A combined optical-acoustical gravitational antenna

V. V. Kulagin, A. G. Polnarev, and V. N. Rudenko

(Submitted 27 February 1986)
Zh. Eksp. Teor. Fiz. **91**, 1553–1564 (November 1986)

We consider a gravitational antenna with acoustical and optical degrees of freedom. Expressions are derived which describe the corresponding facets of the response to gravitational wave bursts of various types. We calculate the sensitivity of a long-baseline interferometric antenna obtained by replacing free test masses with Weber detectors. The level of sensitivity at which such a replacement makes sense is determined. Specifically, displacements $h \sim 10^{-18}$ can be detected without cryogenic cooling of the Weber detectors, and $h \sim 10^{-19}$–10^{-20} with cooling to liquid helium temperatures. We discuss novel properties of such a combined antenna.

INTRODUCTION

The present status of gravitational wave experimentation, with the ultimate goal of detecting bursts of gravitational radiation from relativistic astrophysical objects, may be characterized by two types of gravitational antennas. The first is based on a resonant Weber gravitational detector coupled to an electromagnetic sensor of small acoustic vibrations; the second is a long-baseline detector using free masses, with a laser interferometric position readout system. In the first case, a gravitational wave acts mainly on acoustic degrees of freedom, while in the second, it acts mainly on optical degrees of freedom. For this reason, a Weber-type antenna may be called an acoustical gravitational antenna, and a laser interferometer with free masses, an optical gravitational antenna.

While the Weber-type antenna was historically the first to appear in the laboratory, and has been technically perfected over nearly 15 years, recent achievements in the field of laser interferometer technology indicate that the optical gravitational antenna already possesses competitive capabilities, and is the more promising for the future. Actually, the potential sensitivity of a resonant gravitational detector (with a background of intrinsic thermal noise at temperature T) is given in units of the perturbation of the metric h by

$$h_{min} = \frac{\Delta l}{l} = l^{-1}[kT/m\omega_\mu^2]^{1/2}(\omega_\mu\hat{\tau}/Q_\mu)^{1/2}. \qquad (1)$$

The parameters in Eq. (2) have typical values as follows: detector mass $m = 10^6$ g, length $l = 2\cdot 10^2$ cm, resonant frequency $\omega_\mu = 10^4$ rad/sec, quality factor $Q_\mu = 10^6$. Equation (1) presupposes that the measurement time is equal to the time the detector is affected by an external signal; for $\hat{\tau} = 10^{-3}$, we obtain

$$h_{\min} = 10^{-19} \text{ at } T=300\text{ K}, \quad h_{\min} = 10^{-20} \text{ at } T=3\text{ K}. \qquad (2)$$

The estimates in (2) correspond to detector vibration amplitudes of 10^{-17}–10^{-18} cm. We do not have a way to detect such small displacements by a wideband instrument with $\Delta f = \hat{\tau}^{-1} = 10^3$ Hz. The best experimental result to date[1] enables us to count on measuring displacements of $\sim 5\cdot 10^{-17}$ cm within a considerably narrower passband, $\Delta f = 1$ Hz. (The equivalent noise temperature of the preamplifier in Ref. 1, a microwave amplifier, was 400 K.) This, however, does not provide results at the potential sensitivity indicated in (2). A free-mass antenna with a laser-interferometric readout system is capable of the required precision in measuring relative displacements ΔL of its test mass mirrors; the required $h_{\min}$ is attained by virtue of the large separation L between the test masses. In a Poisson-noise photon background, the sensitivity of such an antenna is given by

$$h_{min} = \frac{\Delta L}{L} = \frac{\lambda}{2\pi l N_0}\left[A\,\frac{2\hbar\omega_e}{\eta W_0}\,\frac{1}{\hat{\tau}}\right]^{1/2}. \qquad (3)$$

For a numerical assessment, we use the parameters of the antenna constructed at the California Institute of Technology (Caltech), which reflect the latest available technology: baseline length $L = 4\cdot 10^3$ cm, interferometer optical driving frequency $\omega_e = 3\cdot 10^{15}$ rad/sec ($\lambda = 5\cdot 10^{-5}$ cm), incident power at the interferometer $W_0 = 1$ mW, photocathode quantum efficiency $\eta = 1/2$, effective number of reflections within the interferometer resonator $N_0 = (1-R)^{-1} \sim 5\cdot 10^3$ (R is the reflection coefficient of the output mirror). The factor A determines the reduction in sensitivity due to a variety of noise sources, relative to the photon noise level. For the Caltech antenna, $A = 10^4$ in the frequency range $f \sim 10^3$ Hz. Then according to Eq. (3), the resolution is $\Delta L = L h_{\min} = 3\cdot 10^{-15}$ cm, which is consistent with experiment.[2] If we further assume that the increase in baseline to $L = 3\cdot 10^5$ cm which is built into the project[3] leaves the fluctuations unchanged, we obtain from (3) in a passband of $\Delta f = \hat{\tau}^{-1} = 10^3$ Hz

$$h_{min} = 3\cdot 10^{-19}, \quad W_0 = 10^{-3}\text{ W}; \quad h_{min} = 3\cdot 10^{-20}, \quad W_0 = 10^{-1}\text{ W}. \qquad (4)$$

A comparison of (4) and (2) shows that the sensitivity of a long-baseline interferometric free-mass antenna [making use of the most technically advanced high-reflectivity mirrors, with $(1-R) \sim 10^{-4}$ (Ref. 4)] should handily achieve the level (2), and with a slight forcing of the parameters (R, A, and W_0) it could reach $h \sim 10^{-21}$–10^{-22}, which would satisfy even the most pessimistic astrophysical predictions.[5]

Thus, in gravitational wave experimentation, the balance has tipped in favor of a free-mass optical gravitational

0038-5646/86/110915-07$04.00

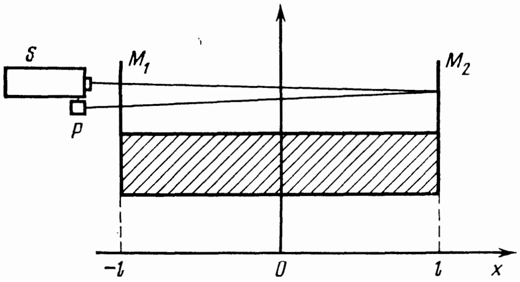

FIG. 1. Weber antenna with optical readout system.

antenna. It must be remembered, however, that this has occurred for purely technical reasons associated with the lack of an adequate detecting element for a resonant Weber detector in the microwave range. Under these circumstances, it is of interest to analyze the possibility of utilizing an optical readout system on a Weber detector. In such an antenna, a gravitational wave interacts with both degrees of freedom, the optical and the acoustical. The problem of separating the acoustical and optical components of the overall antenna response then takes on special interest. From a practical point of view, it is important to investigate the possibility of installing Weber detectors in a long-baseline interferometric antenna. The point of this would be to achieve additional selectivity of gravitational disturbances in an interfering background, and to establish their wave nature from the response of a single antenna. The present paper is devoted to a theoretical analysis of these problems.

§1. WEBER DETECTOR WITH OPTICAL READOUT SYSTEM

We consider a resonant Weber gravitational detector, vibrations of which are detected via an optical interferometer comprised of mirrors fixed to the ends of the detector (Fig. 1). An electromagnetic pump wave (laser beam) propagates from the source S to the photodetector P after being reflected by the mirrors M_1 and M_2.

The phase shift of the electromagnetic wave in propagating from M_1 to M_2 and back again, which is directly induced by a gravitational wave acting on the electromagnetic wave, to first order in the metric perturbation h (Ref. 6; we write $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$)

$$\Delta\varphi_O = \frac{1}{2} K^{(0)\alpha} K^{(0)\beta} \int_{p_1}^{p_2} \left[h_{\alpha\beta}(x^\gamma)\big|_{x^\gamma = K^{(0)\gamma} p + x_1^\gamma} - h_{\alpha\beta}(x^\gamma)\big|_{x^\gamma = K_*^{(0)\gamma} p + x_2^\gamma} \right] dp. \quad (5)$$

Here $K^{(0)\alpha}$ and $K_*^{(0)\alpha}$ are the unperturbed wave vectors of the electromagnetic waves propagating from M_1 to M_2 and back again, respectively (the index 1 corresponds to mirror M_1, and index 2 corresponds to mirror M_2). Subsequent calculations are most conveniently carried out in a synchronous reference frame. In order to calculate the overall phase shift of the wave, it is nexessary to add a term to $\Delta\varphi_O$ in (5) related to the Doppler shift induced by the relative motion of the mirrors, which is not taken into account in (5):

$$\Delta\varphi_A = \frac{\omega_0}{c} \left[-\xi\left(-l, t - \frac{4l}{c}\right) + 2\xi\left(l, t - \frac{2l}{c}\right) - \xi(-l, t) \right], \quad (6)$$

where $\xi(x,t)$ are the displacement coordinates in the extended acoustical system.

Thus, in a synchronous reference frame, the overall phase shift may be represented as a sum of two parts, $\Delta\varphi = \Delta\varphi_O + \Delta\varphi_A$, the first of which, $\Delta\varphi_O$, results from the direct action of the gravitational wave on the electromagnetic wave, permitting us to call this part of the phase shift the optical part, and the second of which, $\Delta\varphi_A$, results from the effect of the gravitational wave on mirror motion. For the free-mass case in the specified reference frame $\xi(-l,\tau) = \xi(l,\tau) = 0$, so the quantity $\Delta\varphi_A$ may be called the acoustical part of the phase shift.

The frequency shift of the electromagnetic wave $\delta\omega/\omega$ can be obtained from (5), (6) by differentiating the phase with respect to time, or directly from the general formula,[7] which in the linear approximation is of the form

$$\delta\omega \approx (U_{\alpha 2} - U_{\alpha 1}) K^{(0)\alpha} + U_\alpha^{(0)} \left(K^\alpha|_{x_2^\gamma} - K^\alpha|_{x_1^\gamma}\right),$$

where $U_{\alpha 1}$ and $U_{\alpha 2}$ are the four-velocities of the mirrors M_1 and M_2. The first term here corresponds to the acoustical response, the second to the optical. It should be noted that this separation is not covariant, but for the above choice of a synchronous system, it has a well-defined meaning.

In order to calculate $\Delta\varphi$ and $\delta\omega/\omega$, we need to find the coordinate displacements of M_1 and M_2 under the influence of a gravitational wave, i.e., $\xi(-l,\tau)$ and $\xi(l,\tau)$.

Assuming sufficiently weak damping $\delta \ll v_s^2/2\omega$, to order $v_s/c \ll 1$, the equations of elastic deformation give[8]

$$\xi(x,t) \approx -\frac{1}{2} \int_{-\infty}^{\infty} \frac{v_s h_\omega}{\omega} e^{i\omega t} d\omega \left[\frac{\cos k_1 l}{\cos \mu l} \sin \mu x + i \frac{\sin k_1 l}{\sin \mu l} (1 - \cos \mu x) \right], \quad (7)$$

where h_ω is a Fourier component in a plane-wave expansion of h_{11}:

$$h_{11}(x,t) = \int_{-\infty}^{\infty} h_\omega \exp(ik_1 x + i\omega t)\, d\omega, \quad k_1 = \frac{\omega}{c} \cos\theta,$$

v_s is the speed of sound in the material of the gravitational detector, θ is the angle between the direction of the gravitational wave and the separation between M_1 and M_2, and $\mu = (\omega/v_s)(1 + 2i\delta\omega/v_s^2)^{-1/2}$.

The first term in square brackets in the integrand of Eq. (7) describes antiphase vibrations of the acoustical system, $\xi(l,\tau) = -\xi(-l,\tau)$, and has resonances at the frequencies

$$\omega_n = (v_s/l)(\pi/2 + n\pi), \quad n = 0, \pm 1, \pm 2 \ldots.$$

Near the nth resonance, the integrand in (7) equals $h_\omega K_1^{(n)}(\omega)$, with

$$K_1^{(n)}(\omega) \approx -\frac{v_s^2 \cos k_1 l}{l\omega_n(\omega_n - \omega + i\omega_n/Q_n)}, \quad (8)$$

where $Q_n = v_s^2/2\delta\omega_n$ is the Q of the nth mode. The total contribution to (7) from antiphase vibrations may be repre-

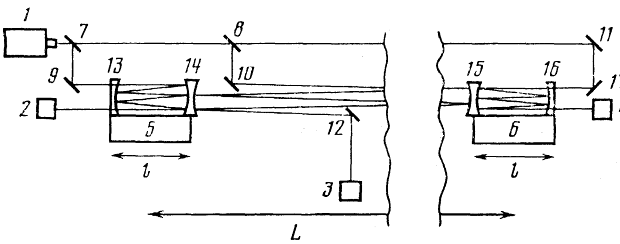

FIG. 2. Simplified diagram of a combined optical/acoustical gravitational antenna. 1) Laser; 2–4) photodetectors; 5, 6) Weber resonators; 7, 8) splitters; 9–12, 17) mirrors; 13–16) mirrors on resonator end-faces (local oscillator beam not shown for simplicity).

sented as a sum of resonances which fall off in amplitude as ω_n^{-3}, and increase in width as ω_n^2. The widths of the resonances equal the frequency intervals between them, starting with the threshold frequency $\omega_{\rm thr} \approx 2\omega_0 Q_0^{1/2}$. In the range $\omega \lesssim \omega_{\rm thr}$, we have $N \approx Q_0^{1/2}$ individual resonances. For $\omega > \omega_{\rm thr}$, the transfer function $K_1(\omega)$ is smooth: $K_1(\omega) \approx iv_s \cos k_1 l/2\omega$.

Analogously, for the in-phase vibrations described by the second term in square brackets in the integrand in (7), $\xi(-l,\tau) = \xi(l,\tau)$, resonances occur at the frequencies $\omega_n = v_s \pi(1+2n)/l$, and near these resonances the transmission function is

$$K_2^{(n)}(\omega) \approx -\frac{iv_s \sin k_1 l}{l\omega_n(\omega_n - \omega + i\omega_n/Q_n)}. \qquad (9)$$

We may also introduce a corresponding transmission function for the optical response:

$$\Delta\varphi_0 = \int_{-\infty}^{\infty} h_\omega K_0(\omega) e^{i\omega t}\, d\omega,$$

which, according to (5), is of the form

$$K_0(\omega) = -\frac{i\omega_e}{\omega} e^{-i2\omega l/c}\left[e^{-ik_1 l}\left(i\sin\frac{2\omega l}{c} + \cos\theta\cos\frac{2\omega l}{c}\right) - \cos\theta e^{ik_1 l}\right] \Big/ \sin^2\theta. \qquad (10)$$

We see from (8)–(10) that there is a threshold frequency $\omega_* \approx \omega_0 (Q_0 v_s/c)^{1/2}$, such that for $\omega_g \lesssim \omega_*$, the envelope of the transmission function of the acoustical response exceeds that of the optical response, and conversely for $\omega_g > \omega_*$. This threshold frequency ω_* differs considerably from an intuitive estimate, according to which for $\omega_g \gg \omega_0$, the response of the combined antenna should behave like that of a free-mass laser antenna. Thus, in the case of sapphire ($Q_0 = 10^9$ and $v_s = 10^6$ cm/sec), the correction factor is $(Q_0 v_s/c)^{1/2} \approx 10^2$. The correction factor appears by virtue of the high resonances which are present in a distributed acoustical antenna: in order for the mirrors to be regarded as free, the frequency of a gravitational wave must exceed the frequency of the highest of the "detectable" resonances.

A. Let us consider the effect on an antenna of a burst of gravitational radiation with finite duration $\hat\tau$. Let the burst consist of m oscillations with period $T_g = (2\pi/\omega_g)(\omega_0 \sim \omega_g)$. From (5)–(10), the ratio of the amplitudes of the acoustical and optical responses is of the form[8]

$$\frac{(\delta\omega/\omega)_A}{(\delta\omega/\omega)_O} \approx \frac{2v_s^2 \tau T_g}{\pi l^2}\left[\frac{\sin(\omega_0-\omega_g)\hat\tau}{(\omega_0-\omega_g)\hat\tau} + \frac{\sin(\omega_0+\omega_g)\hat\tau}{(\omega_0+\omega_g)\hat\tau}\right], \qquad (11)$$

where we have taken only resonant harmonic excitation into account. Since $\omega_0 = \pi v_s/2l$, and under resonant conditions $\omega_g \equiv 2\pi/T_g \approx \omega_0$, the coefficient of the oscillatory part of (11) is obviously of order $16m/\pi \gtrsim 5$.

Accurate numerical calculations using the impulse response have confirmed that higher harmonics may be neglected in the case of resonant long-wavelength bursts of gravitational radiation ($\omega_g \sim \omega_0$), and we may use the simple relation (11) to compare the acoustical and optical responses.

B. In the case of short bursts of gravitational waves, with $\hat\tau < 2l/c$, we must take not only the resonant but the continuous high-frequency portion of the transmission function $K_1(\omega)$ for antiphase vibrations into consideration (the contribution of in-phase vibrations is small). However, the corrections which are involved are of the nature of brief excursions which can be neglected when averaging over times $\gtrsim \omega_0^{-1}$. For gravitational wave bursts with $h_{11}(t) = h_0$, $|t| < \hat\tau$, the acoustical response is

$$\left(\frac{\delta\omega}{\omega}\right)_A \approx -\frac{16h_0\tau v_s^2}{cl}\sum_{n\geq 0}\frac{\sin\omega_n\hat\tau}{\omega_n\hat\tau}\exp\left(-\frac{\omega_n t}{2Q_n}\right)\cos\omega_n t. \qquad (12)$$

It can thus be seen that those harmonics for which $\omega_n\hat\tau \lesssim 1$ are the ones which make a significant contribution to the acoustical response [$N \approx (2\omega_0\hat\tau)^{-1}$ harmonics have effectively been excited]. The response (12) consists of a sequence of pulses with amplitude decreasing and width increasing in time. In essence, we are dealing with an acoustical pulse which propagates the length of the acoustic system and is reflected from its ends. The ratio of acoustical to optical pulse amplitudes is of order $(\delta\omega/\omega)_A/(\delta\omega/\omega)_O \approx 16v_s/\pi c$, although the acoustical response is more prolonged than the optical, which allows for integration.

§2. LONG-BASELINE COMBINED ANTENNA

Let us consider a gravitational antenna which is a combination of a free-mass antenna and of Weber detectors with an optical readout system, a schematic of which is presented in Fig. 2. In this design, there are obviously three possible outputs: one from the long-baseline interferometer (mirrors 14, 15), and two from the readout interferometers of the Weber detectors (mirrors 13, 14, 15, 16). The geometry is such that $L \gg 1$. The availability of three readouts which to a certain extent are independent and mutually complementary provides additional capabilities for extraction of a gravitational wave signal from a background of local interference. This circumstance is the main stimulus for the analysis of the proposed design.

A. We now dwell on some of the features of the reaction of a large interferometer (mirrors 14, 15) to a gravitational wave. The structure of the optical response, as given by Eq. (5), remains the same as before, with l replaced by $L + 1$. For the acoustical response (6), it is also necessary to take the increase in delay into consideration (replacing l by $L + 1$

in the time argument). Moreover, an additional shift in the argument of the second term in square brackets occurs in (6) because of the nonsimultaneity of excitation of the two resonators separated by a distance L, $\Delta t_{\text{add}} = (2L/c)\cos\theta$.

For resonant bursts of a gravitational wave with $\lambda_g/2 > 2L$, the optical response is increased by a factor of L/l over an extended baseline; the acoustical response does not change as L increases (as long as $2L < \lambda_g/2$), and the delay is insignificant. For a short gravitational wave burst, the optical triplet is resolved when $\hat{\tau} \lesssim L/c$.[6] An isolated pulse in the acoustical response also becomes a triplet, with the amplitude of the middle pulse being twice as high as that of the outside pulses, and the spacing not being equidistant. The order of magnitude of the response-amplitude ratio remains as before. Thus, a specific temporal aspect interposes itself here.

Note that the structure of the optical and acoustical responses and the ratio between them are preserved when the radiation makes several passes between the mirrors (the number of reflections is N). The magnitude of these responses then grows by a factor of N, building up over the interval that the sign of the displacement of the mirrors remains the same, until $4N(L+l)/c < T_0/2$.

B. The question of the advantage of including Weber detectors in the design of a laser interferometer free-mass antenna actually depends on a comparison of the sensitivity of the two types. In fact, when there is a sizable difference in sensitivity, the combination is inappropriate. We now carry out this analysis, assuming for simplicity that there is only one degree of freedom in the acoustical component, i.e., assuming the Weber detector to be a one-dimensional oscillator with the frequency of the fundamental mode.

Using Eqs. (5), (6), and (8), we find the phase modulation (at frequency ω_e) of an electromagnetic wave which traverses the large interferometer subject to the influence of a gravitational wave burst. For $\lambda_g \gg 2L$, we have

$$\Delta\psi_S(\omega)=\omega_e N/c[2L+2l\omega^2(\omega^2-\omega_\mu^2-i\omega\omega_\mu/Q_\mu)^{-1}]h_S(\omega), \quad (13)$$

where $h_S(\omega)$ is the spectrum of the gravitational wave signal. There are three main sources of noise with constant spectral density in the proposed setup: thermal noise G_B in the Weber resonators, shot noise G_L in the photodectors, and noise G_p associated with radiation pressure fluctuations:

$$G_B=\frac{2kT\omega_\mu}{\pi m Q_\mu},\quad G_L=A\frac{2\hbar\omega_e}{\eta W_0}\left(\frac{\lambda}{2\pi N}\right)^2,\quad G_P=\frac{2\hbar\omega_e W_0 N^2}{m^2c^2}. \quad (14)$$

Bearing in mind the shape of the transmission function of the combined antenna [see (13)], we obtain the following relations for the equivalent at the output in the usual manner:

$$h_{fl}^2(\omega)=h_B^2(\omega)+h_L^2(\omega)+h_P^2(\omega);$$
$$h_B^2(\omega)=(G_B/4L^2\omega_\mu^2)\{[(1+l/L)\nu^2-1]^2+\nu^2/Q_\mu^2\}^{-1},$$
$$h_P^2(\omega)=(G_P/4L^2\omega_\mu^2)\{[(1+l/L)\nu^2-1]^2+\nu^2/Q_\mu^2\}^{-1},$$
$$h_L^2(\omega)=(G_L/4L^2)[(\nu^2-1)^2+\nu^2/Q_\mu^2]\times\{[(1+l/L)\nu^2-1]^2+\nu^2/Q_\mu^2\}^{-1}, \quad (15)$$

where $\nu=\omega/\omega_\mu$. The maximum signal-to-noise ratio, corresponding to optimal filtering, is given by[9]

$$\mu=(2\pi)^{-1}\int_{-\infty}^{\infty}|h_S(\omega)|^2 d\omega/h_{fl}^2(\omega).$$

For the sake of definiteness, let the spectrum of the gravitational wave be of the form

$$h_S(\omega)=(h_0\hat{\tau}/\pi)[\sin(\omega_\mu-\omega)\hat{\tau}/((\omega_\mu-\omega)\hat{\tau})],\ \omega>0.$$

For the parameter values indicated above, we have $G_B \gg G_P$, and there are three physically distinct cases for which it is straightforward to obtain μ.

1. The case $(G_B+G_P)Q_\mu^2 \ll G_L\omega_\mu^4$ or $G_B Q_\mu^2/\omega_\mu^4 \ll G_L$, or

$$\frac{2\pi N}{\lambda}\left(\frac{kT}{m\omega_\mu^2}\right)^{1/2} \ll \left(A\frac{2\hbar\omega_e}{\eta W_0}\frac{\omega_\mu}{Q_\mu}\right)^{1/2}. \quad (16)$$

This means that the shot noise spectral density exceeds the resonant value of the thermal noise spectral density. In that case, we obtain for μ

$$\mu \approx \frac{4L^2h_0^2\hat{\tau}}{\pi G_L}\left[1+\left(\frac{l}{L}\right)^2 Q_\mu\frac{\omega_\mu\hat{\tau}}{2\pi}\right]. \quad (17)$$

The first term in (17) corresponds to the response of the free masses and the second to the response of the detectors. Assuming that $\omega_\mu\hat{\tau}/2\pi \gtrsim 1$, the equivalent sensitivity condition $Q_\mu(l/L)^2 \simeq 1$ is satisfied for typical parameter values of $Q_\mu \approx 10^6$, $l = 3\cdot10^2$ cm, and $L = 3\cdot10^5$ cm. However, condition (16) for the first case can only be satisfied at room temperature with $W_0 = 1$ mW, $A = 10^4$, and $N = 10^2$ (corresponding to a poor optical sensor). A reduction in optical noise or an increase in the number of reflections can only be balanced by cooling of the Weber detectors, although the margin here is fairly small: for $N = 10^2$, we need a temperature of $T = 3$ K.

A detector to implement (17) would consist of a wideband channel with bandwidth $\Delta f_1 = \hat{\tau}^{-1} = 10^3$ Hz for observing the free-mass response, and a narrow-band channel with bandwidth $\Delta f_2 = 2\delta_\mu/\pi$ (measurement time $\tau_m = Q_\mu/\omega_\mu$) to observe the response of the resonant detectors. When the bandwidth is compressed from Δf_1 to Δf_2, the contribution of the Weber resonators to the signal-to-noise ratio increases to the contribution to μ of a free-mass antenna (buildup of acoustical response). Note that the overall antenna sensitivity is not high in the present case: $h_{\min} = 10^{-17}$ for $T = 300$ K and $h_{\min} = 10^{-18}$ for $T = 3$ K.

2. The case $(G_B+G_P) \gg G_L\omega_\mu^4$ or $G_B/\omega_\mu^4 \gg G_L$, or

$$\frac{2\pi N}{\lambda}\left(\frac{kT}{m\omega_\mu^2}Q_\mu^{-1}\right)^{1/2} \gg \left(A\frac{2\hbar\omega_e}{\eta W_0}\omega_\mu\right)^{1/2}. \quad (18)$$

This is the opposite limit from the one considered: the optical noise spectral density is much less than the thermal noise density, even in a wide band $\sim 2\omega_\mu$. For this case, we obtain

$$\mu \approx \frac{4L^2h_0^2\hat{\tau}}{G_L}\Gamma^{-1}\left(1+\left(\frac{l}{L}\right)^2\omega_\mu\hat{\tau}\right),\quad \Gamma=\frac{G_B}{G_L\omega_\mu^4}. \quad (19)$$

The filtering procedure here consists of "cutting off" the resonant components of the thermal noise, and subsequently passing the mixture of signal and noise through a wideband

TABLE I.

A	T, K	W_0, mW	Q_μ	G_L	$G_B\,\omega_\mu^{-4}$	$(G_L\,\omega_\mu^4/G_B)^{1/2}\cdot 10^{-6}$	h_{min}
10^4	300	1	10^5	$2.5\cdot 10^{-25}$	$3\cdot 10^{-27}$	1.0	$5\cdot 10^{-17}$
10^4	70	100	$5\cdot 10^6$	$2.5\cdot 10^{-27}$	$2\cdot 10^{-38}$	0.4	$5\cdot 10^{-18}$
10^2	3	100	10^7	$2.5\cdot 10^{-29}$	$3\cdot 10^{-40}$	0.3	$5\cdot 10^{-19}$
1	3	100	$3\cdot 10^8$	$2.5\cdot 10^{-31}$	$3\cdot 10^{-42}$	0.3	$5\cdot 10^{-20}$

channel with $\Delta f = \hat{\tau}^{-1}$. Thus, for $\omega_\mu \hat{\tau} \gtrsim \pi$, the sensitivity is limited only by thermal noise (ideal sensor), and can reach $h_{\min} \sim 10^{-19}$–10^{-20} for T between 3 K and 300 K. In this case, however, (18) imposes extremely stringent constraints on the readout system: $N \gtrsim 10^2$, $W_0 > 1$ W, $A = 1$. It should be noted that when (18) is satisfied, the addition of resonant detectors to a free-mass antenna degrades the sensitivity of the latter [see (19)].

3. The case $(G_B + G_P) Q_\mu^2 \gg G_L \omega_\mu^4$, but with $(G_B + G_P) \lesssim G_L \omega_\mu^4$. This situation is apparently the most realistic. The signal-to-noise ratio is

$$\mu = \frac{4L^2 h_0^2 \hat{\tau}}{\pi G_L}\left[1 + \frac{\omega_\mu \hat{\tau}}{2\pi}\left(\frac{l}{L}\right)^2 \Gamma^{-1/2}\right]. \qquad (20)$$

In an equivalent-sensitivity operating mode, the ratio $(l/L)^2 \sim 10^{-6}$ must be balanced by a quantity $\Gamma^{-1/2}$, in which case the filter bandwidth is $\Delta\omega = \omega_0 \Gamma^{-1/2}$. Table I shows the attainable sensitivity of a combined antenna with the parameters indicated ($N = 10^2$, $\hat{\tau} = 10^{-3}$ sec). The data in the table indicate that a combined antenna with sensitivity $h \sim 10^{-19}$–10^{-20} is only feasible when rigorous requirements on the parameters of the constitutent parts have been met. These requirements include, for example, cryogenic cooling of the Weber detectors and small values of the photon noise coefficient at relatively high laser power W_0, which complicates the implementation of the design in practice. One way to solve these problems is to use a so-called displacement transformer installed on the resonant detector.[10–13]

§3. DISPLACEMENT TRANSFORMER IN A COMBINED ANTENNA

The idea of using a displacement transformer to amplify the vibrations of a Weber detector was proposed in Refs. 10, 11, and 14, and was embodied in practical terms in the experiments described in Ref. 15. An elastic low-mass ($m \ll M$) oscillator is affixed to the end-face of a resonant gravitational detector (cylindrical bar), where the frequency of the oscillator is either the same as or close to the frequency of the fundamental mode of the detector. When the detector is excited by a pulse, its vibrational energy is transmitted to the transformer every other half-period, and the amplitude of the latter is significantly greater than the excitation amplitude of the gravitational detector, by a factor of $(M/m)^{1/2} \gg 1$. The thermal variance of the detector and transformer are accordingly distinguished. It is clear, however, that the ratio of thermal to photon noise in a combined antenna also changes. Let us calculate the sensitivity enhancement to be expected in the third case considered in §2, paragraph B, using a displacement transformer. For this calculation, we use a model of the gravitational detector and transformer as coupled oscillators, assuming that a mirror of the large interferometer (Fig. 2) is affixed to the transformer. The equations describing this coupling may be put in the form

$$\ddot{\xi} + 2\delta_M \dot{\xi} + (\omega_M^2 + \omega_m^2 \alpha)\xi - \omega_m^2 \alpha \eta = f_S(t) + f_M(t),$$
$$\ddot{\eta} + 2\delta_m \dot{\eta} + \omega_m^2 \eta - \omega_m^2 \xi = f_S(t) + f_m(t) + f_P(t), \qquad (21)$$

where ξ, η are the coordinates of the gravitational detector and transformer, $\omega_M^2 = k_M/M$, $\omega_m^2 = k_m/m$ are their partial frequencies ($\omega_M \sim \omega_m$), f_M, f_m are thermal fluctuation forces with spectral density G_M and G_m, analogous to G_B in (14), f_P describes fluctuations of the light pressure with spectral density G_P in (14), $f_S(\omega) = -1/2 l\omega^2 h_S(\omega)$ is the gravitational perturbation of the detector, and $\alpha = m/M$ is the transformer ratio.

Taking the Fourier transform of (21), we obtain the spectrum of the signal component of the transformer displacement:

$$\eta_S(\omega) = [\det(i\omega)]^{-1}[\omega_M^2 + \omega_m^2(1+\alpha) - \omega^2 + 2i\delta_M \omega] f_S(\omega),$$
$$\det(i\omega) = (\omega_M^2 + \omega_m^2 \alpha + 2i\delta_M \omega - \omega^2)(\omega_m^2 - \omega^2 + 2i\delta_m \omega) - \alpha \omega_m^4. \qquad (22)$$

The corresponding spectral power of the fluctuations is

$$\eta_{fl}^2(\omega) = \eta_P^2(\omega) + \eta_M^2(\omega) + \eta_m^2(\omega),$$
$$\eta_P^2(\omega) + \eta_m^2(\omega)$$
$$= |\det(i\omega)|^{-2}[(\omega_M^2 + \omega_m^2 \alpha - \omega^2)^2 + 4\delta_M^2 \omega^2](G_P + G_m), \qquad (23)$$
$$\eta_M^2(\omega) = |\det(i\omega)|^{-2} \omega_m^4 G_M.$$

The output signal is proportional to $\eta_S(\omega)$, and besides the fluctuations $\eta_{fl}^2(\omega)$, it also contains the photon noise G_L (14), i.e., the overall noise is

$$\eta_\Sigma^2(\omega) = \eta_{fl}^2(\omega) + G_L.$$

The integral for the signal-to-noise ratio is not evaluated in general. The value of μ can be estimated from the bandwidth Δ_{fl} of the optimal filter, which is practically the same as the bandwidth over which G_M is greater than the other noise, reflected back to the input (we take $G_P \ll G_m$). From (22), (23), we have for the filter bandwidth $\Delta_L \approx \omega_m (\alpha + \Gamma_0^{1/2})^{1/2}$ superposed on G_M and G_L, which is true as long as $\alpha \lesssim \Gamma_0^{1/2}$, where $\Gamma_0 = G_M/(G_L \omega_M^4)$. The bandwidth superposed on G_M and G_m is given by $\Delta_m = \omega_M \alpha^{1/2}$ (for equal Q and temperature of the detector and transformer: $Q_M = Q_m$ and $T_M = T_m$). Finally, $\alpha_{\rm opt} = \Gamma_0^{1/2}$ and

$$\mu \approx \frac{l^2 h_0^2 (\omega_m \hat{\tau})\hat{\tau}\omega_m^4}{32 G_M}\Gamma^{1/4} = \frac{3}{16}\frac{L^2 h_0^2 \hat{\tau}}{G_L}\left(\frac{\omega_\mu \hat{\tau}}{6}\right)\left(\frac{l}{L}\right)^2 \Gamma^{-3/4}. \qquad (24)$$

This same expression is obtained from an approximate evaluation of the integral for μ if we assume that the gravitational wave burst is narrow-band, $\omega_M \tau \gg 1$. Comparing Eqs. (24) and (20), we conclude that a displacement transformer increases the sensitivity of the Weber portion of a combined antenna by a factor $\Gamma^{-1/4}$. Since according to the data in the table Γ^{-1} can be of order $\sim 10^{12}$, the value of the detection threshold $h_{\min}$ at which the combined antenna is still advantageous is reduced by a factor of 30. If in fact $Q_m < Q_M$, then the filter bandwidth with the background of G_M and G_m is reduced by comparison with Δ_m: $\Delta'_m \approx \omega_m (\alpha Q_m / Q_M)^{1/2}$, and μ is reduced accordingly in the ratio $(Q_m/Q_M)^{1/2}$.

A still greater improvement can be obtained by using two transformers with masses m_1 and m_2 connected serially, with $m_1/m_2 = M/m_1$, $\omega_1 \approx \omega_2 \approx \omega_M$. Carrying out a calculation analogous to (21)–(23), we can estimate without difficulty the increase in bandwidth of the optimal filter as compared with the bandwidth of a single-transformer antenna (optimal mass ratio), $\Delta_{f2} = \omega_M \Gamma^{1/6} = \Delta_{f1}\Gamma^{-1/2}$, and the corresponding improvement in sensitivity in the Weber portion of the combined antenna:

$$\mu \approx \frac{3L^2 h_0^2 \hat{\tau}}{16 G_L}\left(\frac{\omega_M \hat{\tau}}{6}\right)\left(\frac{l}{L}\right)^2 \Gamma^{-5/6}, \qquad (25)$$

i.e., $h_{\min}$ is reduced by a factor $\sim 10^2$ as compared with a transformerless antenna. This means that even without liquid-helium cooling of the resonant detector, a sensitivity $h_{\min} \sim 10^{-19}$ becomes attainable.

Reasoning by analogy, we may estimate the sensitivity for n transformers connected serially:

$$\mu \approx \frac{3L^2 h_0^2 \hat{\tau}}{16 G_L}\left(\frac{\omega_\mu \hat{\tau}}{6}\right)\left(\frac{l}{L}\right)^2 \Gamma^{-(2n+1)/(2n+2)}, \qquad (26)$$

and for sufficiently large n, $h_{\min}$ is determined solely by the thermal noise [output at the ultimate sensitivity (1)]. It must be noted, however, that it is technically extremely difficult to make more than two transformers with the optimum mass ratio, although the relative increase in sensitivity for $n \geqslant 2$ is minor, so in practice it is entirely possible to keep $n \leqslant 2$.

CONCLUSION

Recent advances in mirror fabrication technology for laser interferometers give long-baseline free-mass optical antennas an advantage over Weber antennas. This makes it possible to consider a combined optical/acoustical project, replacing the free masses in a laser antenna with Weber detectors.

Our calculations reveal a specific structure to the response of a combined antenna, and indicate that this combination is advantageous at a sensitivity of $h_{\min} \sim 10^{-19}$–10^{-20} (with a displacement transformer), which satisfies the astrophysical requirements completely.[16] This can be done without cryogenic cooling of the Weber resonators.

We stress that even at moderate values of the optical drive power for the interferometer, the thermal noise of the gravitational detector plays a significant role, and for that reason, we have not considered quantum limitations in the present paper.

We now list the novel properties of a combined antenna once more.

1. The occurrence of three independent responses (see Fig. 2, outputs from photodetectors 2, 3, and 4) to a large extent makes it possible to eliminate nongravitational perturbations (as with the third-harmonic veto control on the Stanford antenna[17]).

2. By measuring the phase shift between the relaxation "tails" of the responses from two mutually separated Weber resonators, it is possible to detect the wave nature of a gravitational disturbance; phase-measurement technology is well understood. With a combined-antenna optical baseline of $2L = 3\cdot 10^5$ cm, the phase shift is of order $\Delta\varphi = 2\omega_\mu L / c \approx 10^{-1}$ rad. According to the Cramer-Rao bound,[9] the signal-to-noise ratio determines an upper bound on the phase-measurement accuracy, $\delta\varphi \gtrsim \mu^{-1/2}$, and we must therefore have $\mu \gtrsim 10^2$. This means that we can detect with certainty the wave structure of gravitational radiation for a metric perturbation h which is only an order of magnitude greater than the limiting estimate $h_{\min}$.

In conclusion, we must point out that the addition of Weber resonators to a free-mass laser interferometer antenna leaves the construction cost of the latter practically unchanged if cryogenic cooling is omitted. With cryogenic cooling of the resonant detectors, there will be a small increase in the cost of the overall project. We also stress that the combined-antenna project "extends the lifetime" of the Weber detector in a certain sense, although this does not apply to the two Weber antennas which already exist, since only an optical detection system will give the required sensitivity.

The authors are grateful to V. B. Braginskii and A. V. Gusev for useful discussion.

[1]V. B. Braginskii, V. I. Panov, and V. D. Popel'nik, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 423 (1981) [JETP Lett. **33**, 405 (1981)].

[2]S. E. Whitcomb, D. Z. Anderson, and R. W. P. Drever, in Proc. of the Third Marcel Gross Meeting on GR, 1983, edited by Hu Ning (Scientific Press and North-Holland, 1983), p. 1399.

[3]News in: Lasers and Applications, October, 1984, p.28.

[4]D. Z. Anderson, J. C. Frisch, and C. S. Messer, Appl. Opt. **23**, 1238 (1984).

[5]K. S. Thorne, "Theory of gravitational radiation: an introductory review in gravitational radiation," (Les Houches Summer School), edited by N. Deruelle and T. Piran (North-Holland, Amsterdam, 1983).

[6]V. N. Rudenko and M. V. Sazhin, Kvant. Elektron. **7**, 2344 (1980) [Sov. J. Quantum Electron. **10**, 1366 (1980)].

[7]B. Bertotti, D. Brill, and R. Krotkov, in Gravitation: an Introduction to Current Research (New York and London, 1969), p. 1.

[8]V. V. Kulagin, A. G. Polnarev, and V. N. Rudenko, Preprint No. 933 (1984), IKI Akad. Nauk SSSR.

[9]H. van Trees, The Theory of Detection, Estimation, and Modulation (Russ. Trans., Sovetskoe Radio, Moscow, 1972).

[10]H. J. Paik, in Experimental Gravitation, edited by B. Bertotti (Aca-

demic Press, New York, 1974), p. 515.
[11]H. J. Paik, J. Appl. Phys. **47**, 1168 (1976).
[12]P. F. Michelson and R. C. Taber, J. Appl. Phys. **52**, 4313 (1981).
[13]P. F. Michelson and R. C. Taber, Phys. Rev. D **29**, 2149 (1984).
[14]G. Ya. Lavrent'ev, Zh. Eksp. Teor. Fiz. **39**, 1316 (1969).
[15]S. P. Boughn, W. M. Fairbank, R. P. Giffard, *et al.*, Phys. Rev. Lett. **38**, 454 (1977).
[16]V. B. Braginskii and V. N. Rudenko, Phys. Rep. **46**, 167 (1978).
[17]P. F. Michelson, in Les Houches Proceedings "Gravitational Radiation", edited by N. Deruelle and T. Piran (North-Holland, Amsterdam, 1983), p. 465.
[18]F. B. Estabrook and H. D. Wahlquist, Gen. Rel. Grav. **6**, 439 (1975).

Translated by M. Damashek