Physical meaning of solutions with outgoing sound waves in the theory of shock-wave stability

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It is shown that the solutions, known in the linear theory of shock-wave stability, having shock-wave-front perturbations that neither attenuate nor grow with time and emitting sound waves (the so-called sound-wave generation), do not satisfy the causality principle at $\pi/2<\tau<\pi$, where τ is the angle between the outward normals to the shock- and sound-wave fronts. The same holds also for the analogous solutions for three-wave configuration (unperturbed and perturbed shock waves and an outgoing weak wave). Spontaneous undamped shock-wave perturbations are therefore impossible in the indicated range of the angles τ. The results of a nonlinear analysis of the reflection of perturbations from a shock-wave front (in the quadratic approximation) point to causal phenomena that can be described by the considered solutions of the linear theory.

It is known from the linear theory of shock-wave stability[1–3] that in certain ranges of the parameter $L = J^2(\partial v/\partial p)$ (where J is the flux of matter through the shock-wave front and $(\partial v/\partial p)$ is the derivative of the specific volume v with respect to the pressure along the shock adiabat) there exist stationary (in the moving coordinate frame) solutions that correspond to a perturbed surface (corrugation perturbations) of the shock-wave front with sound waves that arrive at the front or depart from it only at a definite angle. In these solutions, the perturbations of all the quantities (e.g., the pressure) are proportional to a factor of the form

$$\exp\,[i(kx+ly-\omega t)] \tag{1}$$

with real values of k, l, and ω. (The x and y axes are directed respectively along the shock-wave front or perpendicular to it.) The ratio k/l, which defines the orientation of the sound waves, depends on L. The values of L for solutions with outgoing or incoming sound waves are bounded respectively by the conditions

$$-1<L<L_0, \tag{2}$$

$$L_0<L<1+2M, \tag{3}$$

$$L_0\equiv(1-\theta M^2-M^2)/(1+\theta M^2-M^2),$$

where M is the Mach number of the shock wave relative to flow behind it, and θ is the degree of compression in the shock wave. According to the theory of sound reflection by a shock-wave front,[4–6] in case (2), when a sound wave is incident at the same angle as the incoming wave in solution (1), or in case (2) for reflection at the same angle as for the outgoing wave in (1), the ratio p_r/p_f of the pressure p_r of the reflected wave to the pressure p_f of the incident wave becomes zero or infinity, respectively.

In terms of wave-intersection theory, the pattern produced by reflection of a sound wave from a shock-wave front is represented as a configuration of four waves. The conditions under which the reflection coefficient becomes zero or infinity in the linear approximation are also the conditions for the existence of the three-wave configuration produced by an unperturbed and perturbed shock wave and by an infinitely weak (acoustic) incoming or outgoing wave.[7]

The connection between L and the angle γ that characterizes the mutual orientation of the shock and acoustic waves in the three-wave configuration (see Fig. 1) and in solution (1) can be represented in the form[7]

$$\psi(\gamma)\equiv\left[1-\frac{\theta}{A^2}+\frac{2S}{A}-L\left(1+\frac{\theta}{A^2}\right)\right][2(1+A^2)J^2v]^{-1}=0, \tag{4}$$

$$A\equiv(1+\Gamma^2)^{1/2}/M-\Gamma,\quad S\equiv M(1+\Gamma^2)^{1/2}-\Gamma,\quad \Gamma\equiv\operatorname{ctg}\gamma.$$

The possible range of the angles γ for the incoming or outgoing waves 3 (see the Fig. 1) is

$$0<\gamma<\gamma_0\equiv\arccos M, \tag{5}$$

$$\gamma_0<\gamma<\pi. \tag{6}$$

Note the one-to-one relation between the connection (4) of L with γ in the region (3) for L and in the region (6) for γ.[7] The meaning of the stationary solutions for the perturbed shock wave with only incoming sound wave and the meaning of the equation $p_r/p_f = 0$ are quite obvious (see, e.g., Ref. 1). As for solutions with only outgoing waves and for infinite p_r/p_f (resonance, in the terminology of Refs. 4 and 5), it was noted in Refs. 1 and 4 that a special investigation is

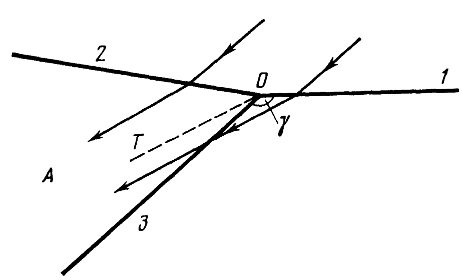

FIG. 1. Three-wave configuration: 1—unperturbed shock wave, 2—perturbed shock wave, 3—weak outgoing wave, T—tangential discontinuity. The arrows indicate the directions of the stream lines in a coordinate frame with immobile point O.

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needed to shed light on the physical meaning of these results. This is indeed the subject of the following part of the article.

§1. UNDAMPED PERTURBATION OF THE FRONT OF A SHOCK WAVE WITH OUTGOING SOUND WAVES, AND THE CAUSALITY PRINCIPLE

The phase and group velocities of sound waves (without dispersion) behind a shock-wave front are equal to c. On the other hand, the propagation velocity of a constant phase along the surface of a shock-wave front, for solutions with only outgoing or with only incoming sound waves, is given by

$$V_t=(c-V\cos\gamma)/\sin\gamma. \quad (7)$$

It can be seen from (7) that $V_t > c$ at

$$\pi/2<\gamma<\pi. \quad (8)$$

(As for the velocity of the point O on Fig. 1 relative to the matter behind the shock-wave front, it is supersonic at all values of γ except $\gamma = \arccos M$. It is important in what follows that $V_t > c$ in the range (8).) It is clear therefore that spontaneous propagation of the "ripples" along the front surface, with velocity (7), should not mean signal transfer in the case (8). By suitable superposition of solutions of type (1) with a constant ratio k/l corresponding to a resonant value of the angle γ, however, it is possible to form a local initial perturbation of the front surface, for example a perturbation in the form of a three-wave configuration (see Fig. 1). Since the sound velocity is independent of frequency, this perturbation would propagate, without change of shape, with velocity (7) (see Ref. 7). The motion of such a perturbation, however, has the character of a signal. It can be concluded from this contradiction that no spontaneous (i.e., not sustained by external action) perturbations of a shock wave front, which are described by solutions (1) with outgoing waves, are possible in the angle range (8). In the case (8) such solutions do not satisfy the causality principle, which is known to play a most important part in shock-wave propagation.

Solution (1) in the linear approximation satisfies formally the hydrodynamic equations and the boundary conditions (the conservation laws) on the shock-wave front. In the angle range (8), however, it contains implicitly a certain error. The point is that one of the necessary conditions for the existence of a shock wave for a flow behind its front is known to be (see, e.g., Refs. 8 and 9) the inequality $V_n < c$ (V_n is the material velocity component normal to the front in the coordinate frame of the front). This condition means that perturbations of the flow behind the front should overtake the front. This condition was assumed satisfied and taken as an initial condition in Refs. 1–3. In case (8), however, this condition does not hold for the point O of the three-wave configuration and accordingly for the front surface in solution (1). Indeed, at the angles (8) the matter-velocity components normal to fronts 2 and 3 at the point O do not belong at all to the region of the flow behind the front (to the sector A in Fig. 1). Under these conditions the perturbations from sector A do not reach the point O, and consequently its motion and the state of matter in it are not causally connected with the parameters of the flow in sector A. In view of this violation of the causel connection between the motions of fronts 2 and 3 at the point O, on the one hand, and the flow behind the front, on the other, the wave configuration initially specified in the form shown in Fig. 1 [or in the form of solution (1)] begins to decay immediately: a rarefaction wave flows from the point O to the interior of the sector A, fronts 2 and 3 become bent, an incoming wave and a reflected wave are produced at the shock-wave discontinuity, etc.

A solution of type (1), obtained in linear stability theory, does not contain an evolution of this kind. As already noted, it does not satisfy the causality principle for the angles (8). Propagation of a supersonic perturbation along the shock-wave front should be due to external factors, i.e., to incoming perturbations that reach the front.

The meaning of solution (1) with outgoing sound waves (and the corresponding solution (4) of Ref. 7) for a three-wave configuration, whether this solution has any physical application, and the meaning of the limit $p_r/p_f = \infty$ will be considered in the next section.

§2. NONLINEAR ANALYSIS. TRANSITION TO THE LIMIT $p_f \to 0$ FOR SOLUTIONS OF THE WEAK AND STRONG FAMILY. PHYSICAL MEANING OF SOLUTIONS (1) WITH OUTGOING WAVES

The answer to the questions raised above can be obtained by analyzing the reflection of weak perturbations from a shock-wave front in the quadratic approximation.[10] It is shown in Ref. 10 that, given a sufficiently low amplitude of the pressure p_f of the incident wave, there are two solutions for p_r in the vicinity of the resonant angle $\gamma = \gamma_{res}$. We shall call them the weak and strong families of solutions, in analogy with the known results for wave reflection from a rigid wall.[8,9] In the weak-family solution, as $p_f \to 0$, the pressure of the reflected wave in the vicinity of the angle γ_{res} tends to zero like $p_f^{1/2}$. With increase of distance from γ to γ_{res} the weak-family solution approaches the corresponding result of the linear theory. The indicated square-root dependence of p_f on p_f is evidence of the stability of the shock wave in region (3) relative to sufficiently small perturbations that reach the front at angles close to γ_{res}. Infinitely small perturbations reflected from the shock-wave front do not produce a finite change of the wave intensity. The reflected-wave amplitude remains infinitely small, and only the order of smallness changes.[1)] This result explains also the possible nature of the solution with outgoing sound wave in the linear theory of stability.[1–3] In the approximation linear in p_r the primary cause of the perturbation, viz., a wave arriving at the resonance angle, has an intensity proportional to p_r^2 and consequently remains "out of sight," as do all other nonlinear effects. Causality of the phenomenon seems absent in the linear approximation, but actually exists, and if the cause vanishes ($p_f \to 0$) the consequence also vanishes ($p_r \to 0$).

Obviously, waves from the flow ahead of the front which are incident on the front at an appropriate angle can also act as a weak external perturbation that is quadratic in the outgoing-wave pressure. In this case the outgoing wave is produced as a result of refraction of the incident wave. (See Ref. 4 concerning refraction of sound by the front of a shock wave in the linear approximation.)

As $p_f \to 0$ the strong-family solution is independent of p_f and does not go over into the result of the linear approximation at incidence angles far from the resonance angle. This solution corresponds at $p_f \to 0$ to a three-wave configuration with a weak outgoing wave. The dependence of the angle γ on the pressure p_3 for such a configuration in the weak outgoing wave 3 is determined by the equation $\psi(\gamma) = -ap_3$ and goes over into (4) at $p_3 = 0$ (the coefficient a depends on the thermodynamic properties of the material ahead and behind the shock-wave front[10]).

The solution (1) with undamped outgoing sound waves, obtained in the linear stability theory for region (3), can be regarded as the limiting case of the strong-family solution (as a corresponding superposition of three-wave configurations in the limit $p_f = 0$). In contrast to the limit transition for the weak-family solutions, in this case one neglects near the limit not the incoming waves, but the dependence of the angle γ on p_3. Since, however, in the considered limit there are no incoming waves at all, not merely in first order in p_3, the question of the causality of the solution again arises. In a more general formulation this is the question of the physical meaning of three-wave configurations not only in the limit as $p_3 \to 0$, but also at a finite intensity of the weak outgoing wave.

One can perform a gedanken experiment, perfectly consistent from the standpoint of causality, in which such three-wave configurations might be realized. Imagine an infinitely thin flat piston freely sliding along the plane of a tangential discontinuity (sector A in Fig. 1). In the plane of the figure, the piston is shown in the form of an infinitely thin needle with one end touching the point O (dashed line in the figure). The pressures on the two sides of the needle are equal, and the needle has no influence whatever on the flow anywhere except at the singular point 0. The length of the needle is immaterial, and it can be, in particular, also infinitely small. The needle orientation in a coordinate frame with an immobile point O, and hence also its velocity in the lab, are uniquely determined by the parameters of the three-wave configuration (for the calculation of the configuration in a linear and in a quadratic approximation see Refs. 7 and 10, respectively).

The solution with outgoing sound waves, obtained in the linear theory of shock-wave stability, can be represented as a superposition of such three-wave configurations with infinitely thin pistons. (The piston thickness should in this case be of higher order of smallness than the distance between the neighboring pistons.)

Thus, if inequalities (3) hold, we can point to three processes that are accompanied by a stationary (in a coordinate frame moving together with front and with velocity (7) along the front) ripple on the surface of a shock wave and with sound waves emitted from the surface. Each of these processes constitutes an external action on the shock-wave front, an action that does not enter explicitly in the linear approximation. Two types of such actions are weak (quadratic in p_r) waves that arrive from the flow regions ahead or behind the front. The third type of action is due to the definite motion of an infinitely thin piston (or system of pistons) along the plane (planes) of the tangential discontinuity.

Waves arriving at angles close to the resonant one[2)] can be caused, of course, also by noise or fluctuations, but the production of shock-wave front perturbations or generation of reflected waves by their action vanish after the cause vanishes, i.e., after the incident wave is damped.

We emphasize that these causal restrictions on the conditions for the existence of solutions (1) pertain the angle range (8). The interval of the values of the parameter L corresponding to such angles is determined, according to (4) by the inequalities

$$L^* < L < 1+2M, \quad L^* = (1+2M^2-\theta M^2)/(1+\theta M^2).$$

[1)]At a fixed difference $\gamma - \gamma_{res}$ and at sufficiently large incident-wave amplitude, regular reflection for one of phases of the wave (compression or rarefaction) is impossible.[10] The more complicated wave configurations produced thereby has not been investigated. It consists apparently of a forward emitted (along the shock-wave front) three-wave configuration with a curved front of wave 3; this configuration perturbs the initial shock wave in such a way that regular reflection again becomes possible. The amplitudes of all waves (except, of course, the initial and perturbed shock waves) tends then in the limit as $p_f \to 0$ to zero at a rate not slower than $p_f^{1/2}$ (Ref. 10).

[2)]The fluctuation-wave energy is proportional to $\Delta\gamma$, and is equal to zero for waves emitted strictly at a definite angle γ.

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