Absorption of longitudinal sound in metals near the Lifshitz topological transition

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The absorption of longitudinal sound in the vicinity of a point where the Fermi-surface topology of a metal is changed is considered for the case of finite temperature and impurity density. It is shown that in the dirty case ($T\tau \ll 1$, where τ is the characteristic relaxation time) the jump of the sound absorption coefficient becomes smeared out near the topological transition both for the long wave ($kl \ll 1$, where l is the mean free path) and for short-wave sound. The laws governing the variation of the coefficient of longitudinal-sound absorption on the right and on the left of the transition point are obtained. The results explain the available experimental material.

1. Electronic topological transitions (ETT) are extensively investigated at present both theoretically and experimentally. It was noted already in I. M. Lifshitz's seminal paper[1] that changes of the topology of the Fermi surface should give rise to various types of singularities in the thermodynamic and kinetic characteristics of a metal. In the "classical" case of zero temperature and in the absence of electron scattering, when the Lifshitz topological transition is in its strict sense an electronic phase transition of order $2\frac{1}{2}$, these singularities are manifested as kinks (density of state, conductivity) or discontinuities (thermoelectric power).

Singularities of this kind were actually repeatedly observed later in a number of compounds ($Li_{1-x}Mg$, $Bi_{1-x}Sb_x$, Bi whiskers, and others) in investigations of galvanomagnetic phenomena,[2] electric conductivity,[3,4] superconducting-transition temperature,[5] and thermoelectric power.[6–10] Under real conditions, however (at nonzero temperatures and in the presence of electron scattering), the kinks become smoothened and the discontinuities joined, but many characteristic features of phase transitions of order $2\frac{1}{2}$ are nonetheless preserved (e.g., the asymmetry of the peak of the thermoelectric power). The elimination of the thermoelectric-power and conductivity singularities was explained in Refs. 11–13 within the framework of a simple model that permits calculation of the kinetic and thermodynamic characteristics of a metal near the ETT at finite temperatures and in the presence of electron scattering.

The behavior of the sound-absorption coefficient Γ in ETT is particularly unusual, in two important respects. Thus, it was first pointed out in Ref. 14 that near the ETT a metal is fundamentally a new nonlinear acoustic medium with elastic properties that are not analytic in the strain. The point is that the pressure change accompanying a sufficiently intense sound wave in a metal near the ETT can itself cause a local transition, and this should undoubtedly influence the sound absorption. The corresponding nonlinear distortions of the sound field in the course of the sound-wave propagation were investigated in Ref. 14.

The particulars of the phase transition of order $2\frac{1}{2}$ manifest themselves in the sound absorption not only by nonlinear effects. The point is that Γ is highly sensitive also to a local change of the structure of the Fermi surface. Kaganov *et al.*[15,16] investigated in detail the absorption of short-wave sound ($kl \gg 1$) as a function of its propagation direction and of the type of ETT. They have shown that near the ETT the sound absorption coefficeint should undergo at $T = 0$ a jump of the same order as that observed in Γ. In the case of a transition of the "neck breaking" type, strong anisotropy should be observed in the dependence of the sound wave vector $\mathbf{k}$ on the direction: in some cases the coefficeint Γ has a jump, just as when a new cavity is formed, and in others it has a logarithmic singularity. Moreover, the analysis of the angular dependence of the absorption coefficient of short-wave sound was generalized in Ref. 16 to include the change when the local geometry of the Fermi surface changes without a change in its connectivity.

In absorption of long-wave sound, the picture is significantly different. The interaction of the electrons with the sound becomes ineffective[17] because of the large velocity difference, and the electrons participating in the interaction come not from an individual "band" only, but from the entire Fermi surface. The question of absorption of long-wave sound near a phase transition of order $2\frac{1}{2}$ at $T = 0$ was considered in Ref. 18 for a model in which the dependence of the strain potential and of the electron mean freepath on the proximity to the transition point was neglected. Under these assumptions it was found that in the case of long-wave sound the absorption coefficeint has an anomaly near the transition point, and when a new cavity of the Fermi surface is formed the change $\Delta\Gamma$ of the absorption coefficient turns out to be of the order of Γ itself. In the case of a "neck-breaking" transition, however, Γ increases logarithmically.

We consider in the present paper the absorption of longitudinal sound in a wide frequency range, all the way to $\omega \sim \tau^{-1}$, where τ is the electron relaxation time in a metal that undergoes an ETT, with account taken of strong impurity scattering and at finite temperatures. We do not confine ourselves to model-dependent premises concerning the deformation potential and the relaxation time, which are independent of the proximity to the ETT, but carry out a consistent microscopic calculation of the sound absorption coefficient using a temperature diagramatic technique. All

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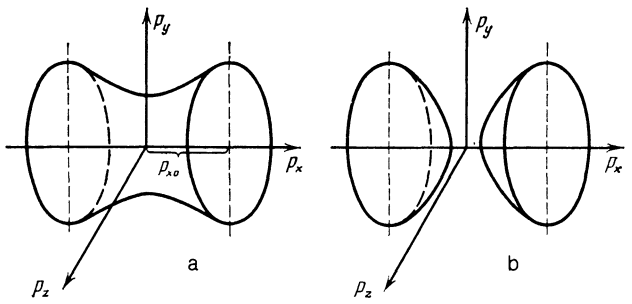

FIG. 1. Topological transition of neck-breaking type. At $z = 0$ the open Fermi surface (a) goes over into a closed one (b); p_{xo}—limiting value of longitudinal momentum.

the information on the ETT is centered in a one-electron Green's function via the electron spectrum $\varepsilon(\mathbf{p},z)$ even prior to averaging over the impurity positions.

We describe the transition using the previously proposed[11–13] model of an ETT of the "neck breaking" type, in which the Fermi surface is chosen in the form of a hyperboloid of revolution (Fig. 1), so that

$$\varepsilon(\mathbf{p},\ z)=\mu+\mathbf{p}_\perp^2/2m_\perp-p_x^2/2m_x-z. \qquad (1)$$

Depending on the sign of the parameter z, which characterizes the proximity of the system to the transition, the hyperboloid has two cavities ($z<0$) or one ($z>0$), meaning a closer or an open Fermi surface. This model is convenient in that it yields similar descriptions on the two sides of the ETT.

To calculate the longitudinal-sound absroption coefficient we use the Tsuneto's method[19] which is valid for an anisotropic Fermi surface and can be easily converted to the formalism of the temperature diagrammatic technique. This approach is quite close to the calculation of the absorption coefficient of sound from the general dynamic equations of elasticity theory in metals[20] within the framework of a strain potential (whose role is assumed here to be the linear response of the system to the lattice velocity field σ_I).

In view of the detailed investigation of the angular dependence of Γ for short-wave sound,[15,16] we confine ourselves to a study of the sound absorption coefficeint directed along the hyperboloid axis. Particular attention will be paid, however, to the question of absorption of long-wave sound ($kl\ll 1$). As will be shown below, the $\Gamma(z)$ dependence exhibits here, too, a "step" smeared out by electron scattering, although the relative size of the step, unlike in Ref. 18, is smaller than in the case of short-wave sound absorption. It is interesting that whereas the main contribution to Γ is proportional to ω^2 only so long as $kl<1$, for the Γ increment due to the ETT this frequency dependence extends all the way to $kl\lesssim\min\{(\varepsilon_0\tau)^{1/2},\ (\varepsilon_0/|z|)^{1/2}\}$, where ε_0 is an energy of the order of ε_F (it is understood that the present analysis is correct only under the assumption $\min\{(\varepsilon_0\tau)^{1/2},\ (\varepsilon_0/|z|)^{1/2}\}\gg 1$, which makes it valid to average over the impurities in the ladder approximation and indicates also that the system is close to an ETT).

2. We proceed to calculate the longitudinal-sound absorption coefficient in a metal near an ETT. The sound absorption coefficient Γ is defined as the ratio of the power Q dissipated per unit volume to the energy flux indent on a unit surface:

$$\Gamma=Q(^1/_2\rho_{ion}v_s|u|^2)^{-1},$$

where ρ_{ion} is the density of the metal, v_s the speed of sound in it, and u is the amplitude of the velocity in the sound wave; we define this amplitude by specifying a velocity field

$$\mathbf{u}(\mathbf{x},\ t)=\mathbf{u}\exp\{i(\mathbf{kx}-\omega t)\}=-i\omega\delta\mathbf{R}(\mathbf{x},\ t), \qquad (2)$$

with wave vector $\mathbf{k}$ and frequency ω.

In the laboratory frame $\{\mathbf{x}'\}$ the electrons interact with the sound wave via an electromagnetic field induced by the wave and having potentials $\mathbf{A}$ and φ, and also via collisions with impurities that move together with the lattice. In this frame, however, in view of the motion of the impurities when the sound wave propagates, the electron scattering from the impurities is inelastic and cannot be treated by the standard technique of averaging over the impurity positions.[21] Therefore, following Ref. 19, we transform to a reference frame (CRF) $\{\mathbf{x}\}$ comoving with the lattice, in which the impurities are immobile:

$$\mathbf{x}=\mathbf{x}'-\delta\mathbf{R}(\mathbf{x},\ t).$$

Here, just as in (2), $\sigma\mathbf{R}(\mathbf{x},t)$ is a slowly varying function of $\mathbf{x}$ and of the time t, and determines the displacement of the lattice ions from their equilibrium positions by the propagating sound wave.

In this reference frame we can already average over the impurity positions by the standard averaging technique,[21] but in view of the inertial character of the CRF we must take into consideration here the additional intertial force $\mathbf{f}$ acting in this frame on the electrons (it corresponds in the lab to electron drag by the moving impurities).

The total electric current flowing in the metal when a sound wave moves in it consists thus in the CRF of two different contributions. The first, $\mathbf{j}(\mathbf{k},\omega)=\sigma(\mathbf{E}(\mathbf{k},\omega)$, is the usual one and is determined by the linear response of the system to the sound-wave electric field $\mathbf{E}=-\operatorname{grad}\varphi$. The second contribution $\mathbf{j}_I(\mathbf{k},\omega)=\sigma_I(\mathbf{k},\omega)\mathbf{f}(\mathbf{k},\omega)/e$ is due to the response to the inertial force $\mathbf{f}=-m_x\mathbf{u}/\tau$ acting on the electron in this frame. Since the electromagnetic field of the sound wave, being defined in a self-consistent manner, is proportional to $\mathbf{u}$, the linear responses $\rho(\mathbf{k},\omega_v)$ and $\rho_I(\mathbf{k},\omega_v)$ to the field $\mathbf{E}$ and to the fictitious force $\mathbf{f}$, which determine the corresponding conductivities $\sigma(\mathbf{k},\omega)$ and $\sigma_I(\mathbf{k},\omega)$, can be calculated independently of each other. In Ref. 19 the longitudinal-sound absorption coefficient Γ was expressed in terms of σ and σ_I:

$$\Gamma=\frac{m_x^2\sigma_0}{\rho_{ion}v_s^2(e\tau)^2\sigma''}\left[\frac{\sigma_I\sigma'}{\sigma''}-\sigma_I''\right], \qquad (3)$$

where

$$\sigma'=\operatorname{Re}\sigma,\quad \sigma''=\operatorname{Im}\sigma,\quad \sigma_I''=\operatorname{Im}\sigma_I,$$

and σ_0 is the usual static conductivity of the metal $\sigma_0=N_e^2\tau/m$, for a metal with a spherical Fermi surface; in

the case of a metal undergoing an ETT it was already calculated earlier.[12]

To find σ and σ_I we present the linear responses $\rho(\mathbf{k},\omega_\nu)$ and $\rho_I(\mathbf{k},\omega_\nu)$ by the temperature of diagrammatic technique. They are expressed in usual fashion via single-electron Green's functions:

$$\begin{pmatrix}\rho(\mathbf{k},\omega_\nu)\\ \rho_I(\mathbf{k},\omega_\nu)\end{pmatrix} = 2Te\left\langle \int \frac{d^3p}{(2\pi)^3}\sum_{\varepsilon_n} G_0(\varepsilon_n,\mathbf{p})\,G_0(\varepsilon_n-\omega_\nu,\mathbf{p}-\mathbf{k})\begin{pmatrix}-e\varphi\\ H_I\end{pmatrix}\right\rangle, \tag{4}$$

where $\omega_\gamma = 2\pi T\nu$ is the external boson frequency, $\varepsilon_n = 2\pi T(n+1/2)$ is the fermion frequency, and

$$H_I = -\frac{i}{m_x\omega_\nu}[\mathbf{k}(\mathbf{p}+{}^1/_2\mathbf{k})][\mathbf{u}(\mathbf{p}+{}^1/_2\mathbf{k})]-\mathbf{u}(\mathbf{p}+{}^1/_2\mathbf{k}) \tag{5}$$

is the vertex corresponding to the action of the inertia force $\mathbf{f}$ on the electron (this expression can be easily obtained by a trivial generalization of the results of Ref. 19). The angle brackets denote averaging of the corresponding expression over the position of the impurities, which is now carried out in standard fashion and reduces to averaging of the Green's functions themselves and to renormalization of the corresponding scalar vertex $\zeta(\mathbf{k},\omega_\nu)$ satisfying the usual ladder equation

$$\zeta(\mathbf{k},\omega_\nu) = 1+\zeta(\mathbf{k},\omega_\nu)I, \tag{6}$$

where

$$I = n_i|U|^2\int G(\mathbf{p},\varepsilon_n)\,G(\mathbf{p}-\mathbf{k},\varepsilon_n-\omega_\nu)\frac{d^3p}{(2\pi)^3}. \tag{7}$$

Here n_i is the impurity density, and U the amplitude of electron scattering by the impurities (for simplicity, the scattering is assumed isotropic).

The linear responses $\rho(\mathbf{k},\omega)$ and $\rho_I(\mathbf{k},\omega)$ are connected with $\sigma(\mathbf{k},\omega)$ and $\sigma_I(\mathbf{k},\omega)$ by the relations

$$\begin{aligned}\sigma(\mathbf{k},\omega) &= (-i\omega/k^2\varphi)[\rho(\mathbf{k},\omega)+\rho(\mathbf{k},0)],\\ \sigma_I(\mathbf{k},\omega) &= (-\omega/k)(-e\tau/m_x u)[\rho_I(\mathbf{k},\omega)-\rho_I(\mathbf{k},0)].\end{aligned} \tag{8}$$

By calculating ρ and ρ_I in accordance with (4)–(7), we can thus find in principle the longitudinal-sound absorption coefficient in a metal with an arbitrary electron spectrum. Thus, for a meal with a spherical Fermi surface the procedure described leads to the well-known Pippard result[17]

$$\Gamma = \frac{mN}{\rho_{ion}v_s\tau}\left[\frac{1}{3}\frac{(kl)^2\operatorname{arctg} kl}{kl-\operatorname{arctg} kl}-1\right]. \tag{9}$$

3. To calculate Γ near an ETT it is therefore necessary to find the linear responses $\rho(\mathbf{k},\omega)$ and $\rho_I(\mathbf{k},\omega)$ with a one-electron Green's function corresponding to the spectrum (1) and averaged over the impurity positions. It was obtained earlier in Ref. 12, and in the case of a sufficiently dirty metal ($T\tau \ll 1$), which is of interest to us, it takes the form

$$G^{-1}(\varepsilon_n,\mathbf{p}) = i\varepsilon_n-\mathbf{p}_\perp^2/2m_\perp+p_x^2/2m_x+z+(i/2\tilde{\tau})\operatorname{sign}\varepsilon_n, \tag{10}$$

where

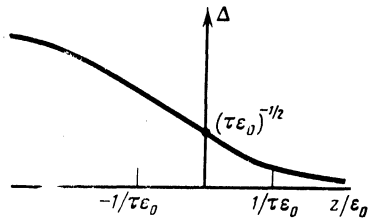

FIG. 2. Dependence [Eq. (12)] of Δ on the parameter z.

$$1/\tilde{\tau} = [1-{}^1/_2\Delta(z)]/\tau, \tag{11}$$

$$\Delta(z) = 2^{1/2}\{[1/4\tau^2\varepsilon_0^2+z^2/\varepsilon_0^2]^{1/2}-z/\varepsilon_0\}^{1/2}. \tag{12}$$

In expression (11) we have neglected the dependence of $\tilde{\tau}$ on the frequency ε_n, shown in Ref. 12 to be immaterial for the calculation of the conductivity near an ETT. Figure 2 shows a plot fo Δ vs the parameter z/ε_0.

We begin with calculation of the Fourier component $\rho(\mathbf{k},0)$, whose appearance in (8) is due to change of the order of the summation and integration in (4). Calculating in standard fashion the sum over the frequencies in Eq. (4) (Ref. 21) and then integrating with respect to the transverse component of the momentum, we get

$$\rho(\mathbf{k},0) = \frac{e^2m_\perp p_{x0}\varphi}{2\pi^2}\left[1-\left(\frac{2T}{\varepsilon_0}\right)^{1/2}\int_{z/2T}^{\infty}\frac{(y-z/2T)^{1/2}}{\operatorname{ch}^2 y}dy\right], \tag{13}$$

whence

$$\rho(\mathbf{k},0) = \frac{e^2m_\perp p_{x0}\varphi}{2\pi^2}\begin{cases}1-2(|z|/\varepsilon_0)^{1/2}, & z\ll -T,\\ 1-0{,}76(2T/\varepsilon_0)^{1/2}+z(2\varepsilon_0T)^{-1/2}, & |z|\ll T,\\ 1-(\pi T/\varepsilon_0)^{1/2}\exp(-z/T), & z\gg T.\end{cases} \tag{14}$$

To find the renormalized vertex $\rho(\mathbf{k},\omega_\nu)$ in (7) we first integrate over the transverse component of the momentum, after which the expression for I takes the form

$$I = \frac{\tilde{\tau}}{2\tau}\int_{-1}^{1}\frac{dt}{i\omega_\nu\tilde{\tau}+i+lkt}\times\left\{i-\frac{1}{2\pi}\ln\frac{-\varepsilon_0t^2-z-i(\varepsilon_n+1/2\tilde{\tau})}{-\varepsilon_0t^2+lkt/\tilde{\tau}-z-i(\varepsilon_n-\omega_\nu-1/2\tilde{\tau})}\right\}.$$

Next, separating the real and imaginary parts of the logarithm and neglecting lkt/τ compared with ε_0t^2 in its argument, we get

$$I = \frac{\tilde{\tau}}{\tau}\left[\frac{\operatorname{arctg} lk}{lk}-J\right], \tag{15}$$

where

$$J = \frac{1}{\pi}\int_0^1\frac{dx}{1+(kl)^2x^2}\operatorname{arctg}\frac{1}{2\varepsilon_0\tilde{\tau}x^2+2\tilde{\tau}z}. \tag{16}$$

This integral can already be directly calculated for different values of kl:

$$J=\begin{cases}\dfrac{\Delta(z)}{2}-\dfrac{kl\,\mathrm{arctg}\,kl}{2\varepsilon_0\tau}, & kl\ll\min\left\{(\varepsilon_0\tau)^{1/2},\ \left(\dfrac{\varepsilon_0}{|z|}\right)^{1/2}\right\},\\ \dfrac{1}{2kl}\,\mathrm{arctg}\,\dfrac{1}{2\tau z}, & kl\gg\min\left\{(\varepsilon_0\tau)^{1/2},\ \left(\dfrac{\varepsilon_0}{|z|}\right)^{1/2}\right\},\end{cases} \tag{17}$$

and, in accordance with the cut in the region where the logarithm is defined, the arctan x branch is chosen such that $\arctan x\in[0,\pi]$.

In the calculation of $\rho(\mathbf{k},\omega_\nu)$ we encounter the same integral I as in the determination of the vertex $\zeta(\mathbf{k},\omega_\nu)$. Inasmuch as in the dirty limit the final result (15)–(17) for I depends on ε_n only via the theta function $\theta(\varepsilon_n(\omega_\nu-\varepsilon_n)$, that determines the locations of the poles in the complex plane, the summation over ε_n in (4), which remains after integrating over the momenta, is elementary and reduces to multiplication by $\omega_\nu/2\pi T$. Thus, for $\rho(\mathbf{k},\omega_\nu)$ we get

$$\rho(\mathbf{k},\omega_\nu)=-\frac{e^2\omega_\nu\tau m_\perp p_{x0}}{\pi^2}\varphi\frac{I}{1-I}.$$

The usual analytic continuation of this expression with respect to frequency ($i\omega_\nu\to\omega$) yields

$$\sigma(\mathbf{k},\omega)=\frac{e^2m_\perp p_{x0}\omega^2\tau}{k^2\pi^2}\frac{I}{1-I}-i\frac{\omega e^2}{k^2}\rho(\mathbf{k},0). \tag{18}$$

As shown by Tsuneto,[19] to calculate $\sigma_I(\mathbf{k},\omega)$ there is no need to calculate explicitly the response $\rho_I(\mathbf{k},0)$—only the terms proportional to $-m\mathbf{u}/e\tau$ contribute to the conductivity $\sigma_I(\mathbf{k},\omega)$. Calculating $\rho_I(\mathbf{k},\omega)$ from (4) in analogy with the calculation of $\rho(\mathbf{k},\omega)$ above but with vertex (5), and recognizing that expression (3) for Γ contains only the imaginary part σ_I'', we get

$$\sigma_I''=\frac{\omega}{k}\frac{e\tilde{\tau}}{m_x u}\operatorname{Im}\rho_I'=-\frac{e^2m_\perp p_{x0}\omega}{\pi^2k^2}(1-\Delta(z)/2). \tag{19}$$

Defining $\sigma_0=\rho(\mathbf{k},0)p_{x0}^2\tilde{\tau}/3\varphi m_x^2$ as in Ref. 12, we ultimately obtain for the sound absorption coefficient

$$\Gamma=\frac{m_\perp p_{x0}^3}{3\pi^2\rho_{i0n}v_s\tau}\left[\frac{k^2l^2}{3}\frac{I}{1-I}-\left(1-\frac{\Delta(z)}{2}\right)\right]. \tag{20}$$

4. We analyze first the case of long-wave sound ($kl\lesssim 1$). So long as $kl\lesssim\min\{(\varepsilon_0\tau)^{1/2},(\varepsilon_0/|z|)^{1/2}\}$, the ratio $I/(1-I)$ in (20) can be represented in the form

$$\frac{I}{1-I}=\left(\frac{\mathrm{arctg}\,lk}{lk}-\frac{\Delta(z)}{2}\right)\left(1-\frac{\mathrm{arctg}\,lk}{lk}\right)^{-1}. \tag{21}$$

Expanding arctan lk in powers of lk up to fifth order, we obtain after simple transformations

$$\Gamma(z)=\Gamma_n(kl\ll 1)(1-\Delta(z)/8), \tag{22}$$

where

$$\Gamma_n(kl\ll 1)=\frac{4}{15}\frac{p_{x0}^3m_\perp(kl_0)^2}{3\pi^2\rho_{i0n}v_s\tau},\qquad l_0=\frac{p_{x0}\tau}{m_x}.$$

We see that $\Gamma(z)$ varies smoothly near the ETT by virtue of the change of Δz (see Fig. 2. Thus, in the case of strong impurity scattering, the absorption anomaly is preserved even for absorption of sound of sufficiently large wavelength. In contrast to Ref. 18, however, the relative size of the jump turns out here to be small in the parameter $\min\{(\varepsilon_0\tau)^{-1/2},(\varepsilon_0/|z|)^{-1/2}\}$. This discrepancy is due to the assumption made in Ref. 18 that the mean path is constant near the ETT. Let us examine this assumption in greater detail.

In the case of a solitary Fermi sphere, this assumption obviously corresponds to the presence of s-scattering of the electrons by the impurities. When the cavity produced in the ETT is modeled in this fashion, the relaxation time $\tau=l/v\to\infty$ with decrease of the cavity size, and it is this which leads to the appearance of the finite jump $\Delta\Gamma\sim\Gamma_n$ in Ref. 18 (for a transition of the neck-breaking type this assumption leads to a logarithmic singularity in $\Delta\Gamma$). It must be noted, however, that this model can be realized only, say, for degenerate semiconductor → dielectric transitions, since a cavity is formed in ETT only in the presence of other—extended—sections of the Fermi surface. Scatterings of the electrons of the produced cavity and their departure to a large neighboring section of the Fermi surface is therefore possible, and it is this which leads to a perfectly finite relaxation time.

In the Fermi-surface model considered above the electron relaxation time turns out to be finite automatically[12] because the electrons scattered by impurities can go over from the neck region to remote sections of the Fermi surface. Thus, in the case of a neck-breaking transition the logarithmic divergence obtained in Ref. 18 for does not take place.

In the case $1\lesssim kl\lesssim\min\{(\varepsilon_0\tau)^{1/2},(\varepsilon_0/|z|)^{1/2}\}$ the ratio $I/(1-I)$ is determined as before by (21), but it can no longer be expanded in powers of kl. The strong cancellations of the preceding case no longer occur in expression (20). The physical reason for this is the different character of the electron interaction with the sound-wave field. Thus, whereas in the case $kl\lesssim 1$ all the electrons interact effectively with the sound wave, in the collisionless region ($kl\gtrsim 1$) the only electrons that take part in the sound absorption are those moving in phase with the sound wave, i.e., those for which the condition $\mathbf{k}\cdot\mathbf{v}\approx\omega$ is satisfied.[20,22] For the second absorption coefficient near the ETT we have the region $1\lesssim kl\lesssim\min\{(\varepsilon_0\tau)^{1/2},(\varepsilon_0/|z|)^{1/2}\}$

$$\Gamma(z)=\Gamma_n(kl\gg 1)[1-(kl_0/\pi)\Delta(z)], \tag{23}$$

where

$$\Gamma_n(kl\gg 1)=2m_xm_\perp\varepsilon_0^2\omega/9\pi\rho_{i0n}v_s^2$$

coincides, apart from a coefficient that depends on the Fermi-surface geometry, with the expression that follows from Pippard's formula (9) for the case $kl\gg 1$. Note that even though the coefficient $\Gamma_n(kl\gg 1)$ ceases to depend on l in this region and is linear in the frequency ω, the correction $\Delta\Gamma$ necessitated by the proximity of the system to the ETT remains in the case $kl\ll 1$ proportional to ω^2l_0 as before. The relative value of the jump in this wavelength region depends thus on the frequency, and this apparently makes it possible to observe this region in experiment.

The case $kl\gtrsim\min\{(\varepsilon_0)^{1/2},(\varepsilon_0/|z|)^{1/2}\}$ seems to be difficult to observe in experiment for real compounds in which ETT takes place (since it corresponds to very high frequen-

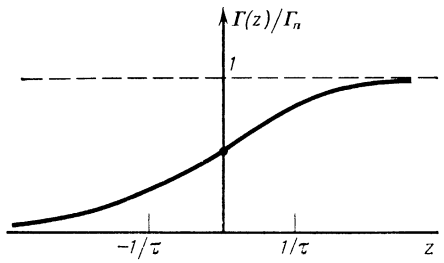

FIG. 3. Relative absorption coefficient $\Gamma(z)/\Gamma_n$ of longitudinal sound vs the parameter z that characterizes the proximity of the system to a topological transition in the case $kl \gtrsim \min\{(\varepsilon_0\tau)^{1/2}, (\varepsilon_0/|z|)^{1/2}\}$.

cies). Nontheless, to complete the picture we present an expression, which follows from (20), for $\Gamma(z)$ near an ETT in this case:

$$\Gamma(z)=\Gamma_n(kl\gg 1)\begin{cases} 1-1/2\pi\tau z, & z\gg 1/\tau, \\ {}^1/_2+2\tau z/\pi, & |z|\ll 1/\tau, \\ 1/2\tau\pi|z|, & -\varepsilon_0^{1/3}\tau^{-2/3}<z\ll -1/\tau. \end{cases} \quad (24)$$

We see that in this case, in accordance with the results of Ref. 15, the jump of the absorption coefficeint near the ETT turns out to be of the order of its value far from the transition. The corresponding $\Gamma(z)$ dependence for this case is shown in Fig. 3. Note that in this wavelength region the relative magnitude of the ETT-induced jump again ceases to depend on the sound-wave frequency, since both contributions to (24) are already proportional to ω in this case.

We discuss in conclusion the available experimental results. The absorption coefficient of longitudinal sound of frequency $f = 30$ MHz was measured in Ref. 23 in a BiSb alloy near an ETT. The experimental conditions corresponded to the case $kl \ll 1$. Near the ETT there was actually observed a spread-out jump in the absorption coefficient, about 5% in relative value. If, following Ref. 23, the mean free path is assumed to be $l_0 \approx 0.01$ mm, expression (22) leads to a satisfactory agreement with the experimental data for both the magnitude of the jump and the width of its spread.

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