low. However, the photoconductivity which is then observed would be of considerable interest for research and applications because of other parameters of these materials. These difficulties can be overcome in superlattices and two-dimensional systems because in fields

$H = H_s$  corresponding to the filling of an integral number of minibands (Landau levels) the values of  $\sigma_{xx}$ ,  $\sigma_{yy}$ , and  $\sigma_{zz}$  vanish, as demonstrated above. In other words, under these conditions the dark conductivity is sufficiently small in spite of the high total carrier density and this is true even if allowance is made for the effects of background radiation.

In such "insulating" intervals the photoconductivity of structures of this kind should be very high because any optically induced transfer to a higher miniband creates a free electron in that band and a hole in the filled miniband, which increases  $\sigma_{xx}$ ,  $\sigma_{yy}$ , and  $\sigma_{zz}$ . The Hall conductivity is then practically unaffected.

Another favorable circumstance is the considerable reduction in the noise at the points corresponding to  $H = H_s$ . Numerical values of the noise characteristics, like the values of  $\sigma_{xx}$ ,  $\sigma_{yy}$ , and  $\sigma_{zz}$  are governed by the miniband occupancy (or by the occupancy of Landau levels in two-dimensional systems).

The photosensitivity spectrum should depend strongly on the polarization of the incident light. When this light is polarized in the  $xy$  plane, it gives rise to transitions between different Landau levels so that the photosensitivity maximum occurs at the cyclotron resonance frequency ( $\omega = \omega_c$ ). When light is polarized along the  $z$  axis, it induces transitions between minibands (in two-dimensional systems between levels) corresponding to different values of  $i$  in Eq. (4). In this case the resonances are described by the condition  $\hbar\omega = \epsilon_i - \epsilon_i'$ , when in the case of a mirror-symmetric potential of the superlattice the minibands  $i'$  and  $i$  should have different parities. The photosensitivity spectrum is then identical with the spectrum of the absorption coefficient calculated for a superlattice in Ref. 14.

We shall now obtain an expression for the photoconductivity in the case of light polarized in the  $xy$  plane. If the depth of the skin layer is large compared with the dimensions of the sample, the absorption of light is governed by the imaginary part of the permittivity  $\epsilon''$  and is due to transitions between magnetic minibands (cyclotron resonance):

$$\epsilon'' = \frac{4\pi Ne^2}{\omega m_\perp} \frac{\gamma}{(\omega - \omega_c)^2 + \gamma^2}. \quad (23)$$

Here,  $\gamma$  is the width of a resonance and we are assuming that in the most interesting case the upper Landau level is almost completely filled, i.e., it contains  $\approx N$  electrons. Then, the density of nonequilibrium electrons at the higher level is

$$\Delta n = \frac{4\pi I \tau_{lf}}{c \epsilon^{1/2}} \frac{Ne^2}{m_\perp} \frac{\gamma}{[(\omega - \omega_c)^2 + \gamma^2]}, \quad (24)$$

where  $I$  is the radiation intensity;  $\epsilon$  is the permittivity; and  $\tau_{lf}$  is the lifetime.<sup>4)</sup>

We can go over from  $\Delta n$  to the quantity  $\Delta\sigma$  of interest to us by using the relationship between the carrier density and the electrical conductivity which in the case of a superlattice is not as trivial as for a homogeneous sample. However, near the metal-insulator transition points, when  $n = sN + \delta n$ , where  $s$  is an integer and  $\delta n \ll N$ , we find that at absolute zero we have  $\sigma \sim |\delta n|$  (we shall omit the indices of  $\sigma$ , because all this and the subsequent discussion applies both to  $\sigma_{xx,yy}$  and to  $\sigma_{zz}$ ). Therefore, we have  $\Delta\sigma/\sigma_0 = \Delta n/|(\delta n)_0|$  (the index "0" represents the dark values of the identified quantities). We finally obtain from Eq. (24) the photoconductivity

$$\frac{\Delta\sigma}{\sigma_0} = \frac{4\pi I e^2 \tau_{if} \gamma}{e^{1/2} s c m_{\perp} [(\omega - \omega_c)^2 + \gamma^2] | (H_s/H) - 1 |}. \quad (25)$$

The above formula describes the magnetic field dependence of the photosensitivity with singularities at  $H = H_s$  and a maximum at  $H = \omega m_{\perp} c/e$ , corresponding to the cyclotron resonance frequency. The most convenient for experimental observation is the case when these two conditions coincide, i.e., when the frequency of the incident light is  $\omega = 2\pi\hbar n d / m_{\perp}$ .

A similar formula for the photosensitivity can be written also for light polarized along the  $z$  axis if  $\omega_c$  is replaced by the corresponding resonance frequency. In contrast to  $\omega_c$ , it is independent of the magnetic field, which alters somewhat the nature of the dependence of  $\Delta\sigma/\sigma_0$  on  $H$ , but the singularities at  $H = H_s$  still remain.

The presence of localized states in the system may alter considerably its photoelectric properties. In particular, optically excited carriers may be found in localized states and the photoconductivity is then due to activation to the mobility edge or due to hopping. This alters the nature of the dependences of  $\Delta\sigma$  on the temperature of a sample and on the Fermi level position (i.e., in the final analysis, on the magnetic field). More specific conclusions cannot be drawn in the absence of information on the localization criteria and on the mobility edges of a superlattice representing an intermediate case between two- and three-dimensional systems the localization properties of which are known to differ very strongly.

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<sup>11</sup>It is very difficult to attain the condition of a strong magnetic field in this geometry because the effective mass  $m_{\parallel}$  is large.

<sup>21</sup>Information on the de Haas-van Alphen effect in layer structures can be found also in the work of Gvozdkov.<sup>5</sup>

<sup>31</sup>The arguments given later were formulated with the assistance of L. P. Pitaevskii to whom the authors are deeply grateful.

<sup>41</sup>In contrast to quasi-two-dimensional systems in the absence of a magnetic field, when the lifetime  $\tau_{if}$  is of the order of the momentum relaxation time  $\tau_r$ , i.e., when it is anomalously short (see Ref. 15), in our case  $\tau_{if}$  has its usual value and becomes anomalously short only at  $\omega = \omega_0$  ( $\omega_0$  is the optical phonon frequency), when the one-phonon recombination process is possible.

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