Interference current in nonequilibrium superconductors

A. M. Gulyan, G. F. Zharkov, and G. M. Sergoyan

Institute of Physical Research, Academy of Sciences of the Armenian SSR; P. N. Lebedev Physics Institute, Academy of Sciences of the USSR, Moscow; Institute for Research in the Physics of Condensed Media, Erevan State University

(Submitted 14 May 1986)
Zh. Eksp. Teor. Fiz. **92**, 190–198 (January 1987)

When a superconductor is described by the time-dependent Ginzburg-Landau equations, the current consists of normal, superfluid, and interference components. It is demonstrated in the example of weak links that an interference current can play an important role in describing the physics of nonequilibrium superconductors.

1. INTRODUCTION

The behavior of electrons in nonequilibrium superconductors is fundamentally different from that in the equilibrium case, in which there is only a superfluid motion, which can be described by the Ginzburg-Landau order parameter.[1,2] In nonequilibrium cases, the normal component also becomes involved in the motion.

It can be shown that the simultaneous involvement of an electron system in two types of motion gives rise to a specific (interference) component of the current. The existence of this component, which is the subject of the present study, not only underscores the arbitariness of partitioning a superfluid system into two components (as was pointed out even by Landau[3]) but also demonstrates the restrictiveness of the two-fluid description (since in addition to the normal motion and the superfluid motion interference between these two motions occurs).

An expression for the current in the nonstationary case is included in the system of time-dependent Ginzburg-Landau equations. On the other hand, the additional interference term is not found in the final expressions for the current which have been used in several studies in which time-dependent equations have been derived for $T\sim T_c$ (Refs. 4–9). As we will show below, the interference component of the current must be taken into account, since it is generally not small and may play a fundamental role.

The equations of nonequilibrium superconductivity underlie a microscopic derivation of an expression for the current in the time-varying case.[10,11] General relations for the current were derived in Refs. 4–9 in the "dirty" limit (in the sense of Ref. 12) under the customary conditions of a semiclassical treatment.[11] Schmid's results[9] contain a term not present in Refs. 4–8. Since the various results must be rechecked and compared, and since the expressions which have been derived for the current in all of these studies have been calculated in a gauge with a real order parameter, it is worthwhile to rederive a general expression for the current. This procedure, which is carried out in an aribtrary gauge, is the content of Section 2. In Section 3 we make the transition to a closed expression for the current (i.e., an expression which does not explicitly contain distribution functions of the excitations). Like the time-dependent equation for the order parameter,[8] the expression for the current becomes the expression for a gapless superconductor[13] when the gap in the excitation spectrum is "smeared out" (formally—when the excitation decay values are quite large). In this case the current consists exclusively of normal and superfluid components. It can thus be asserted that the interference term which arises in a "gap" superconductor is a consequence of a strong correlation between the system of one-particle excitations and condensate pairs (this correlation turns out to be suppressed in the gapless case).

The expression derived for the current can be used to reanalyze certain well-known time-dependent problems. In this connection, we examine in Section 4 the behavior of weakly linked superconductors in a resistive state on the basis of the model of Aslamazov and Larkin.[14] In our approach, phase-dependent dissipative terms appear in the current even if we do not use a time-dependent equation for the order parameter.[1)] It turns out that instead of the "cosine" term[16] there is another term, large in magnitude and of the same nature.

Further analysis, in Section 4, shows that it is the interference term which is responsible for the well-known "excess" current in weakly linked superconductors. The structure of the resulting expression is reminiscent of that found in the theory of Artemenko *et al.*[17] near T_c. It is shown that the excess current in weakly linked structures oscillates in time (periodically vanishing, at the same time as the modulus of the order parameter).

In Section 5, the final section, we also discuss certain other aspects of manifestations of an interference current, in particular, in a problem related to the "cos φ-term paradox" in weak-link systems.

2. GENERAL EXPRESSION FOR THE CURRENT

According to the microscopic equations of nonequilibrium superconductivity,[10,11] the current density is given by the expression ($e=\hbar=c=1$)

$$\mathbf{j}(\mathbf{r},t)=-\frac{1}{4}N(0)\int_{-\infty}^{\infty}d\varepsilon\int\frac{dO_\mathbf{p}}{4\pi}\frac{\mathbf{p}}{m}\,\mathrm{Sp}\Big[\hat{\tau}_3\frac{\mathbf{p}}{m}\hat{g}_p(\mathbf{r},t)\Big]$$

$$=-\frac{N(0)p_F^2}{12m^2}\int_{-\infty}^{\infty}d\varepsilon\,\mathrm{Sp}\,\hat{\tau}_3\hat{\mathbf{g}}_p(\mathbf{r},t),\qquad(1)$$

where $\hat{\mathbf{g}}_p(\mathbf{r},t)$ is the vector Keldysh part of $\hat{g}$, the matrix Green-Gor'kov function integrated over the energy, which is itself a matrix element:

$$\check{g}=\begin{pmatrix}\hat{g}^R & \hat{g}\\ \hat{0} & \hat{g}^A\end{pmatrix}.\qquad(2)$$

In the Usadel approximation,[12] the function $\check{g}$ can be written

 0038-5646/87/010107-05$04.00

in the form

$$\check{g}=\check{g}_s+\frac{\mathbf{p}}{m}\check{g}_p. \tag{3}$$

In expression (1), $N(0)$ is the density of the levels of normal electrons at the Fermi surface, and $\hat{\tau}_3$ is the Pauli matrix. The function $\check{g}$ given by (2) satisfies a kinetic equation (which we will not reproduce here, to avoid overburdening the derivation; see Refs. 9–11) and also the normalization condition

$$\check{g}*\check{g}=\check{1}. \tag{4}$$

In the semiclassical approximation, which is understood to be employed below, the asterisk (*) in this equation has the following meaning:

$$A*B=AB+\frac{i}{2}\left\{\frac{\partial A}{\partial\varepsilon}\frac{\partial B}{\partial t}-\frac{\partial A}{\partial t}\frac{\partial B}{\partial\varepsilon}\right\}, \tag{5}$$

where the terms in braces correspond to so-called convolution corrections (cf. Ref. 8). Taking these terms into account corresponds to the approximation of the next higher order in the semiclassical theory.[11] In the normalization (4), the solution of the kinetic equation for the vector harmonic $\check{g}_p$ is

$$\check{g}_p=-\tau(\check{g}_s*\check{\mathbf{d}}*\check{g}_s-\check{\mathbf{d}}), \tag{6}$$

where τ is the transport mean free time of an electron in the metal, and the isotropic part $\hat{g}_s$ is given by

$$\check{\mathbf{d}}=\check{1}\frac{\partial}{\partial\mathbf{r}}-i\check{\tau}_3\mathbf{A},\quad \check{1}=\begin{pmatrix}\hat{1} & \hat{0}\\ \hat{0} & \hat{1}\end{pmatrix},\quad \check{\tau}_3=\begin{pmatrix}\hat{\tau}_3 & \hat{0}\\ \hat{0} & \hat{\tau}_3\end{pmatrix}, \tag{7}$$

$$\hat{g}_s=\hat{g}^R*\hat{F}-\hat{F}*\hat{g}^A,\quad \hat{F}=f_1\hat{1}+f_2\hat{\tau}_3. \tag{8}$$

From (1)–(8) we find

$$\mathbf{j}=\frac{N(0)D}{4}\int_{-\infty}^{\infty}d\varepsilon\,\mathrm{Sp}\,\hat{\tau}_3\{\hat{g}^R*\hat{\mathbf{d}}*\hat{g}^R*\hat{F}-\hat{g}^R*\hat{\mathbf{d}}*\hat{F}*\hat{g}^A+\hat{g}^R*\hat{F}*\hat{\mathbf{d}}*\hat{g}^A-\hat{F}*\hat{g}^A*\hat{\mathbf{d}}*\hat{g}^A\}, \tag{9}$$

where

$$\hat{\mathbf{d}}=\hat{1}\frac{\partial}{\partial\mathbf{r}}-i\mathbf{A}\hat{\tau}_3, \tag{10}$$

D is the diffusion coefficient in the normal state, and the spectral functions $\hat{g}^{R(A)}$ are defined by[2)]

$$\hat{g}^{R(A)}=\begin{pmatrix}g & f\\ -f^+ & \bar{g}\end{pmatrix}^{R(A)}$$
$$=\overset{+}{(-)}[(\varepsilon\overset{+}{(-)}i\gamma)^2-|\Delta|^2]^{-1/2}\begin{pmatrix}\varepsilon\overset{+}{(-)}i\gamma & \Delta\\ -\Delta^* & -(\varepsilon\overset{+}{(-)}i\gamma)\end{pmatrix}$$
$$\equiv\overset{+}{(-)}\begin{pmatrix}N_1\overset{+}{(-)}iR_1 & e^{-i\theta}(R_2\overset{-}{(+)}iN_2)\\ e^{i\theta}(R_2\overset{-}{(+)}iN_2) & -(N_1\overset{+}{(-)}iR_1)\end{pmatrix}, \tag{11}$$

Here $\gamma=2\tau_\varepsilon^{-1}$ is the energy decay of the electrons, and the complex order parameter is introduced by means of

$$\Delta=|\Delta|e^{-i\theta}. \tag{12}$$

In the further manipulations of (9), we assume that T is close to T_c, so that the following inequalities hold:

$$(\gamma,\ |\Delta|)\ll T. \tag{13}$$

Under these conditions, we can assume, in particular, that the function γ does not depend on ε. We also adopt the following approximations: The "distribution functions" f_1 and f_2, which satisfy kinetic equations, depend on the time only implicitly (this is the so-called local equilibrium approximation). We omit terms with the derivatives $R_{i,\varepsilon}$ ($\equiv\partial R_i/\partial\varepsilon$), $N_{i,\varepsilon}$, and $\nabla f_{1,\varepsilon}$ ($\equiv\partial^2 f_1/\partial\mathbf{r}\partial\varepsilon$) and also terms with derivatives of higher orders and their products, which make only a small contribution to the current. We take into account the symmetry properties of the expression in the integrand (R_i is an odd function of ε, and N_i is an even function of ε). We also note that when we take the trace in (9) several of the terms reduce to total differentials, which vanish upon integration. Furthermore, it follows directly from (11) that the following identities hold:

$$N_1^2+N_2^2-R_1^2-R_2^2=1,\quad R_1N_1+R_2N_2=0. \tag{14}$$

On the basis of the above arguments and after some calculations, we find an expression for the even significant part of the trace in (9):

$$\mathrm{Sp}(\ldots)=4\{(\mathbf{A}+{}^1/_2\nabla\theta)[4R_1N_1f_1-f_{1,\varepsilon}(\dot{R}_2^2-\dot{N}_2^2)]-(\dot{\mathbf{A}}+{}^1/_2\nabla\dot{\theta})f_{1,\varepsilon}(N_1^2+R_2^2)+(\nabla f_2+{}^1/_2f_{1,\varepsilon}\nabla\dot{\theta})(N_1^2+N_2^2)\}. \tag{15}$$

The superior dot denotes a partial derivative with respect to the time, and $\dot{a}^2\equiv\partial(a^2)/\partial t$. Defining the "superfluid momentum"

$$\mathbf{Q}=2m\mathbf{v}_s=-\nabla\theta-2\mathbf{A}, \tag{16}$$

we can write expression (9) for the current as

$$\mathbf{j}=\sigma\int_{-\infty}^{\infty}d\varepsilon\Big\{\mathbf{Q}R_2N_2f_1+\frac{1}{4}\dot{\mathbf{Q}}(N_1^2+N_2^2)f_{1,\varepsilon}+\frac{1}{2}(N_1^2+N_2^2)\Big(\nabla f_2+\frac{1}{2}f_{1,\varepsilon}\nabla\dot{\theta}\Big)+\frac{1}{4}f_{1,\varepsilon}\frac{\partial}{\partial t}[\mathbf{Q}(R_2^2-N_2^2)]\Big\}, \tag{17}$$

where the normal conductivity σ is defined by

$$\sigma=2N(0)D={}^2/_3N(0)v_F^2\tau. \tag{18}$$

In this step we find that in the $\dot{\theta}=0$ gauge expression (17) is the same as Schmid's result.[9] The last term in (17 with the time derivative), which was omitted from Refs. 4–8, vanishes if we ignore the dispersive dependence of $f_{1,\varepsilon}$. Substitution of the equilibrium value $f_1=f_1^0(\varepsilon)$ into this term leads to a nonzero value containing an additional small factor $|\Delta|/T$. Since this term is also small in proportion to the parameter ω/T, we will omit it below. Expression (17) is the basis for the analysis below. In contrast with Refs. 4–9, it has been derived here in an arbitrary gauge, so that we can be convinced that this calculation approach is self-consistent. The functions $f_1(\varepsilon)=(1-n_\varepsilon-n_{-\varepsilon})\cdot\mathrm{sign}\,\varepsilon$ and $f_2(\varepsilon)=-(n_\varepsilon-n_{-\varepsilon})/N_1\cdot\mathrm{sign}\,\varepsilon$ in (17) should be determined in general by a kinetic equation for the distribution function of the nonequilibrium electron-hole excitations, n_ε. In many cases, however, it is sufficient to substitute the equilibrium function $n_\varepsilon=n_\varepsilon^0=[\exp(|\varepsilon|/T)+1]^{-1}$ into (17). We believe that this procedure was not carried out altogether correctly in Refs. 4–9, with the result that certain terms whose contributions are not small were left out of the expression for the current. At this point we turn to an analysis of these terms.

3. "EQUILIBRIUM" AND "LOCALLY EQUILIBRIUM" APPROXIMATIONS FOR THE CURRENT

To transform the terms containing $\dot{\theta}$ and ∇f_2 in (17), we introduce the gauge-invariant potential

$$\mu={}^1/_2\dot{\theta}-\varphi \tag{19}$$

and the associated electric field

$$\mathbf{E}=-\dot{\mathbf{A}}-\nabla\varphi={}^1/_2\dot{\mathbf{Q}}+\nabla\mu. \tag{20}$$

When there is a potential φ, the function f_2 is nonzero; under the condition $\varepsilon\gg|\Delta|$ it is

$$f_2=-\varphi f_{1,\varepsilon} \tag{21}$$

[relation (21) is discussed in more detail in Ref. 18]. Substitution of (21) into (17) leads to

$$\mathbf{j}=\sigma\int_{-\infty}^{\infty}d\varepsilon\{\mathbf{Q}R_2N_2f_1+{}^1/_2f_{1,\varepsilon}(N_1^2+N_2^2)\mathbf{E}\}. \tag{22}$$

In the equilibrium theory, the current in a "dirty" superconductor is given by the first term in (22), where we should set $f_1=f_1^0(\varepsilon)=\text{th}(\varepsilon/2T)$ (cf. Ref. 19). In the nonequilibrium case, two more groups of terms arise when we substitute the equilibrium function $f_1^0(\varepsilon)$ into (22). The reason for this result is the following relation, which can be established directly from (11):

$$N_1^2+N_2^2=\frac{1}{2}\left\{1+\left[1-\left(\frac{2\varepsilon|\Delta|}{\varepsilon^2+\gamma^2+|\Delta|^2}\right)^2\right]^{-1/2}\right\}. \tag{23}$$

Under inequalities (13), the integral (22) together with (23) can be evaluated by quadrature.[3] In the time-dependent theory it is necessary to evaluate this interval for an arbitrary relation between $|\Delta|$ and γ (although the equilibrium value of $|\Delta|$ in a "gap" superconductor is large in comparison with γ, in the time-varying case $|\Delta(\mathbf{r},t)|$ may in fact vanish!). As a result of a direct integration in which we make use of the definitions of N_i and R_i in (11), we find, omitting small terms of higher order,

$$\mathbf{j}=\frac{\pi\sigma}{4T}\mathbf{Q}|\Delta|^2+\sigma\mathbf{E}\left\{1+\frac{(|\Delta|^2+\gamma^2)^{1/2}}{2T}\left[K\left(\frac{|\Delta|}{(|\Delta|^2+\gamma^2)^{1/2}}\right)-E\left(\frac{|\Delta|}{(|\Delta|^2+\gamma^2)^{1/2}}\right)\right]\right\}, \tag{24}$$

where the functions $K(x)$ and $E(x)$ are complete elliptic integrals of the first and second kinds.

Expression (22) can be written in the form

$$\mathbf{j}=\mathbf{j}_s+\mathbf{j}_n+\mathbf{j}_{int}, \tag{25}$$

where the superfluid and normal components of the current are given by the standard relations

$$\mathbf{j}_s=\frac{\pi\sigma}{4T}\mathbf{Q}|\Delta|^2,\quad \mathbf{j}_n=\sigma\mathbf{E}, \tag{26}$$

and the "interference" component is

$$\mathbf{j}_{int}=\sigma\mathbf{E}\left\{\frac{(|\Delta|^2+\gamma^2)^{1/2}}{2T}\left[K\left(\frac{|\Delta|}{(|\Delta|^2+\gamma^2)^{1/2}}\right)-E\left(\frac{|\Delta|}{(|\Delta|^2+\gamma^2)^{1/2}}\right)\right]\right\}. \tag{27}$$

The terms in (27) are characterized by quantities which are properties of both the superconducting condensate and the normal metal. Actually, these terms include some interference of the motions which occur in the electron system of the superconductor.

Comparison of (27) with (26) shows that the interference component of the current is not small. Using the well-known asymptotic expression for the elliptic integrals, we can easily show that (24) takes the following form in the specified limiting cases:

$$\mathbf{j}=\frac{\pi\sigma}{4T}\mathbf{Q}|\Delta|^2+\sigma\mathbf{E}\left\{1+\frac{|\Delta|}{2T}\left(\ln\frac{4|\Delta|}{\gamma}-1\right)\right\}\quad\text{for}\quad\gamma\ll|\Delta|, \tag{28}$$

$$\mathbf{j}=\frac{\pi\sigma}{4T}\mathbf{Q}|\Delta|^2+\sigma\mathbf{E}\quad\text{for}\quad\gamma\gg|\Delta|. \tag{29}$$

Using (28) and (29), we can write the following rough approximation for the behavior of the functions in braces in (24):

$$\{\ldots\}\approx\left\{1+\frac{|\Delta|}{2T}\ln\frac{4|\Delta|+\gamma}{\gamma}\right\}. \tag{30}$$

This approximation turns out to be convenient for practical calculations.

We used the equilibrium approximation for the functions f_1 and f_2 above. In the time-dependent theory,[4–9] nonequilibrium contributions are taken into account in the determination of these functions:

$$f_1=f_1^0(\varepsilon)+\delta f_1(\varepsilon),\qquad \delta f_1(\varepsilon)=-f_{1,\varepsilon}{}^0\frac{R_2}{N_1}\frac{2}{\gamma}\frac{\partial|\Delta|}{\partial t}, \tag{31}$$

$$f_2=-\varphi f_{1,\varepsilon}{}^0+\delta f_2(\varepsilon),\qquad \delta f_2(\varepsilon)=-2\mu\frac{N_2\tau_\varepsilon|\Delta|}{N_1+2N_2\tau_\varepsilon|\Delta|}f_{1,\varepsilon}{}^0. \tag{32}$$

The current component due to the function $\delta f_2(\varepsilon)$ in (32) is vanishingly small and can be ignored. In contrast, the function $\delta f_1(\varepsilon)$ in (31) contributes a current component which, although small in comparison with $\mathbf{j}_s$, is not of a dissipative nature. In general, this component may not be small in comparison with $\mathbf{j}_n$. As a result, the current is given by the following expression in the local equilibrium approximation:

$$\mathbf{j}=\frac{\pi\sigma}{4T}\mathbf{Q}\left(|\Delta|^2-\frac{1}{\gamma}\frac{\partial|\Delta|^2}{\partial t}\right)+\sigma\mathbf{E}\left\{1+\frac{(|\Delta|^2+\gamma^2)^{1/2}}{2T}\left[K\left(\frac{|\Delta|}{(|\Delta|^2+\gamma^2)^{1/2}}\right)-E\left(\frac{|\Delta|}{(|\Delta|^2+\gamma^2)^{1/2}}\right)\right]\right\}. \tag{33}$$

This expression should appear in the Ginzburg-Landau equations instead of the expression presented in Refs. 4–9 [by way of comparison we note that in those other papers relation (29) was given instead of (33) for the current in the local-equilibrium approximation].

4. CURRENT IN WEAKLY LINKED SUPERCONDUCTORS

To illustrate the distinctive features of the expression derived here for the current, we consider the resistive state which arises in weakly linked superconductors. To describe it, we make use of the Aslamazov-Larkin model.[14] In accordance with that model, we assume that the order parameter near the weak link can be written

$$\Delta(x,t)=\frac{1}{2}\Delta_0\left\{\left(1-\frac{x}{a}\right)e^{-i\theta_1}+\left(1+\frac{x}{a}\right)e^{-i\theta_2}\right\}. \qquad (34)$$

The picture is assumed to be one-dimensional here; the coordinate x in the vicinity of the weak link varies from $-a$ to a; $\Delta_0=\text{const}$ is the modulus of the order parameter in the bulk superconductor; and θ_1 and θ_2 are the time-dependent values on the left and right "banks," respectively, of the phases of the order parameter [we recall that the phase θ was introduced by (12)].

Since the model is one-dimensional, it is sufficient to calculate the current at the point $x=0$. It follows from (34) that we have (in the $\mathbf{A}=0$ gauge and at $x=0$)

$$|\Delta|^2=\Delta_0^2\cos^2[(\theta_1-\theta_2)/2], \qquad (35)$$
$$|\Delta|^2Q=-|\Delta|^2\nabla\theta=\mathrm{Im}(\Delta^*\,\mathrm{grad}\,\Delta)=(\Delta_0^2/2a)\sin(\theta_1-\theta_2).$$

In the bulk "banks" we can set $\mu=0$; assuming $a\ll\xi(T)$, we find from (19) and (20)

$$\dot\theta_{1(2)}=2\varphi_{1(2)},\quad E=-\mathrm{grad}\,\varphi\approx V/2a,\quad V=\varphi_1-\varphi_2. \qquad (36)$$

The condition $a\ll\xi(T)$, which is typical of experiments on weak-link structures, means that an analysis based on equations of the Ginzburg-Landau type (which assume that the spatial derivatives are quite small) would not be applicable here. Nevertheless, the expression for the current in the form in (24) is applicable in this situation, since $f_1(\varepsilon)$ in (22) can be assumed to be an equilibrium function because of the rapid diffusive dissipation of the nonequilibrium excitations by the banks. Substituting (35) and (36) into (26), (27), we find

$$j=j_s+j_n+j_{int},\quad j_s=j_0\sin(2Vt+\theta_0),\quad j_n=\frac{\sigma V}{2a}, \qquad (37)$$
$$j_{int}=\frac{\sigma V}{2a}\frac{\Delta_0}{2T}\left|\cos\left(Vt+\frac{\theta_0}{2}\right)\right|\ln\frac{4\Delta_0|\cos(Vt+\theta_0/2)|+\gamma}{\gamma}, \qquad (38)$$

where θ_0 is a constant phase difference, and $j_0=\pi\sigma\Delta_0^2/8aT$. It follows from (38) that the interference current does indeed have a phase-dependent dissipative nature (as was stipulated in Section 3).[4)]

The relationship between the interference contribution (38) and the familiar phenomenon of an "excess current" is a curious one. Taking an average of (37) over the time, we write the result in the first approximation as

$$j=\sigma^*\frac{V}{2a},\quad \sigma^*\approx\sigma\left(1+\frac{\Delta_0}{\pi T}\ln\frac{4\Delta_0}{\gamma}\right), \qquad (39)$$

where σ^* is an effective conductivity. The second term in σ^* describes an excess conductivity which stems from the interference component of the current. An excess current has been observed in several experiments on weak-link structures. Artemenko *et al.*[17] have explained this phenomenon using the model of short bridges. Equation (39), which we have derived in another model and by another method, has much in common with the expression given in Ref. 17 under the conditions $V\gtrsim\Delta_0$ and $T\sim T_c$. Specifically, the temperature dependence $[j_{exc}\propto\Delta_{BCS}(T)]$ is essentially the same, as are the absolute values of the excess current. At the same time, other aspects of this phenomenon can be seen: primarily, the fact that the excess current is actually of a "pulsating" nature, as can be seen from (38). Furthermore, while it was concluded previously (cf. Ref. 21) on the basis of Ref. 17 that the excess current in the bridges of a weak link arises because of the massive banks (i.e., the excess current is more of the nature of a boundary effect), in our own analysis we see that the effect should also occur in the interior. It thus may also be manifested in experiments with bulk samples.

According to (39), experiments carried out to determine the excess conductivity would make it possible to evaluate the energy relaxation time $\tau_\varepsilon=2\gamma^{-1}$ of single-particle excitations in superconductors.

5. DISCUSSION

The presence of an interference term in nonequilibrium superconductors may be related to qualitative features in other time-dependent phenomena, e.g., the "cos φ-term paradox" (Ref. 22, for example). For weak-link bridges this paradox arises when one attempts to interpret experimental data on the basis of the expression found for the current by the method of a tunnelling Hamiltonian.[16] This problem must obviously be reexamined in light of the expression derived in Section 4.

As an example, we consider the interpretation of the experiment reported in Ref. 23. The fluctuating value of the derivative of the voltage with respect to the current was measured in that study. A theoretical analysis of that quantity is based on a Fokker-Planck equation,[24] which can be written for fluctuations in weak-line superconductors by analogy with the motion of a Brownian particle in an external potential field.[25] As a result, the following expression is found for the quantity $(\partial\mathscr{V}/\partial x)_{x=0}$

$$\left.\frac{\partial\mathscr{V}(y)}{dx}\right|_{x=0}\approx4\pi^2\left\{\int_0^{2\pi}f(\theta)\,d\theta\int_0^{2\pi}\frac{d\theta'}{f(\theta')}\left[1+\alpha_1\cos\theta+\alpha_2\left|\cos\frac{\theta'}{2}\right|+\alpha_3p_1\left(\frac{\theta'}{2}\right)\right]\right\}^{-1}. \qquad (40)$$

Here $\mathscr{V}=V/jSR$, $x=j/j_0$, $f(\theta)=\exp(1/2y\cos\theta)$, $y=j_0S/T$, S is the cross-sectional area of the weak link, R is its resistance in the normal state, and the parameters α_i are,[5)] according to Section 4,

$$\alpha_1\approx-\frac{\Delta_0^2}{T^2}\approx0,\quad \alpha_2=\frac{\Delta_0}{2T}\ln\frac{4\Delta_0}{\gamma},\quad \alpha_3=\frac{\pi\Delta_0^2}{4T\gamma}. \qquad (41)$$

For a comparison with experimental results, Falco[23] used, instead expressions (40) and (41), the quantities which arise when the ordinary Josephson expression is used for the current [in the latter case, we should set $\alpha_1=1,\alpha_2=\alpha_3=0$ in (41)]. The experimental data were found to lie well below the theoretical curve (they were close to the curve with $\alpha_1=-1$), and that result was preceived as paradoxical. If we instead use expressions (40) and (41) for the comparison with the experimental results, we find that theoretical curve (40) runs well below that in the case $\alpha_1=1$, $\alpha_2=\alpha_3=0$, and it may be close to the experimental results of Ref. 23. There are thus grounds for expecting that a paradox will not occur in this case. On the other hand, it would be difficult to claim accurate agreement with experimental results at this point, since Falco *et al.*,[23] did not report sufficiently detailed data on the quantities determining the voltage-current characteristic of the weak link. It is not clear, for example, how the measured voltage compared with γ; this is an important question, since it determines the applicability of the theoretical expressions.

The use of expression (33) for the current may also affect the results of a theory for the resistive state of "one-dimensional" (see the review by Ivlev and Kopnin[26]) and "two-dimensional"[27] superconductors. That question, however, requires further study.

In conclusion we would like to point out that a logarithmic renormalization of the conductivity analogous to (39) has also arisen previously in the theory for the linear (Refs. 28 and 29, for example) and nonlinear[30] responses of a superconductor to a time-varying external electromagnetic field of frequency ω_0. Those studies, however, dealt with frequencies $\omega_0 \gg \gamma$. For this reason, the logarithmic factors which appeared, reflecting the occurrence of interference between normal and superfluid motions, had a slightly different structure.

We wish to thank S. N. Artemenko, A. G. Volkov, B. I. Ivlev, A. M. Kogan, K. K. Likharev, Yu. N. Ovchinnikov, and G. M. Éliashberg for useful discussions.

[1)] Similar terms in the current were found previously[15] from equations for gapless superconductors through the use of an equation for the order parameter. For short bridges, that approach would not be justified, since equations of the Ginzburg-Landau type do not apply in this case.

[2)] In writing the spectral functions (11) we have completely ignored the effect of the external fields A and φ on them, since expression (11) actually correspond to the $\varphi = 0$ gauge. For an arbitrary gauge with $\varphi \neq 0$, the functions $\hat{g}^{R(A)}$ (and, in particular, N_1) change.[18] This change, however, produces no substantial effect in the expression for the current.

[3)] The difference which we mentioned between our result and that reported in Refs. 4–9 stems from the circumstance that the quantity $N_1^2 + N_2^2$, given by (23), was set equal to one in the corresponding integral in Refs. 4–9.

[4)] Near the weak link ($|x| < a$), there is some number of "captured" nonequilibrium excitations with energies[20] $\varepsilon < \Delta_0$. Those excitations lie outside the scope of our analysis. As Schmid *et al.*[21] have shown, the presence of these excitations at voltages $V \ll \gamma$ contributes a current increment $\delta j = j_0 p_1[(\theta_1 - \theta_2)/2] V\gamma^{-1}$, where p_1 is some nonnegative even function of period π with a maximum value of 2/5. The magnitude of the same dissipative phase-dependent structure could also have been derived formally by substituting (35) and (36) into the second term in (33). The interference term which would arise in this approach would not be small in comparison with j_n in (37) ($\delta j \sim j_n |\Delta|^2/\gamma T$). As was mentioned earlier, there should be no "locally equilibrium" increments in the current in the case of short bridges ($a \ll \xi(T)$) (because of the fast diffusion processes associated with the presence of the banks). In other cases, however, this term may prove important.

[5)] The value given here for α_1 is found in the case in which a term $\approx -j_0 VT^{-1}\cos(2Vt + \theta_0)$, deriving from the last term in (17), is restored in (37). Incorporating this term in the expressions for the current would go beyond the accuracy of this treatment, however, so that the "cosine" term should be discarded.

[1] V. L. Ginzburg and L. D. Landau, Zh. Eksp. Teor. Fiz. **20**, 1064 (1950).
[2] L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **36**, 1918 (1959) [Sov. Phys. JETP **9**, 1364 (1954)].
[3] L. D. Landau, Zh. Eksp. Teor. Fiz. **11**, 592 (1941).
[4] A. A. Golub, Zh. Eksp. Teor. Fiz. **71**, 341 (1976) [Sov. Phys. JETP **44**, 178 (1976)].
[5] L. Kramer and R. J. Watts-Tobin, Phys. Rev. Lett. **40**, 1041 (1978).
[6] G. Schön and V. Ambegaokar, Phys. Rev. **B19**, 3515 (1979).
[7] C.-R. Hu, Phys. Rev. **B21**, 2775 (1980).
[8] R. J. Watts-Tobin, U. Krähenbühl, and L. Kramer, J. Low Temp. Phys. **42**, 459 (1981).
[9] A. Schmid, NATO Adv. Study Inst. Ser. **B65**, 423 (1981).
[10] G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **61**, 1254 (1971) [Sov. Phys. JETP **34**, 668 (1972)].
[11] A. I. Larkin and Yu. N. Ovshinnikov, Zh. Eksp. Teor. Fiz. **73**, 299 (1977) [Sov. Phys. JETP **46**, 155 (1977)].
[12] K. D. Usadel, Phys. Rev. Lett. **25**, 507 (1970).
[13] L. P. Gor'kov and G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **54**, 612 (1968) [Sov. Phys. JETP **27**, 328 (1968)].
[14] L. G. Aslamazov and A. I. Larkin, Pis'ma Zh. Eksp. Teor. Fiz. **9**, 150 (1969) [JETP Lett. **9**, 87 (1967)].
[15] K. K. Likharev and A. A. Yakobson, Zh. Eksp. Teor. Fiz. **68**, 1150 (1975) [Sov. Phys. JETP **41**, 570 (1975)].
[16] B. D. Josephson, Phys. Lett. **1**, 251 (1962).
[17] S. N. Artemenko, A. F. Volkov, and A.V. Zaitzev, Solid State Commun. **30**, 771 (1979).
[18] A. M. Gulyan, G. F. Zharkov, and G. M. Sergoyan, Kratkie Soobschen-iya po Fizike **10**, 45 (1986).
[19] A. A. Abrokosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Methods of Quantum Field Theory in Statistical Physics), Fizmatgiz, Moscow, 1962, p. 441.
[20] L. G. Aslamazov and A. I. Larkin, Zh. Eksp. Teor. Fiz. **70**, 1340 (1976) [Sov. Phys. JETP **43**, 698 (1976)].
[21] A. Schmid, G. Schön, and M. Tinkham, Phys. Rev. **B21**, 5076 (1980).
[22] A. V. Zorin, I. O. Kulik, and K. K. Likharev, Fiz. Nizk. Temp. **5**, 1138 (1979) [Sov. J. Low Temp. Phys. **5**, 537 (1979)].
[23] C. M. Falco, W. H. Parker, and S. E. Trullinger, Phys. Rev. Lett. **31**, 933 (1973).
[24] H. A. Kramers, Physica **7**, 284 (1940).
[25] V. Ambegaokar and B. I. Halperin, Phys. Rev. Lett. **22**, 1364 (1969).
[26] B. I. Ivlev and N. B. Kopnin, Usp. Fiz. Nauk **142**, 435 (1984) [Sov. Phys. Usp. **27**, 206 (1984)].
[27] S. V. Lempitskiĭ, Zh. Eksp. Teor. Fiz. **90**, 793 (1986) [Sov. Phys. JETP **63**, 462 (1986)].
[28] A. A. Abrikosov and I. M. Khalatnikov, Usp. Fiz. Nauk **65**, 551 (1958).
[29] Yu. N. Ovshinnikov, Zh. Eksp. Teor. Fiz. **59**, 128 (1970) [Sov. Phys. JETP **32**, 72 (1971)].
[30] L. P. Gor'kov and G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **56**, 1297 (1969) [Sov. Phys. JETP **29**, 698 (1969)].

Translated by Dave Parsons