Anomalous penetration of longitudinal alternating electric field into a degenerate plasma with an arbitrary specularity parameter

V. M. Gokhfel'd, M. I. Kaganov,[1] and G. Ya. Lyubarskiĭ[2]

Donetsk Physicotechnical Institute, Ukrainian Academy of Sciences
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The penetration of an alternating electric field from a vacuum into a degenerate electron plasma of a conductor in a direction perpendicular to the interface is analyzed with allowance for electron scattering by the conductor surface. The asymptote of the inhomogeneous part of the field in the interior of the plasma is formed by carriers with maximum velocity components along the normal to the interface, and constitutes in the collisionless limit a nonexponentially damped quasiwave. The dependence of its amplitude on the specularity parameter that characterizes the intensity of the surface scattering of the electrons is obtained. A new method is proposed for the required calculations.

1. The structure of an alternating electric field in the transition layer between the sample surface and the plasma interior in which the microscopic description is valid depends substantially on the dynamics of the plasma particles. The problem of the behavior of the field in such a layer was first solved by Landau[1] for a half-space filled with a nondegenerate Maxwellian plasma. It was assumed in Ref. 1 that the electrons are specularly reflected by the surface. An important result of Ref. 1 was detection of a longitudinal-field component E_{x1} with a slower than exponential damping. It is this component which determines the asymptotic behavior of the field in the transition layer (the approach to the macroscopic value $E_0/\varepsilon(\omega)$, where $\varepsilon(\omega)$ is the dielectric constant of the plasma). According to [1],

$$E_{x1} \propto X^{-2/3} \exp[-{}^3/_4(\omega X/\omega_0 a)^{2/3}], \tag{1}$$

where ω is the field frequency, $\omega_0 = (4\pi ne^2/m)^{1/2}$ the plasma frequency, $a = (T/4\pi ne^2)^{1/2}$ the electronic Debye-Hückel radius (the remaining notation is standard and X is the coordinate along the normal to the surface).

Degeneracy of the plasma alters substantially the field structure in the transition layer. In particular, the existence of a maximum electron velocity $(v_x)_{\max}$ (v_0 is the Fermi velocity) leads to the appearance of a field component that is damped non-exponentially (almost as a power law). This component is customarily called *quasiwave*.[2] From the physical standpoint a quasiwave is the result of the penetration of ballistic electrons, moving perpendicular to the sample surface, to a depth on the order of the mean free path l. At distances $x \ll l$ the quasiwave has a power-law attenuation. Mathematically, the quasiwave is due to the presence of branch points of the dielectric constant of the metal (as a function of the complex wave vector). If the Fermi surface of a degenerate plasma has a complicated shape, this is "felt" by the field in the transition layer, viz., E_{xt} consists of several terms, each generated by a separate value of v_x.

A collisionless plasma is a medium with clearly pronounced spatial dispersion that leads to a nonlocal connection between the current density and the electric field strength. Determination of this connection (derivation of the material equation calls for solution of the kinetic equation for the electron distribution function f, an impossible task without formulating the boundary conditions for f. This is a special complicated problem whose solution starts out with more-or-less model-dependent description of the surface and of the character of its interaction with the electrons (see Ref. 3 and the citations therein).

To calculate the electric conductivity of a thin plate, Fuchs[4] formulated a phenomenological boundary condition that takes into account the nonspecularity of electron reflection by a sample surface ($X \geqslant 0$):

$$f_1 \Big|_{\substack{v_x>0\\X=0}} = q f_1 \Big|_{\substack{v_x<0\\X=0}}, \quad 0 \leqslant q \leqslant 1, \tag{2}$$

$f_1 = f - f_0$ is a nonequilibrium increment to the equilibrium distribution function of the electrons. Condition (2) should not contradict the natural requirement that the current not leak through the boundary. In the problem of the conductivity of a thin plate[4,5] and also in the calculation of the surface impedance of a metallic isotropic half-space[6], the nonleakage condition was automatically satisfied because f_1 is odd in the transverse (relative to the normal to the metal surface) components of the electron velocity. Encountering (in a study of the high-frequency properties of an anisotropic metal) a violation of the non-leakage condition, Azbel' and Peschanskiĭ[7] generalized the condition (2) as follows:

$$f_1 \Big|_{\substack{v_x>0\\X=0}} = \left\{ q f_1 \big|_{v_x=0} + (1-q) \frac{\int d^3p v_x f_1}{\int d^3p v_x} \right\} \Bigg|_{X=0}, \tag{3}$$

where the integration is over that part of the Fermi surface on which $v_x < 0$.

Although recent studies have shown that the phenomenological boundary conditions (2) or (3) do not cover all the situations, considerable interest attaches to solution of electrodynamics problems with an arbitrary Fuchs parameter q. First, as will be shown below, it requires a significant generalization of the Wiener-Hopf method, a generalization that will undoubtedly be useful in many branches of theoretical physics. Second (and foremost), an analysis of the solution can identify the electrons responsible for some particular electrodynamic property of an electron plasma, while comparison of the phenomenological boundary conditions with the microscopic ones can determine the effective value of the physically lucid parameter q. Such a comparison, however, requires a solution for arbitrary q.

Our present task is the find the distribution of the longitudinal electric field in a transition layer of an extremely

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degenerate electron plasma ($T=0$) with boundary condition (3) at an arbitrary value of the specularity parameter $q (0 \leqslant q \leqslant 1)$. The electron dispersion law is assumed isotropic:

$$\varepsilon = p^2/2m. \tag{4}$$

This, of course, is a simplifying assumption. It is appropriate to mention, however, that metals exist with spherical Fermi surfaces (Na, K, Rb, Cs,...). In addition, paying principal attention to the dependence of the solution on the specularity parameter q, it is natural to simplify to the utmost the dynamic properties of the particle in the interior of the plasma.

We emphasize, finally, that the problem of penetration of a longitudinal electric field into a plasma is one of those problems whose solution adds to our knowledge of the behavior of a many-body system under external action. The solution of this problem is of importance for the understanding various processes that evolve in electron systems.

2. We consider thus the penetration of an alternating electric field $E(X,t) \equiv \{E(X)\exp(-i\omega t); 0; 0\}$ from a vacuum $X<0$, where $E(X) = E_0$, into a plasma occupying the half-space $X \geqslant 0$. The connection between the field in the conduction current

$$J_x = j_x e^{-i\omega t}, \quad j_x = e^2 \langle v(\psi_+ - \psi_-)\rangle_+ \tag{5}$$

is obtained directly from Maxwell's equation

$$\frac{\partial}{\partial X}(4\pi j_x - i\omega E) = 0. \tag{6}$$

Here $v \equiv |v_x|$, the subscript $+$ or $-$ coincides eith the sign of v_x, and the angle brackets denote averaging over the Fermi surface:

$$\langle \ldots \rangle_+ \equiv \frac{4\pi m^2}{h^3} \int_0^{v_0} \ldots dv$$

($v_0 = (2\varepsilon_F/m)^{1/2}$ is the Fermi velocity). The function $\psi_\pm$ determines the nonequilibrium increment $-e\psi_\pm (\partial f_0/\partial\varepsilon) e^{-i\omega t}$ to the Fermi distribution function f_0 of the degenerate electron gas and satisfies the kinetic equation

$$\partial\psi_\pm/\partial X \pm \mu\psi_\pm = E(X), \tag{7}$$

where $\mu \equiv (\nu - i\omega)/v$, and ν (the electron relaxation frequency) is the smallest problem parameter with dimension of frequency, introduced to make the corresponding integrals convergent. We are actually interested in the collisionless limit (we could introduce the parameter of the adiabatic turning-on of the field in place of the collision frequency, see Ref. 1). In terms $\psi_\pm$ the boundary condition (3) takes the form

$$\psi_+(0) = q\psi_-(0) + C \qquad (C = \text{const}).$$

The solution of Eq. (7) takes the form

$$\psi_+ = Ce^{-\mu X} + \int_0^X dY\, E(Y) e^{\mu(Y-X)} - q \int_0^\infty dY\, E(Y) e^{-\mu(X+Y)},$$
$$\psi_- = -\int_X^\infty dY\, E(Y) e^{\mu(X-Y)}. \tag{8}$$

since $j_x = 0$ in vacuum, the condition that the total current (6) be constant means that

$$E(X) + (4\pi i/\omega) j_x = E_0.$$

This in fact the equation for the distribution of the field $E(X)$ in the plasma. Using the definition (5) of the current, the solution (8) of the kinetic equation, and the non-leakage condition, we express the field in explicit form:

$$E(X) - E_0 = \int_0^\infty dY E(Y) \left\{ T(|X-Y|) - qT(X+Y) - (1-q)\frac{T(X)T(Y)}{T(0)} \right\}$$
$$T(X) \equiv (4\pi e^2/i\omega) \langle v \exp(-\mu X)\rangle_+. \tag{9}$$

We introduce now the constant

$$\beta = 2\int_0^\infty dX T(X) = \frac{8\pi}{3}\left(\frac{mv_0}{h}\right)^3 \frac{4\pi e^2}{m\omega(\omega+i\nu)} \equiv \frac{\omega_0^2\omega^{-2}}{1+i\nu/\omega}; \tag{10}$$

It is equal to the difference $1-\varepsilon(\omega)$ between the dielectric constants of the vacuum and the plasma. The plasma frequency ω_0 is the largest parameter with dimension of frequency in the problem. For metals (and also semimetals and even degenerate semiconductors) it is natural to assume a field frequency $\omega \ll \omega_0$, and β is then a large parameter of the problem. The solution given below uses essentially the fact that

$$|\beta| \gg 1. \tag{11}$$

It follows from (9), as can be easily shown (see also Ref. 1), that $E(+\infty) = E_0(1-\beta)^{-1}$. We shall consider only the inhomogeneous part of the field, $E(X) - E_0(1-\beta)^{-1}$, which vanishes at infinity. We introduce its dimensionless value through the equation

$$E(X)/E_0 = (1-\beta\mathscr{E})/(1-\beta),$$
$$(\mathscr{E}(0) = 1, \quad \mathscr{E}(+\infty) = 0). \tag{12}$$

Transforming in (9) to the dimensionless variables $x \equiv \omega X/v_0$, $u \equiv v/v_0$, we obtain the following equation for $\mathscr{E}(x)$:

$$\mathscr{E}(x) - \beta\int_0^\infty dy\, \mathscr{E}(y) \left\{ L(|x-y|) - qL(x+y) - (1-q)\frac{L(x)L(y)}{L(0)} \right\} = F(x),$$
$$L(x) \equiv -\frac{3\alpha}{2}\int_0^1 du\, u e^{\alpha x/u}, \tag{13}$$

$$F(x) \equiv \int_0^1 du\, u\left[1-q+3u\frac{1+q}{2}\right] e^{\alpha x/u}, \quad \alpha = i - \nu/\omega \quad (\nu/\omega \ll 1).$$

The kernel of this equation is not a difference kernel as for $q=0$, nor can it be transformed into a difference kernel, as in the case $q=1$, by an odd continuation of the sought function to the semi-axis $x<0$. It is difficult to obtain an approximate solution of (13) because its kernel is unbounded at large values of the parameter $|\beta|$. It is necessary to find for the equation an equivalent form that would facilitate an investigation of the solution at large $|\beta|$.

3. The use of the Laplace transform

$$\mathscr{E}_L(p)=\int_0^\infty dx\, e^{-px}\mathscr{E}(x), \quad \mathrm{Re}\, p>0,$$

yields in place of (13)

$$\mathscr{E}_L(p)\varepsilon(p)+3\alpha\beta\int_0^1 du\,\mathscr{E}_L(-\alpha/u)B(p,u)=F_L(p), \quad (14)$$

where

$$\varepsilon(p)=1+3\beta\frac{\alpha^2}{p^2}\left(1+\frac{\alpha}{2p}\ln\frac{\alpha-p}{\alpha+p}\right) \quad (15)$$

is the dielectric function of the metal and coincides at $p=0$ with $\varepsilon(\omega)$. It is convenient to introduce in the complex p plane two cuts $(\alpha, \alpha\infty)$ and $(-\alpha, -\alpha\infty)$. The function $\varepsilon(p)$ is even outside the cuts and has no singular points other than the branch points $p=\pm\alpha$. It vanishes at

$$p=\pm p_0 \quad (|\beta|\gg 1: \quad p_0\approx i\alpha(3\beta)^{1/2}). \quad (16)$$

The function

$$F_L(p)\equiv\frac{1}{p}\left\{1+3\alpha\frac{1+q}{4p}+\left(1-q+3\alpha\frac{1+q}{2p}\right)\times\left[\frac{\alpha}{p}+\frac{\alpha^2}{p^2}\ln(1-p/\alpha)\right]\right\} \quad (17)$$

has only one singular point—a branch point $p=\alpha$ in the plane with a cut $(\alpha,\alpha\infty)$, we note that $\lim_{p\to\infty} pF_L(p)=1$. The function

$$B(p,u)\equiv\frac{u}{2}\left\{\frac{1}{p+\alpha/u}-\frac{q}{p-\alpha/u}-(1-q)\left[\frac{1}{p}+\frac{2\alpha}{p^2}+\frac{2\alpha^2}{p^3}\ln(1-p/\alpha)\right]\right\} \quad (18)$$

has the same branch point and two poles $p=\pm\alpha/u$;

$$\lim_{p\to\infty} p^2B(p,u)=-\alpha\left(\frac{1+q}{2}+u(1-q)\right).$$

Relation (14) shows thus that the sought function $\mathscr{E}_L(p)$ can be continued into the left half-plane $\mathrm{Re}\, p<0$ with a cut $(\alpha,\alpha\infty)$, and has there only one simple pole p_0 and one branch point α. Consequently, the solution $\mathscr{E}(x)$ has the same structure as in the limiting cases $q=0$ and 1 (see Ref. 2): a rapidly damped term $A\exp(p_0x)$ plus a quasiwave, i.e., the contribution of the branch point. In the general case, however, when $q\neq 0$ or 1, the derivation of relations that can determine both contributions (and their dependencies on q) is not trivial and requires modification of the known Wiener-Hopf method.[8]

Namely, dividing (14) by $\varepsilon(p)$, we represent the functions

$$\gamma(p,u)\equiv B(p,u)/\varepsilon(p)$$
$$\text{и}\quad \varphi(p)\equiv F_L(p)/\varepsilon(p)$$

in the form of Cauchy integrals along the contour shown in Fig. 1. If we now let the radius of the circle go to infinity, each of these functions breaks up into a sum of four terms

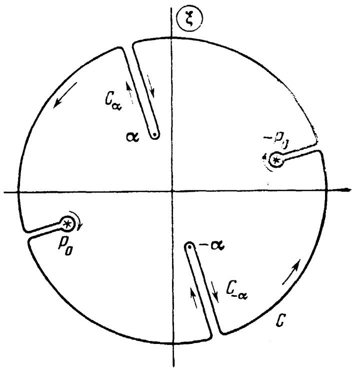

FIG. 1.

$[\gamma_{+p0}, \gamma_{-p0}, \gamma_\alpha, \gamma_{-\alpha}$; similarly for $\varphi(p)]$, having in the cut plane only singular point each—identified by the subscript. These terms are

$$\gamma_{\pm p_0}(p,u)=\frac{\pm B(\pm p_0,u)}{\varepsilon'(p_0)(p\mp p_0)}, \quad \gamma_{\pm\alpha}(p,u)=\int_{C_{\pm\alpha}}\frac{d\xi}{\xi-p}\frac{B(\xi,u)}{2\pi i\varepsilon(\xi)},$$
$$\varphi_{\pm p_0}(p)=\frac{\pm F_L(\pm p_0)}{\varepsilon'(p_0)(p\mp p_0)}, \quad \varphi_{\pm\alpha}(p)=\int_{C_{\pm\alpha}}\frac{d\xi}{\xi-p}\frac{F_L(\xi)}{2\pi i\varepsilon(\xi)}. \quad (19)$$

Using these expansions, we can rewrite (14) in the form

$$\mathscr{E}_L(p)+3\alpha\beta\int_0^1 du\,\mathscr{E}_L(-\alpha/u)\gamma_\alpha(p,u)-\varphi_\alpha(p)-A(p-p_0)^{-1}$$
$$=-3\alpha\beta\int_0^1 du\,\mathscr{E}_L(-\alpha/u)\gamma_{-\alpha}(p,u)+\varphi_{-\alpha}(p)-A_1(p+p_0)^{-1}, \quad (20)$$

where the prime denotes differentiation with respect to p, and

$$\binom{A}{A_1}=\frac{1}{\varepsilon'(p_0)}\left[F_L(\pm p_0)-3\alpha\beta\int_0^1 du\,\mathscr{E}_L(-\alpha/u)B(\pm p,u)\right]. \quad (21)$$

The left-hand side of (20) is analytic in the right-hand plane ($\mathrm{Re}\, p\geqslant 0$), and the right part in the left half ($\mathrm{Re}\, p\leqslant 0$), i.e., both are equal in the entire (uncut) p plane to one and the same analytic function, zero in this case [see (17) and (18)]. In addition, the point $p=-p_0$ is not singular for the function $\varphi_{-\alpha}$; all this leads to three equalities:

$$\mathscr{E}_L(p)+3\alpha\beta\int_0^1 du\,\mathscr{E}_L(-\alpha/u)\gamma_\alpha(p,u)-\varphi_\alpha(p)=\frac{A}{p-p_0},$$
$$3\alpha\beta\int_0^1 du\,\mathscr{E}_L(-\alpha/u)\gamma_{-\alpha}(p,u)=\varphi_{-\alpha}(p), \quad A_1=0. \quad (22)$$

4. Identity transformations (left out for brevity) of (22) lead to the following formulation of the problem:

$$\mathscr{E}(x)=Ae^{p_0x}-\alpha\int_1^\infty dz\frac{W(z)+\Phi(z)}{r(z)}e^{\alpha xz}, \quad (23)$$

where

$$r(z)\equiv\left(\frac{z^2}{3\beta}+1+\frac{1}{2z}\ln\frac{z-1}{z+1}\right)^2+\frac{\pi^2}{4z^2}, \quad (24)$$

$\Phi(z)$ is a bounded (also as $|\beta| \to \infty$) function of z, the form of which is known, while the unknown function $W(z)$ satisfies the integral equation

$$W(z)+\int_1^\infty \frac{dy}{yr(y)} Q(z,y)[W(y)+\Phi(y)]=0 \tag{25}$$

with the symmetric kernel

$$Q(z,y)=\frac{1-q^2}{4yz}\left\{\frac{\ln(1+y)}{1-y/z}+\frac{\ln(1+z)}{1-z/y}\right.$$
$$\left.-2\left(1-\frac{\ln(1+y)}{y}\right)\left(1-\frac{\ln(1+z)}{z}\right)\right\}, \tag{26}$$

which is independent of β. It is bounded by the inequality

$$\max_{z\geqslant 1}|Q(z,y)|<\frac{1-q^2}{2y}\left(1-\frac{\ln(1+y)}{y}\right).$$

5. No use was made so far of condition (11) in the reformulated problem. Equation (23) and (26) are suitable just at $|\beta| \gg 1$. An investigation based on an estimate of integrals of the form

$$\int_1^\infty dz\, z^n/r(z) \qquad (n\leqslant 2),$$

shows that at $|\beta| \gg 1$, neglecting remainders that are small in the parameter $\ln|\beta|/|\beta|^{1/2} \ll 1$, the functions $r(z)$ and $\Phi(z)$ in expressions (23) and (25) can be replaced by

$$r_0(z)=\left(1+\frac{1}{2z}\ln\frac{z-1}{z+1}\right)^2+\frac{\pi^2}{4z^2}, \tag{27}$$

$$\Phi_0(z)=\frac{1+q}{6\alpha\beta}\left\{1+\frac{1-q}{2z}\left[\frac{3}{2}-\frac{3}{z}+\left(\frac{3}{z^2}-1\right)\ln(1+z)\right]\right\}. \tag{28}$$

The constant

$$A\approx 1+\frac{1}{4i(3\beta)^{1/2}}\left(1-q+i\pi\frac{1+q}{2}\right) \tag{29}$$

is calculated with the same accuracy. We have thus determined the first term of (23). Note the weak dependence of the ordinary wave on the specularity parameter. Clearly, this is a result of the kinetic correction to the hydrodynamic solution (actually macroscopic and therefore insensitive to the character of the electron reflection from the surface).

The ordinary wave attenuates rapidly in the interior of the conductor (in real coordinates—at a depth on the order of the Debye screening radius $v_0/3^{1/2}\omega_0$), so that the second term of (23) predominates already at $x \gg \ln|\beta|/|\beta|^{1/2}$. This term can be found by numerical integration of (25) (with (27) and (28) taken into account).

The asymptotic form of the field $\mathscr{E}(x)$ at $x \gg 1$, i.e., in real coordinates at depths larger than the distance v_0/ω negotiated by the electron during the period of the field, can be approximately obtained analytically. From (23) we have

$$\mathscr{E}(x)\approx -\alpha e^{\alpha x}[\Phi_0(1)+W(1)]\int_0^\infty \frac{dy}{r_0(y+1)}e^{\alpha xy}. \tag{30}$$

The asymptotic form of the integral in this equation is easy to calculate:

$$\int_0^\infty dy\frac{e^{\alpha xy}}{r_0(y+1)}\approx -\frac{1}{\alpha x}\left(\frac{2}{\ln x}\right)^2 \qquad (x\gg 1).$$

The quantity $W(1)$ can be estimated from Eq. (25), using the explicit forms of the functions $\Phi_0(z)$, $r_0(z)$ and $Q(z,y)$:

$$|W(1)|\leqslant\frac{(1-q^2)\cdot 0{,}0038}{1-(1-q^2)\cdot 0{,}22}\frac{1+q}{6|\beta|}. \tag{31}$$

This quantity is less than half of one percent of $\Phi_0(1)$ [see (28)], and can be simply neglected.

Thus,

$$\mathscr{E}(x,q)\approx 2\frac{1+q}{3\alpha\beta}[1+(1-q)(\ln 2-3/4)]\frac{e^{\alpha x}}{x\ln^2 x}$$
$$\times(x\gg 1,\ |\beta|\gg 1). \tag{32}$$

Recall that $\alpha = i - \nu/\omega$, and $x = \omega X/v_0$, i.e., the quasiwave (32) propagates with Fermi velocity v_0 to depth on the order of the electron mean free path $l = v_0/\nu$; in the collisionless limit (or at $X \ll l$) it attenuates non-exponentially. Complete uniformity of the field

$$E(X)=E_0(1-\beta)^{-1},$$

is reached, as is clear from (32), relatively slowly (in proportion to $x^{-1}\ln^{-2}x$).

6. Estimating the role of the asymptotic behavior of the function $\mathscr{E}(x)$, we must bear in mind the following: its smallness is due only to the fact that $x \equiv \omega X/v_0 \gg 1$. The function $\mathscr{E}(x)$ itself does not contain any special smallness whatever, since the factor β^{-1} in (32) is contained both in the value of the macroscopic field in the interior of the conductor, $E_0/\varepsilon(\omega) \approx -E_0/\beta$ (cf. Refs. 9 and 10, in which it is shown that the amplitude of an acoustic quasiwave contains an "extra" small factor—the ratio of the electron and ion masses—compared with the amplitude of an ordinary sound wave).

The character of the surface reflection of the carriers (the value of q) influences, as we see, the constant coefficient in the quasiwave amplitude. Its dependence (albeit weak) on the specularity parameter q is nonlinear; the ratio of the amplitudes for the limiting cases $q = 0$ and $q = 1$ is approximately

$$1/8+1/2\ln 2\approx 0{,}47,$$

which agrees, naturally, with the result of Ref. 2.

It is clear from the analysis of the solution (32) that, independently of q, the asymptotic form of the quasiwave is governed by electrons that move into the interior of the sample with maximum velocity v_x (i.e., in the present case, normal to its boundary). This can, on the one hand, help choose the corresponding effective value of the parameter q when solving this (or a similar) problem), but with allowance for the real interaction of the electron with the surface, and on the other, shows that in the case of a complicated electron dispersion law, at any type of scattering from the surface, the spectrum of the quasiwave is determined by the extremal values of v_x on the Fermi surface (see Refs. 2 and 11).

Note that solutions of problems of high-frequency electrodynamics of an electron plasma for arbitrary q are rarely encountered in the literature. We know only Refs. 12 and 13, in which a concrete form of the integral-equation kernel is used. The method proposed [transition to Eq. (25)] can conceivably find use in numerical integrations of equations containing a large parameter, similar to the one considered in the present communication.

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[1)] Institute of Physics Problems, USSR Academy of Sciences.
[2)] Kharkov Physicotechnical Institute, Ukrainian Academy of Sciences.

[1]L. D. Landau, Zh. Eksp. Teor. Fiz. **16**, 574 (1946).
[2]V. M. Gokhfel'd, M. A. Gulyanskiĭ, M. I. Kaganov, and A. G. Plavenek, *ibid.* **89**, 985 (1985) [Sov. Phys. JETP **62**, 566 (1985)].
[3]V. I. Okulov and V. V. Ustinov, Fiz. Nizk. Temp. **5**, 213 (1979) [Sov. J. Low Temp. Phys. **5**, 101 (1979)].
[4]K. Fuchs, Proc. Cambridge Phil. Soc. **34**, 100 (1938).
[5]M. I. Kaganov and M. Ya. Azbel', Zh. Eksp. Teor. Fiz. **27**, 62 (1954).
[6]G. E. Reuter and E. H. Sondheimer, Proc. Roy. Soc. **A195**, 336 (1948). M. I. Kaganov and M. Ya. Azbel', Dokl. Akad. Nauk SSSR **102**, 29 (1955).
[7]M. Ya. Azbel' and V. G. Peschanskiĭ, Zh. Eksp. Teor. Fiz. **49**, 572 (1965) [Sov. Phys. JETP **22**, 399 (1966)].
[8]B. Noble, *Methods Based on the Wiener-Hopf Technique for the Solution of Partial Differential Equations*, Pergamon, 1958.
[9]G. Ivanovski and M. I. Kaganov, Zh. Eksp. Teor. Fiz. **83**, 2320 (1982) [Sov. Phys. JETP **56**, 1345 (1982)].
[10]V. M. Gokhfel'd and M. I. Kaganov, Fiz. Nizk. Temp. **10**, 863 (1985); **11**, 517 (1985) [Sov. J. Low Temp. Phys. **10**, 270 (1985); 11 (1985)].
[11]I. P. Ipatova, M. I. Kaganov, and A. V. Subashiev, Zh. Eksp. Teor. Fiz. **84**, 1830 (1983) [Sov. Phys. JETP **57**, 1066 (1983)].
[12]L. E. Hartmann and J. M. Luttinger, Phys. Rev. 151, 430 (1983).
[13]B. E. Meierovich, Zh. Eksp. Teor. Fiz. **57**, 1445 (1969); **58**, 324 (1970); **59**, 276 (1971) [Sov. Phys. JETP **30**, 782 (1970); **31**, 175 (1970); **32**, 149 (1971)].

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