Coherent interaction of light with a discrete periodic resonant medium

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Soliton solutions are derived for the first time for the two-wave Maxwell-Bloch equations, which describe the coherent nonlinear interaction of light with a resonant medium under conditions of diffraction in a one-dimensional structure. We discuss the conditions for formation and the dynamic properties of a new type of localized excitation, the "two-wave" soliton. A stable time-dependent solution for the field and excitation of the medium, a two-wave breather, is found for the superradiance problem. We predict self-induced nonlinear suppression of diffractive reflection of the field at a boundary, and scattering in the medium, as well as the "retardation" of a field pulse reflection.

1. INTRODUCTION

Nonlinear coherent processes involving the interaction of radiation with matter have for many years greatly interested researchers. This is primarily because qualitatively new physical effects appear which do not exist in the linear case, such as the spontaneous formation and coherent decay of a macroscopic collective state of a system of oscillators (superradiance, abbreviated SR)[1–3]; the appearance of a long-lived pseudospin system memory (light echo)[2,4]; and the formation of unattenuated field pulses and excitations of the medium, optical soliton (self-induced transparency (SIT)).[2,5] Until now, theoretical and experimental investigations of nonlinear effects have been carried out mainly using the weak-interaction approximation for the field modes (field in a continuous medium). References 6 and 7 are exceptions, having considered the dynamics of SR under diffractive conditions in the quantum mode[6] and semiclassical[7] descriptions, for the special case of short systems. However, as implied by a previous letter,[8] taking coherent nonlinear effects into account while investigating diffractive processes in extended resonant media leads to new physical behavior, in particular, to self-induced suppression of diffractive scattering of a field pulse, and to its localization in the medium.

In the present paper, we derive the full system of two-wave Maxwell-Bloch equations to describe nonlinear diffraction in a one-dimensional resonant periodic medium. For the first time, the full multisoliton solution is derived for an infinite medium. Using numerical integration, we also make a detailed study of the one-soliton solution, describing the formation, propagation, and decay processes of two-wave solitons in an infinite medium. Retardation of the reflected signal has been detected during modeling of the nonlinear diffractive reflection of field pulses. We also show for the first time that the evolution of a superradiant extended system can lead to a nontrivial stable state of the excited medium and field.

2. THE TWO-WAVE MAXWELL-BLOCH EQUATIONS

We use the semiclassical picture to describe the interaction of the field with a resonant medium, wherein the classical field interacts with a set of quantized oscillators.[2] A discrete one-dimensional periodic resonant medium consists of a set of periodically positioned thin layers of thickness $a \ll \lambda$ (λ is the wavelength of the radiation) containing two-level atoms. To ensure discreteness, the period d of the structure should satisfy the condition $d \gtrsim \lambda$. A one-dimensional medium makes it possible to choose the simplest form of solution of the field equations for SR when the sources of emission lie within the medium. It is necessary first that the plane-wave approximation apply, and second, that there be as few waves as possible. One medium which satisfies these requirements is a bar of length l and cross-sectional area s, for which the Fresnel number is $F \equiv s/l\lambda \approx 1$. The resonant planes lie perpendicular to the axis of the bar. In the SIT problem, where an external field interacts coherently with an unexcited medium, it is sufficient to require the plane-wave condition $F > 1$ (disk-shaped sample), and to limit the number of Bragg modes to two diametrically opposite points on the Ewald sphere).

Thus, we seek a solution of Maxwell's wave equation

$$\nabla^2 \mathbf{A}(\mathbf{r},t) - c^{-2}\mathbf{A}_{tt}(\mathbf{r},t) = -\frac{4\pi}{c} J(\mathbf{r},t) \qquad (1)$$

for the vector potential **A** and bound-charge current density **J** in the form of two plane waves with complex, slowly-varying amplitudes, propagating in opposite directions:

$$\mathbf{A}(x,t) = \mathbf{A}^+(x,t)\exp[i(kx-\omega t)] + \mathbf{A}^-(x,t)\exp[-i(kx+\omega t)] + \text{c.c.}, \qquad (2)$$

$$|A_x^\pm| \ll |kA^\pm|, \quad |A_t^\pm| \ll |\omega A^\pm|, \qquad (3)$$

where $k = \omega/c$, and ω is the resonant transition frequency.

Let us convert to equations for the slow amplitudes. To do so, we substitute Eq. (2) into Eq. (1) and ignore second derivatives, assuming them to be small compared to first derivatives; we then average over a time $\Delta t \gg \omega^{-1}$ (which is still less than the characteristic time of A and J) using the temporal condition (3), thereby extracting the equations for $A^\pm$ and $(A^\pm)^*$. As a result, we obtain

$$(A_x^+ + c^{-1}A_t^+)e^{ikx} + (-A_x^- + c^{-1}A_t^-)e^{-ikx} = (2\pi i/\omega)\langle Je^{i\omega t}\rangle_t, \qquad (4)$$

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where the angular brackets denote the averaging over Δt mentioned above. We have omitted vector symbols from Eq. (4), since for simplicity we consider only a single field polarization and transition current. Exactly at resonance, the quantum mechanical mean current density for a dipole transition may be expressed in the form[9,10]

$$\langle J(t,\mathbf{r})e^{i\omega t}\rangle_t = \sum_j a_0^*(t,\mathbf{r}_j)a_1(t,\mathbf{r}_j)J_z\delta(\mathbf{r}-\mathbf{r}_j), \quad (5)$$

where the slowly-varying functions $a_0(t, \mathbf{r}_j)$ and $a_1(t, \mathbf{r}_j)$ define the superposition state of the jth two-level atom, which is described by the wave function

$$\Psi_j(t) = a_1(t,\mathbf{r}_j)\Psi_1 + a_0(t,\mathbf{r}_j)\Psi_0,$$

where Ψ_1, Ψ_0 are the unperturbed wave functions of the upper and lower atomic states; J_z is the matrix element of the projection of the atomic dipole transition current in the direction of polarization of the vector potential $\mathbf{A}$ and $\delta(\mathbf{r}-\mathbf{r}_j)$ is the Dirac delta function.

To separate out the amplitudes A^+ and A^-, we must multiply Eq. (4) by $\exp(\pm ikx)$ and average over a volume $V_0 \sim \lambda^3$. In the one-dimensional model, the atoms located at each resonant plane with surface density σ must be assumed to be in identical states. Furthermore, the condition $d \gtrsim \lambda$ eliminates averaging over the polarization and the inversion of adjacent planes, and we take the system, as before, to be a discrete set of planes designated by i. After substituting (5) into (4) and averaging with (3) taken into account, we obtain

$$\pm A_x^\pm(x,t) + c^{-1}A_t^\pm(x,t) = \frac{2\pi i}{\omega}\frac{\sigma}{\lambda}J_z\sum_i \exp(\mp ikx_i)a_0(x_i,t)a_1(x_i,t)\tilde{\delta}(x-x_i). \quad (6)$$

The function $\tilde{\delta}(x-x_i) = 1$ for $x \in (x_i \pm \lambda/2)$, and is zero otherwise.

It is not hard to derive the optical Bloch equations for an atom in the ith plane in the field (2) from the Schrodinger equation in the usual manner (see Ref. 11, for example):

$$P_t(x_i,t) = \alpha n(x_i,t)[A^+(x_i,t)\exp(ikx_i) + A^-(x_i,t)\exp(-ikx_i)] - T_2^{-1}P(x_i,t), \quad (7)$$

$$n_t(x_i,t) = -(\alpha/2)\,\mathrm{Re}\{P^*(x_i,t)[A^+(x_i,t)\exp(ikx_i) + A^-(x_i,t)\exp(-ikx_i)]\} - T_1^{-1}(n(x_i,t)+1),$$

where $P(x_i, t) = -2|J_z|^{-1}(a_0^*a_1J_z)_i$ is a dimensionless characteristic of the atomic "polarization"; $n(x_i, t) = |a_1(x_i,t)|^2 - |a_0(x_i,t)|^2$ is the population inversion of the atoms; $\alpha \equiv 2i|J_z|/\hbar c = 2i\omega\mu_z/\hbar c$; μ_z is the matrix element of the transition dipole moment; T_1 and T_2 are the longitudinal and transverse relaxation times of the Bloch vector $\mathbf{R}_i = \{\mathrm{Re}\,P_i, \mathrm{Im}\,P_i, n_i\}$. Finally, multiplying Eq. (6) by α, we may rewrite the entire self-consistent system of two-wave Maxwell-Bloch equations (6), (7) in a form which is convenient for further analysis (neglecting spontaneous incoherent relaxation of the Bloch vector):

$$c\Omega_x^+(x,t) + \Omega_t^+(x,t) = \tau_c^{-2}\sum_i \exp(-ikx_i)P(x_i,t)\tilde{\delta}(x-x_i), \quad (8a)$$

$$-c\Omega_x^-(x,t) + \Omega_t^-(x,t) = \tau_c^{-2}\sum_i \exp(ikx_i)P(x_i,t)\tilde{\delta}(x-x_i), \quad (8b)$$

$$P_t(x_i,t) = n(x_i,t)[\Omega^+(x_i,t)\exp(ikx_i) + \Omega^-(x_i,t)\exp(-ikx_i)], \quad (8c)$$

$$n_t(x_i,t) = -\mathrm{Re}\{P^*(x_i,t)[\Omega^+(x_i,t)\exp(ikx_i) + \Omega^-(x_i,t)\exp(-ikx_i)]\}, \quad (8d)$$

where $\Omega^\pm(x,t) \equiv \alpha A^\pm(x,t) = 2(\mu_z/\hbar)E_0^\pm$, $E_0^\pm$ are the complex amplitudes of the electromagnetic field, $\tau_c^2 = 8\pi T_1/3c\rho\lambda^2$ (for $d=\lambda$), and ρ is the density of the resonant atoms. The cooperative time τ_c is an important parameter of coherent interaction, characterizing the mean photon lifetime in the medium preceding resonant absorption,[12] and must be distinguished from the reciprocal of the Rabi frequency,[2] which corresponds to the mean excitation time of an atom in a resonant field. The Rabi frequency is independent of ρ.

3. SELF-INDUCED TRANSPARENCY IN THE PRESENCE OF TWO-WAVE DIFFRACTION

Self-induced transparency of a continuous medium occurs when there is coherent interaction of a light pulse with a resonant medium. A strong pulse of area 2π propagating along a sample expends a portion of its energy exciting resonant atoms, and is then responsible for their induced decay, where because there is only one mode the problem (propagating coherently as it pumps the continuous medium), the energy is returned to the exciting pulse. As a result, the energy and area of the pulse remain unchanged, and the medium becomes transparent. Such a situation cannot occur in the Bragg case, where the energy reradiated by the atoms is in fact not returned to the single field mode, but is divided beween two counterpropagating, strongly interacting traveling Bragg waves. At first glance, this inevitably leads to spreading of the pulse within the medium. It is shown below that for a sufficiently strong incident pulse, the Bragg reflection at the boundary is selectively (for a portion of the pulse) suppressed, while stable field pulse and excitation of the medium, differing from the ordinary 2π pulse, propagate in the sample.

Let us first consider the interaction between a coherent resonant field and an infinite discrete periodic medium. The two-wave equations (8) are considerably simplified when the Bragg condition is fulfilled exactly, and allow for an analytic solution. After we average over a region $\Delta V \gg d^3$ when $\tau_p \gg d/c$, where τ_p is the characteristic time of a field pulse, Eq. (8) takes the form

$$(\pm c\partial/\partial x + \partial/\partial t)\Omega^\pm(x,t) = \tau_c^{-2}P(x,t), \quad (9a)$$

$$P_t(x,t) = n(x,t)(\Omega^+ + \Omega^-), \quad (9b)$$

$$n_t(x,t) = -P(x,t)(\Omega^+ + \Omega^-). \quad (9c)$$

This set of equations can be reduced to a single equation in the quantity

$$\theta(x,t)=\int_{-\infty}^{t}\Omega(x,t')\,dt',\tag{10}$$

where $\Omega=\Omega^{+}+\Omega^{-}$. To do so, we successively add and subtract the two forms of Eq. (9a), yielding the equivalent equations for the function Ω and $\Omega'=\Omega^{+}-\Omega^{-}$. We then differentiate the first of these with respect to t and the second with respect to x, add, and expressing Ω'_x in terms of Ω, we obtain an equation in Ω and P:

$$-c^2\Omega_{xx}+\Omega_{tt}=2\tau_c^{-2}P_t.\tag{11}$$

Integrating Eq. (11) with respect to time, and taking into account both (10) and the solution of the Bloch equations (9b) and (9c),

$$P=-\sin\theta,\quad n=-\cos\theta,\tag{12}$$

we obtain an equation for $\theta(x,t)$:

$$c^2\theta_{xx}-\theta_{tt}=2\tau_c^{-2}\sin\theta.\tag{13}$$

The unperturbed sine-Gordon equation (13) has a complete set of localized solutions in an infinite medium. Each new solution is obtained from the previous one by application of a Bäcklund transformation,[13] and describes the interaction dynamics of a different number of stable solutions (solitons and breathers). Let $\theta^{(n)}(x,t)=\hat{B}^{(n)}\theta^{(0)}(x,t)$ be a solution of Eq. (13) obtained by n-fold application of the Bäcklund transformation operator. Then from (9a), we obtain for the wave amplitudes

$$\Omega^{\pm}(x,t)=(\theta_t^{(n)}\mp c\theta_x^{(n)})/2.\tag{14}$$

Let us examine the single-soliton solution $\theta^{(1)}$, the most physically interesting of the simple solutions to the sine-Gordon equation. It is just this solution for a single wave in the case of a continuous medium that describes the SIT phenomenon. The soliton

$$\theta(\xi)=4\,\mathrm{arctg}\exp(\xi/\tau)\tag{15}$$

is a stationary solution which depends only on $\xi=t-x/v$ (v is the constant displacement speed in the x-direction), and satisfies the following boundary conditions at infinity: $\theta(\xi=-\infty)=0$, $\theta(\xi=\infty)=2\pi$. The soliton halfwidth is

$$\tau=(\tau_c/2^{1/2}u)(1-u^2)^{1/2},\tag{16}$$

where $u=v/c$. Substituting (15) into (12), we can show without difficulty that the excitation pulse is localized and stationary, while the state of the medium remains unchanged after the pulse passes. $P(\xi=\pm\infty)=0$, $n(\xi=\pm\infty)=-1$.

In the problem under discussion, the quantity $\Omega(\xi)=\theta_t$ has the sense of a total rotation rate of a Bloch vector at the point x and t subject to the fields Ω^{+} and Ω^{-}, but the function $\alpha^{-1}\Omega$ is not the complete field amplitude, in contrast to the traditional single-wave case. To find the actual amplitudes of both modes $\Omega^{\pm}$, we substitute Eq. (15) into (14), obtaining

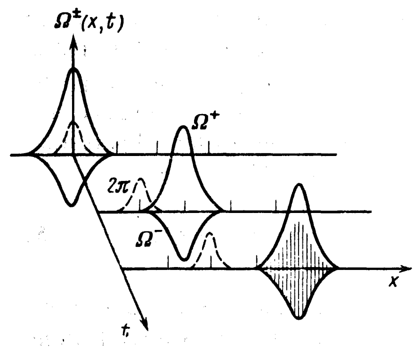

FIG. 1. Dynamics of a 2π pulse in a continuous medium (dashed curves) and a T-soliton in a discrete medium with the same mean density of resonant atoms. The speed $v=2v_{2\pi}$ corresponds to the values $\varkappa=\varkappa_{2\pi}=2.5$, $\tau=\tau_{2\pi}$.

$$\Omega^{\pm}(\xi)=\pm[(1\pm u)/2u]\Omega(x,t),\tag{17}$$

where

$$\Omega(x,t)=2\tau^{-1}\,\mathrm{sech}[(t-x/v)/\tau].\tag{18}$$

The field pulse (17) and the excitation of the medium as well are both localized and move along the medium at the constant speed

$$v=c/(1+2\tau^2/\tau_c^2)^{1/2}.\tag{19}$$

The pulse halfwidth τ is given by Eq. (16) (see Fig. 1).

Thus, solutions (15) and (17) describe the SIT of a resonant Bragg structure. Note that this phenomenon differs fundamentally from the illumination effect on a Bragg mirror predicted previously in Refs. 14 and 15, in which weak Bragg modulation of the medium's dielectric constant is cancelled by modulation induced by the field of a Bragg standing wave because of the quadratic nonlinear interaction between the field and the medium.

4. PROPERTIES OF A "TWO-WAVE" SOLITON

The main feature of the solitary pulse in a Bragg medium is the two-wave character of the field (17), and it is therefore appropriate to refer to it for the sake of brevity as a "two-wave" soliton, or T-soliton.

The absolute area under the rightward-propagating wave $|\theta^{+}|>2\pi$, which is easy to verify by direct substitution of the expression (17) for Ω^{+} into (10) at $t=\infty$. The net effect of the T-soliton is the same as that of the 2π pulse (18). This is formally due to the opposite signs on the angular rates of $\Omega^{\pm}$ in (17), which leads to a reduction in the overall rotation rate of the Bloch vector, $|\Omega|<|\Omega^{+}|$. The physical mechanism for "excluding" part of the field of a T-soliton from the interaction process with the medium, as well as the explanation of the oppositely-directed velocities of the pulse and the wave vector of one of the components of this Bragg wave (Ω^{-} in the present instance), become perfectly clear if we consider the total field in the medium:

$$E(x,t)=\frac{\hbar}{2\mu}\{\Omega^+\exp[i(kx-\omega t)]+\Omega^-\exp[i(-kx-\omega t)]\}+\text{c.c.}=\frac{\hbar}{\mu}[\Omega(x,t)\cos(kx-\omega t)-2\Omega^-(x,t)\sin(kx)\sin(\omega t)]. \quad (20)$$

The field of the $(-k)$-mode is associated in the "standing" wave of amplitude $2\Omega^-$ (20) with a part of the field of the forward wave (shaded region in Fig. 1). The nodes of the "standing" wave are located at the resonant planes of the structure, so this part of the field effectively fails to interact with the medium. Interaction only occurs between the "traveling" component of the field (20) and the amplitude Ω (18).

The physical properties of a T-soliton in a continuous medium differ considerably from those of an ordinary 2π pulse. Formally, the latter is described by (15) and (18), allowing for a pulsewidth which differs from (16) (Ref. 2):

$$\tau_{2\pi}=\tau_c(1-u)^{1/2}/u^{1/2}. \quad (21)$$

Making use of Eqs. (16) and (21), we can write the ratio of the velocities of the T- and 2π pulses when all other parameters are equal:

$$\frac{v}{v_{2\pi}}=\frac{1+\varkappa^2}{(1+2\varkappa^2)^{1/2}}\bigg|_{\varkappa\gg1}\approx\frac{\varkappa}{2^{1/2}},$$

where $\varkappa\equiv\tau/\tau_c$. Thus, when the temporal widths of the pulses are equal ($\tau=\tau_{2\pi}$), the T-soliton speed is higher, and its spatial dimensions therefore exceed the width of the 2π pulse by a factor of $\varkappa/2^{1/2}$ (at optical wavelengths, we can have[2] $\varkappa>10$). To make comparison more convenient, we have shown both types of soliton in Fig. 1. Note also that in the present paper, we do not consider the effects of inhomogeneous broadening on the dynamics of the process. Taking it into account does not lead to any fundamental qualitative differences, just as in the case of a continuous medium.

One characteristic feature of a T-soliton is a captive part of the field which effectively fails to interact with the medium, and which obviously possesses a certain additional energy over and above that of the 2π pulse. Let us estimate its magnitude. The excitation energy of the medium may be expressed in terms of the inversion function $n(x,t)$ in the following manner:

$$\mathscr{E}_m=\rho\frac{\hbar\omega}{2}\int[n(x,t)+1]dV,$$

Then taking the solution of (12) $n=-\cos\theta=2\,\text{sech}^2(\xi/\tau)-1$ into account we obtain

$$\mathscr{E}_m=2v\tau\rho s\hbar\omega. \quad (22)$$

Making use of (20), we find the field energy of the T-soliton, averaged over the periods of the rapidly varying functions. The total average energy of the T-soliton may be written in the form

$$\mathscr{E}=(1+\varkappa^2)\mathscr{E}'+\varkappa^2\mathscr{E}',\quad \mathscr{E}'=2\varkappa^{-2}v\tau s\rho\hbar\omega. \quad (23)$$

Analogously, we obtain for the 2π pulse

$$\mathscr{E}_{2\pi}=\mathscr{E}_{2\pi}'+\varkappa_{2\pi}^2\mathscr{E}_{2\pi}',\quad \mathscr{E}_{2\pi}'=2\varkappa_{2\pi}^{-2}v_{2\pi}\tau_{2\pi}s\rho\hbar\omega, \quad (24)$$

where $\varkappa_{2\pi}=\tau_{2\pi}/\tau_c$. The first terms on the right-hand sides of Eqs. (23), (24) account for the field energy, and the second terms describe the excitation energy of the medium for the corresponding pulses. Comparing (23) and (24), we come to the following basic conclusions: 1) the field energy of the standing wave of a T-soliton equals the excitation energy $\mathscr{E}_m$ of the medium, so the total field energy is always greater than $\mathscr{E}_m$; 2) when the basic pulse and medium parameters $(\tau,\varkappa,\rho)$ are the same, a discrete Bragg medium can produce and transmit stationary pulses without attenuation at higher energies than a continuous medium:

$$\mathscr{E}/\mathscr{E}_{2\pi}|_{\varkappa\gg1}=2\mathscr{E}'/\mathscr{E}_{2\pi}'\approx2^{1/2}\varkappa\gg1, \quad (25)$$

the field energy of the T-soliton being $\varkappa^3\gg1$ times greater than the field energy of the 2π pulse.

Using Eqs. (16), (19), and (23), we can write the speed of the pulse in the form $c/v=(1+\tilde{\mathscr{E}}/\mathscr{E}')^{1/2}$, where $\tilde{\mathscr{E}}=2\varkappa^2\mathscr{E}'$ is the mean energy in the captive field of the "standing" wave and excitation of the medium, and $\mathscr{E}'$ is the mean energy of the effective field of the T-soliton. For a 2π pulse,[13] $c/v_{2\pi}=1+\mathscr{E}_m/\mathscr{E}'$.

It is interesting to consider the relativistic properties of the T-soliton treated as a particle. Its spatial size $l_u=\tau v$ can be expressed in terms of the Fitzgerald-contracted characteristic interaction length $l_c=\tau_c c/2^{1/2}$:

$$l_u=l_c(1-v^2/c^2)^{1/2},$$

and the total energy can be represented by the Einstein formula

$$\mathscr{E}=\frac{m_0}{(1-u^2)^{1/2}}c^2, \quad (26)$$

where the "rest mass" is $m_0=4l_c s\rho\hbar\omega/c^2$. The energy of a 2π pulse cannot be represented in the form (26), as it is equal to

$$\mathscr{E}_{2\pi}=\frac{2^{1/2}l_c s\rho\hbar\omega}{c^2}\frac{u^{1/2}}{(1-u)^{1/2}}c^2.$$

5. PRODUCTION AND DECAY OF A T-SOLITON IN A FINITE MEDIUM: THE AREA THEOREM

The properties of a T-soliton considered above relate to stationary pulses in which the area under the "traveling" component of the field is 2π. An area theorem has been proven[2] which provides a description of the evolution of the area of a pulse $\theta(x)$ in the single-wave case, stating that the change in $\theta(x)$ is governed by the equation of the damped pendulum

$$\theta_x(x)=\tfrac{1}{2}\alpha_0\sin\theta(x),$$

with α_0 the resonant damping coefficient. The quantity $\theta(x)$ always tends to the nearest stable multiple of 2π. The pulse spreads if the initial area is $|\theta(0)|<\pi$. How does the pulse area evolve in the two-wave case?

Let the fields $\Omega^+(x,t)$ and $\Omega^-(x,t)$ be localized in space and time. Then there is always a t_0 such that when $t\geqslant t_0$ at any point x, the fields satisfy $\Omega^\pm(x,t)=0$, and the pulse area

$$\theta(x,t>t_0)\equiv\theta_0(x)=\int_{-\infty}^{t_0}\Omega(x,t')\,dt' \qquad (27)$$

is time-independent. Therefore, for any $t\geqslant t_0$, Eq. (13) is transformed into

$$d^2\theta_0(x)/dx^2=l_c^{-2}\sin\theta_0(x). \qquad (28)$$

This is the equation for the behavior of the pulse area as it propagates along a Bragg medium. It has the form of an undamped pendulum equation in the area θ_0, with a stable equilibrium at $\theta_0=\pi$. In the event of a small initial displacement $\Delta\ll 1$ from a position of unstable equilibrium, $\theta_0(x_0)=2\pi n-\Delta$, the solution is of the form

$$\theta_0(x)=2\pi n+4\,\mathrm{arctg}\exp[(-x+l_D)/l_c] \qquad (29)$$

in the range $4ml_D\leqslant x\leqslant 2(2m+1)l_D$, and

$$\theta_0(x)=2\pi n+4\,\mathrm{arctg}\exp[(x-l_D)/l_c] \qquad (30)$$

in the range $2(2m+1)l_D<x<4(m+1)l_D$; $n, m=0, 1, \ldots$

In Fig. 2, we have plotted curves of solutions of (29) (solid curves) and (30) (dashed curves) for $n=0$, 1 and $m=0$, 1. These are the curves which formally correspond to the case $(2\pi n+\Delta)$, but with the initial value of x_0 shifted to $2l_D$. For $n=0$, the solution describes a two-wave pulse moving with an active component of the field having area $\theta_0(x=x_0)=2\pi-\Delta$. If we translate along the x-axis away from the point x_0, the pulse maintains its area over a segment of length $\Delta x\ll l_D$, and then spreads as it approaches the point $x=l_D=l_c\ln|4/\Delta|$. By definition (27), Eq. (28) does not describe the dynamics of pulse spreading, since this process is nonstationary, and the concept of area in the sense of (17) becomes undefined due to the appearance of nonlocalized fields $\Omega^{\pm}$. However, Eq. (28) and its solutions (29) and (30) can therefore describe the asymptotic dynamics of $\theta_0(x)$ in the region $|x-l_D|\gtrsim l_D$, where the area of the pulse is relatively stable.

In order to treat the actual pulse dynamics and verify Eqs. (29) and (30), we performed a numerical integration of the system of equation (9) with boundary conditions

$$\Omega^+(x=0,\,t)=\Omega_0^+(t),\quad \Omega^-(x=l,\,t)=0, \qquad (31)$$
$$\Omega^{\pm}(x,\,t=0)=0,\quad \theta(x,\,t=0)=0,$$

corresponding to an external pulse $\Omega_0^+(t)$ incident on the boundary $x=0$ of an unexcited medium of length l. Solving this problem also enables us to answer another important question having to do with the possibility of exciting a T-soliton in a bounded medium by means of an external field.

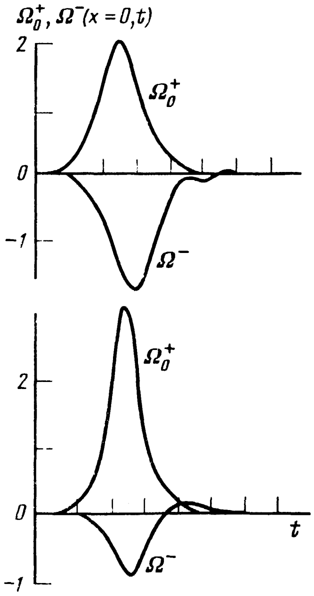

FIG. 3. Time dependence of the amplitude envelope of the reflected field $\Omega^-(x=0,t)$ arising upon incidence upon the medium of a pulse $\Omega_0^+(t)$ of area $\theta_0^+=4\pi$ and width $\tau_0=1.5\tau_c$ (upper curves), and $\tau_0\approx\tau_c$ (lower curves).

The upper curves of Fig. 3 show the amplitude of an incident pulse $\Omega_0^+(t)=\Omega_0\,\mathrm{sech}[(t-t_0)/\tau_0]$ of area $\theta_0^+=4\pi$ and halfwidth $\tau_0=1.5\tau_c$, and the reflected field $\Omega^-(0,t)$. Total Bragg reflection is observed, as in the linear interaction case. After the interaction, the medium remains completely unexcited. Reducing the duration of the incident pulse to $\tau_0=\tau_c$ leads to a sharp reduction in the reflected signal, that is, suppression of the Bragg reflection (lower curves in Fig. 3). A T-soliton is produced in the medium. Numerical modeling of nonlinear Bragg reflection for pulses $\Omega_0^+(t)$ with various values of θ_0^+ and τ_0 enables us to conclude that to produce a T-soliton, it is sufficient that the pulse parameters satisfy

$$\theta_0^+>4\pi,\quad \tau_0\leqslant\tau_c. \qquad (32)$$

We make two remarks regarding the conditions (32), First, we note that for transillumination of a resonantly absorbing Bragg medium, it is not enough to have a pulse with a certain area (as in the case of a continuous medium for $\theta_0^+>\pi$); it

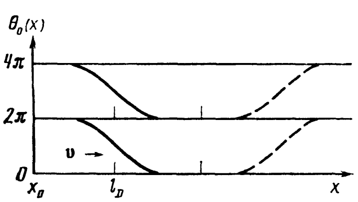

FIG. 2. Evolution of the area of two-wave pulses.

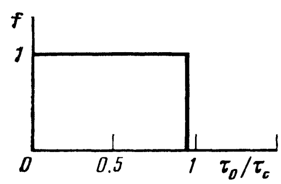

FIG. 4. Dependence of the T-soliton "production function" on the width τ_0 of the incident pulse Ω_0^+ for constant area $\theta_0^+=4\pi$. The function $f=1$ if a T-soliton is produced, and $f=0$ if there is just reflection of the field.

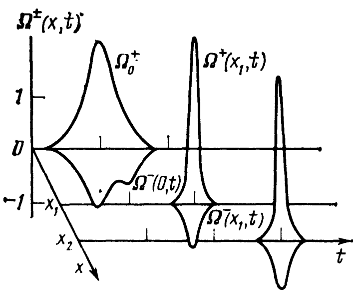

FIG. 5. Generation of a T-soliton by a wide pulse (are $\theta_0^+ = 10\pi$) of relatively low amplitude ($\tau_0 = 3.7\tau_c$).

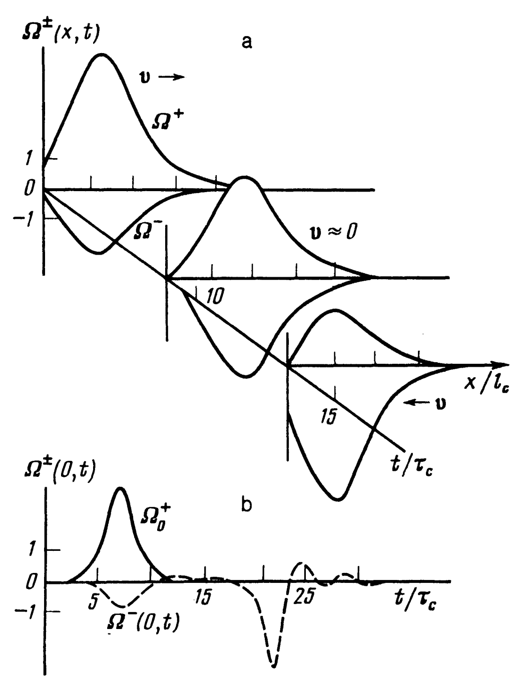

FIG. 6. a) Evolution of a pulse of area $\theta_0 = (2\pi - 0.6)$ propagating along a Bragg medium; b) amplitude envelope of an incident external pulse $\Omega_0^+(t)$ and the radiation field $\Omega^-(x=0, t)$ at the entrance boundary.

is also necessary that the pulsewidth satisfy (32). This is related to the requirement that the Bloch vector rotate rapidly through $\sim 2\pi$ and that a T-soliton develop while the field has not yet managed to leave the medium as a result of Bragg reflection. Thus, a resonant Bragg medium selects pulses not only on the basis of area, but intensity as well (Fig. 4). The second important feature of the Bragg-reflection suppression process is the possibility that it may even occur when (32) is satisfied for only part of a pulse (Fig. 5).

Let us return to the area theorem. By varying the parameters of the external field pulses during numerical modeling, we can trace the evolution of two-wave pulses in the medium for a variety of initial areas. Analysis of these results brings us to the following conclusion, which reflects the content of the area theorem in the two-wave case: pulses of area $\theta_0 = \pm 2\pi n \mp \Delta$ $(0 < \Delta < 2\pi, n = 2, 3, ...)$ propagating in a resonant Bragg medium evolve to stable two-wave localized pulses with a total area $\theta_0(x = \pm\infty) = 2\pi(\pm n \mp 1)$, where for $\Delta \ll 1$, the pulse dynamics in the range $x \ll l_D$ and $x \gg l_D$ are governed by Eq. (29) $(\theta_0 > 0)$. The pulse spreading, $(2\pi n - \Delta) \to 2\pi(n-1)$ is accompanied by energy loss through two channels, Bragg scattering and residual excitation of the medium. The process becomes irreversible, and Eq. (3) implies that an "inverse" rotation of the Bloch pendulum, i.e., reversion to the original pulse areas, does not occur.

There is an interesting phenomenon which occurs when the external field Ω_0^+ produces a two-wave pulse of area $\theta_0 = 2\pi - \Delta$ in the medium. The latter propagates through the sample, undergoing an inversion in shape and direction of motion (Fig. 6a). As a result, most of the external field of the pulse is radiated after some delay which depends on the magnitude of the area decrease and reflection is also delayed (Fig. 6b).

The T-soliton field also shows a nontrivial infrastructure in its interaction with inhomogeneities of the medium. The most important of these is the medium-vacuum boundary. We have also investigated T-soliton behavior upon leaving the medium by numerically integrating (9) with the boundary conditions (31). It is intuitively clear that inhomogeneity of the medium at the edge of the sample must lead to a breakdown of the steady-state collective interaction of the field and medium, and as a result, liberate the "captive" mode Ω^-. In fact, this actually occurs. Upon reaching the boundary $x = l$, the T-soliton dissociates, the direct wave leaves the medium, and the liberated field Ω^- propagates in the opposite direction (along the intrinsic wave vector); in the case of a short medium, after a time delay equal to the photon time-of-flight through the medium, it is radiated in the form of low-intensity damped oscillations at the boundary $x = 0$. In an extended medium $(l \gg l_{c0})$, the Ω^- mode is attenuated.

6. SUPERRADIANCE AND THE DISCRETE BRAGG MEDIUM

The adequacy of modeling of the SR process, or more precisely, of the initial stage of spontaneous decay, is governed in the semiclassical description by the choice of initial conditions. The method we have used to numerically integrate the system (8) makes it possible to follow the dynamics of SR more completely when we have stochastic initial conditions:

$$P(x_i, 0) = \sin\theta_0 \exp(i\varphi_i), \quad n(x_i, 0) = \cos\theta_0,$$
$$\Omega^\pm(x, 0) = \Omega^+(0, t) = \Omega^-(l, t) = 0. \tag{33}$$

The initial Bloch angle is $\theta_0 = 2/N^{1/2}$ (N is the total number of radiators in the system),[16,17] while the stochastic initial polarization $P(x_i, 0)$ is specified independently at each point i by a random phase φ_i in the closed interval $[0, 2\pi]$. There is no mean atomic polarization of the system at the initial instant of time $(\langle P_i \rangle_V = 0)$, and the atoms radiate independently. Furthermore, all the dipoles are in phase in the collective radiation field (when the Bragg condition is met precisely). The system shows a macroscopic mean polarization at some random phase which is unpredictable for any given realization of the initial conditions $\{\varphi_i\}$. Evolution of the collective pseudospin leads the system to a coher-

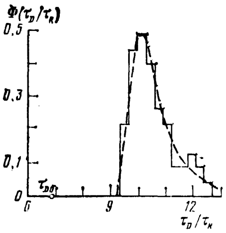

FIG. 7. Probability density function for the SR time-delay distribution $\Phi(\tau_D/\tau_R)$ obtained by numerical modeling of SR for one hundred different realizations of the stochastic initial conditions $\{\theta_0, \varphi_i\}$. The parameters are $\theta_0 = 10^{-2}$, $l = l_c$. The SR characteristic time is $\tau_R = \tau_c^2/\tau_p$, $\tau_p = l/c$.

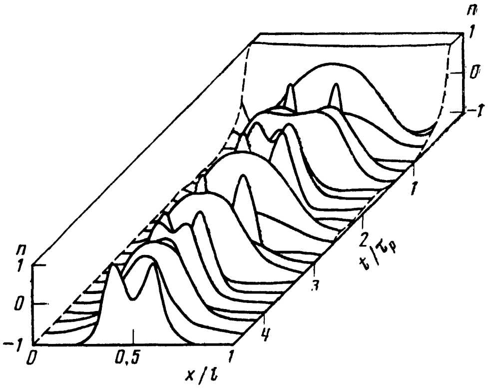

FIG. 8. Evolution of inversion for superradiance of an extended medium in the two-wave case.

ent SR state, and it radiates strongly. Repeating the numerical experiment with various $\{\varphi_i\}$ enables us to conclude that the stochastic specification of the initial polarization in a Bragg system only has the effect of making the distribution of SR delays τ_D random, and increasing them relative to the time delays observed for coherent initial conditions $\varphi_i = \text{const}$ (τ_{D_0}, Fig. 7). Moreover, the shape and maximum intensity of SR pulses is practically the same (the changes are no more than 4%). SR pulses resulting from coherent and stochastic initial conditions are practically indistinguishable.

As is well known, the SR pulse forms at the nonlinear stage of the radiation process, when the angle $\theta(t)$ is varying rapidly.[3] For this reason, the constancy of the pulse shape suggests perfect phasing of all x-radiators i, and formation of a coherent collective state at the linear stage of field-medium interaction, a result which holds only for a Bragg system. The breakdown of spatial coherence ($d \neq \lambda$) under random initial conditions induces a significant unpredictable change in the shape of the SR pulses, as well as a lack of correlation of the radiation propagating in opposite directions. Similar shape fluctuations are seen in SR modeling in continuous media,[18,19] where there is also only weak interaction between oppositely propagating waves, and perfect phasing of all the dipoles with the fields of both waves is not possible.

In numerical modeling of SR in an extended Bragg system of length $l \gg l_c$, we have detected localized stable nonstationary excitations of the field and the medium. Figure 8 depicts the dynamics of the inversion $n(x,t) = \cos\theta(x,t)$ corresponding to such an excitation, obtained by integrating the equations (8) with boundary conditions (33) for the following process parameters: $\tau_p \equiv l/c = 10\tau_c$, $d = \lambda$, $\varphi_i = 0$, $\theta_0 = 10^{-5}$.

With random initial conditions $\{\varphi_i\}$, the form of the solution obtained is similar, but with a narrower excitation region which is stochastically shifted relative to the center of the medium.

The possibility of localized excitations in a Bragg system has been investigated in detail in the previous sections, although the solution discussed there (the T-soliton) clearly differs from that obtained in the SR problem. It is necessary to study the non-soliton stable solutions of the sine-Gordon solution (13) and find analogs among these of the solution shown in Fig. 8. The simplest stable nonstationary solution of Eq. (13) is the breather

$$\theta(\tilde{x}, \tilde{t}) = 4 \operatorname{arctg}\{\operatorname{tg}\nu \sin(\tilde{t}\cos\nu)\operatorname{sech}(\tilde{x}\sin\nu)\}, \qquad (34)$$

where the parameter $0 < \nu < 2\pi$ determines the characteristic breather width and amplitude, $\tilde{x} \equiv x/l_c$, $\tilde{t} \equiv t/(\tau_c/2^{1/2})$. A direct comparison which we performed of the plots of the function $\cos\theta(\tilde{x}, \tilde{t})$ computed from (34) and the solution of the SR problem in Fig. 8 enables us to say that the localized nonstationary excitation appearing as a result of evolution of a superradiant Bragg system is described by the breather solution (34) of Eq. (13), and by the corresponding expressions for the fields $\Omega^{\pm}$ of (14). The time-narrowing of the excitation region observed in Fig 8 is related to energy dissipation due to field radiation at the same extremities.

7. CONCLUSION

The experimental detection of a two-wave soliton, as well as the observation of delay and the suppression of coherent-field pulse reflection under conditions of nonlinear two-wave diffraction in a one-dimensional resonant medium do not require, in our view, any fundamental changes in the customary experimental procedures presently used in coherent nonlinear optics.[2–5,20] Any resonant medium in which SIT or SR is observed is a potential candidate for study. It is only necessary to produce a Bragg structure in which the atoms of the active material are concentrated periodically in narrow planes separated by a nonabsorbent transparent medium. Simple estimates indicate that the formation of a T-soliton at optical wavelengths requires a structure with a large number of layers ($N \gtrsim 100$). The pulses should not exceed 10–100 nsec in duration.

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